B-SPLINE SURFACES

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UNIFORM B-SPLINE SURFACES

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As with Bézier surfaces: grid of control points

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Example: **uniform biquadratic B-spline surface**

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Example: **uniform biquadratic B-spline surface** → (3×3 control points)

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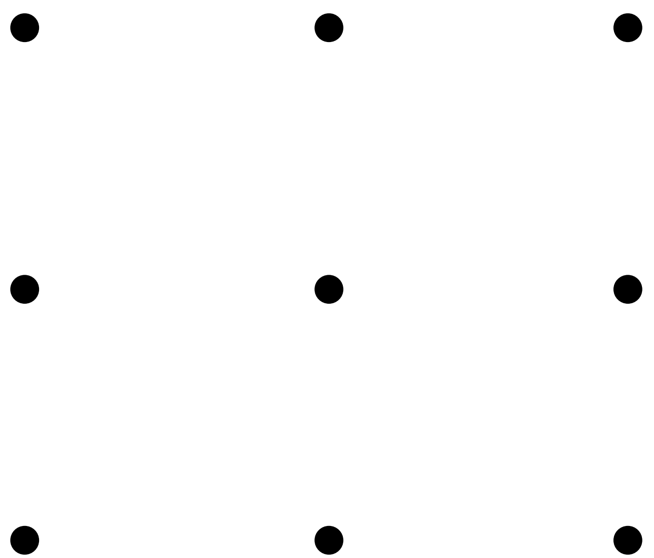
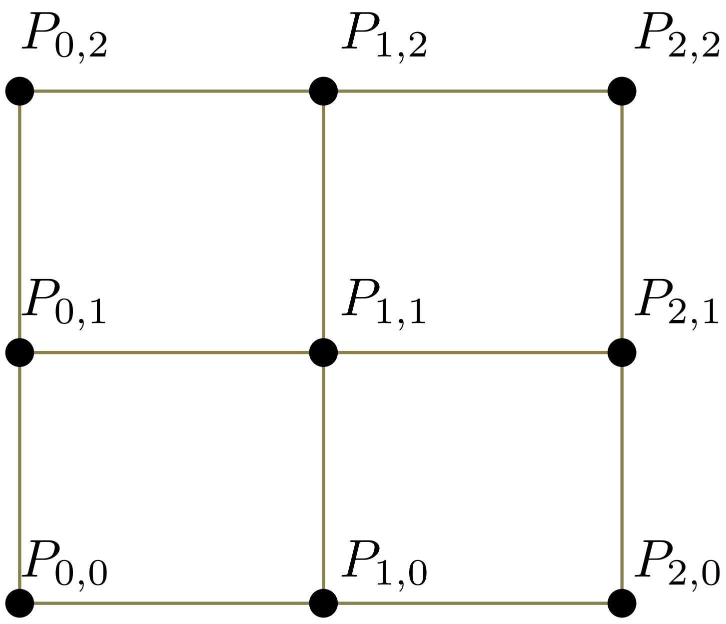


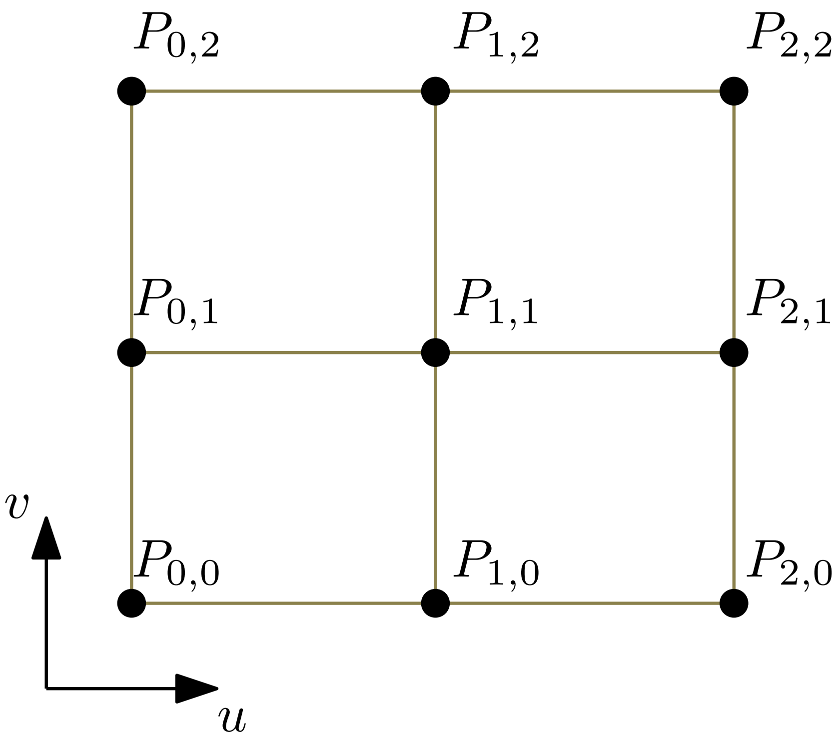
As with Bézier surfaces: grid of control points

Example: **uniform biquadratic B-spline surface** → (3×3 control points)



As with Bézier surfaces: grid of control points

Example: **uniform biquadratic B-spline surface** → $(3 \times 3$ control points)

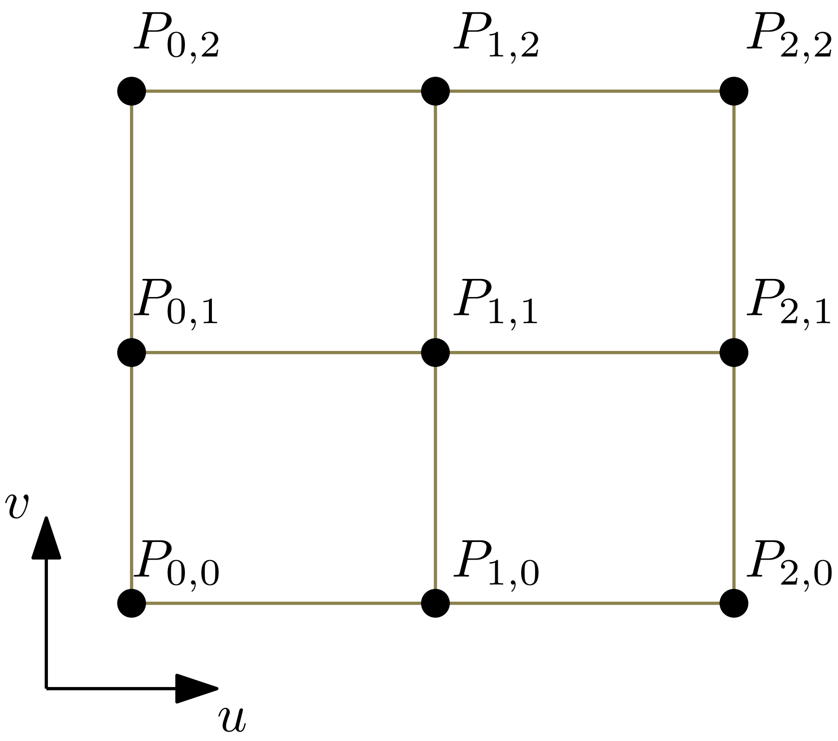


As with Bézier surfaces: grid of control points

Example: **uniform biquadratic B-spline surface** → (3×3 control points)

$$\mathbf{P}(u,v) = \left(\frac{1}{2}\right)^2 (u^2, u, 1) \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{P}_{00} & \mathbf{P}_{01} & \mathbf{P}_{02} \\ \mathbf{P}_{10} & \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{20} & \mathbf{P}_{21} & \mathbf{P}_{22} \end{pmatrix} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}^T \begin{pmatrix} v^2 \\ v \\ 1 \end{pmatrix}$$

$$0 \le u, v \le 1$$



As with Bézier surfaces: grid of control points

Example: **uniform biquadratic B-spline surface** (3 × 3 control points)

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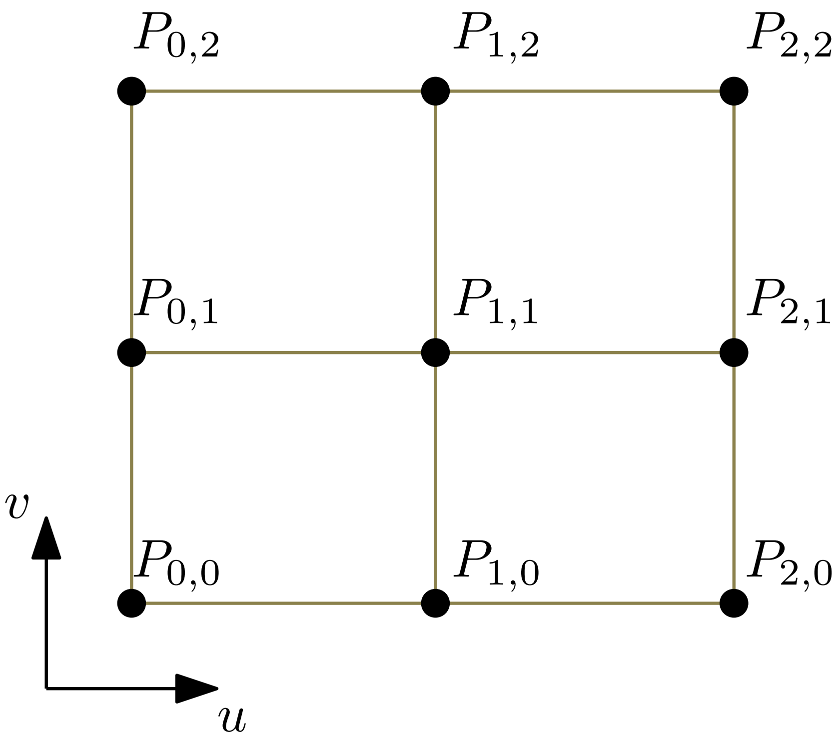
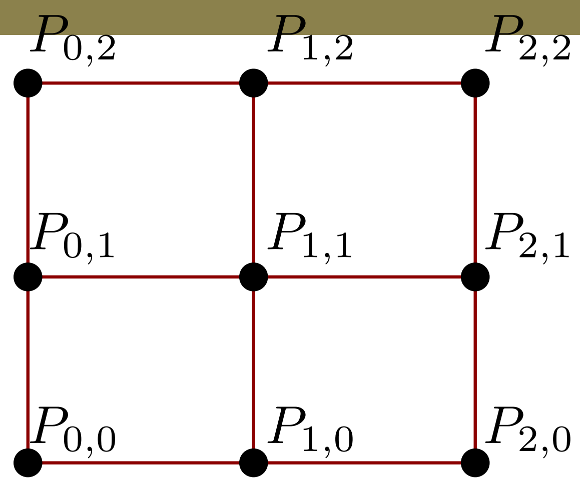


Written as affine combination of the control points:

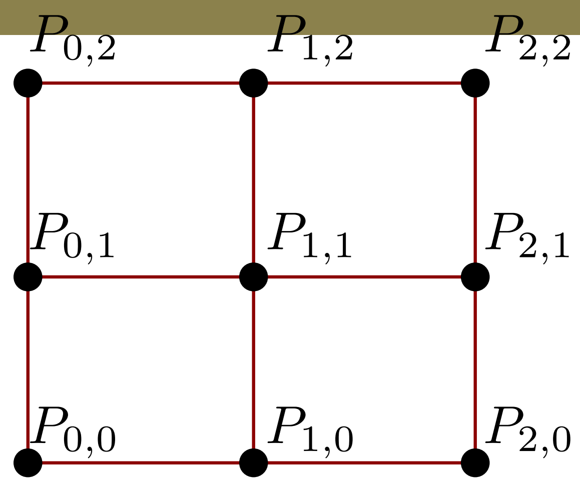
$$\mathbf{P}(u,v) = \sum_{i=0}^{2} \sum_{j=0}^{2} N_{i,3}(u) N_{j,3}(v) \mathbf{P}_{ij}$$

Corner points

What are the corner points of this surface patch?



Corner points



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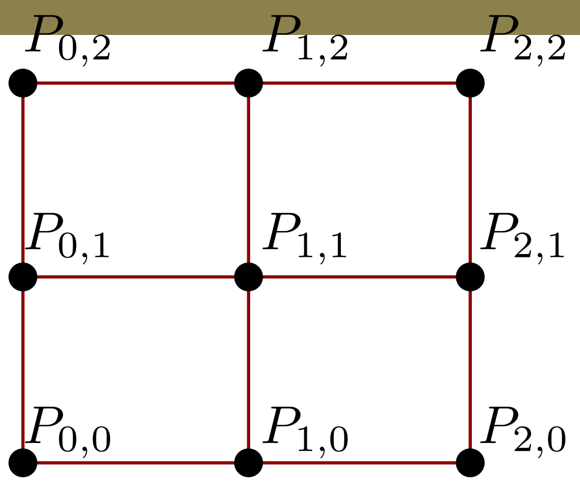

Corner points

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Bottom-left corner:

$$\mathbf{P}(0,0) = \left(\frac{1}{2}\right)^2 (0, 0, 1) \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{P}_{00} & \mathbf{P}_{01} & \mathbf{P}_{02} \\ \mathbf{P}_{10} & \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{20} & \mathbf{P}_{21} & \mathbf{P}_{22} \end{pmatrix} \begin{pmatrix} 1 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

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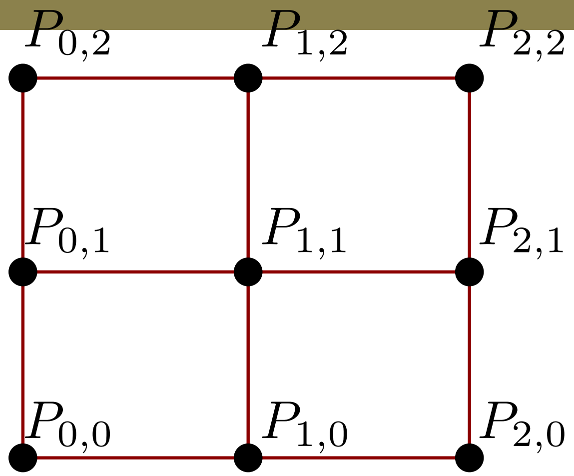

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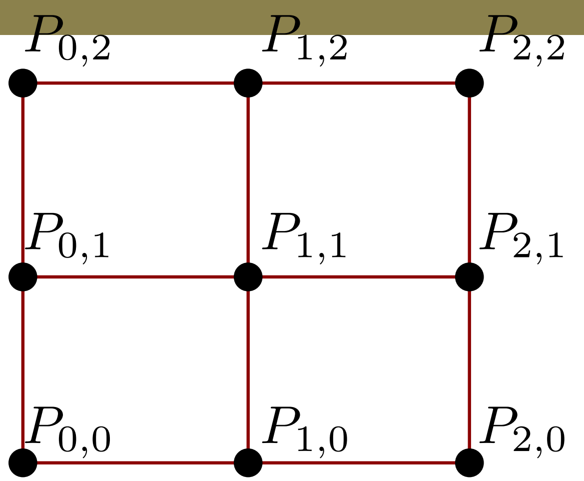

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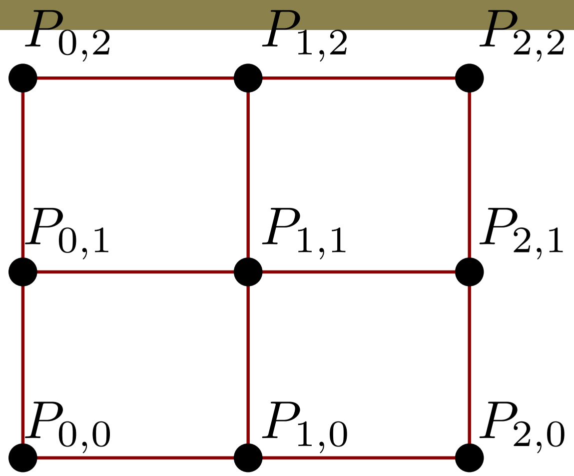

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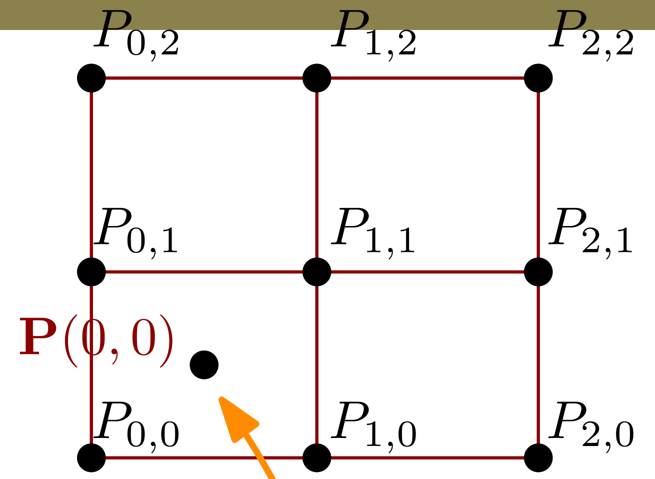

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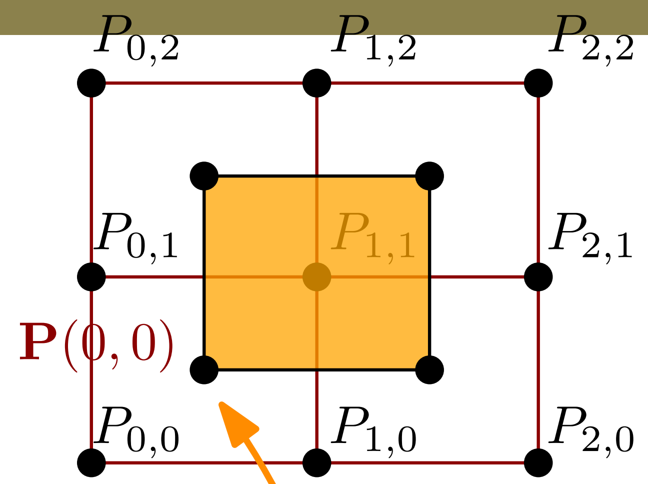

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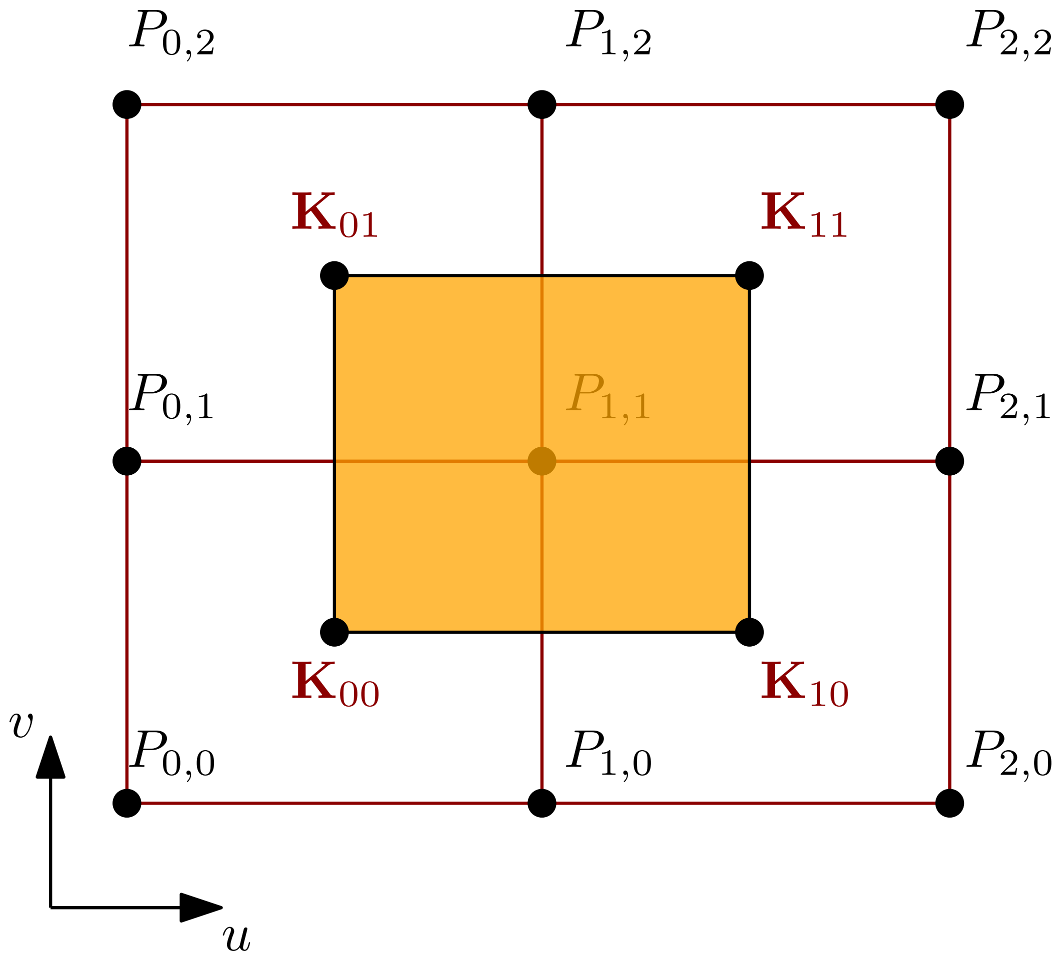
Corner points of biquadratic patch

$$\mathbf{K}_{00} = \mathbf{P}(0,0) = \frac{1}{4}(\mathbf{P}_{00} + \mathbf{P}_{01} + \mathbf{P}_{10} + \mathbf{P}_{11})$$

$$\mathbf{K}_{01} = \mathbf{P}(0,1) = \frac{1}{4}(\mathbf{P}_{01} + \mathbf{P}_{02} + \mathbf{P}_{11} + \mathbf{P}_{12})$$

$$\mathbf{K}_{10} = \mathbf{P}(1,0) = \frac{1}{4}(\mathbf{P}_{10} + \mathbf{P}_{11} + \mathbf{P}_{20} + \mathbf{P}_{21})$$

$$\mathbf{K}_{11} = \mathbf{P}(1,1) = \frac{1}{4}(\mathbf{P}_{11} + \mathbf{P}_{12} + \mathbf{P}_{21} + \mathbf{P}_{22})$$



Example of biquadratic B-Spline patch

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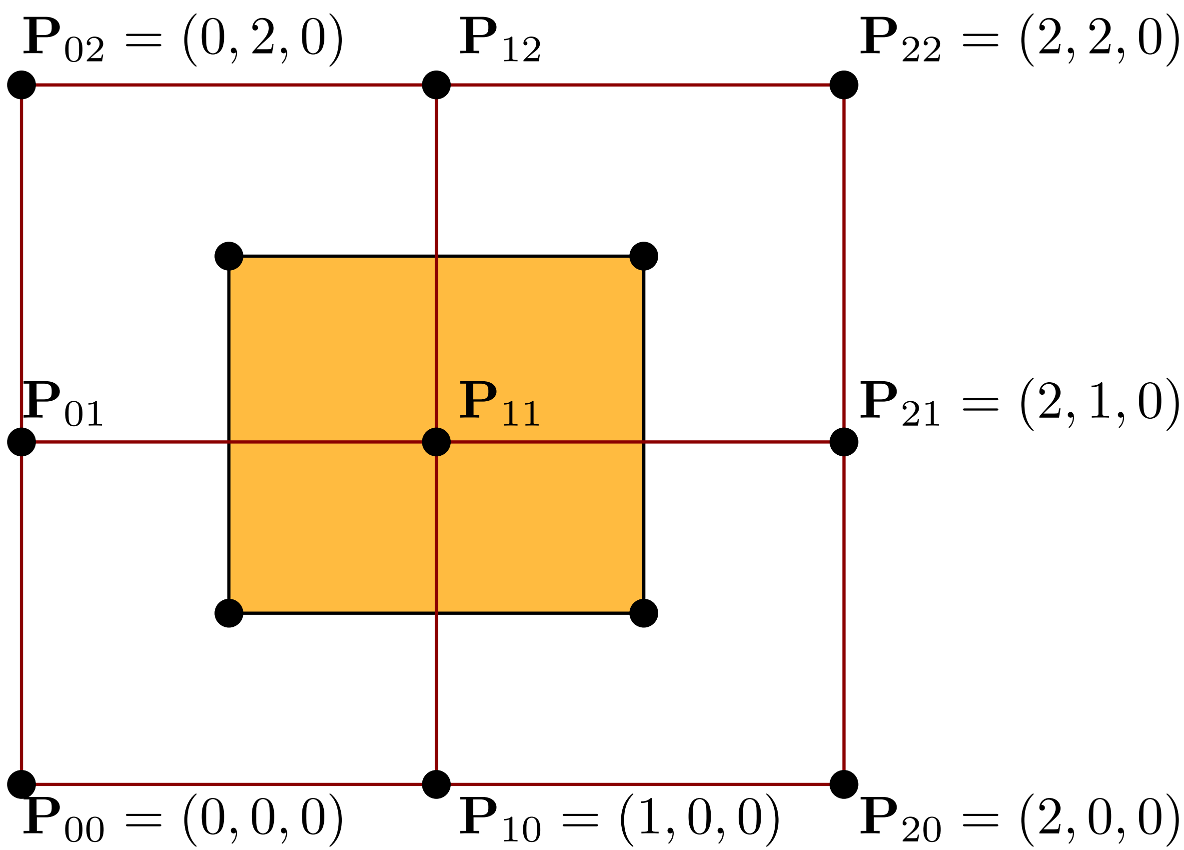

$$\mathbf{P}_{00} = (0, 0, 0), \quad \mathbf{P}_{01} = (0, 1, 0), \quad \mathbf{P}_{02} = (0, 2, 0),$$
$$\mathbf{P}_{10} = (1, 0, 0), \quad \mathbf{P}_{11} = (1, 1, 1), \quad \mathbf{P}_{12} = (1, 2, 0),$$
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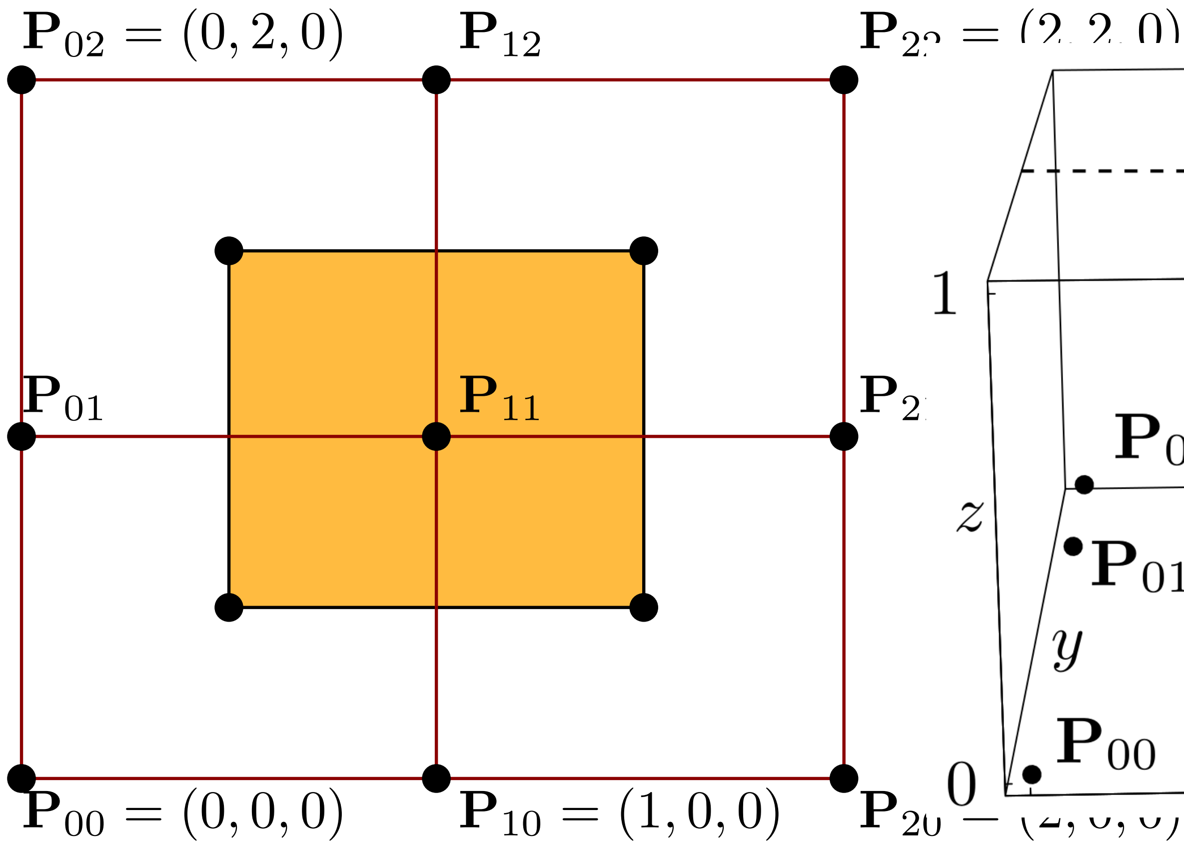

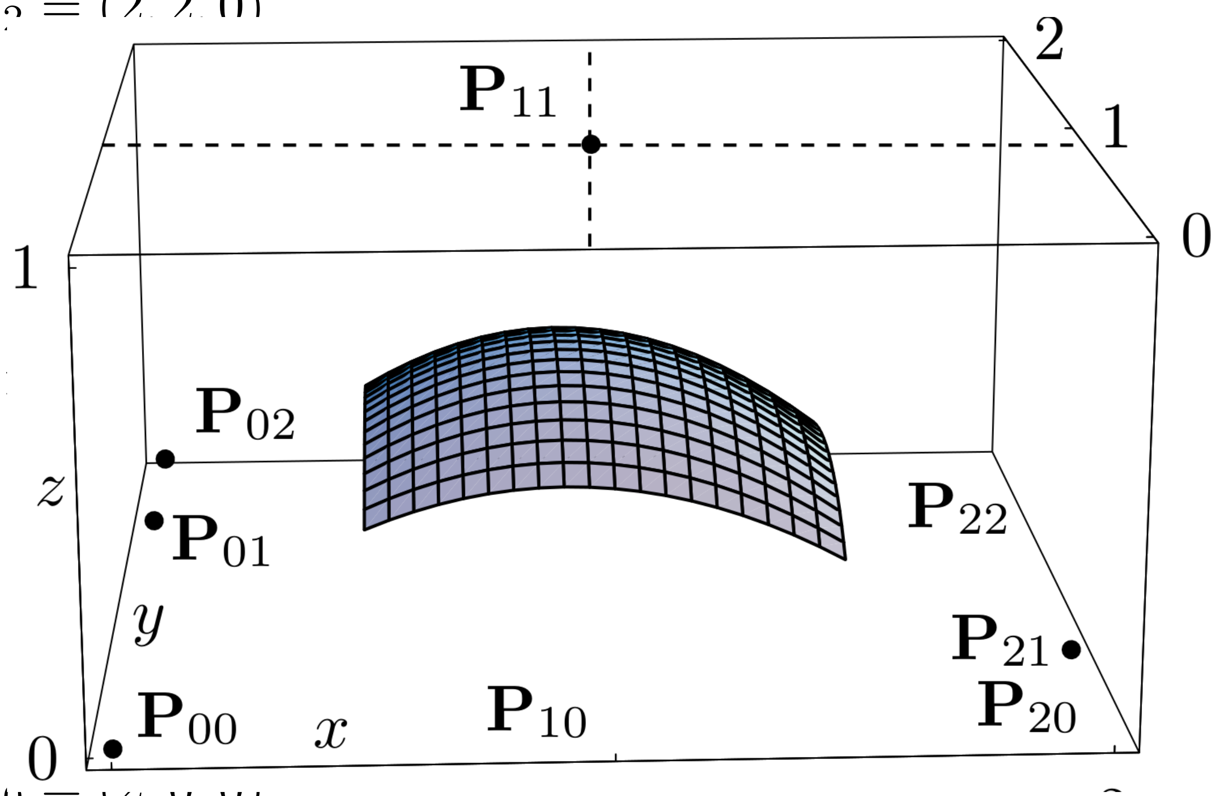

[Fig. 7.25 from Salomon]

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General formula

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$$\mathbf{P}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} \mathbf{P}_{ij} N_{i,p}(u) N_{j,q}(v)$$

$$N_{i,1}(t) = \begin{cases} 1, & \text{if } t \in [t_i, t_{i+1}) \\ 0, & \text{otherwise} \end{cases} \qquad N_{i,k}(t) = \frac{t - t_i}{t_{i+k-1} - t_i} N_{i,k-1}(t) + \frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} N_{i+1,k-1}(t)$$

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For example, for the *open* version, we can use

$$U = \{\underbrace{0,\ldots,0}_{p+1}, u_{p+1},\ldots,u_{r-p-1},\underbrace{1,\ldots,1}_{p+1}\} \quad r+1 \text{ knots with } r = n+p+1$$

$$V = \{\underbrace{0,\ldots,0}_{q+1}, v_{q+1},\ldots,v_{s-q-1},\underbrace{1,\ldots,1}_{q+1}\} \quad s+1 \text{ knots with } s = m+q+1$$

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$$\mathbf{P}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} \mathbf{P}_{ij} N_{i,p}(u) N_{j,q}(v)$$

In order to compute $S(u,v)$ for a given pair (u,v), we need:

1. Find the parameter subinterval for u. Say $u \in [u_i, u_{i+1})$
2. Compute $N_{i-p,p}(u), \ldots, N_{i,p}(u) = (N_{k,p}(u))_{i-p \leq k \leq i}$
3. Find the parameter subinterval for v. Say $v \in [v_j, v_{j+1})$
4. Compute $N_{j-q,q}(v), \ldots, N_{j,q}(v) = (N_{l,q}(v))_{j-q \leq l \leq j}$
5. Then we can compute the patch for parameters in $[u_i, u_{i+1}) \times [v_j, v_{j+1})$ using the formula for $\mathbf{P}(u,v)$

Properties of basis functions

$$\mathbf{P}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} \mathbf{P}_{ij} N_{i,p}(u) N_{j,q}(v)$$

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- In $[u_{i_0}, u_{i_0+1}) \times [v_{j_0}, v_{j_0+1})$, at most $(p+1)(q+1)$ basis functions are non-zero, namely $N_{i,p}(u)N_{j,q}(v)$ for all $i_0 - p \leq i \leq i_0$ and $j_0 - q \leq j \leq j_0$.

RATIONAL B-SPLINE SURFACES

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- $w_{i,j} \geq 0$: The weights associated with each control point
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Most properties extend from curves to surfaces in an analogous way