

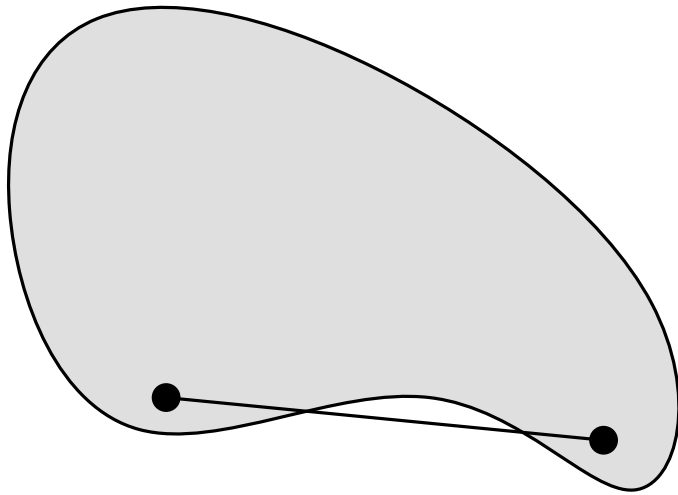
CONVEX HULL IN 2D

Vera Sacristán
Rodrigo Silveira

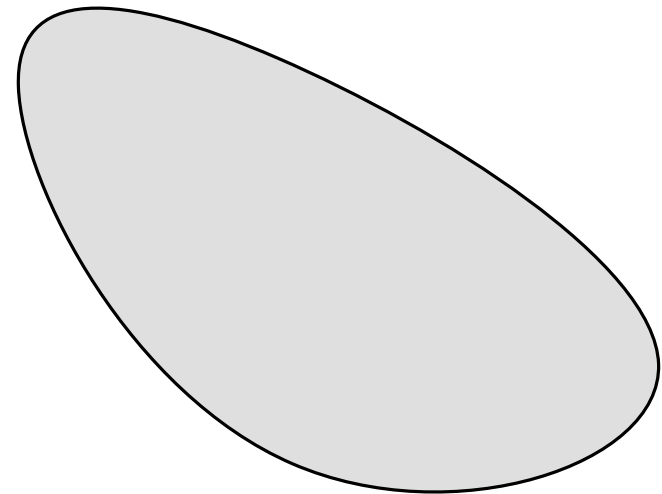
Discrete and Algorithmic Geometry Geometry
Facultat de Matemàtiques i Estadística
Universitat Politècnica de Catalunya

CONVEX HULL IN 2D

A set C is said to be **convex** if $\forall p, q \in C$ the segment \overline{pq} is enclosed in C .



not convex



convex

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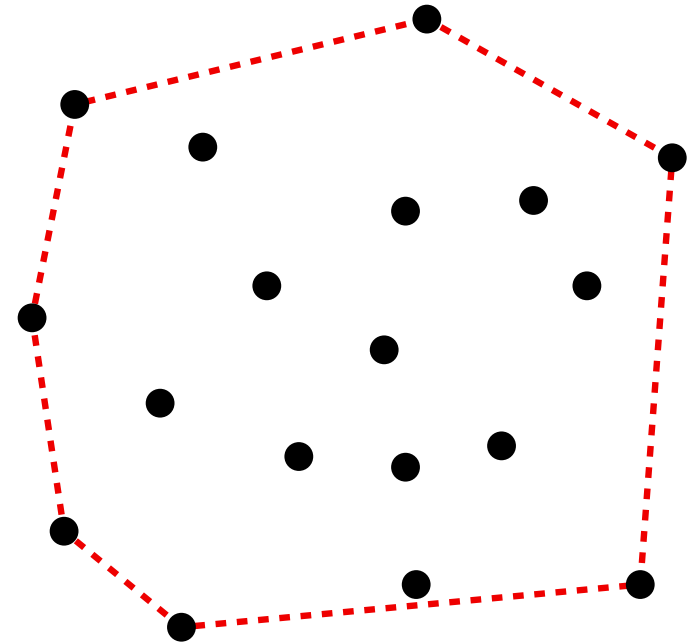
Convex hull of a set of points

The **convex hull** of a set X is the smallest convex set C enclosing X .

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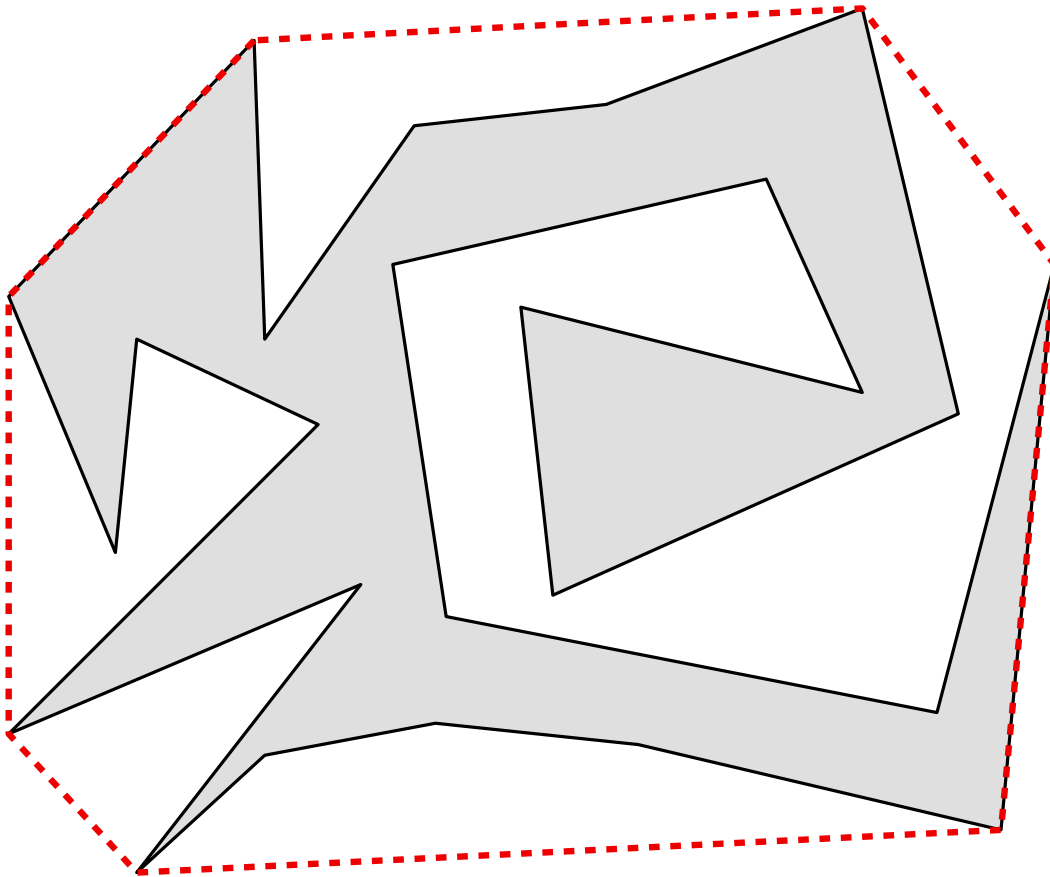


The convex hull of a finite set of points in the plane is a convex polygon.

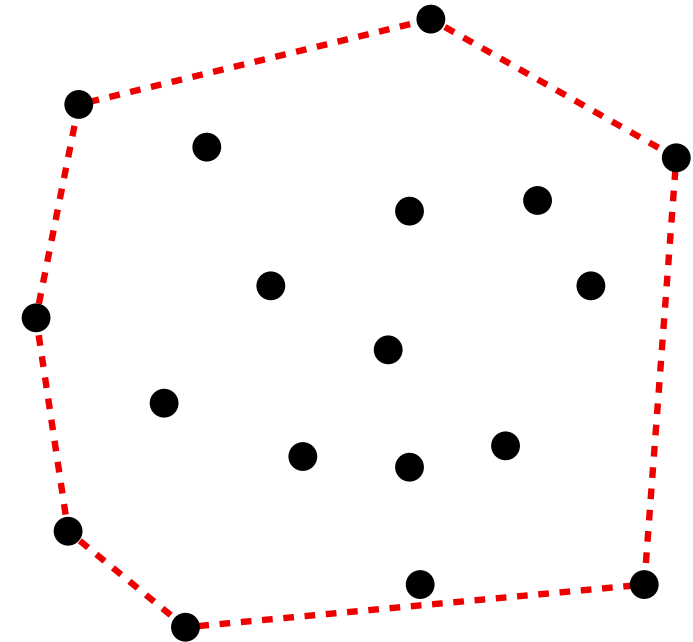
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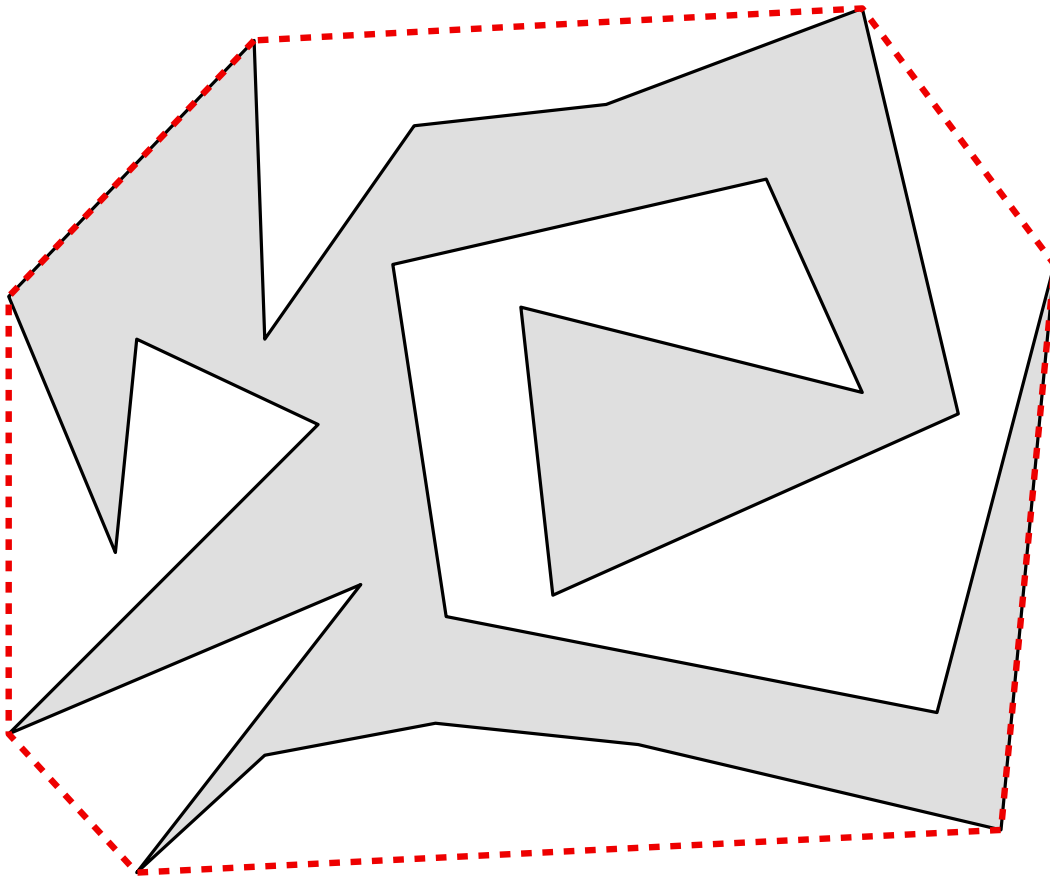


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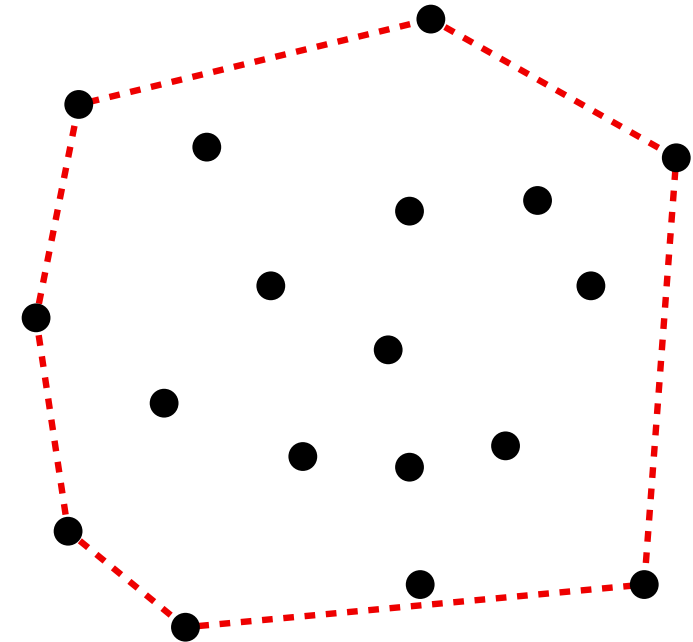
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The **convex hull** of a set X is the smallest convex set C enclosing X .



The convex hull of a simple polygon is a convex polygon.



The convex hull of a finite set of points in the plane is a convex polygon.

In both cases, the vertices of $ch(X)$ are points of X .

Why are convex hulls important?

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Alright, let's see how to **compute them**

CONVEX HULL IN 2D

Computing the extreme points

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Computing the extreme points

Characterization

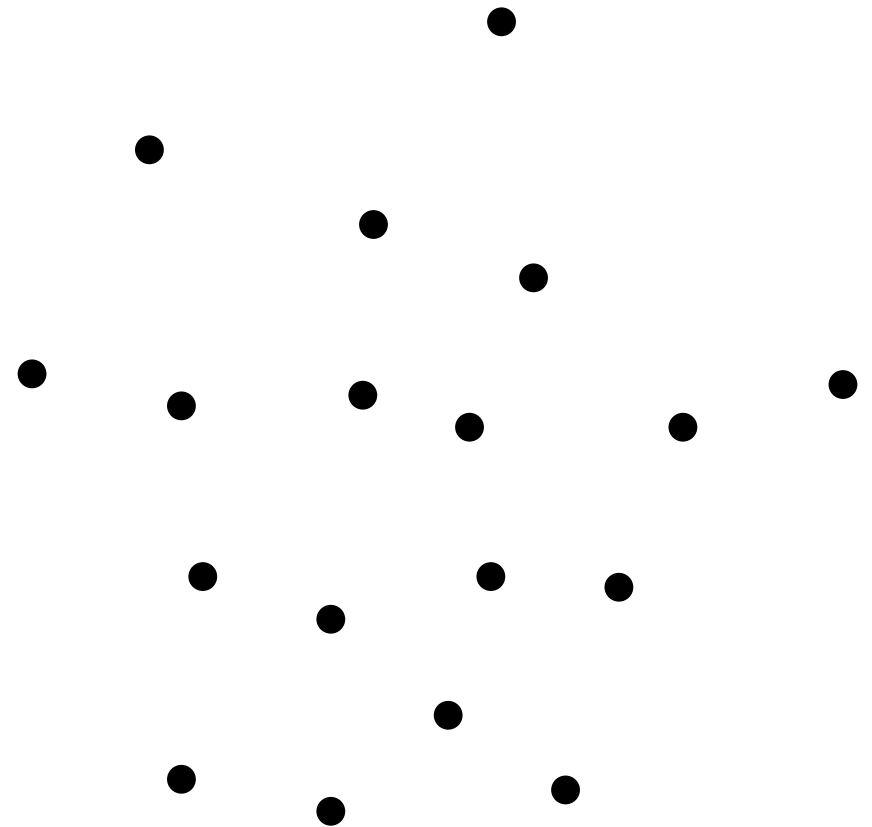
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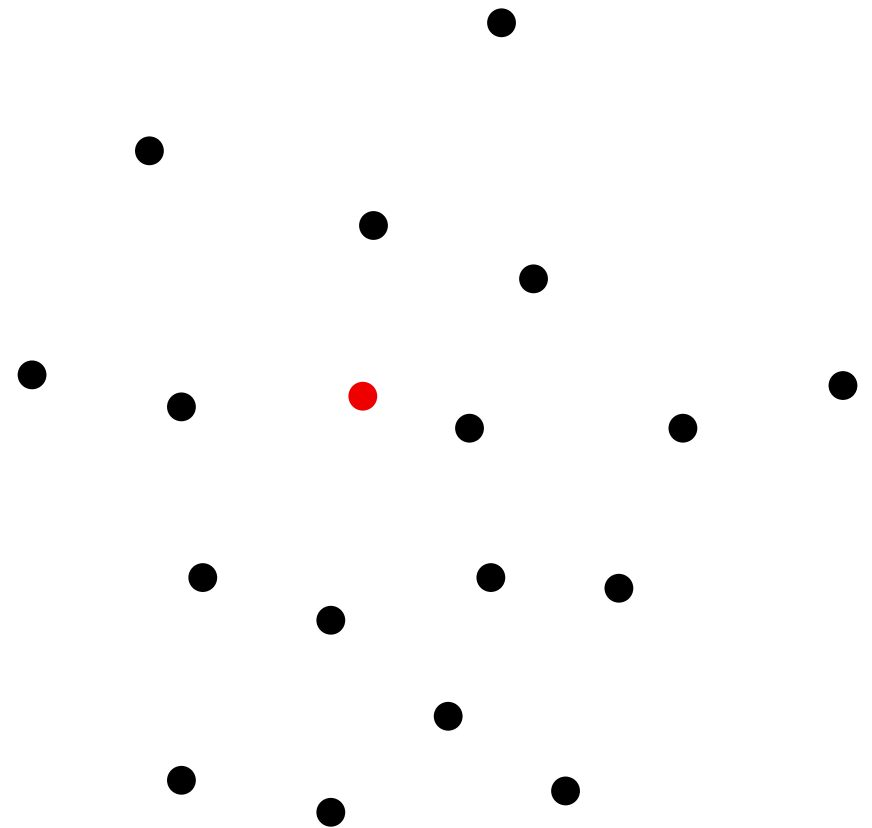


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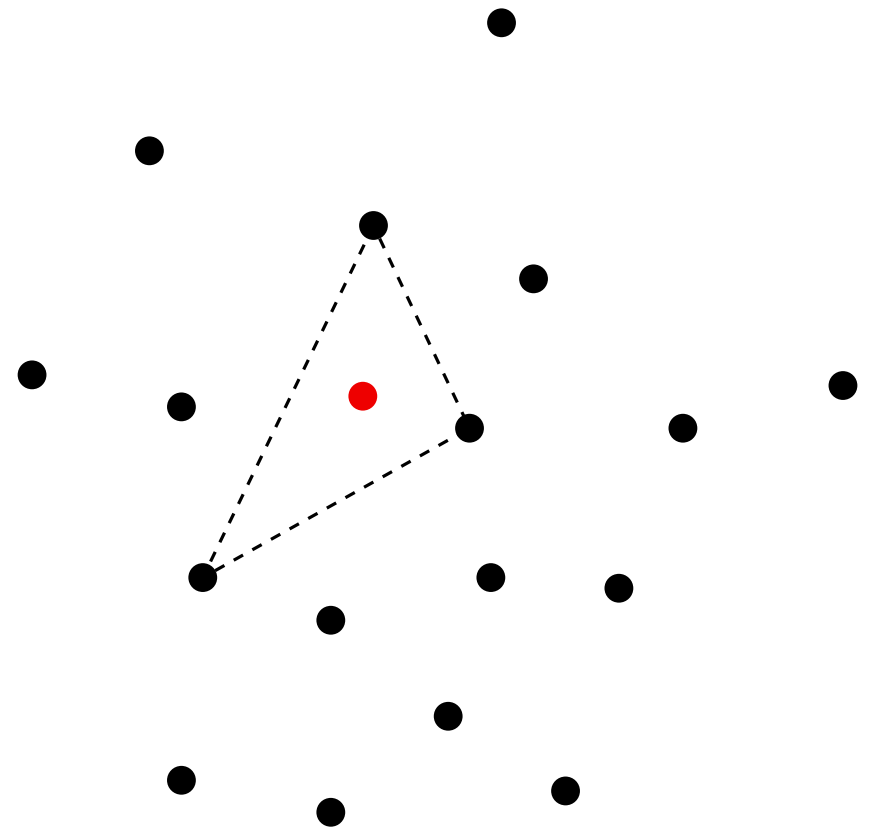


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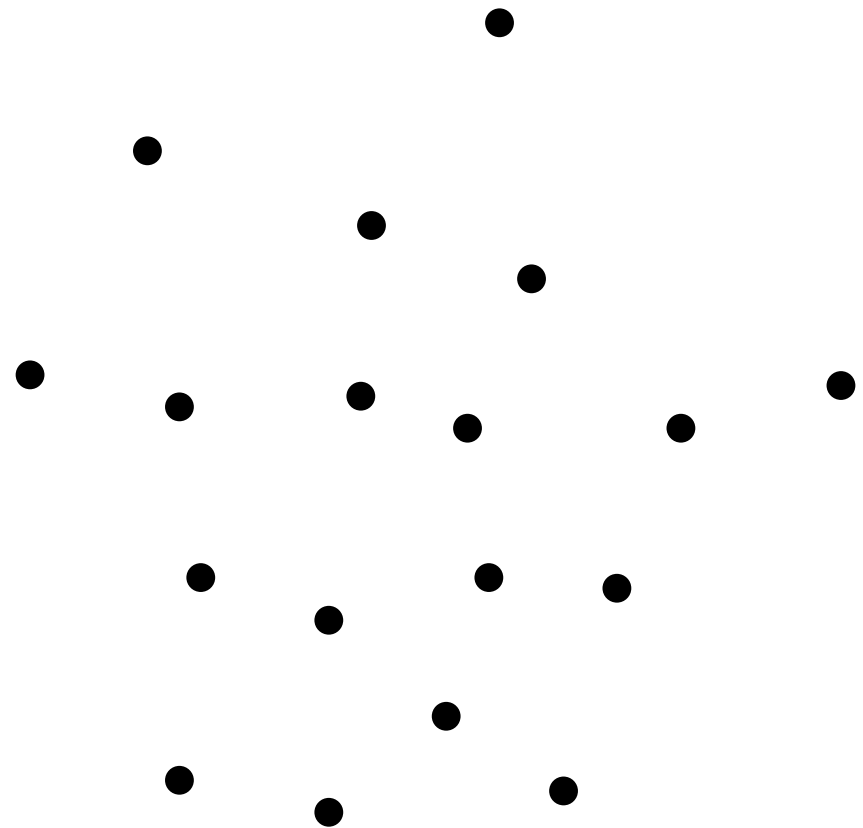
Procedure:

For each i ,

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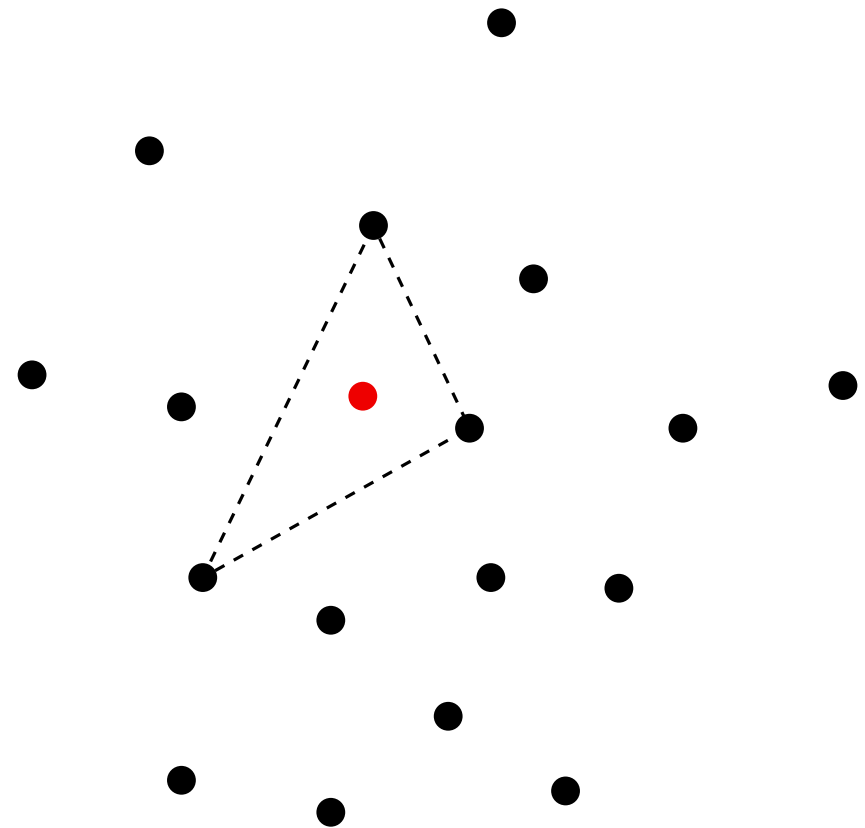
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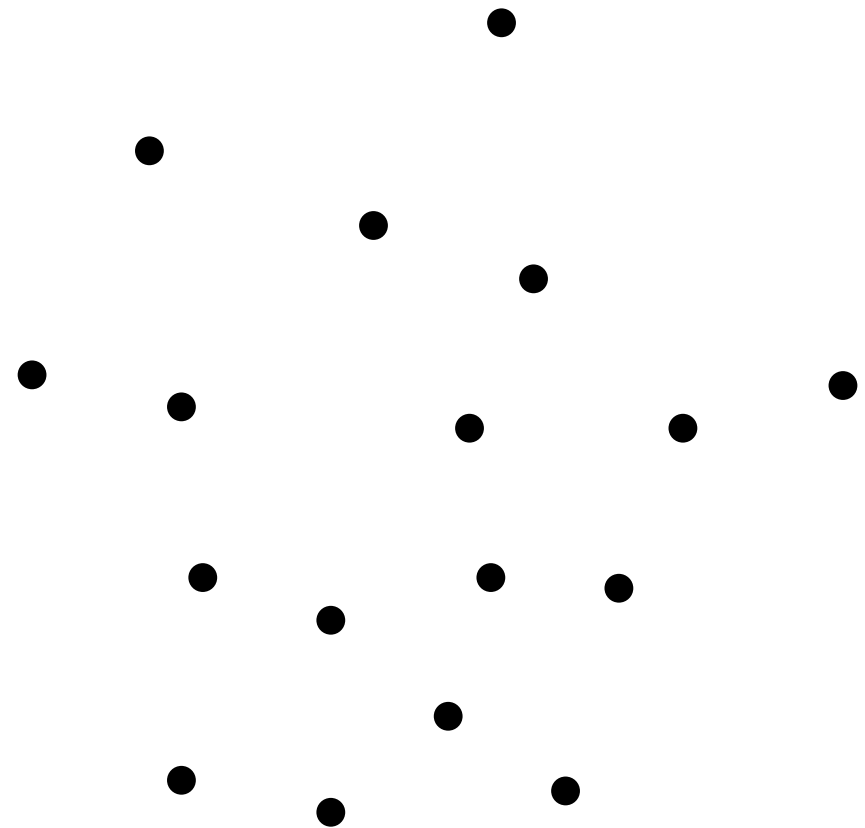
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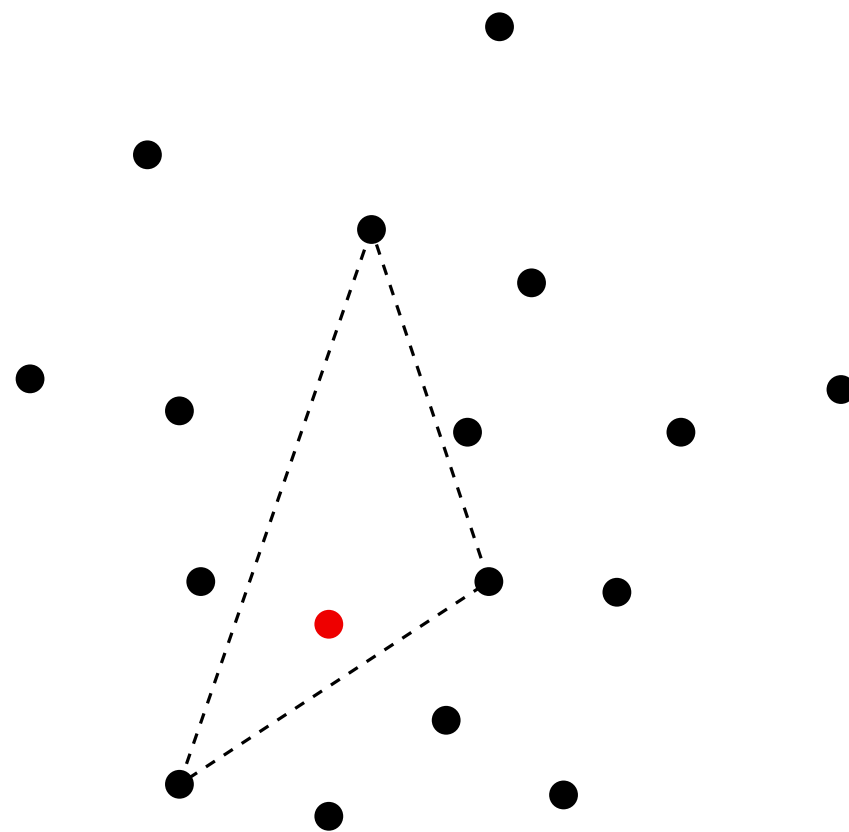
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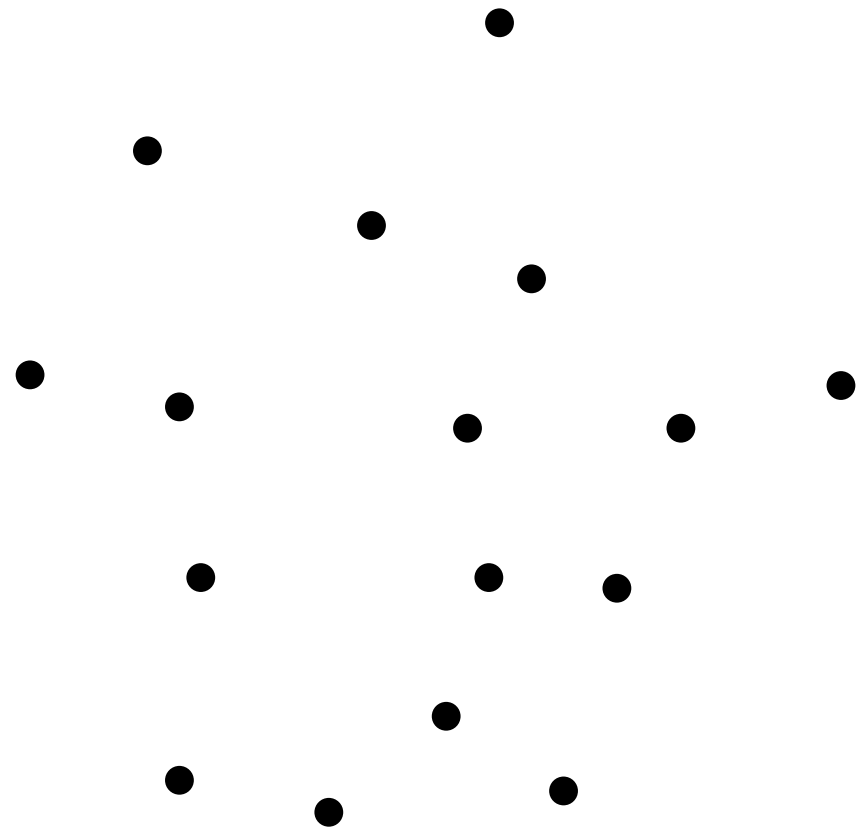
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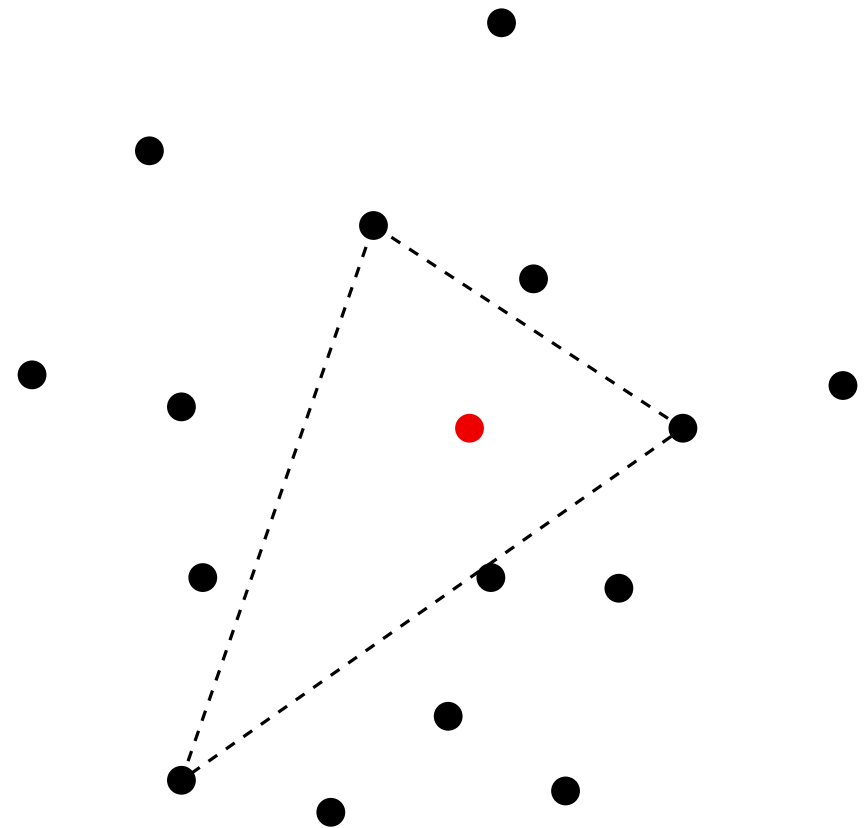
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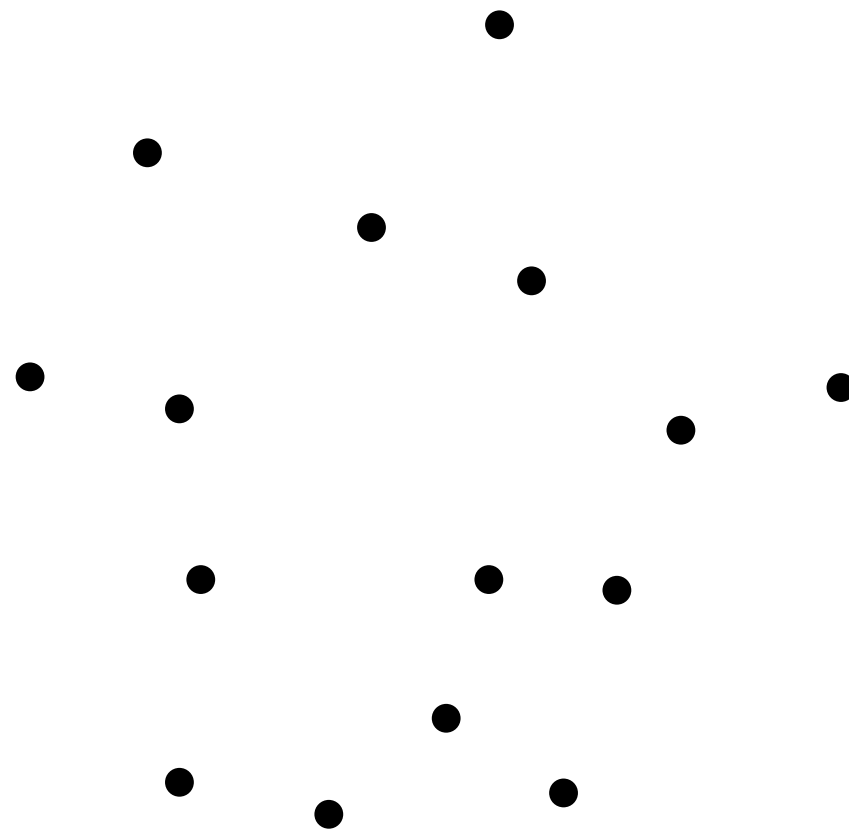
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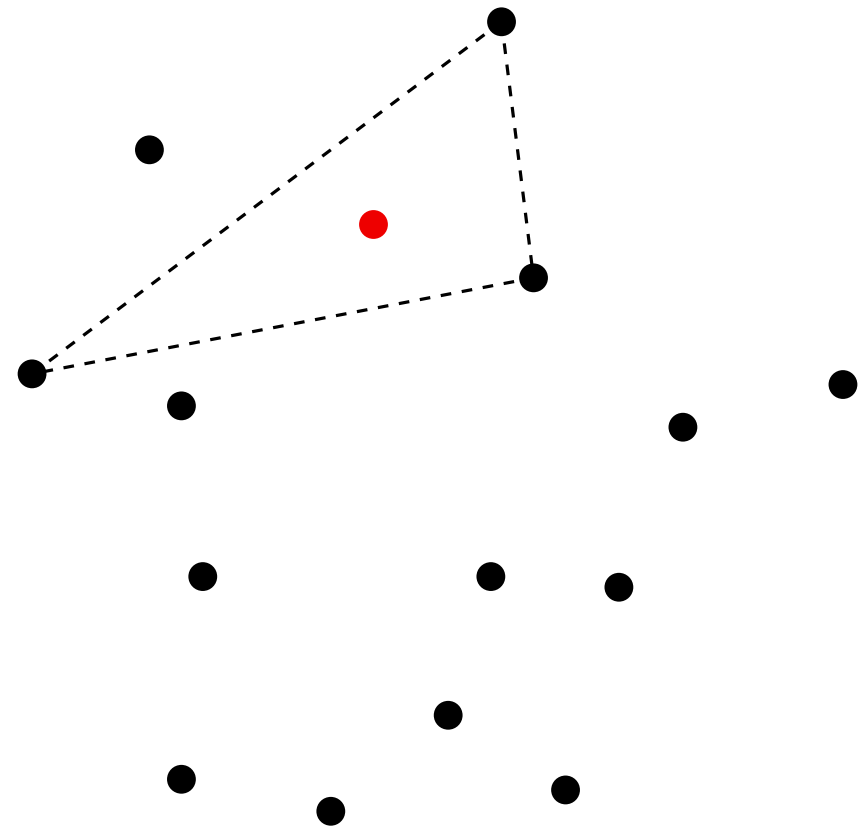
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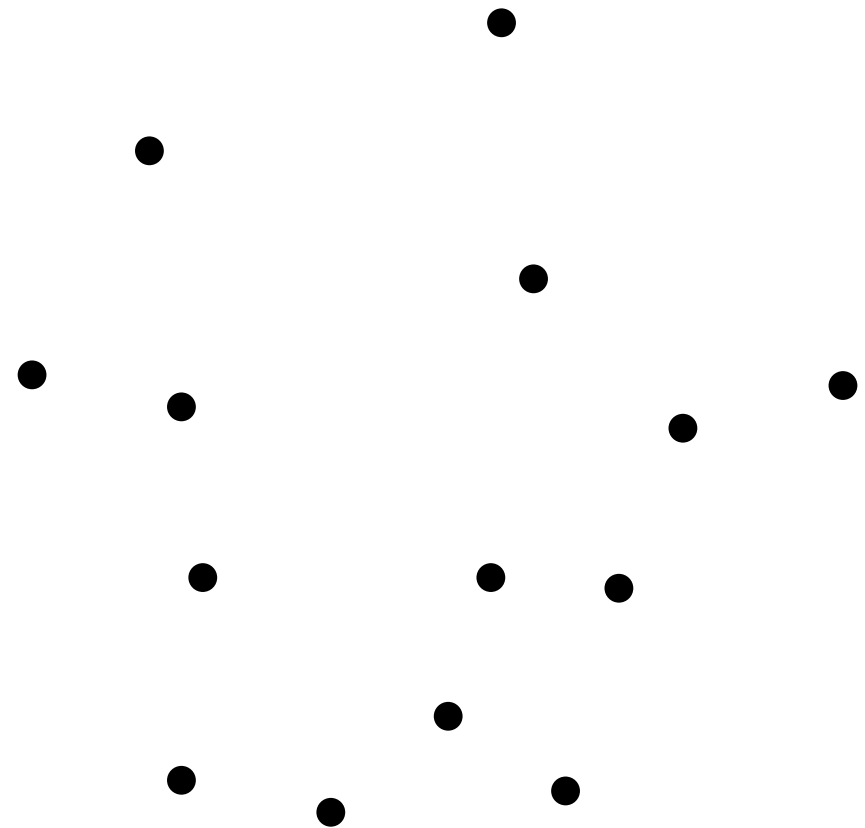
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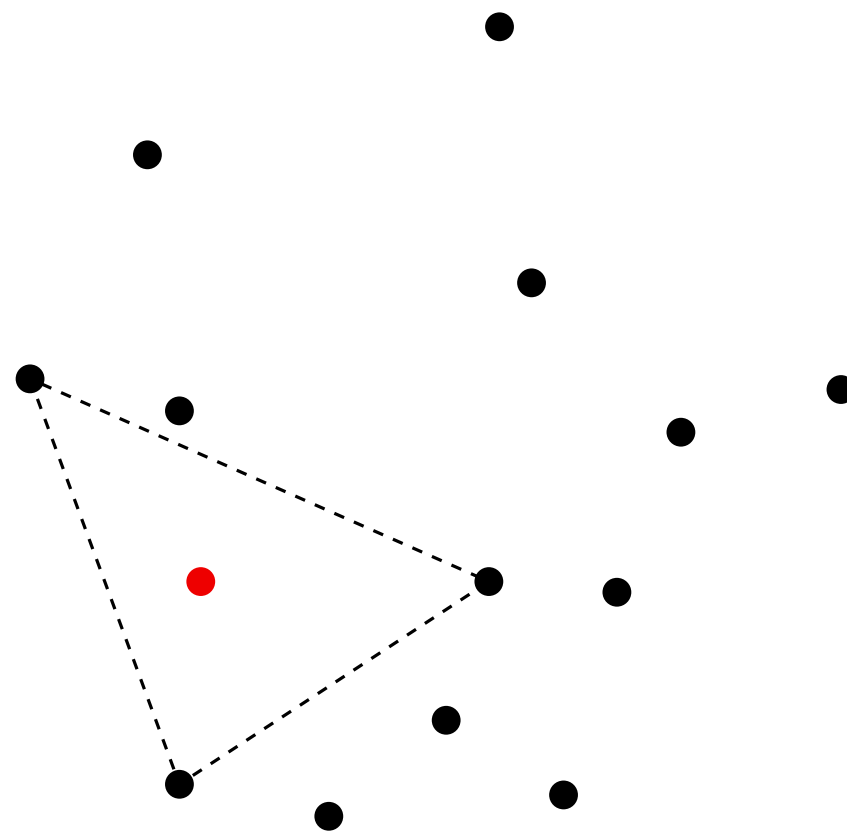
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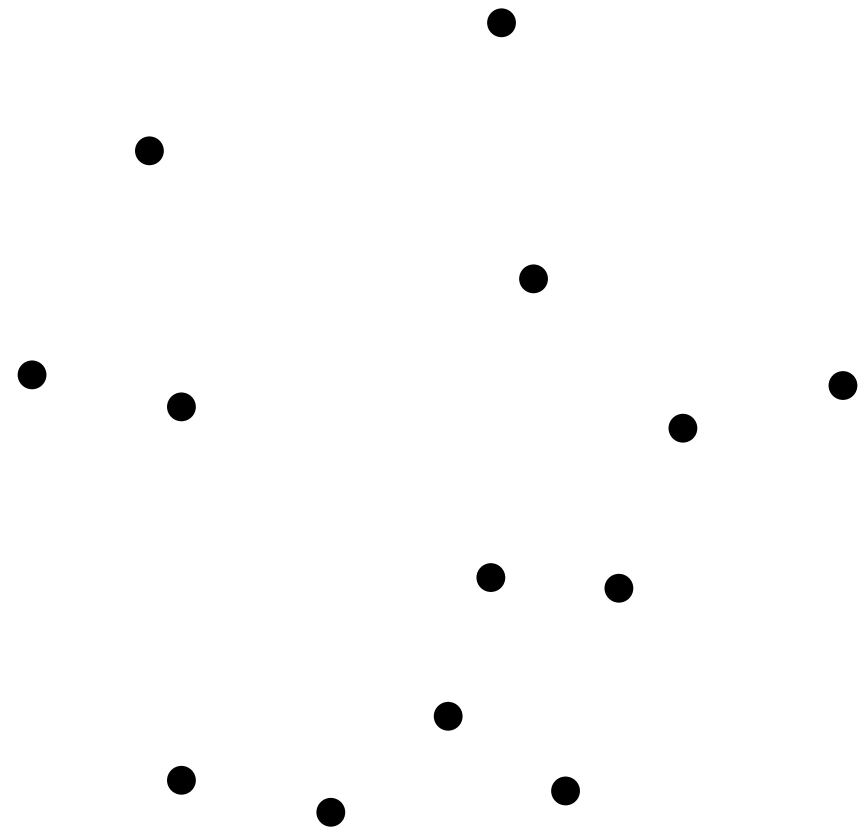
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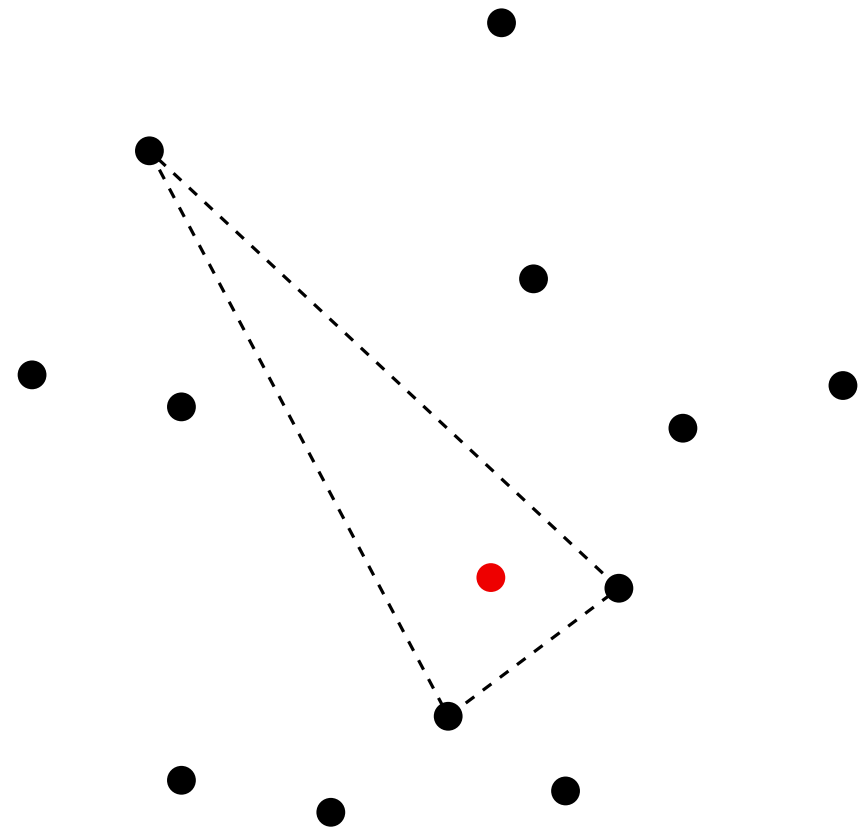
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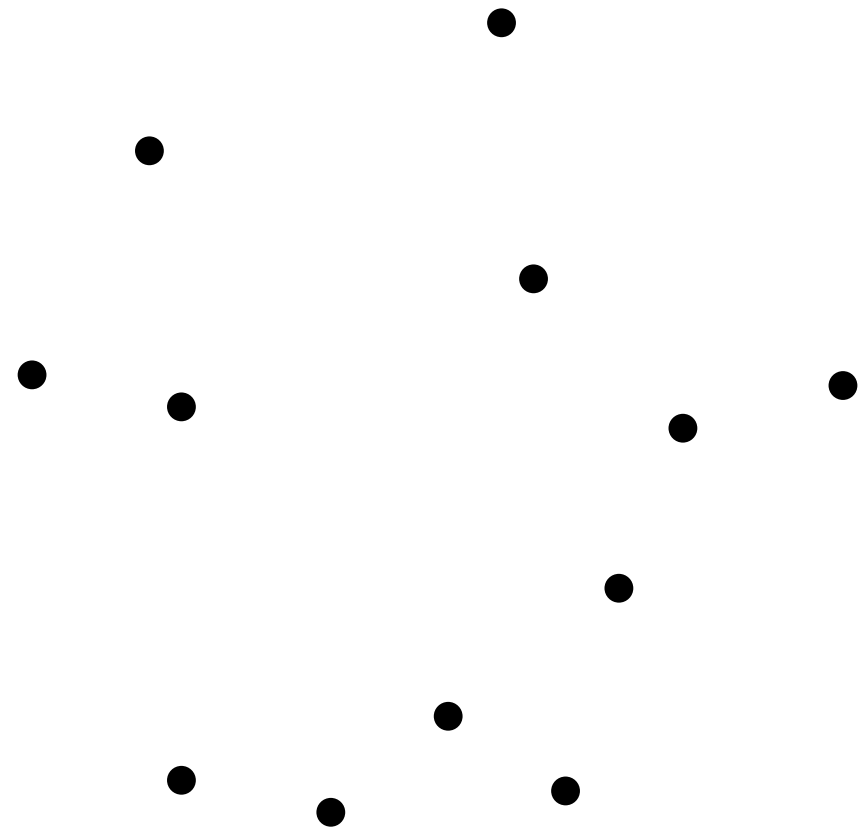
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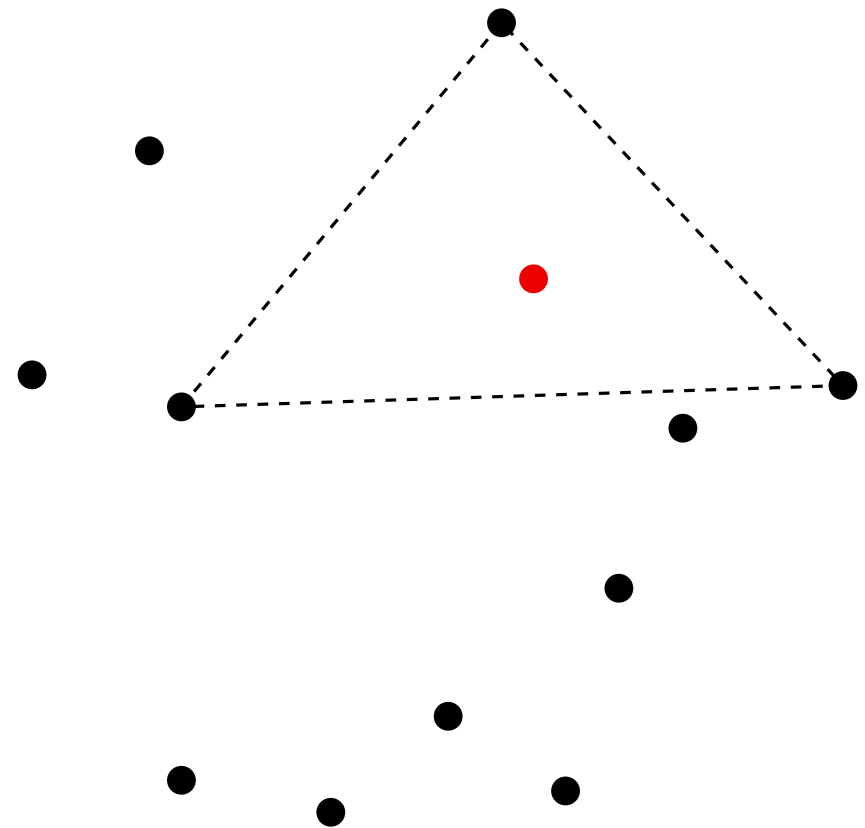
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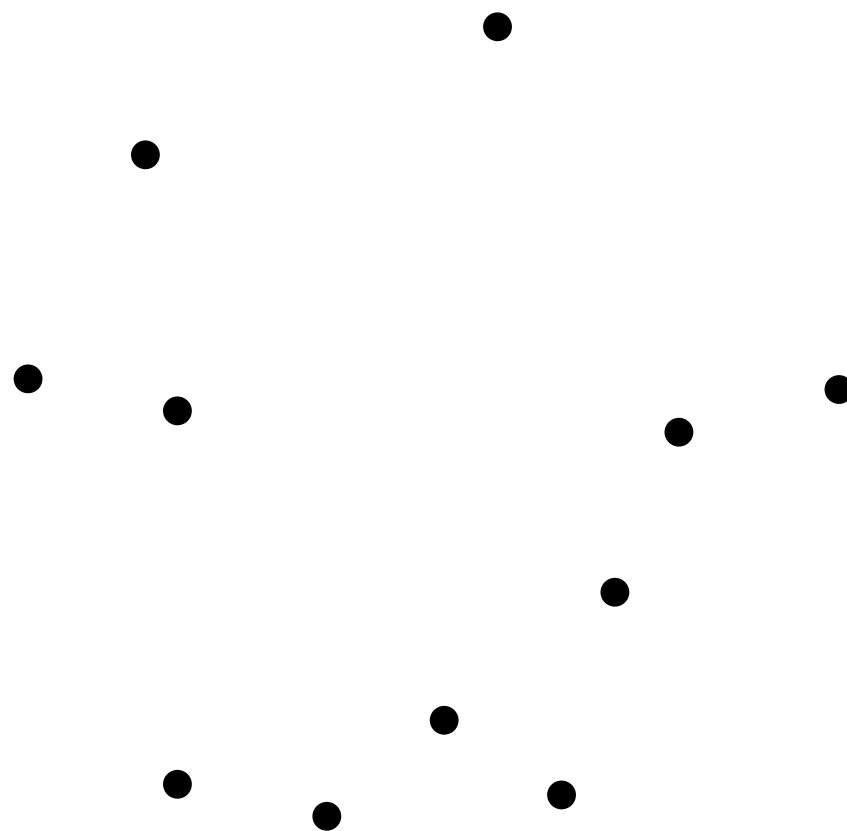
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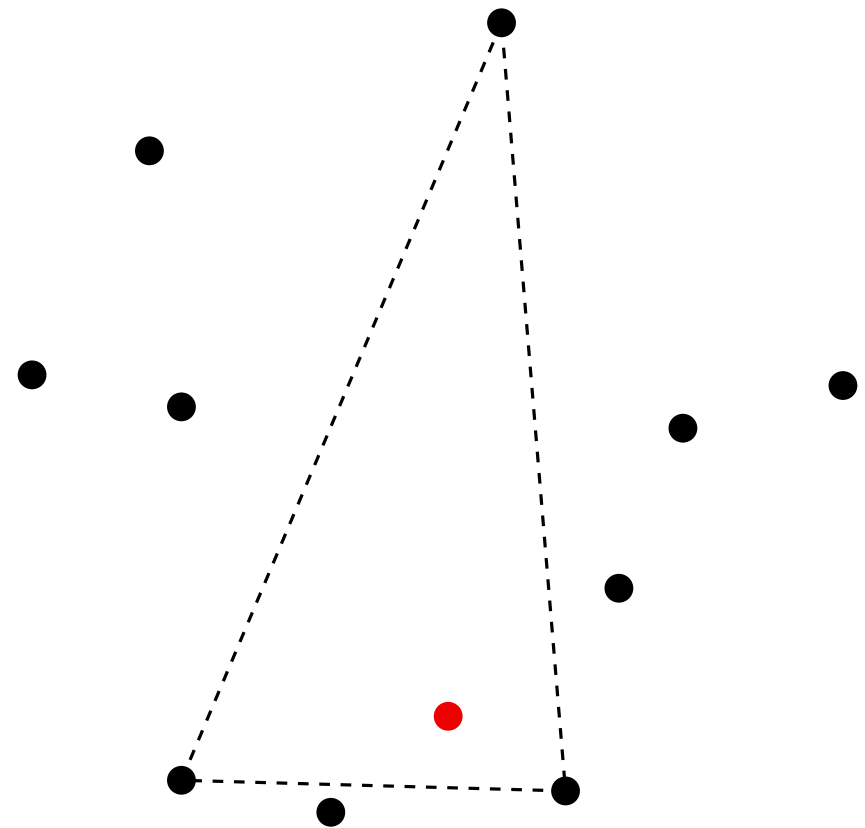
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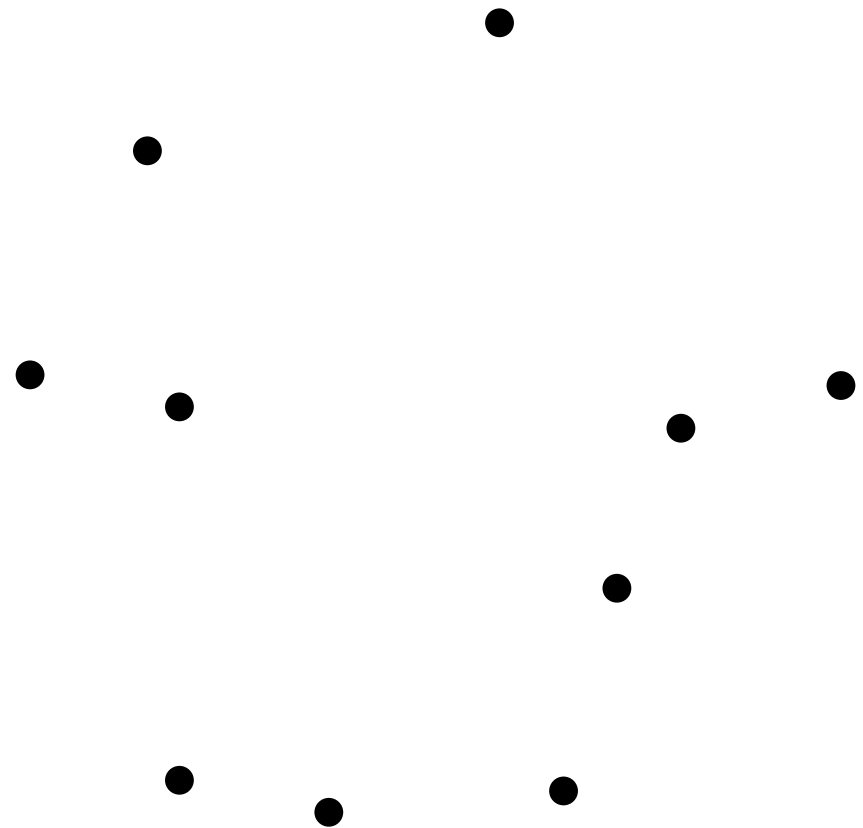
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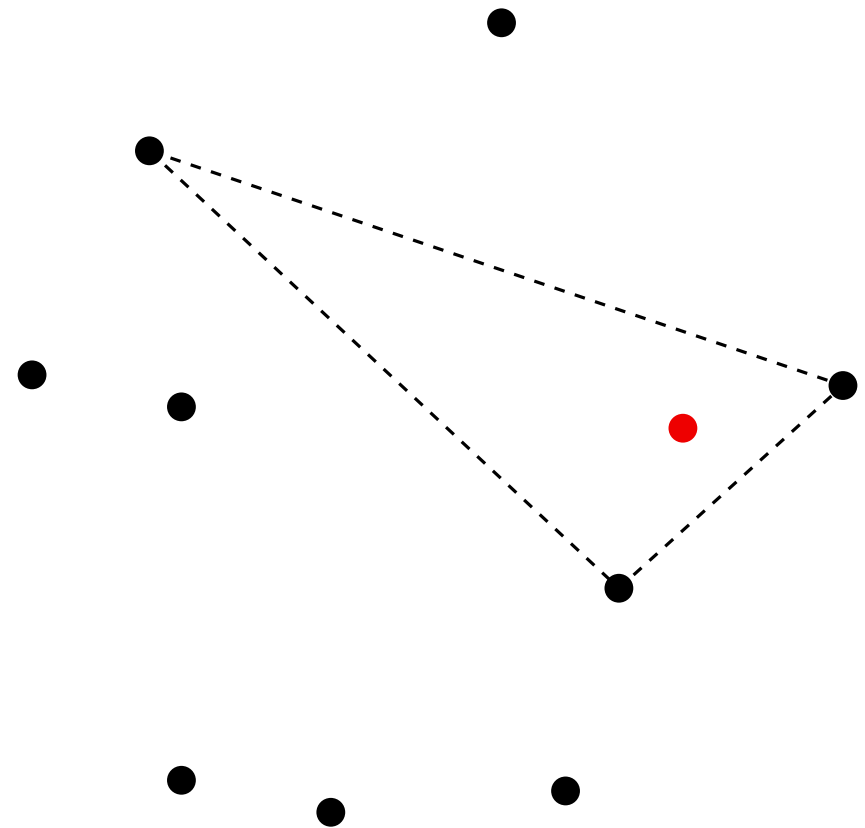
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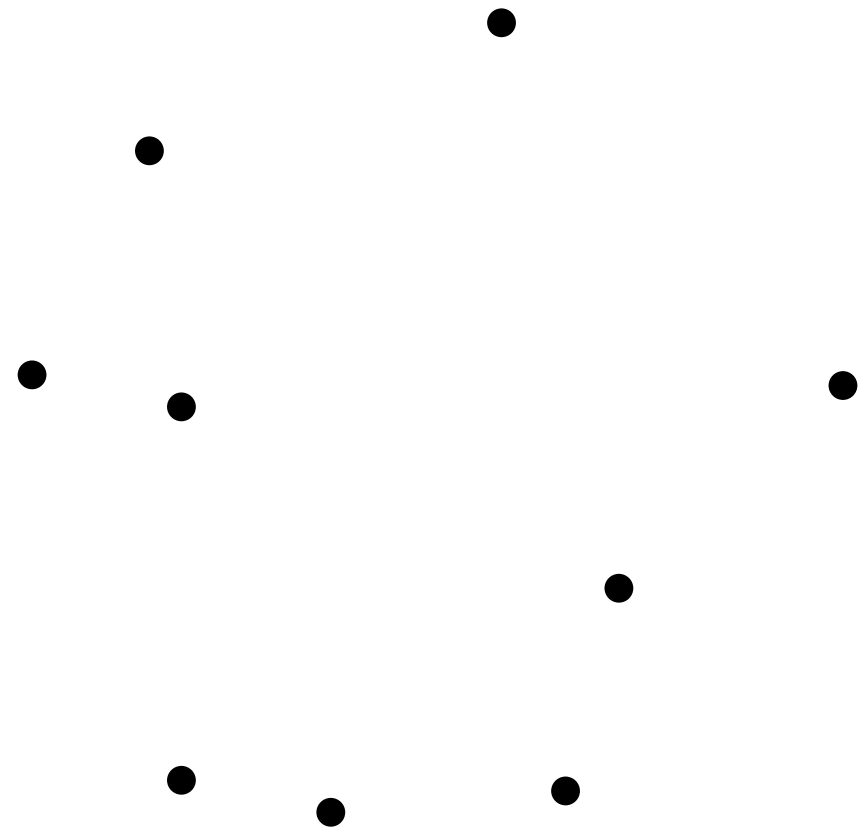
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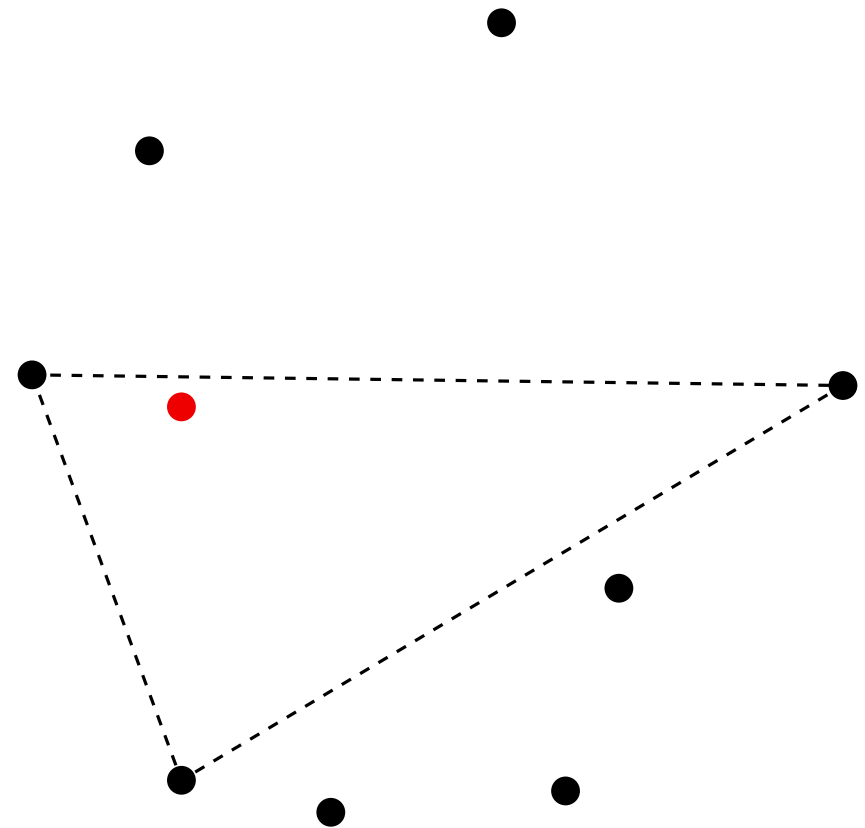
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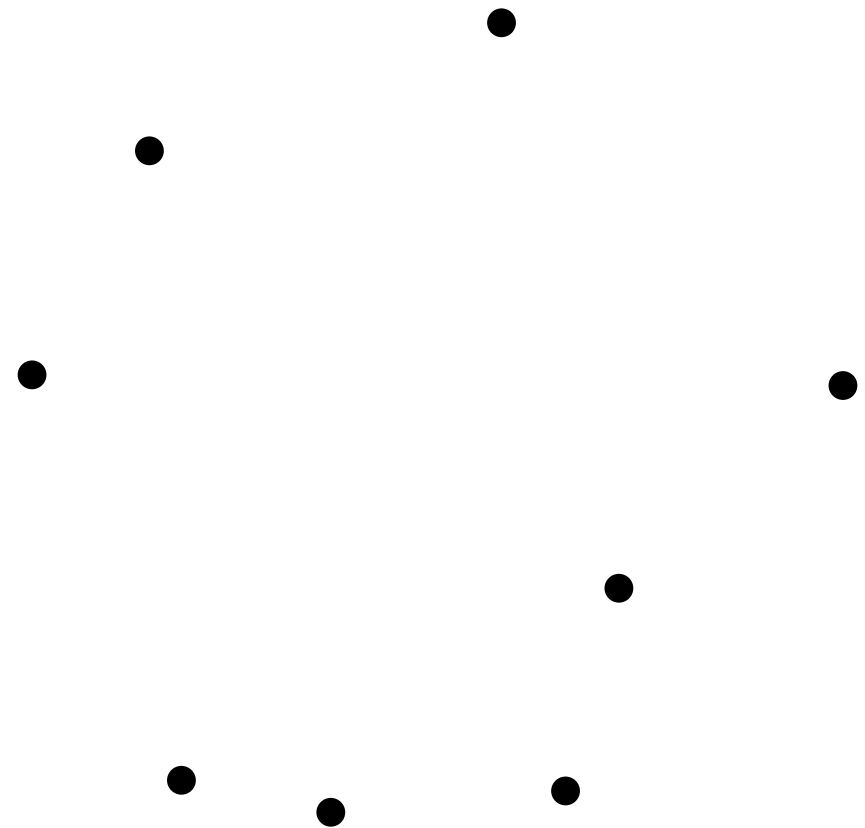
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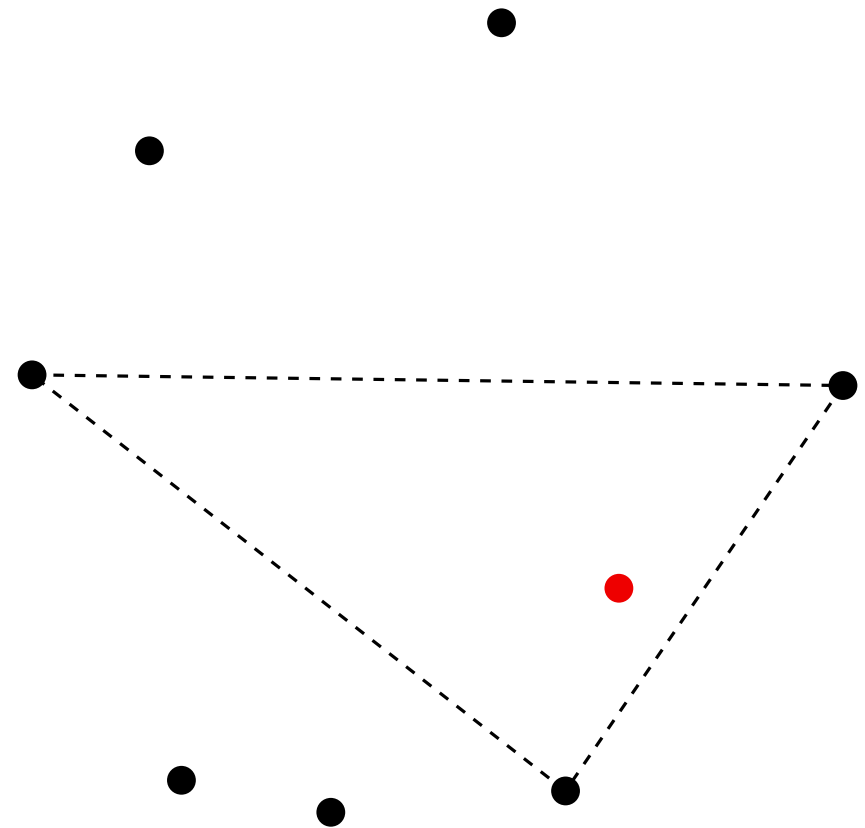
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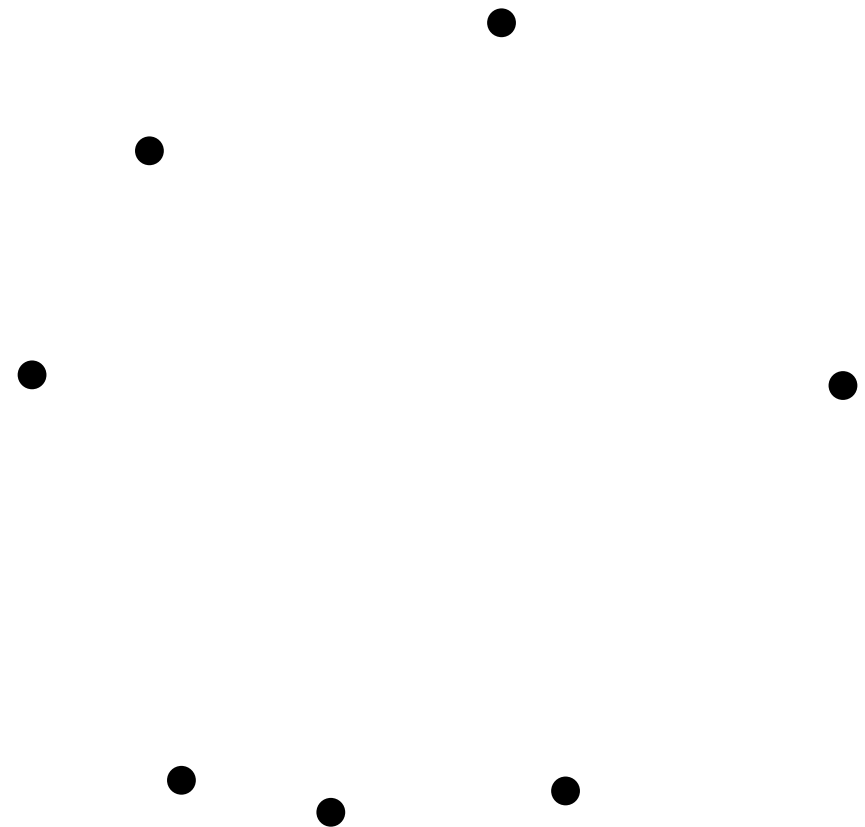
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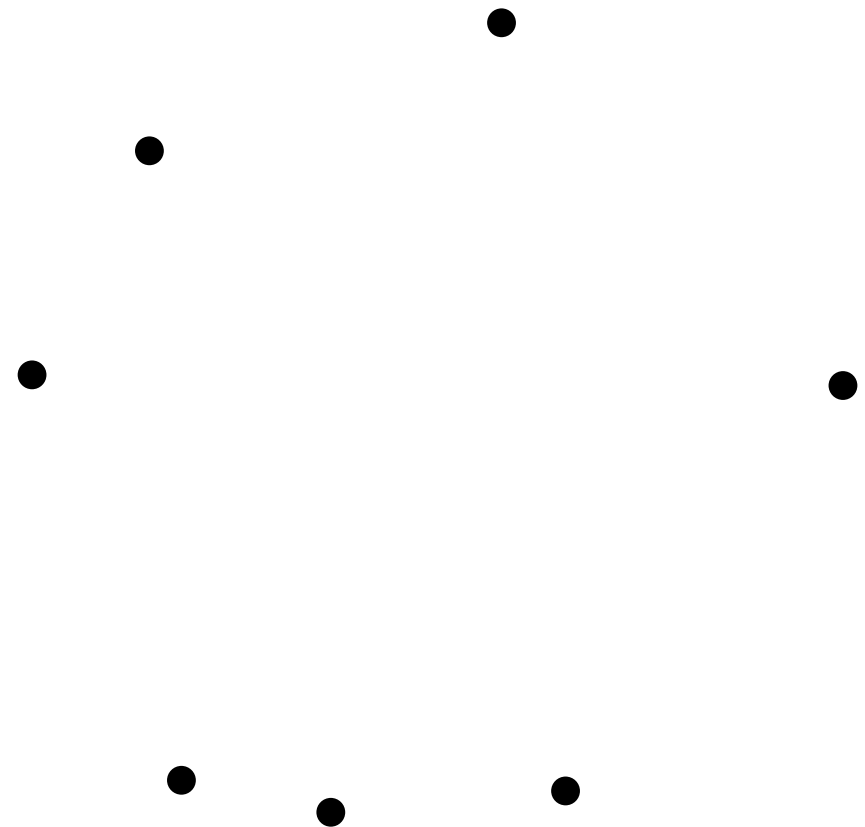
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Running time: $\Theta(n^4)$



CONVEX HULL IN 2D

Computing the extreme segments

CONVEX HULL IN 2D

Computing the extreme segments

Characterization

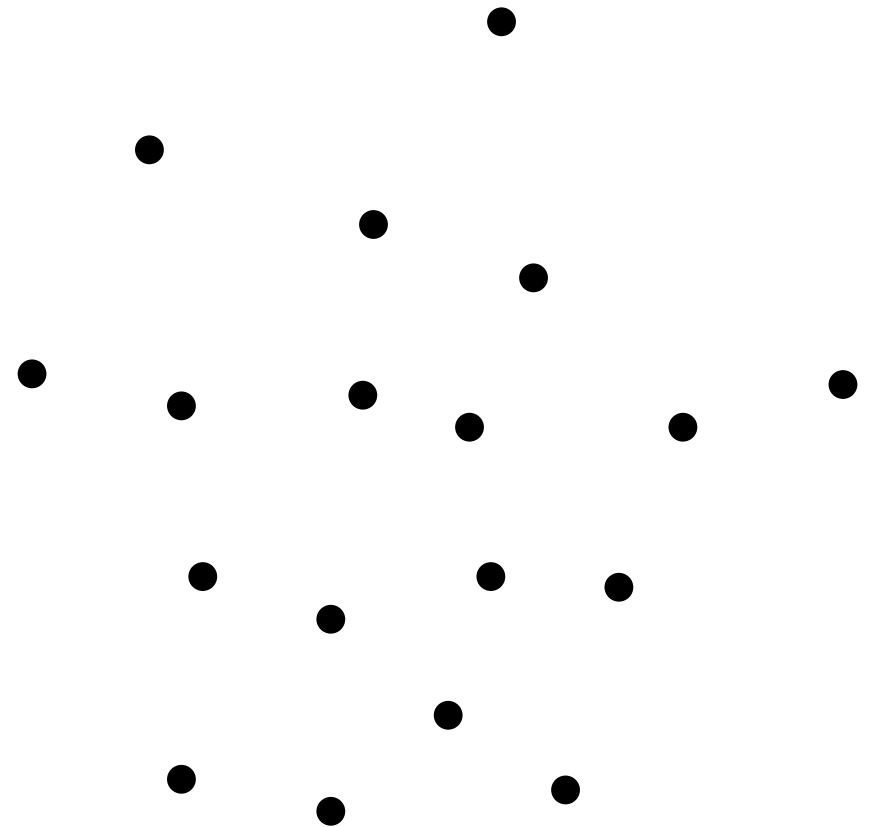
Given $X = \{p_1, \dots, p_n\}$, the segment $p_i p_j$ is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line $p_i p_j$.

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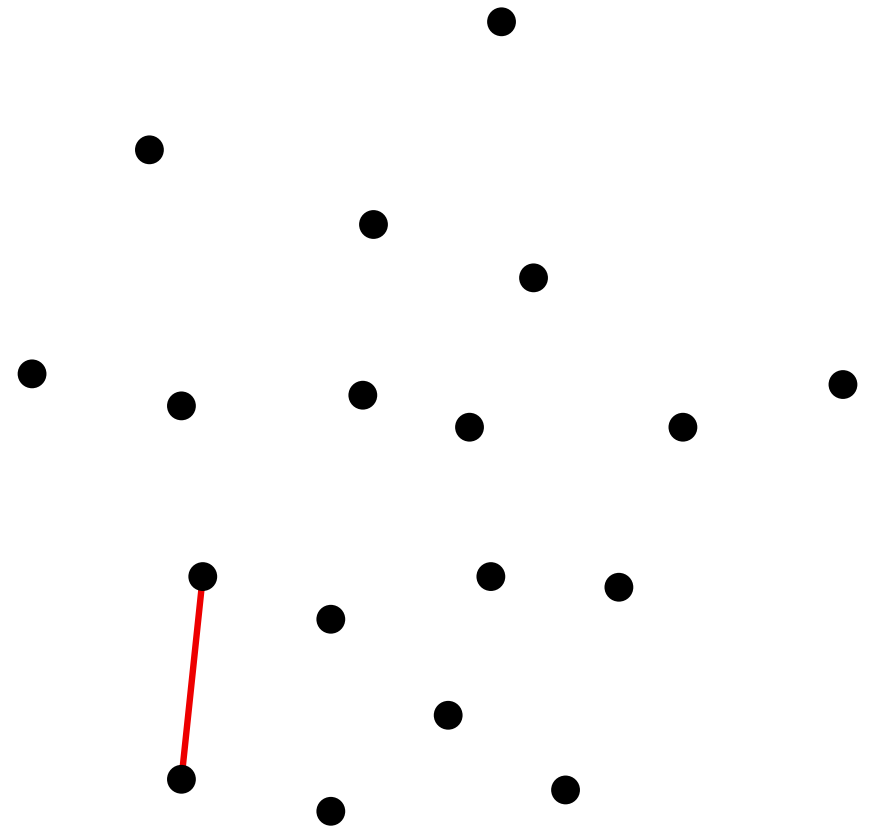


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Computing the extreme segments

Characterization

Given $X = \{p_1, \dots, p_n\}$, the segment $p_i p_j$ is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line $p_i p_j$.

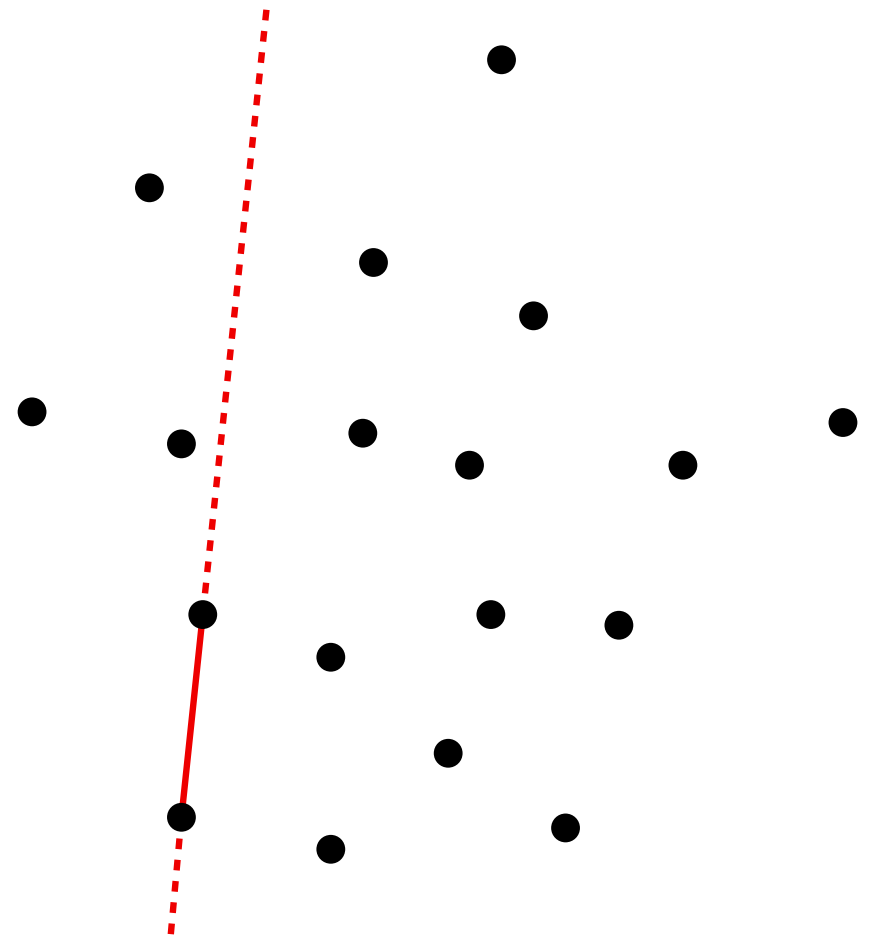


CONVEX HULL IN 2D

Computing the extreme segments

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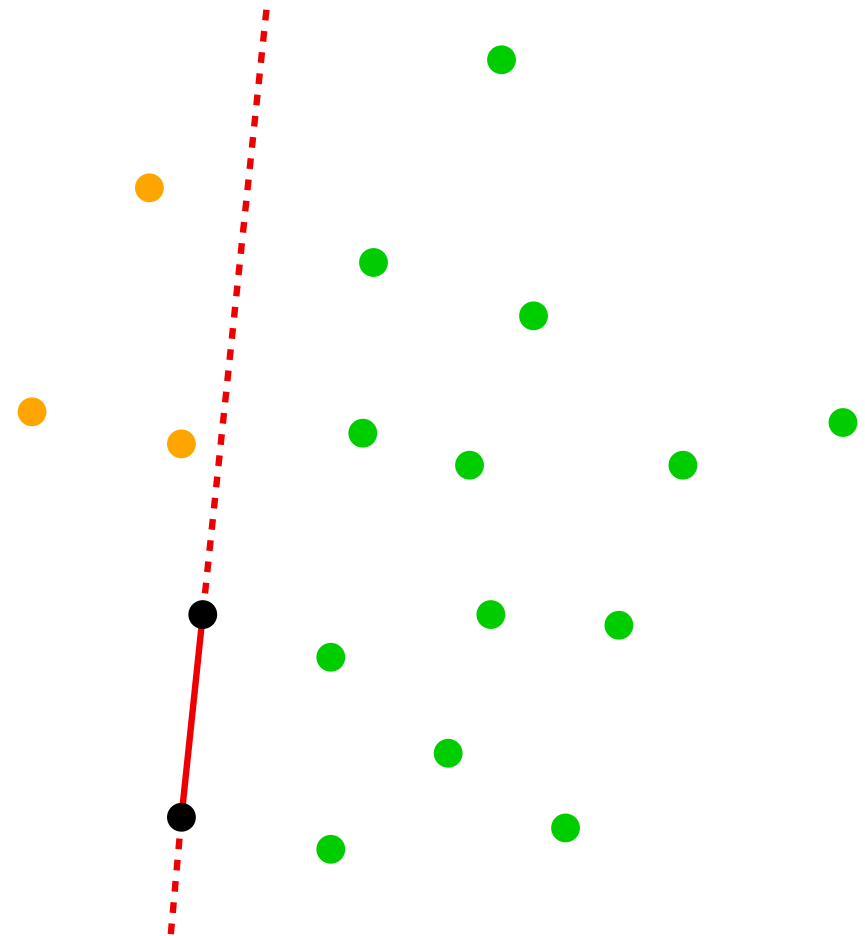


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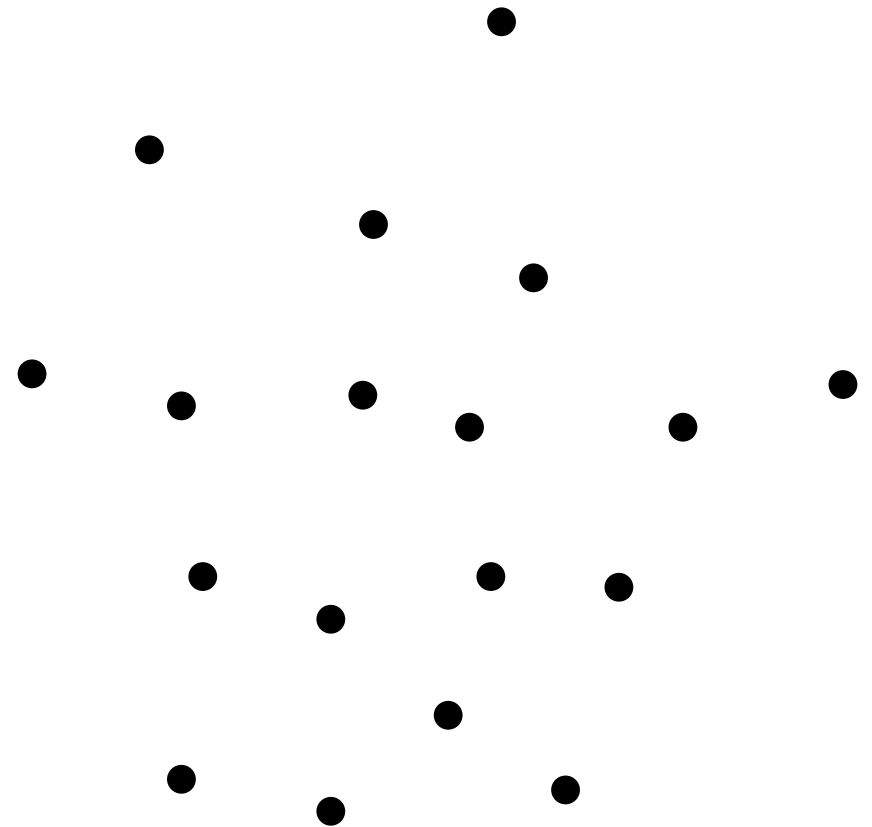


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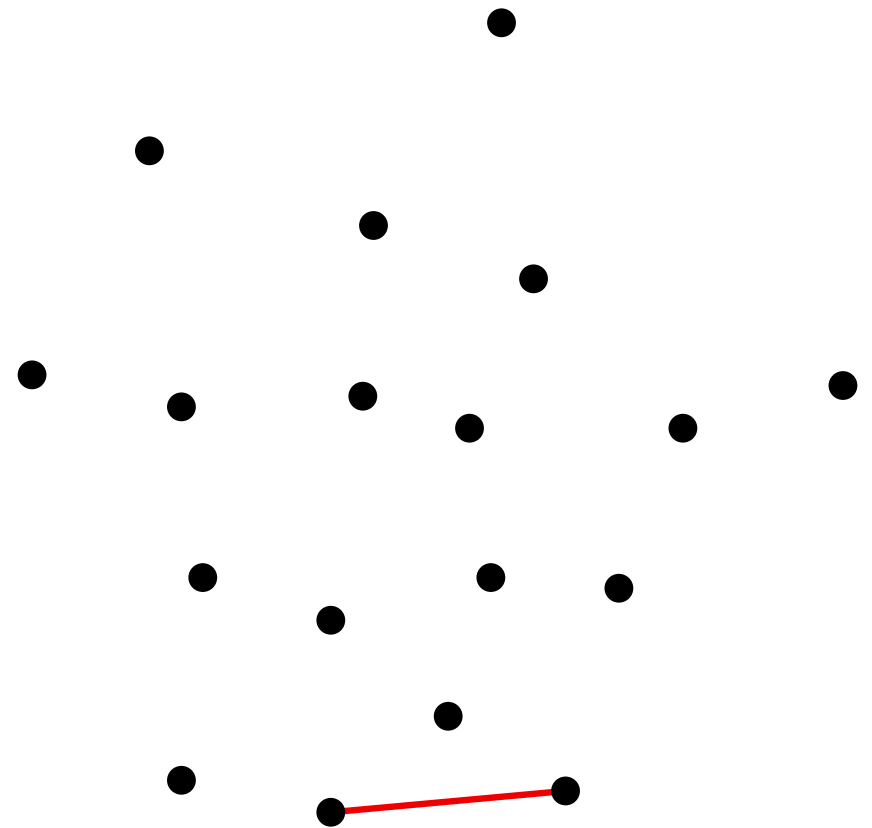


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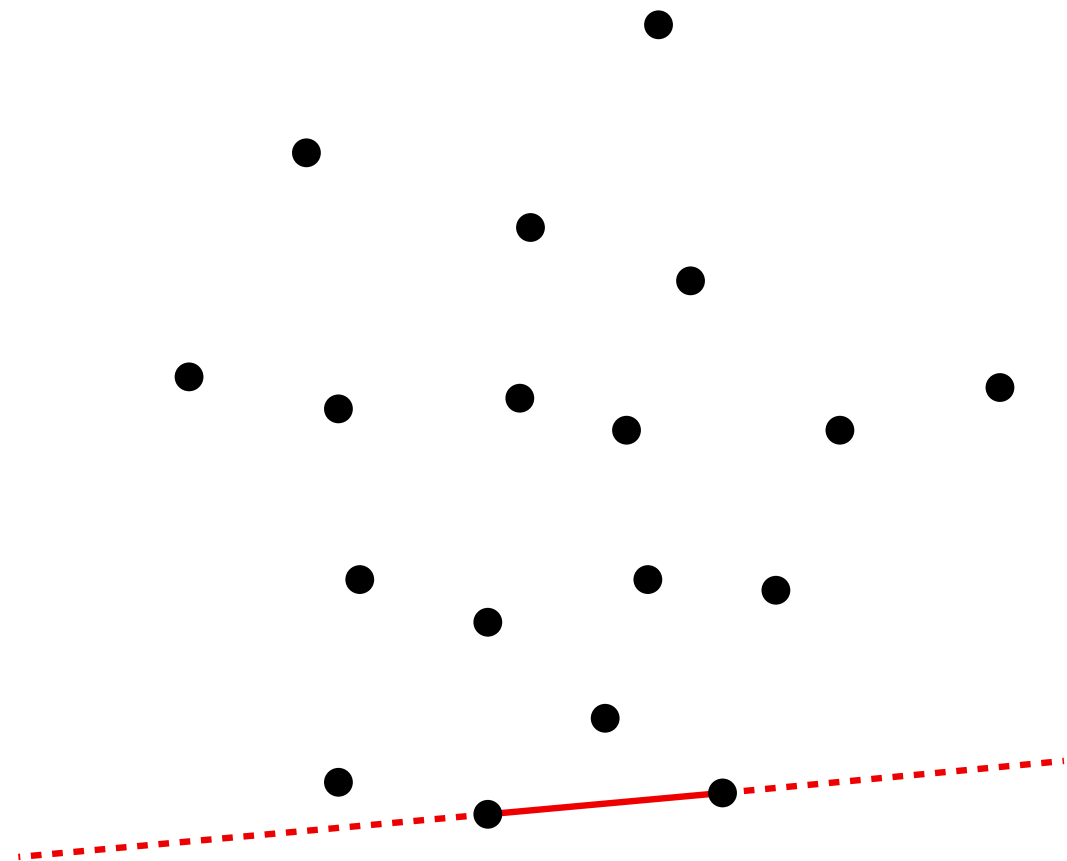


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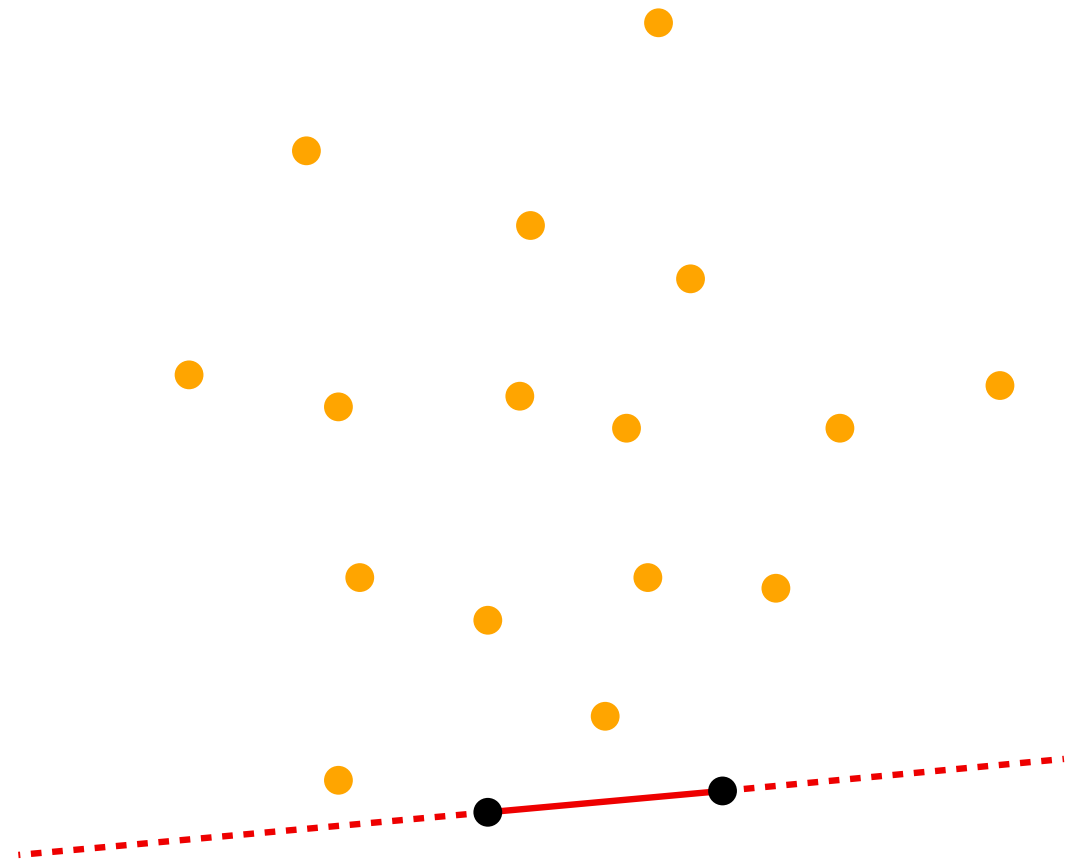


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Algorithm

Input: p_1, \dots, p_n

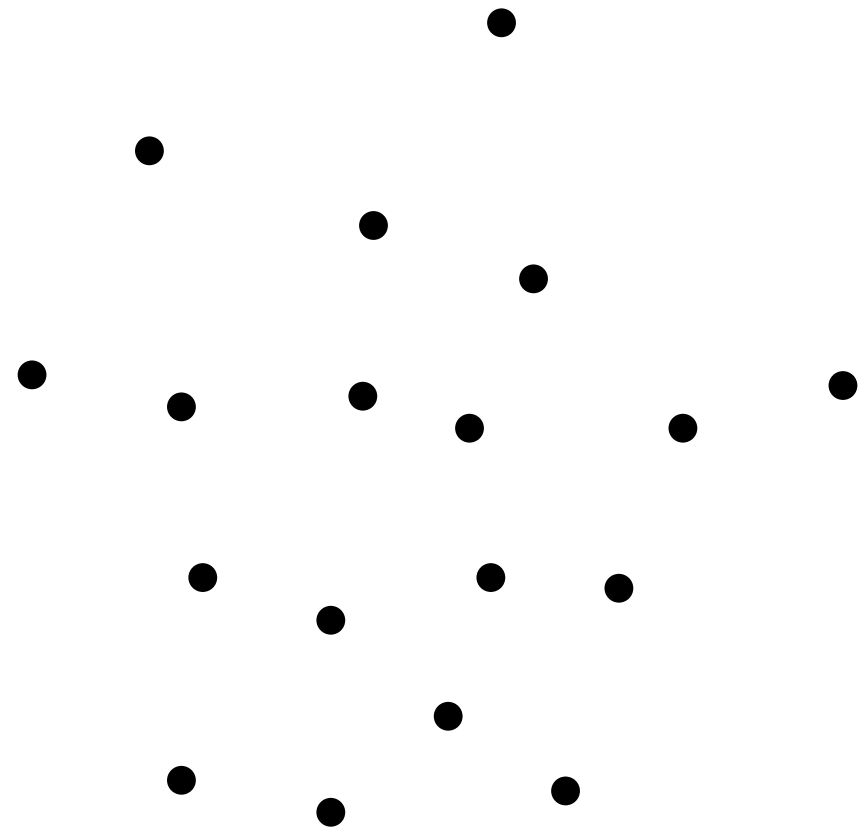
Output: set of the extreme segments

Procedure:

For each i, j ,

Check whether all p_k with $k \neq i, j$
lie in the same halfplane defined by $p_i p_j$.

In the affirmative, return the segment $p_i p_j$.



CONVEX HULL IN 2D

Computing the extreme segments

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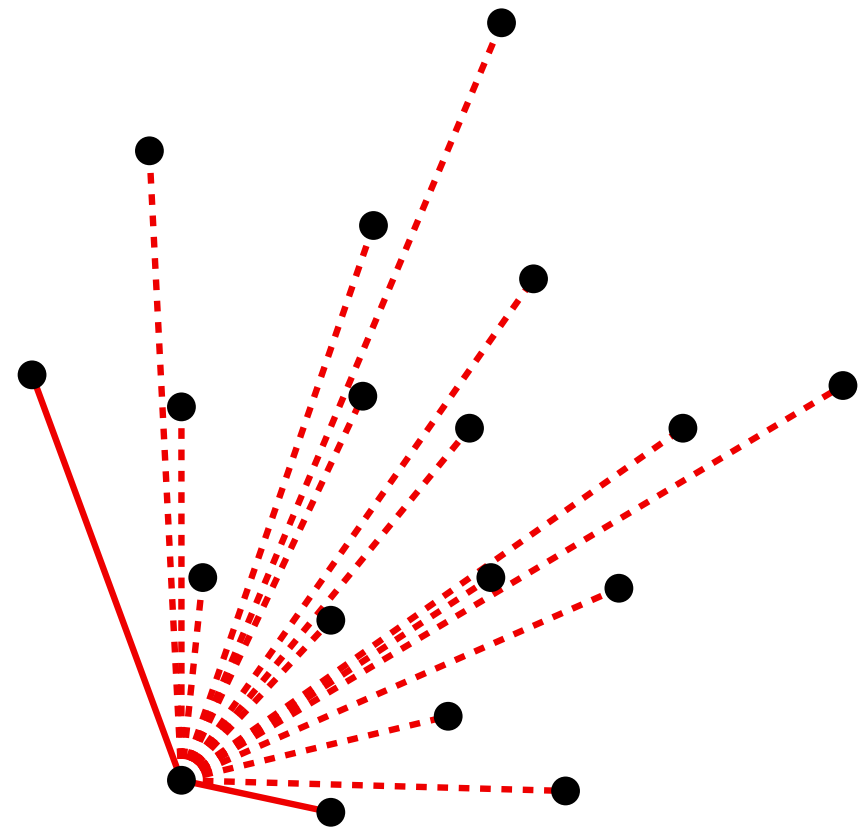
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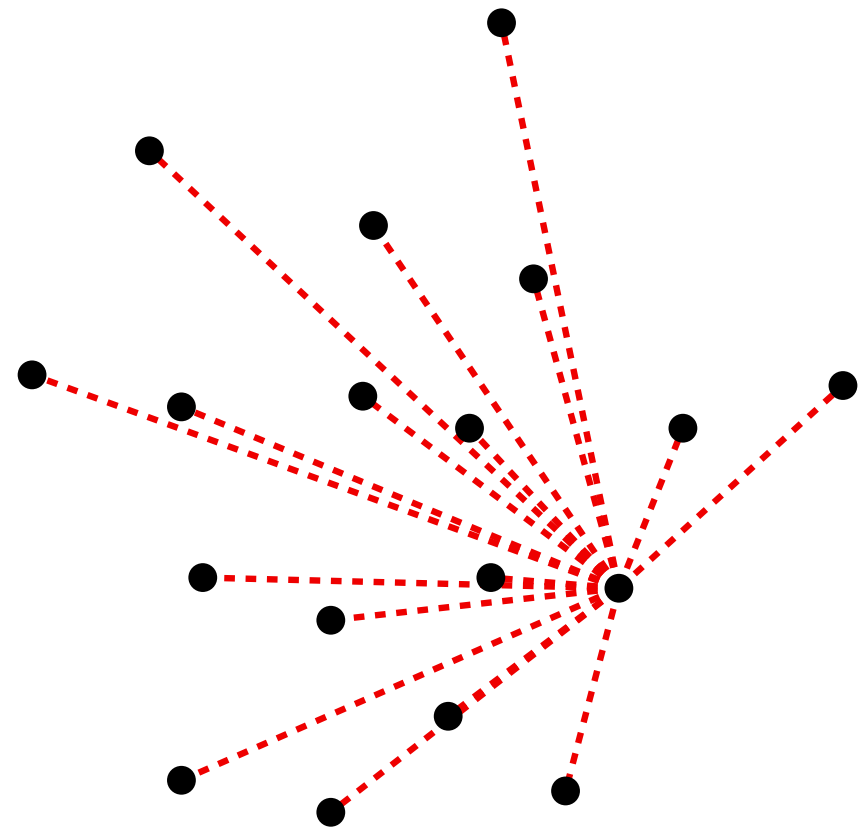
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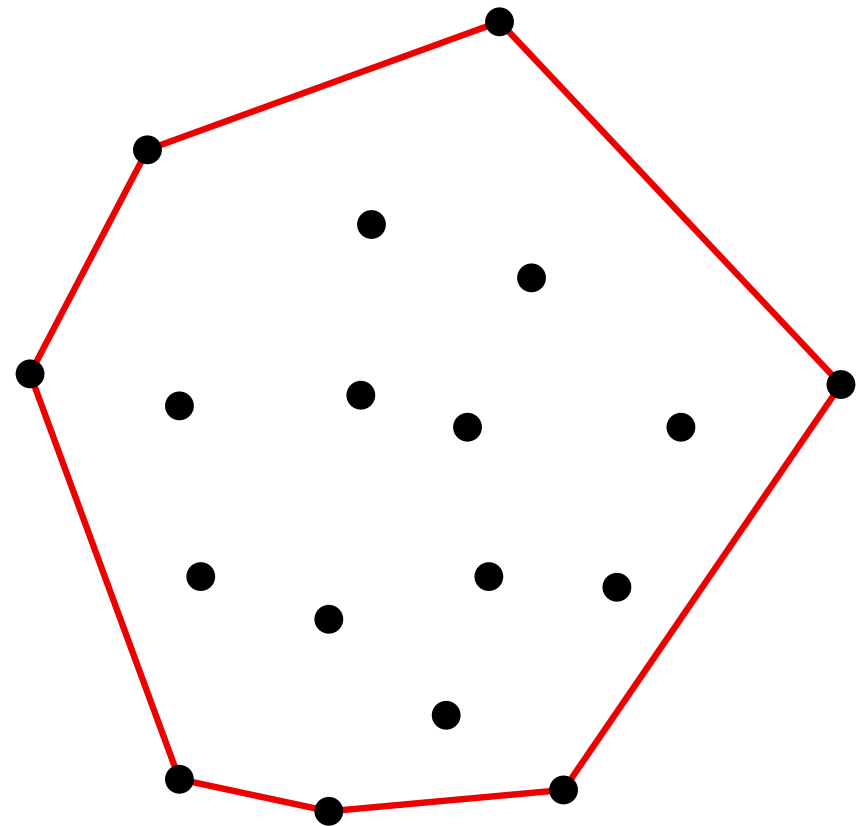
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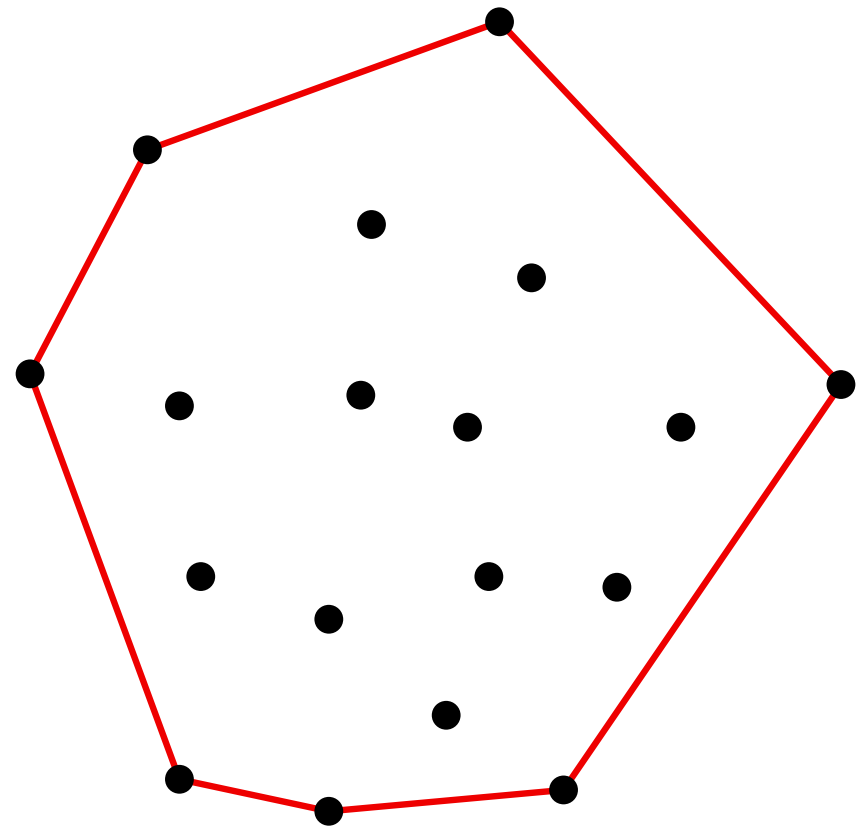
Procedure:

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In the affirmative, return the segment $p_i p_j$.

Running time: $\Theta(n^3)$



CONVEX HULL IN 2D

Computing the convex hull

CONVEX HULL IN 2D

Computing the convex hull (sorted list of its vertices)

CONVEX HULL IN 2D

Computing the convex hull (sorted list of its vertices)

Input:

$P = \{p_1, \dots, p_n\} \subset \mathbb{R}^2$ a set of n points in the plane

Output:

l , the list of the vertices of $ch(P)$ sorted in counterclockwise order

CONVEX HULL IN 2D

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Input:

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Output:

l , the list of the vertices of $ch(P)$ sorted in counterclockwise order

Characterization

Given $X = \{p_1, \dots, p_n\}$, the segment $p_i p_j$ is an edge of the convex hull of X if and only if all the points p_k with $k \neq i, j$ lie to the left of the oriented line $p_i p_j$.

CONVEX HULL IN 2D

Jarvis march (also known as gift wrapping)

CONVEX HULL IN 2D

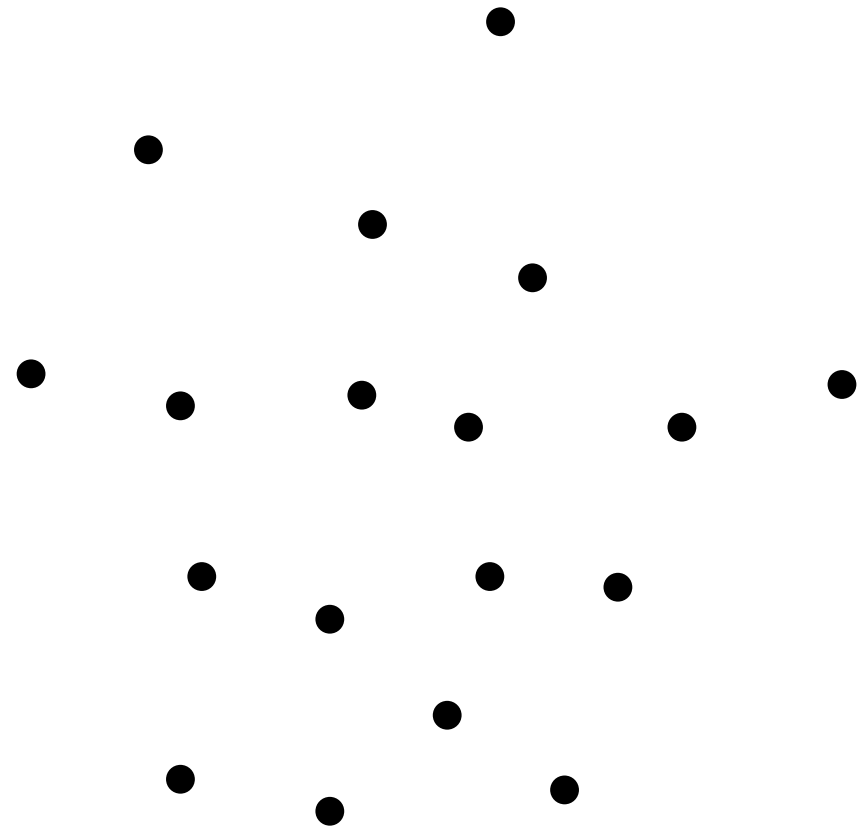
Jarvis march (also known as gift wrapping)

1. Find a vertex of $ch(P)$ (for example, the lexicographically smaller point $p_i \in P$) and add it to l
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CONVEX HULL IN 2D

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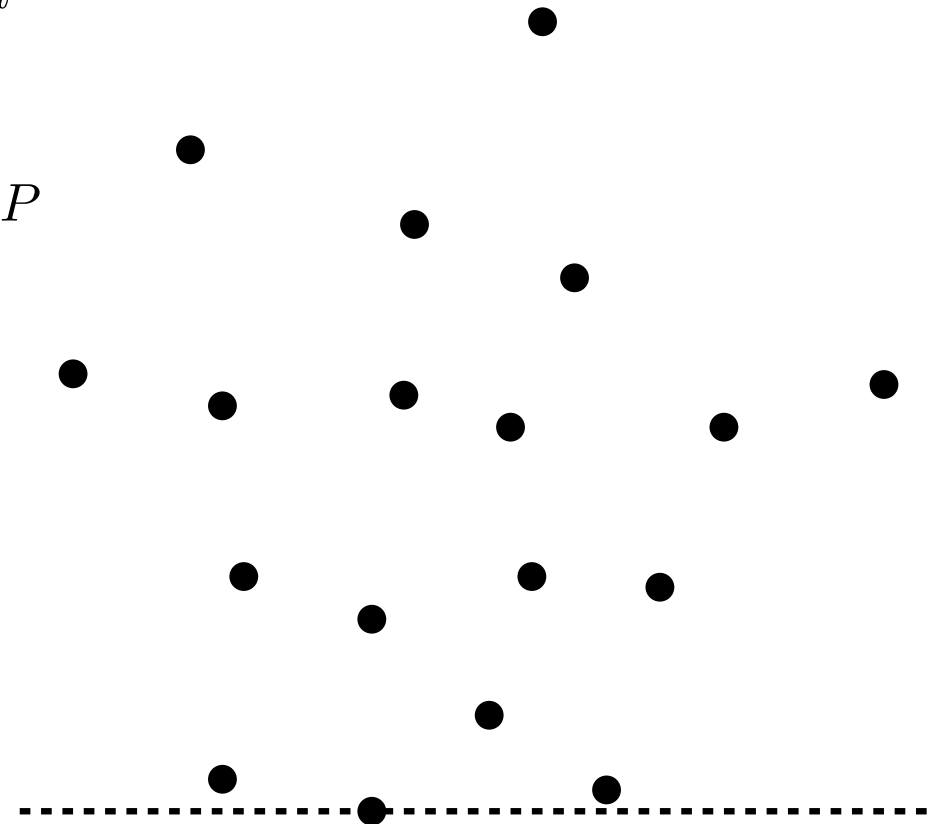
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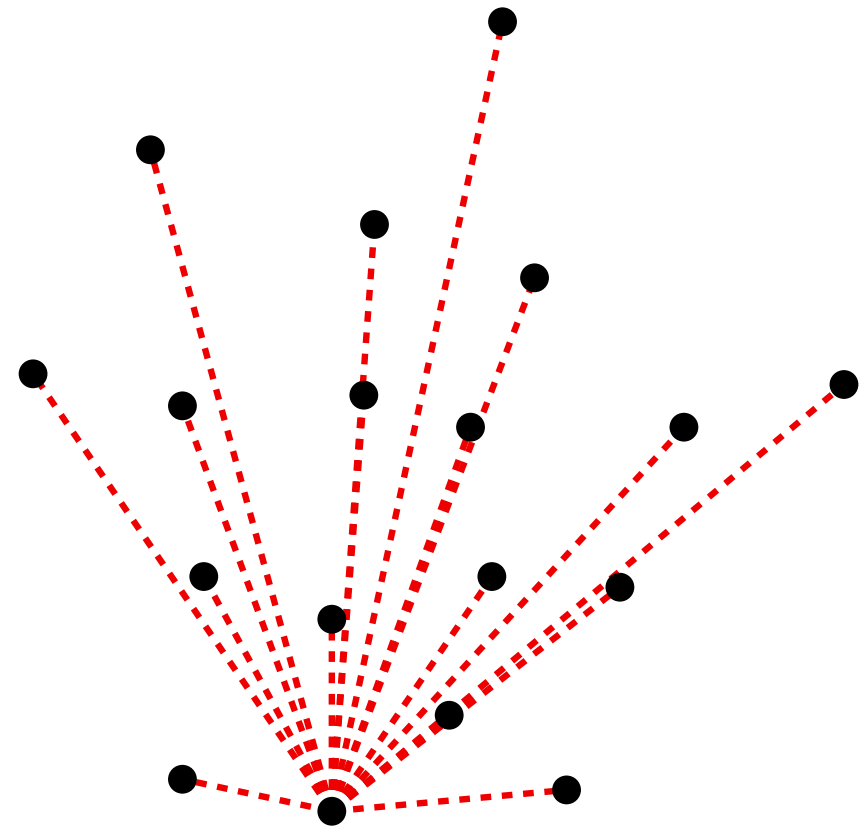
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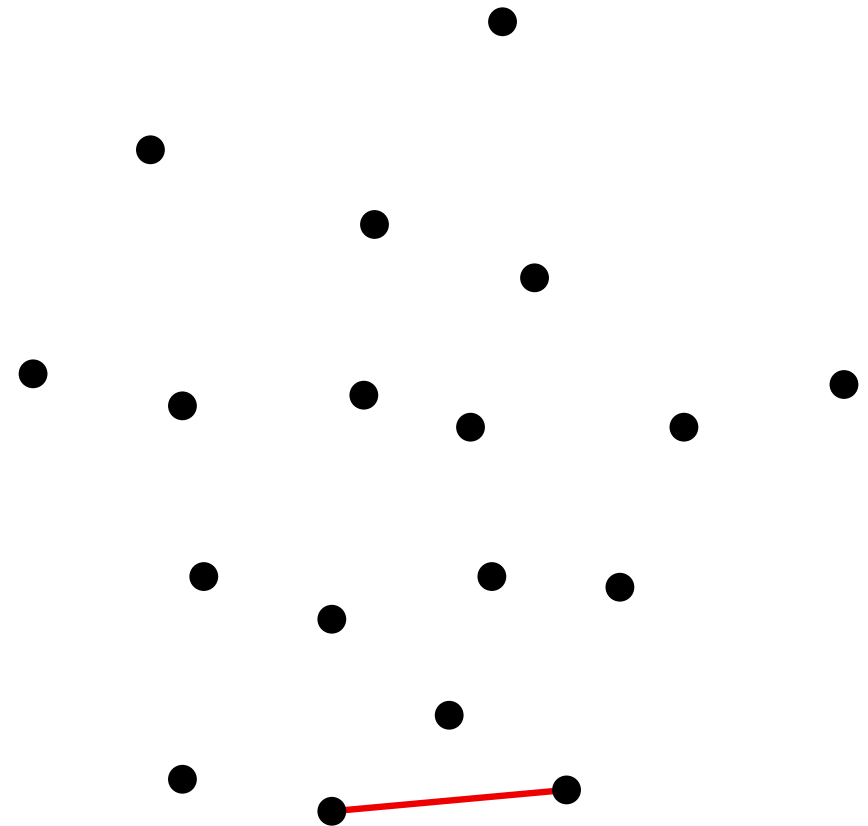
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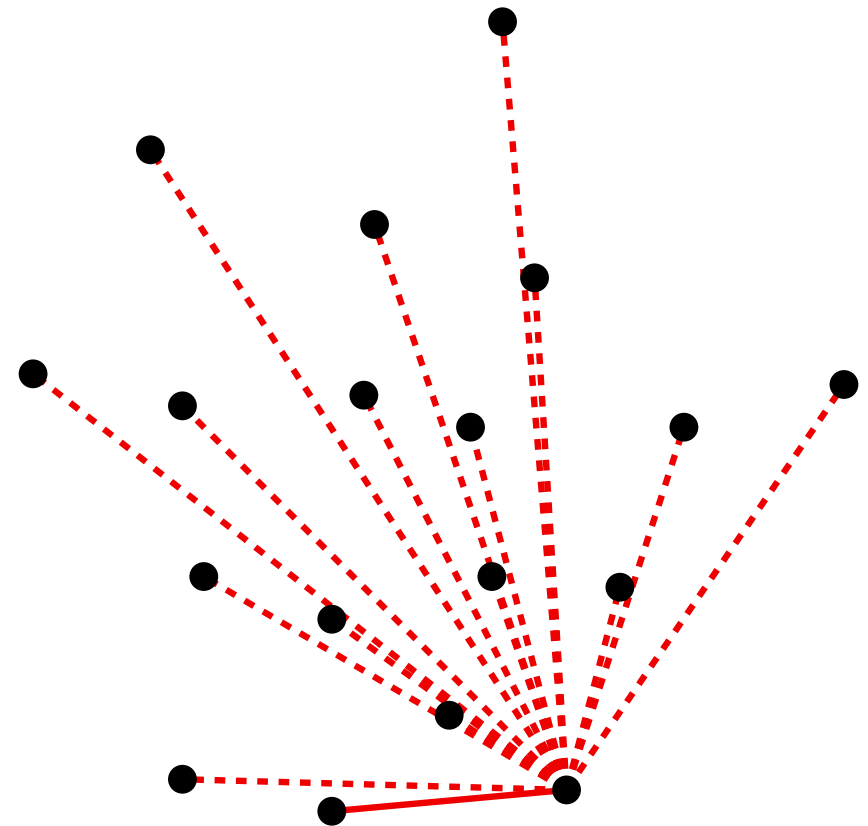
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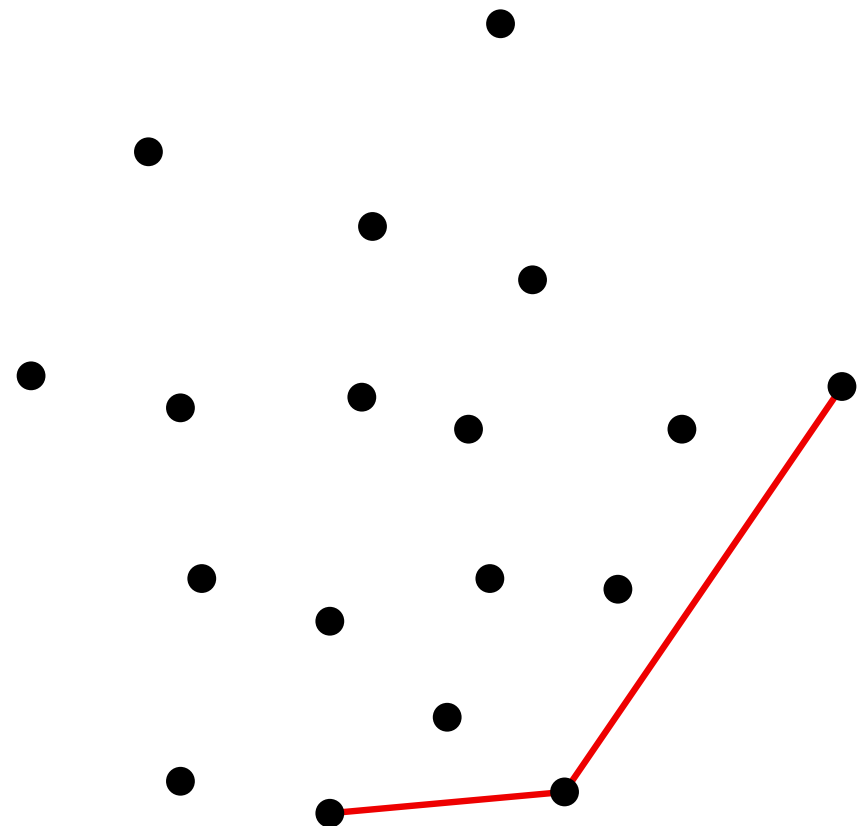
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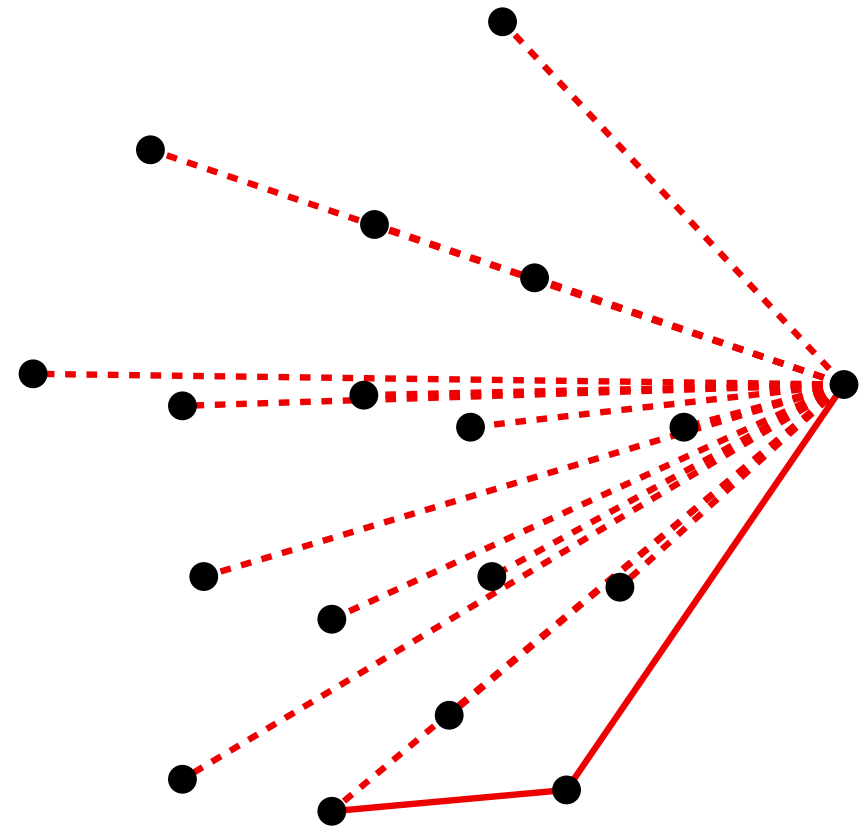
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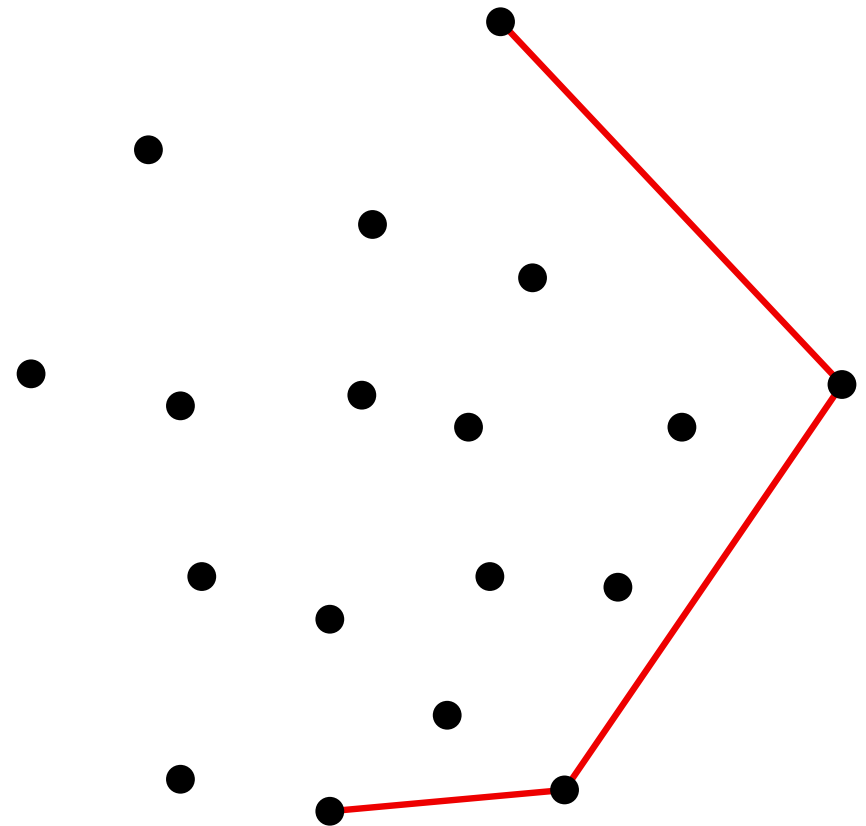
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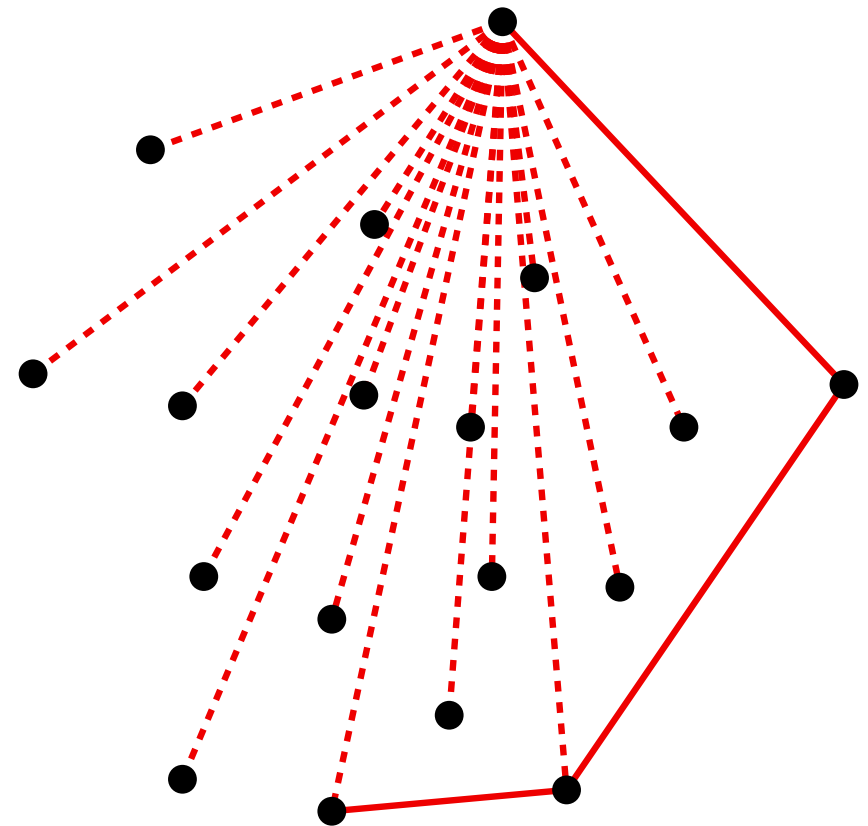
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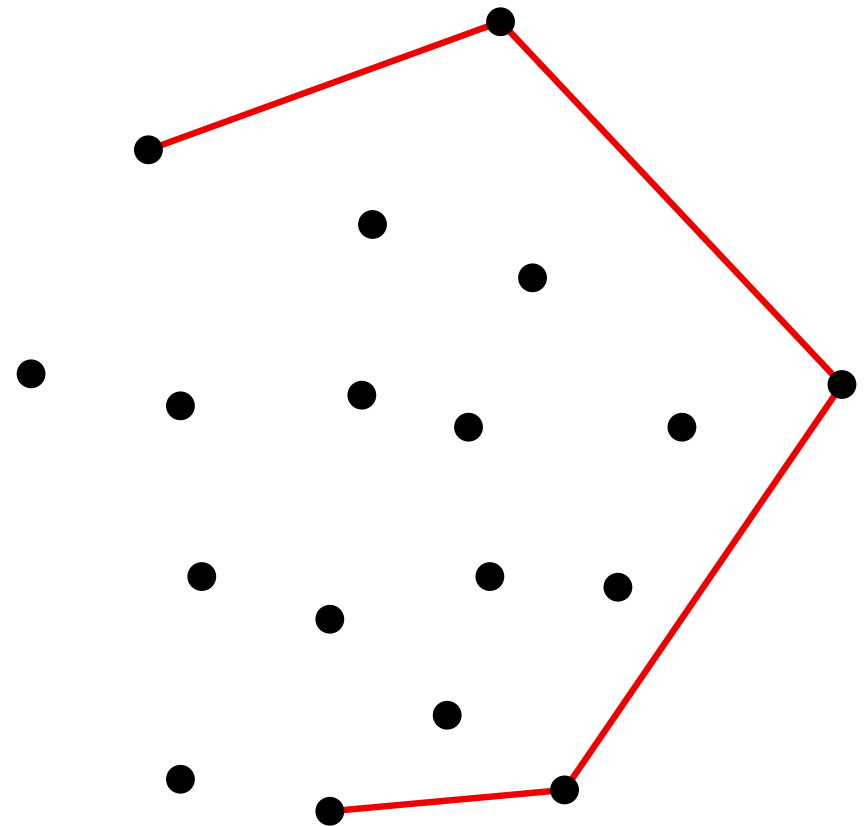
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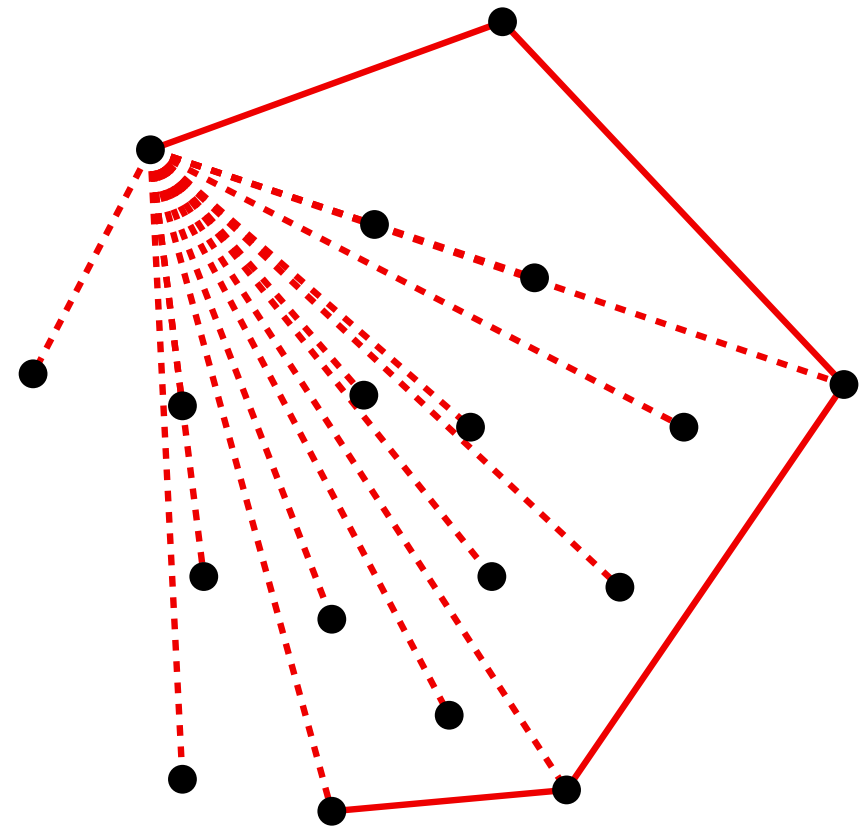
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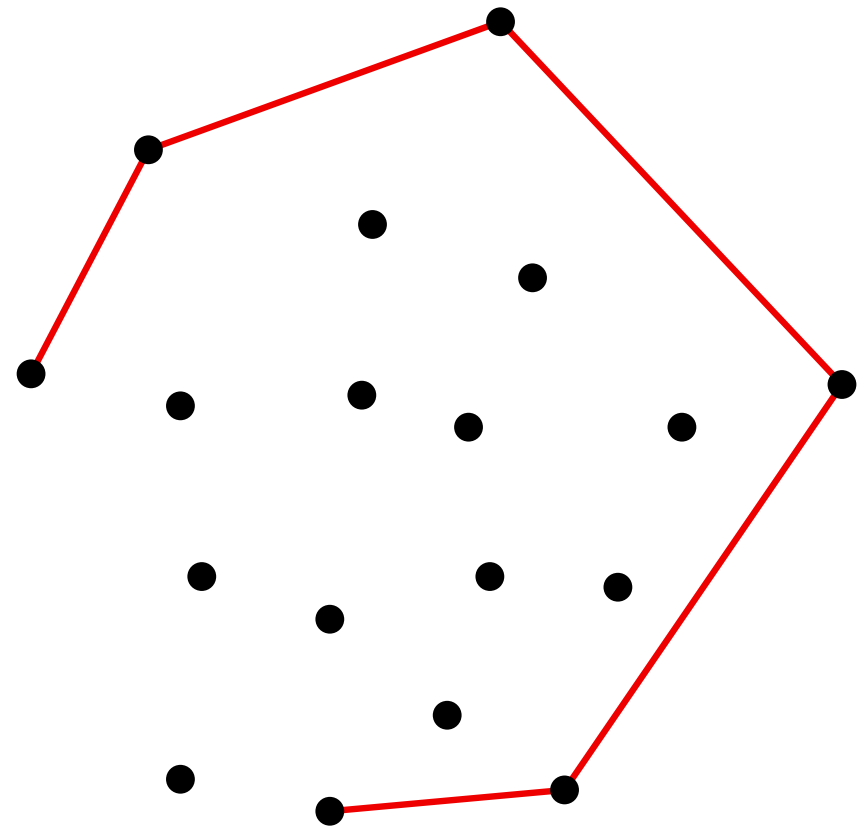
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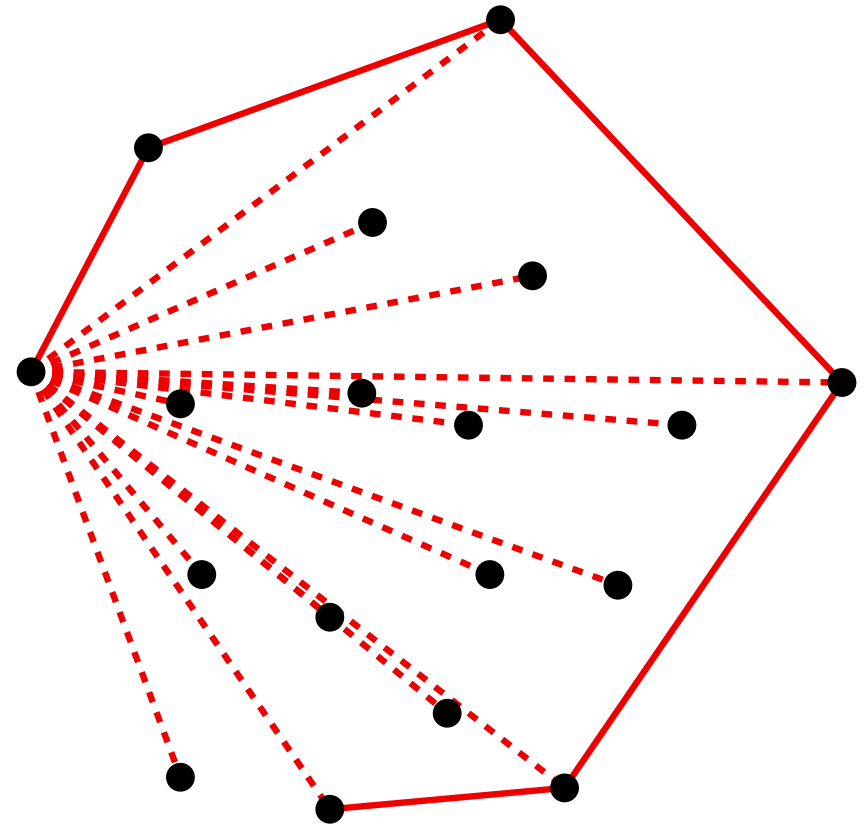
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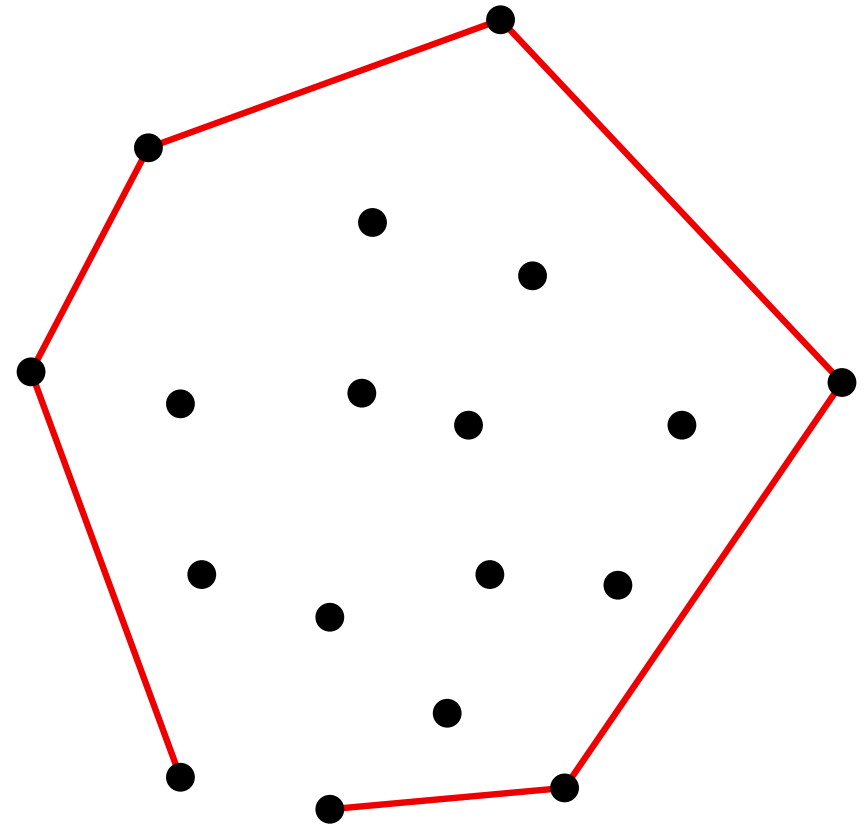
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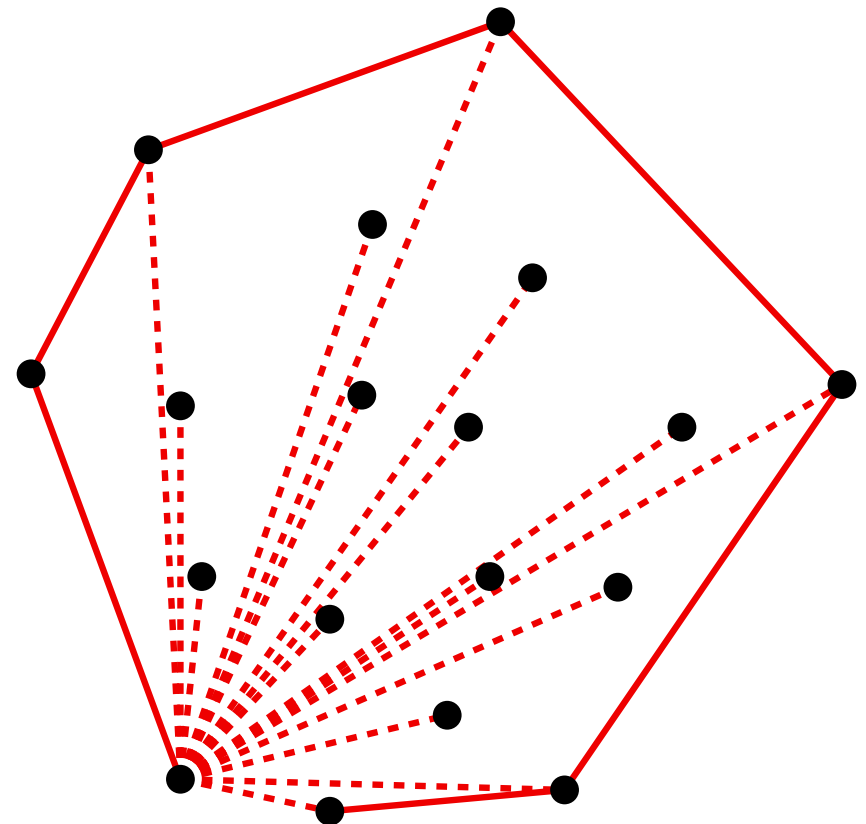
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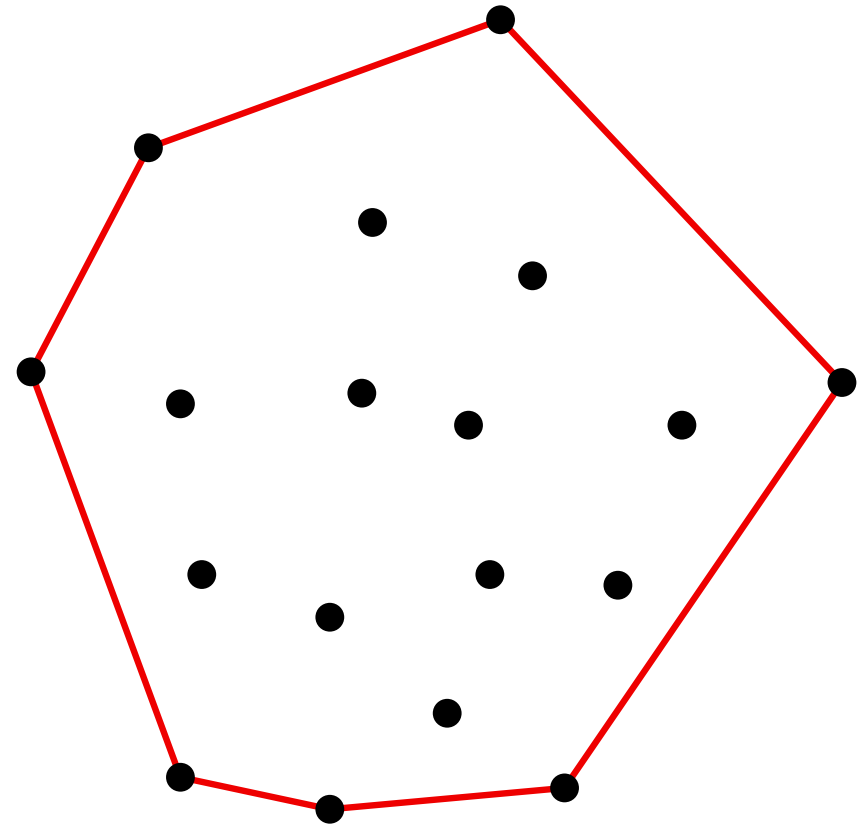
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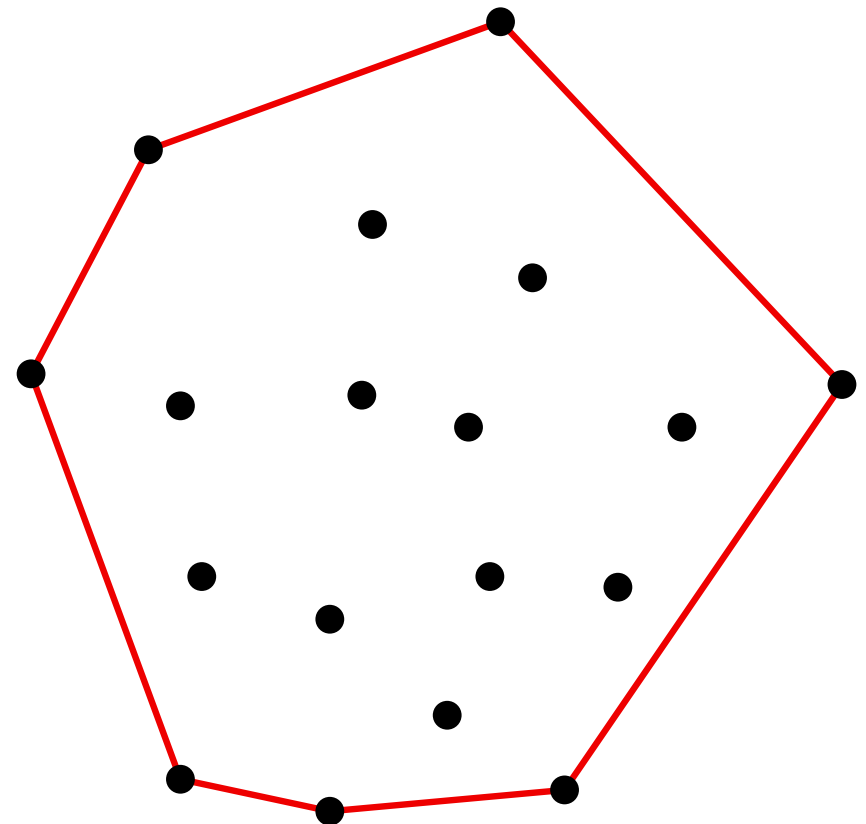
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Time cost: $\Theta(hn) = O(n^2)$

Output sensitive!



CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

Initialization

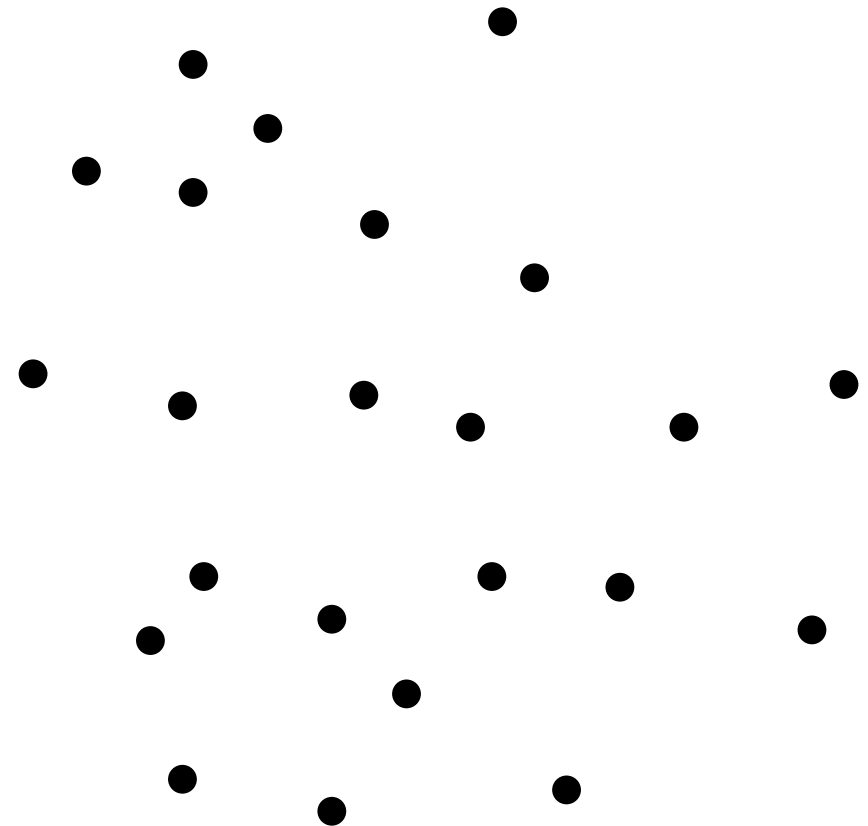
1. Find the extreme points in the horizontal and vertical directions.
2. Compute the convex hull of these (between 2 and 8) points.
3. Test all the remaining points, and classify them according to their position (NE, SE, SW, NW) or eliminate them if they lie in the interior.

CONVEX HULL IN 2D

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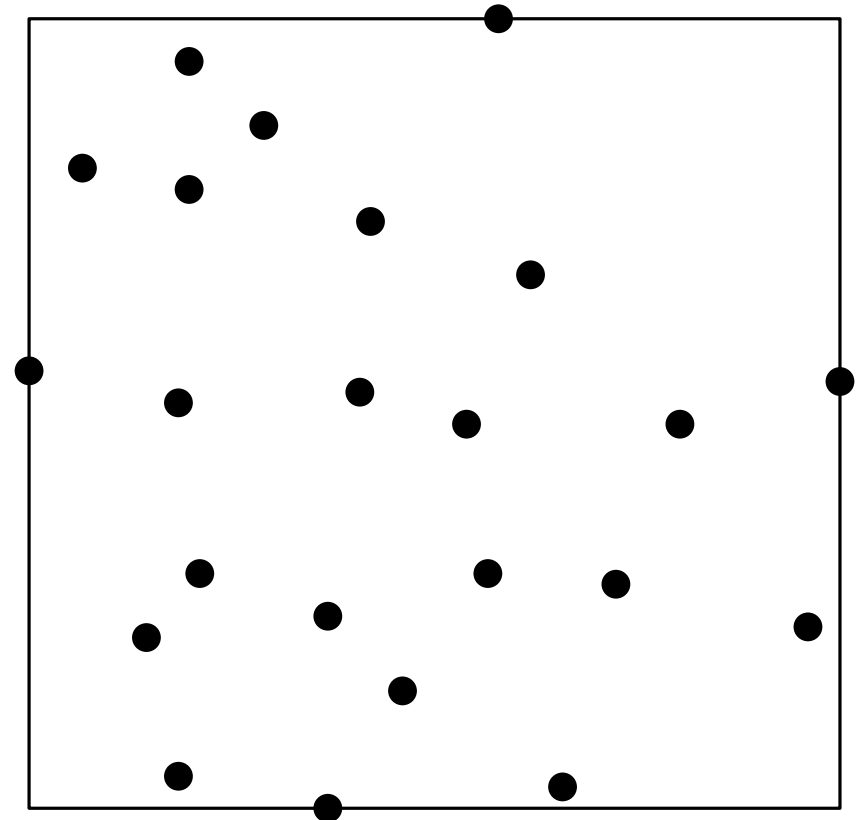


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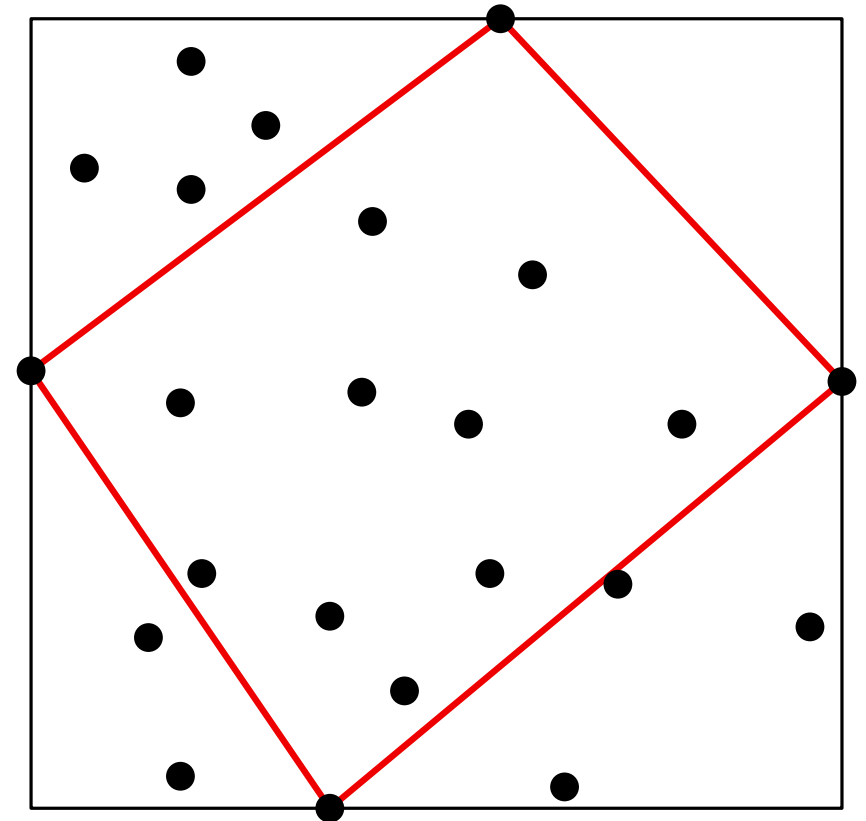


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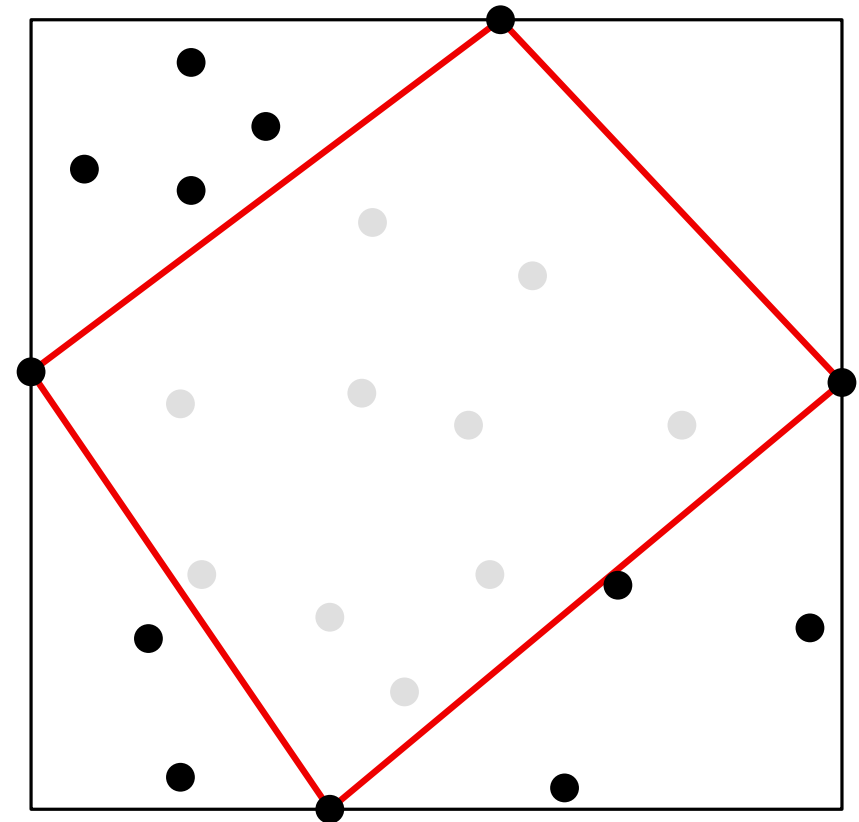


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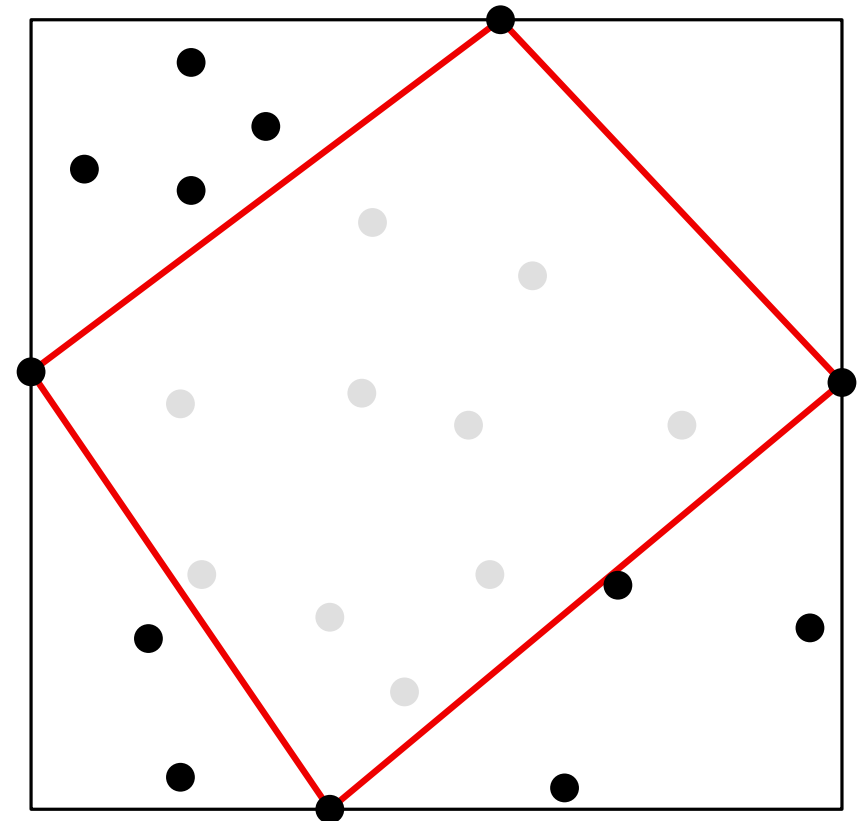
CONVEX HULL IN 2D

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Running time of this step: $O(n)$



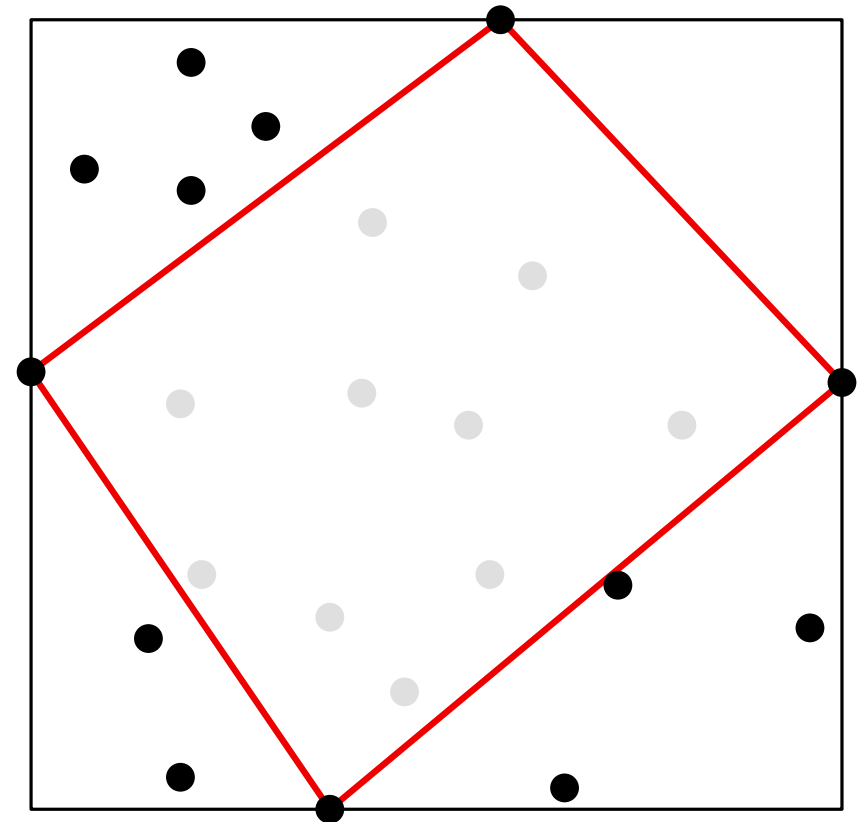
CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

Advance

Recursively, do:

1. Among all points lying in each region, find the extreme point in the direction orthogonal to the edge that determines the region.
2. Connect the extreme point with the endpoints of the edge, and update the convex hull.
3. Test all the remaining points of each region, and classify them according to their position (left or right) or eliminate them if they lie in the interior of the newly created triangle.



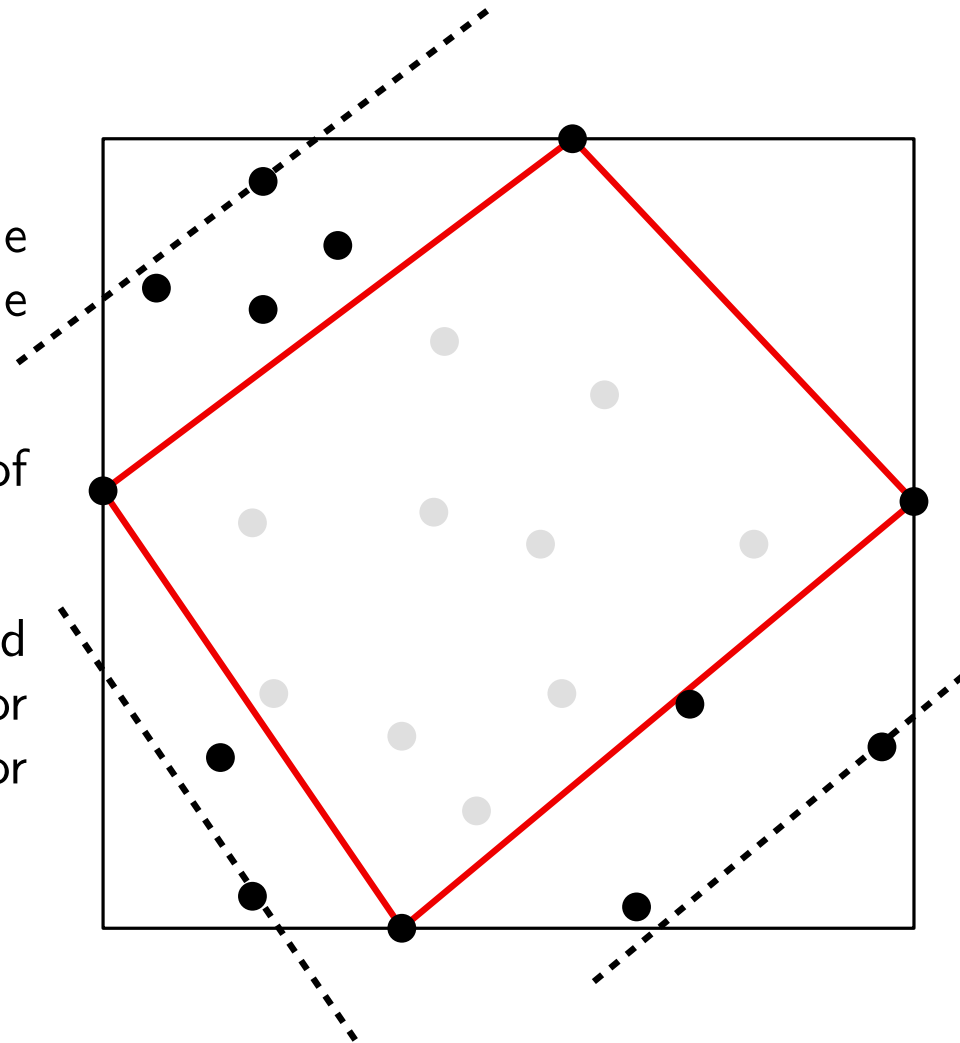
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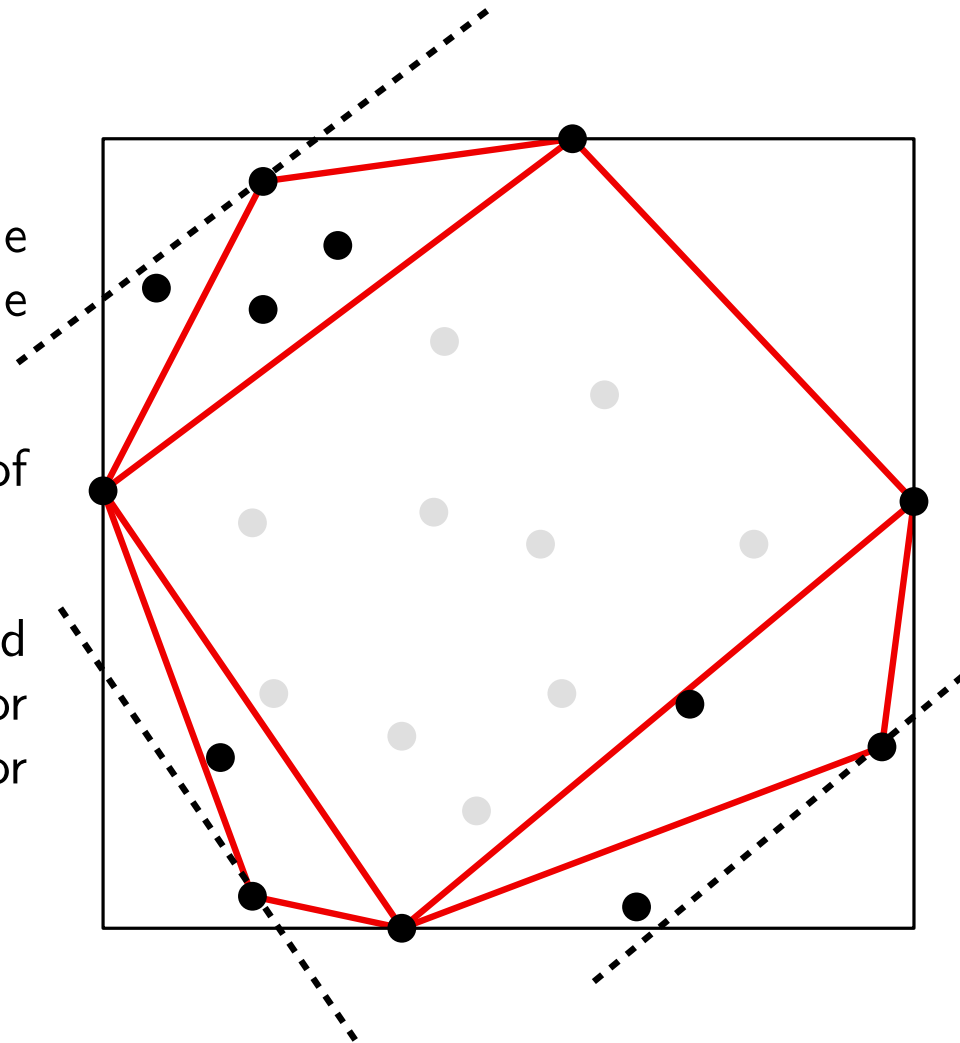
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QuickHull algorithm (by prune-and-search)

Advance

Recursively, do:

1. Among all points lying in each region, find the extreme point in the direction orthogonal to the edge that determines the region.
2. Connect the extreme point with the endpoints of the edge, and update the convex hull.
3. Test all the remaining points of each region, and classify them according to their position (left or right) or eliminate them if they lie in the interior of the newly created triangle.



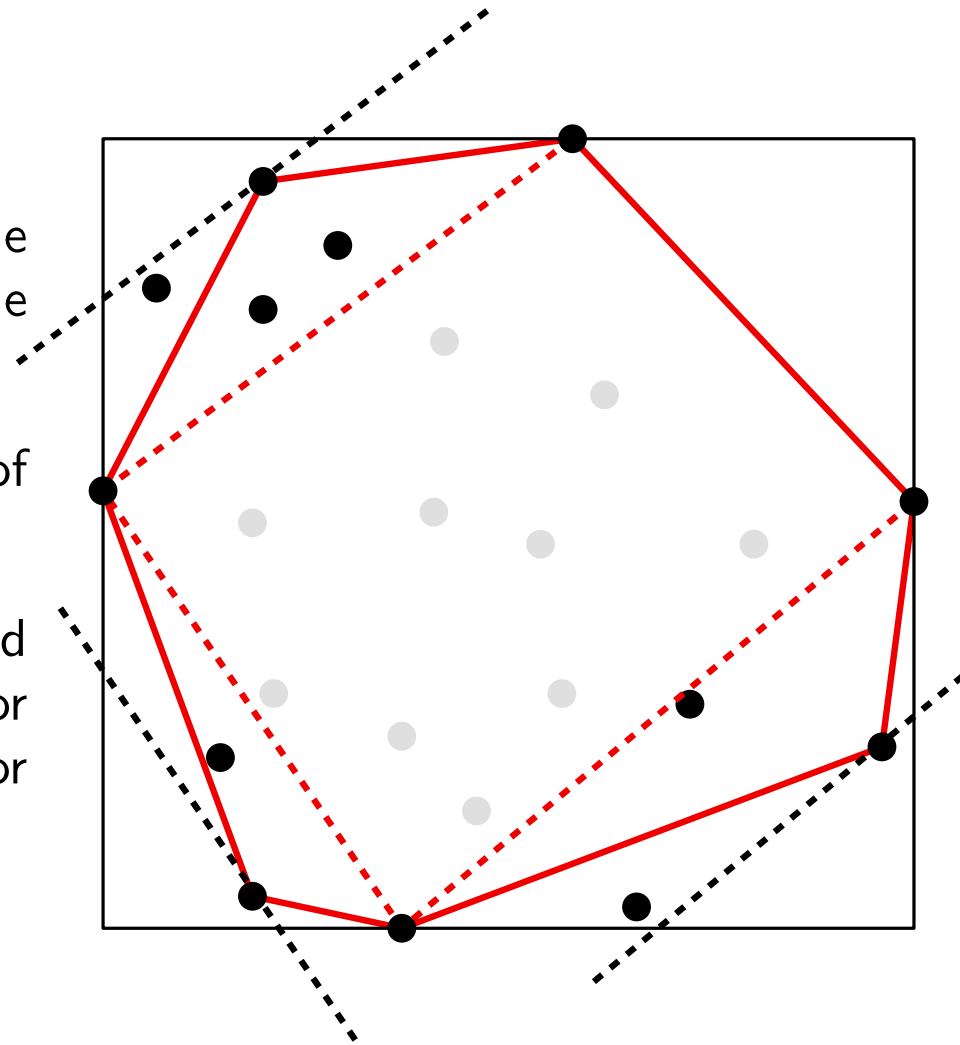
CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

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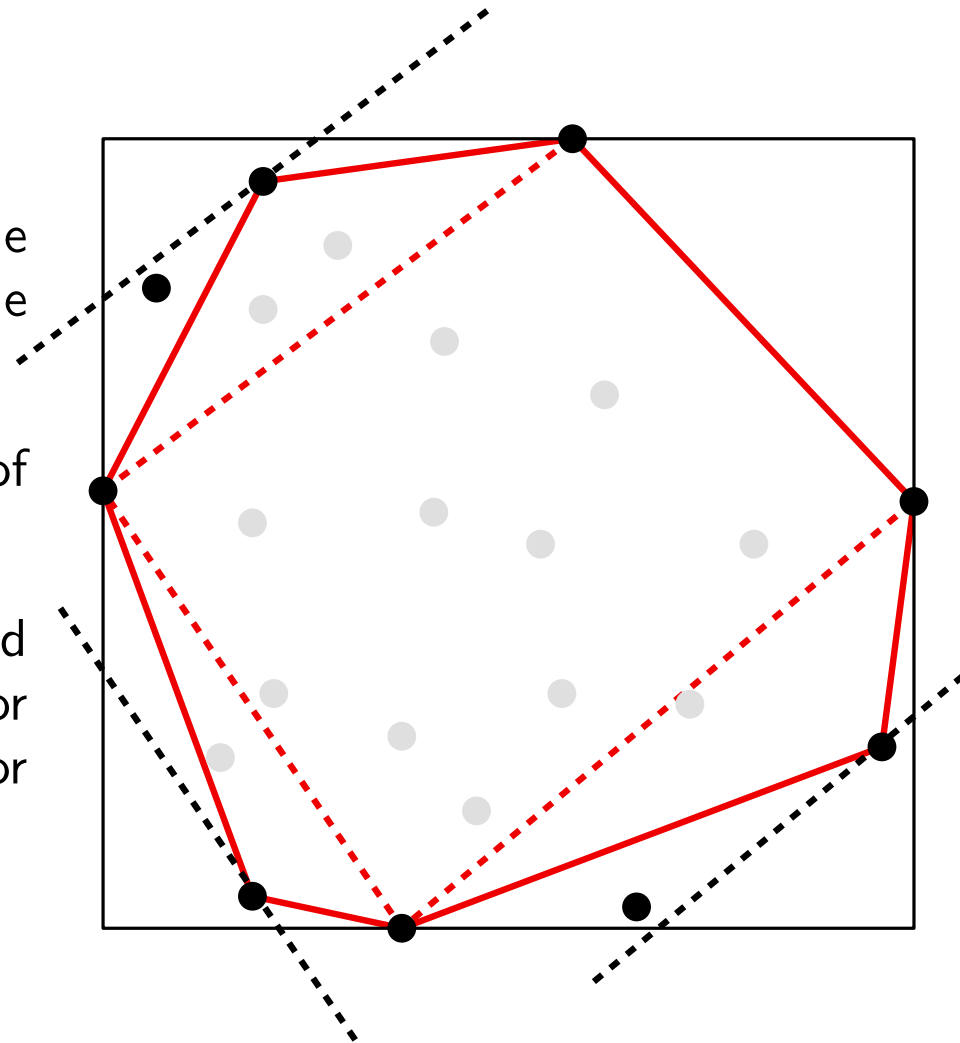
CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

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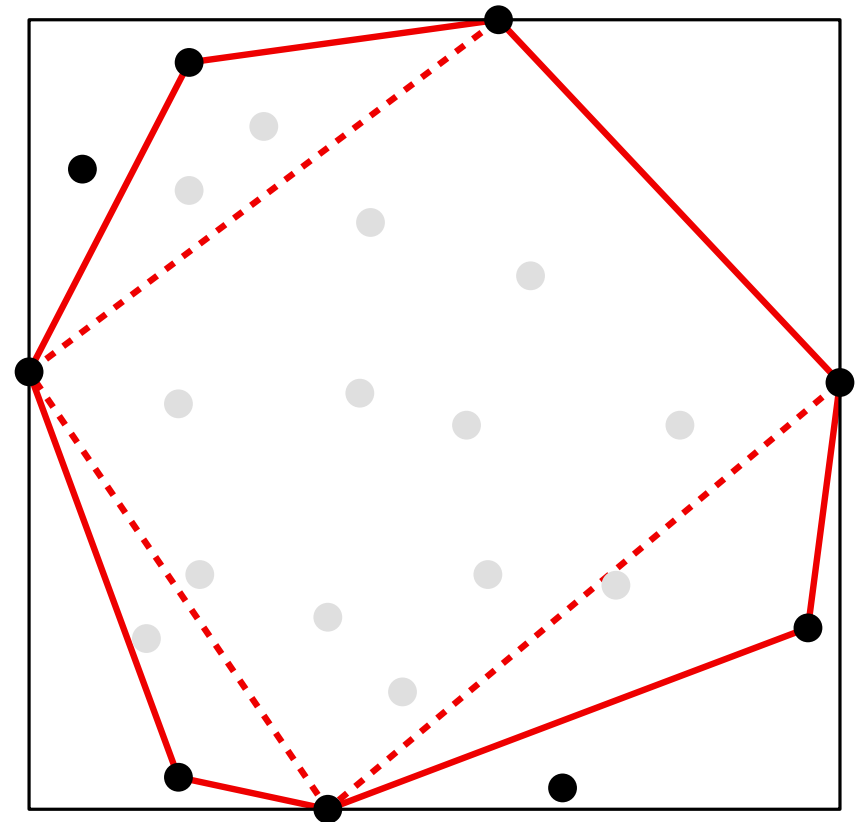
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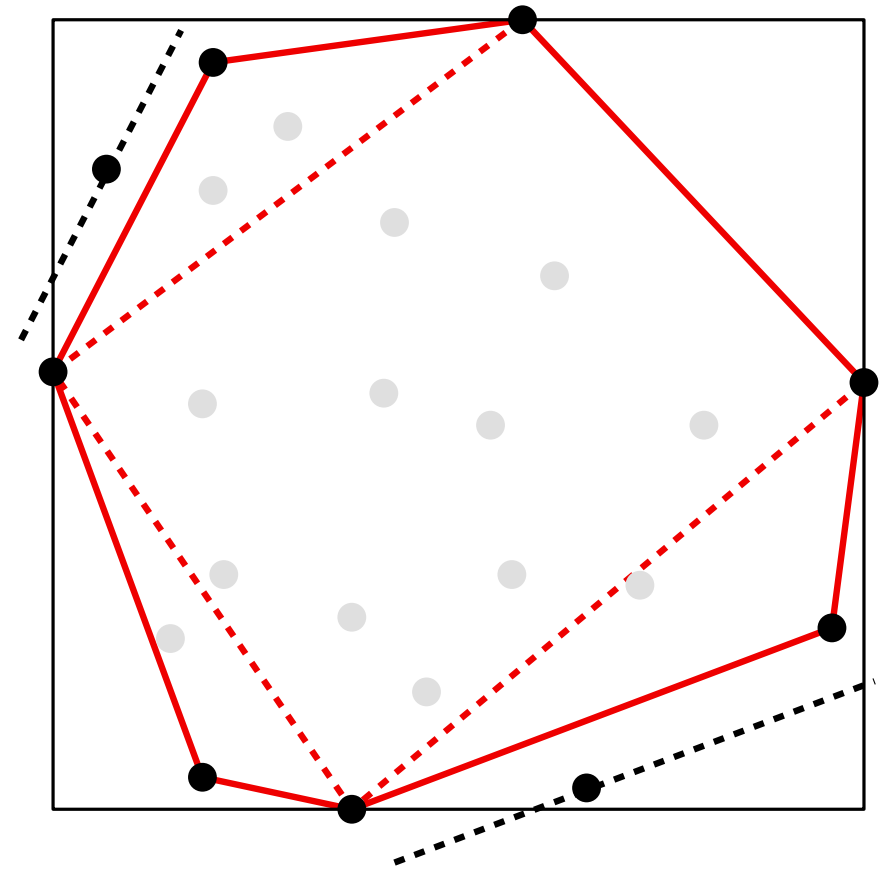
CONVEX HULL IN 2D

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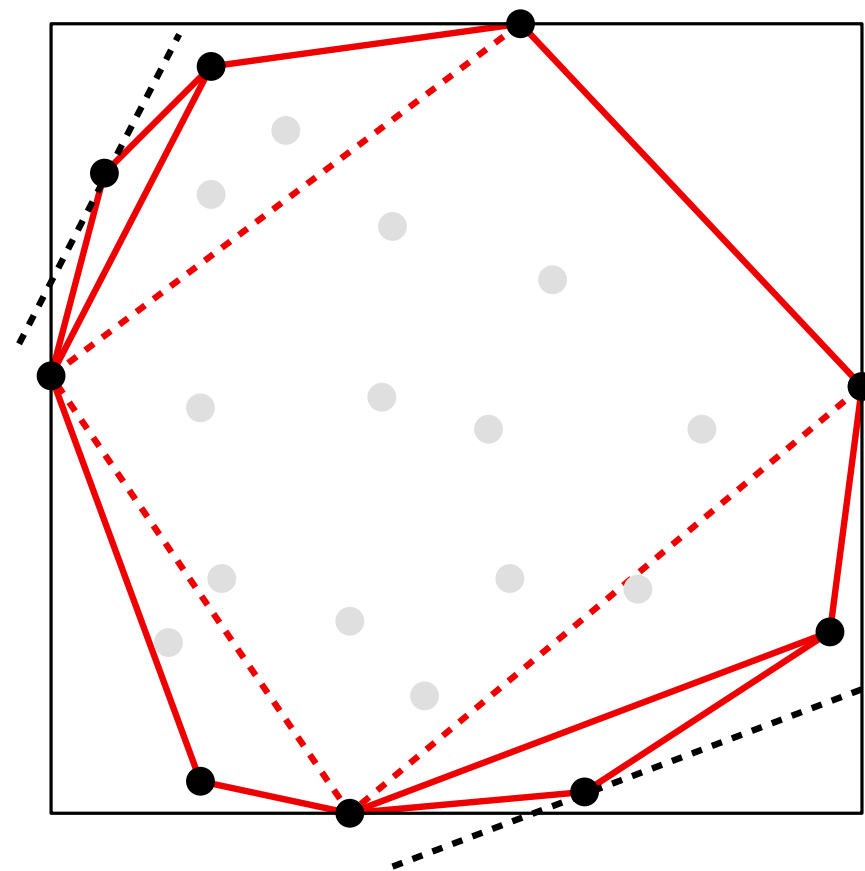
CONVEX HULL IN 2D

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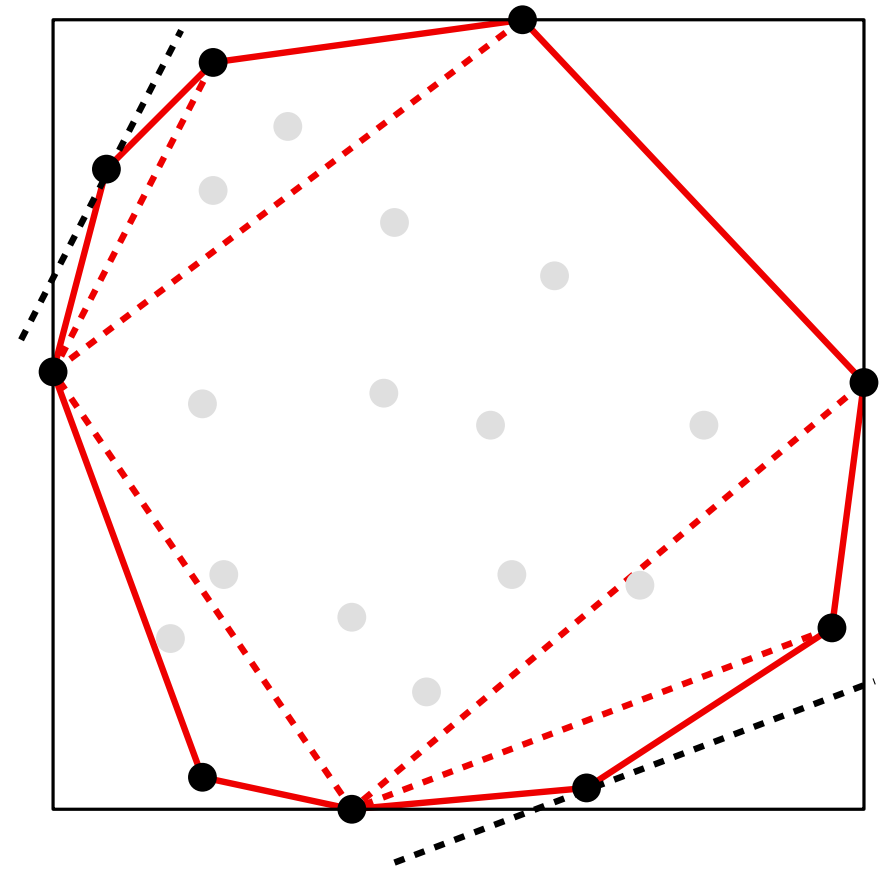
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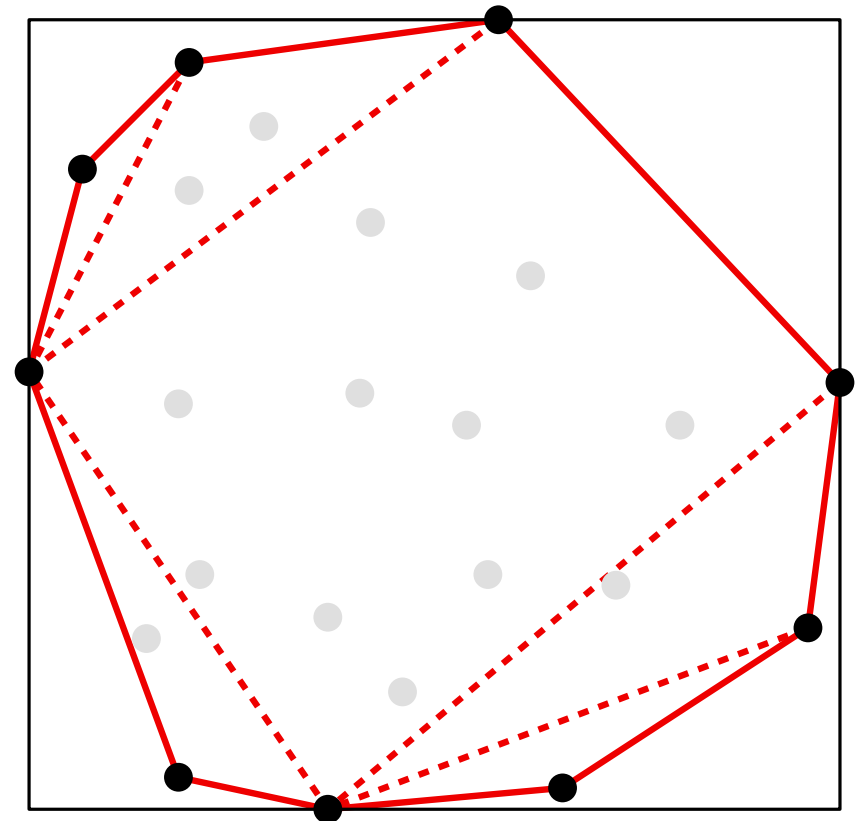
CONVEX HULL IN 2D

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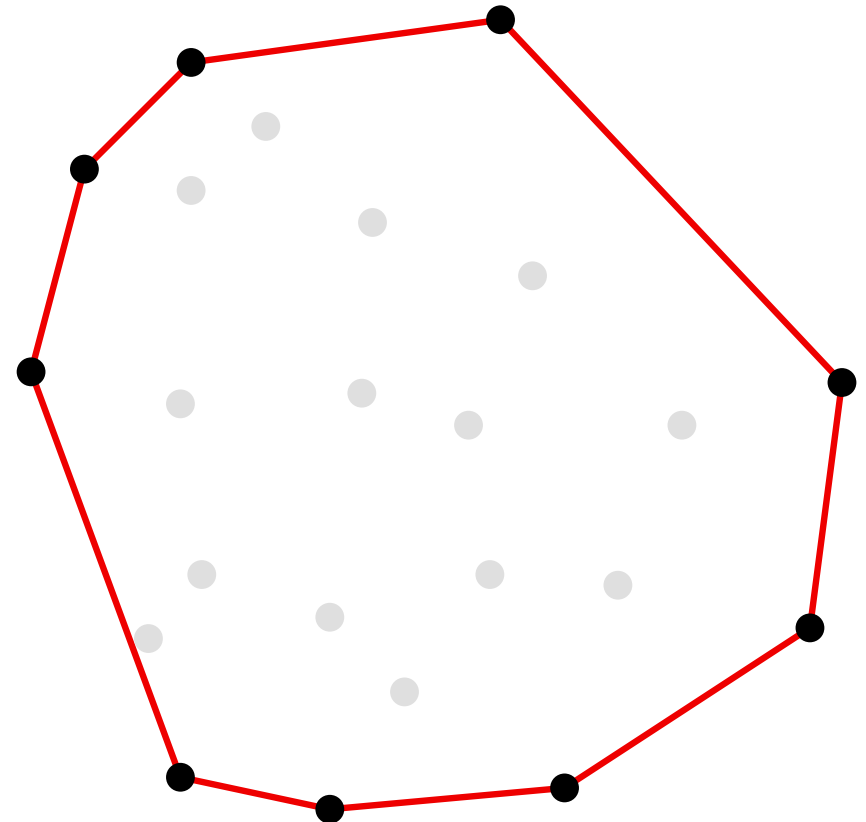
CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

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CONVEX HULL IN 2D

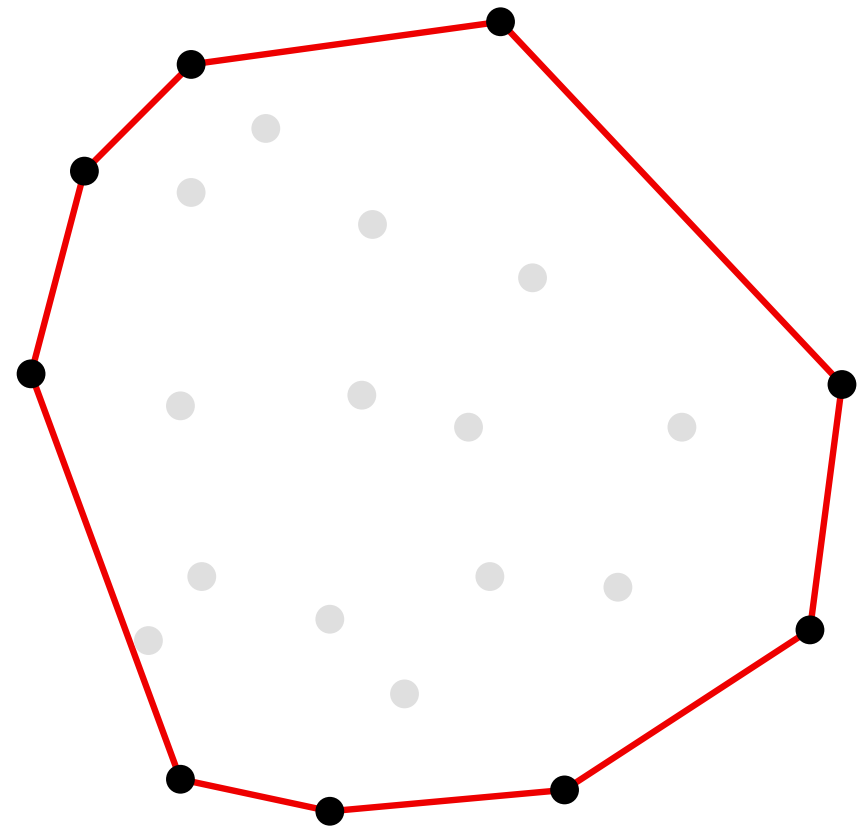
QuickHull algorithm (by prune-and-search)

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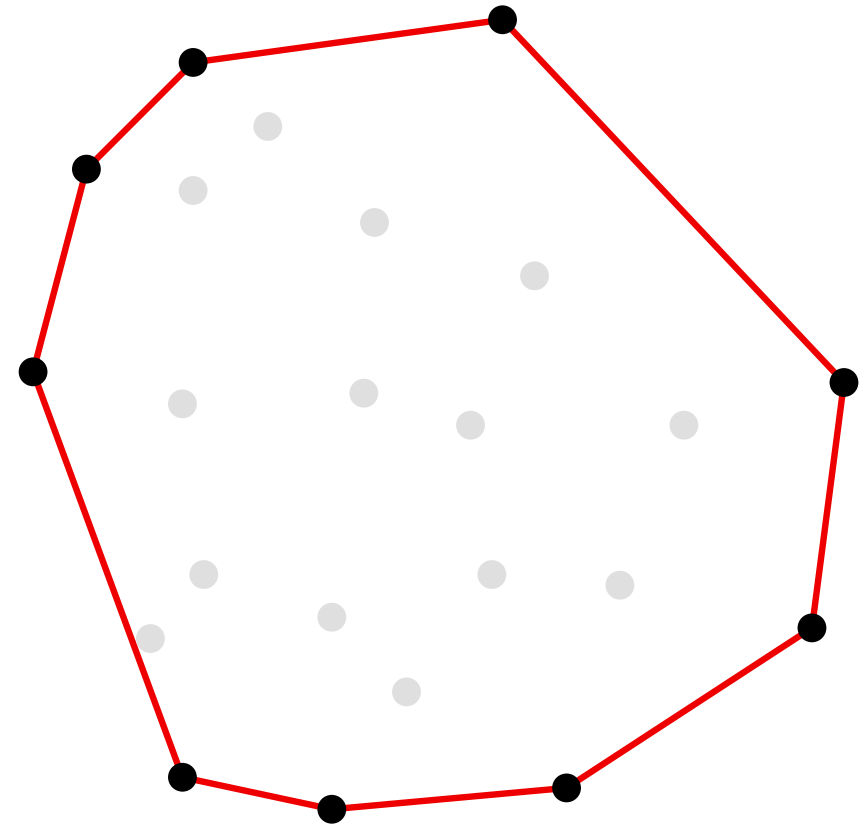
Running time of this step: $O(n^2)$



CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

Overall running time: $O(n^2)$



CONVEX HULL IN 2D

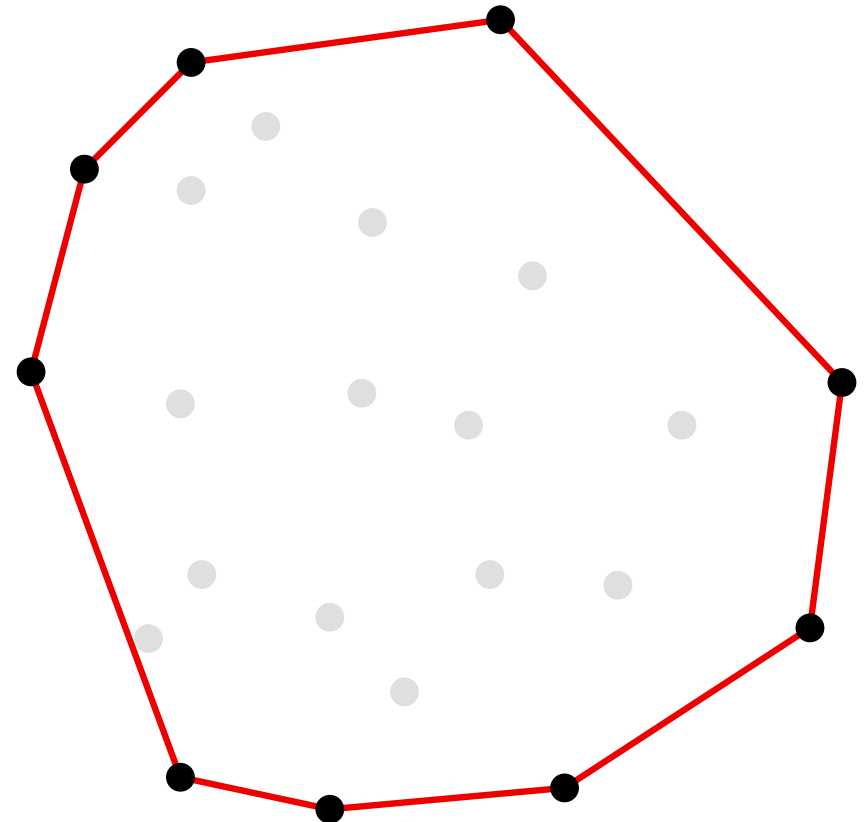
QuickHull algorithm (by prune-and-search)

Overall running time: $O(n^2)$

Nevertheless, the running time of this algorithm depends on the position of the input points.

For example:

- If the input points are in convex position, the running time is $\Theta(n^2)$.
- If the points are such that each prune step eliminates half of the current points, then the algorithm runs in $\Theta(n \log n)$ time.
- If the convex hull is triangular, the algorithm runs in $\Theta(n)$ time.



CONVEX HULL IN 2D

Graham's algorithm

CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:

- Push p_i in l

- Advance i

- Else:

- Pop p from l

Return l

CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

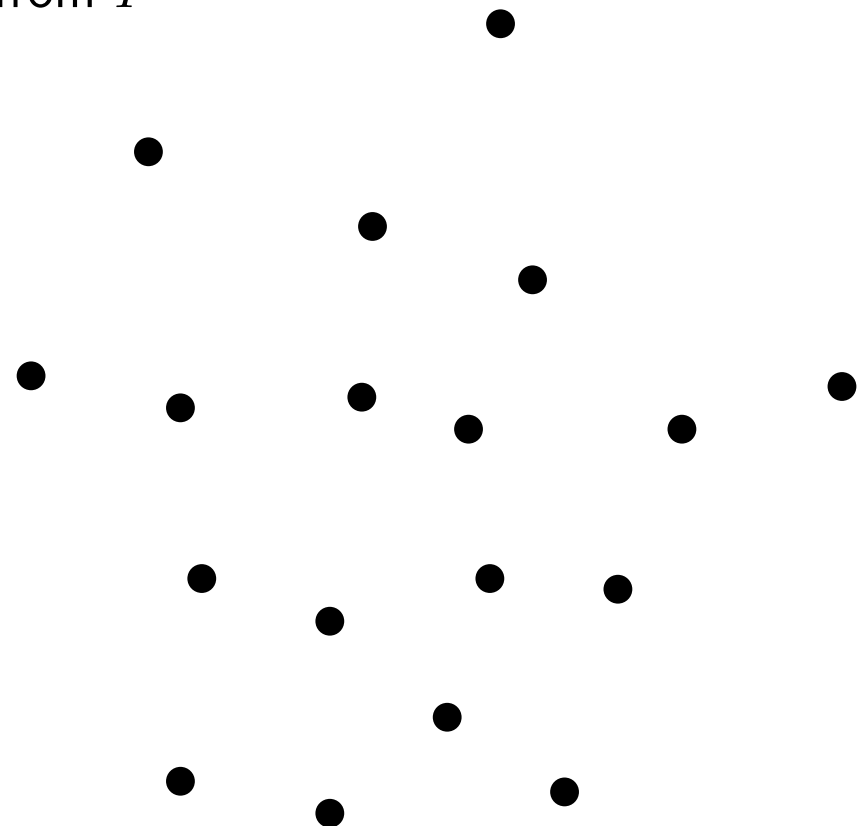
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$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

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 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

While there exist points $p_i \in P$ to be explored, do:

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- If $p^- p p_i$ is a left turn:

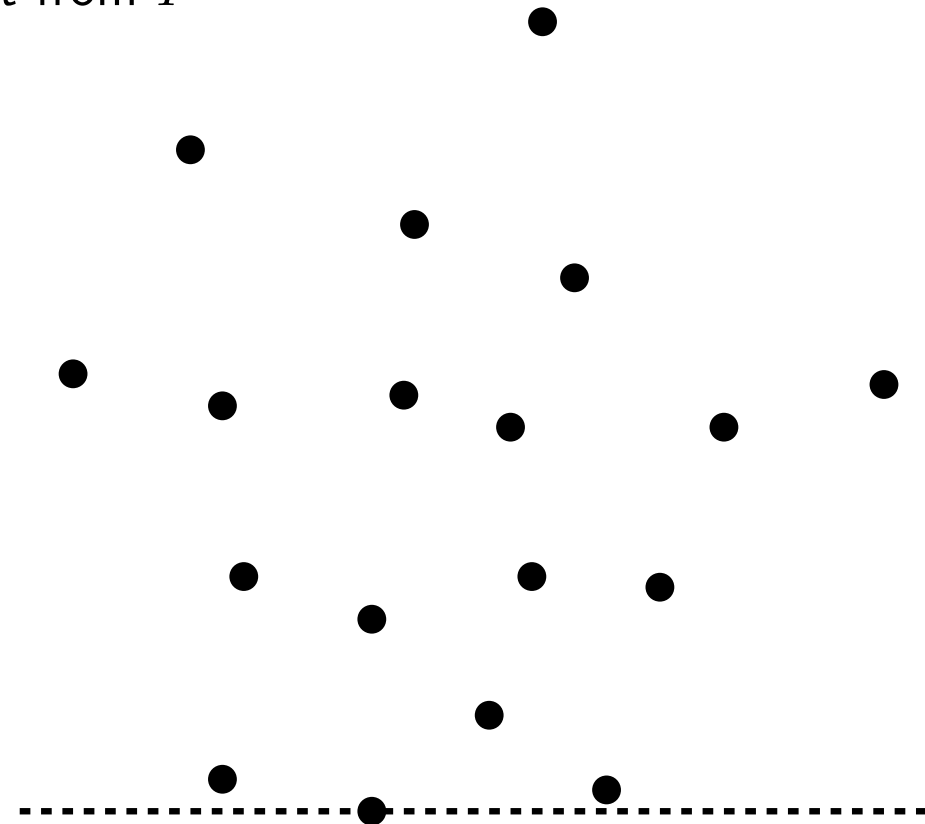
- Push p_i in l

- Advance i

- Else:

- Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

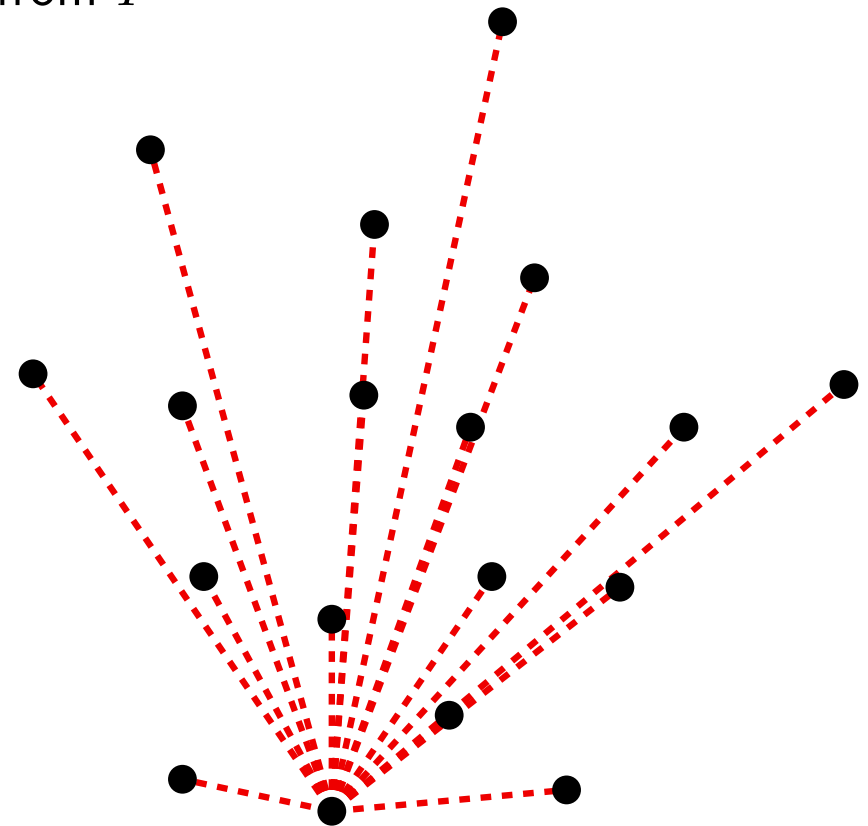
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

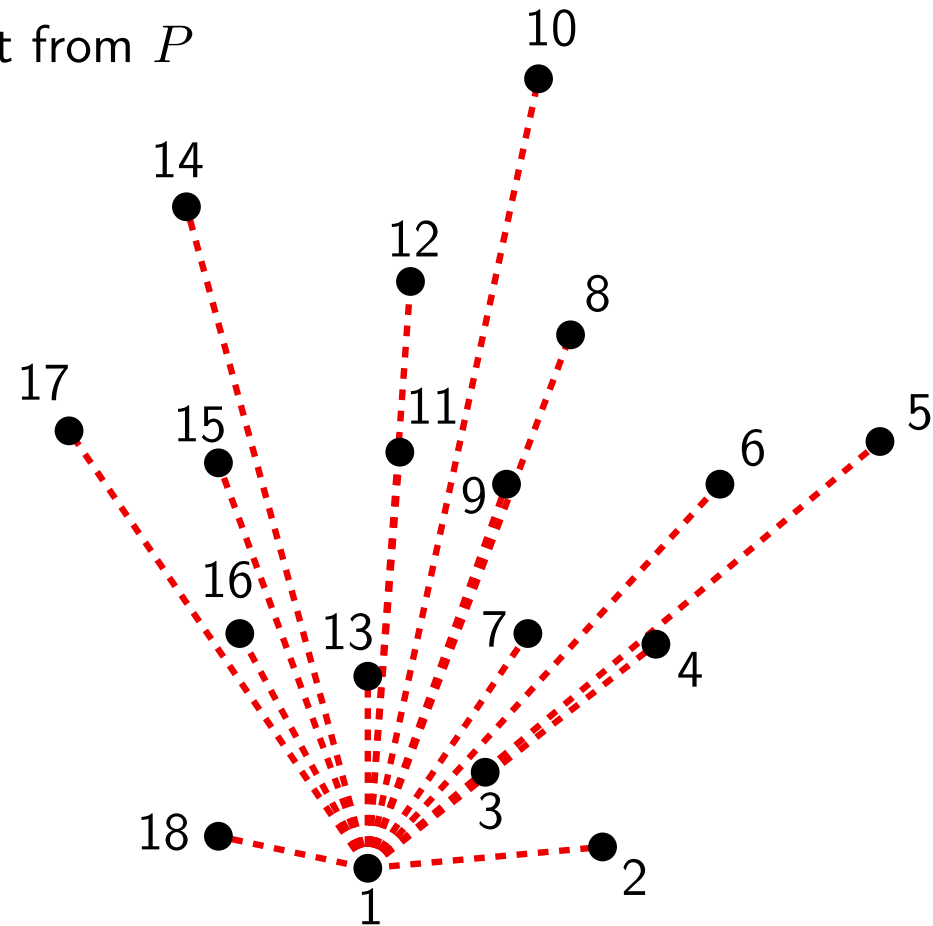
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

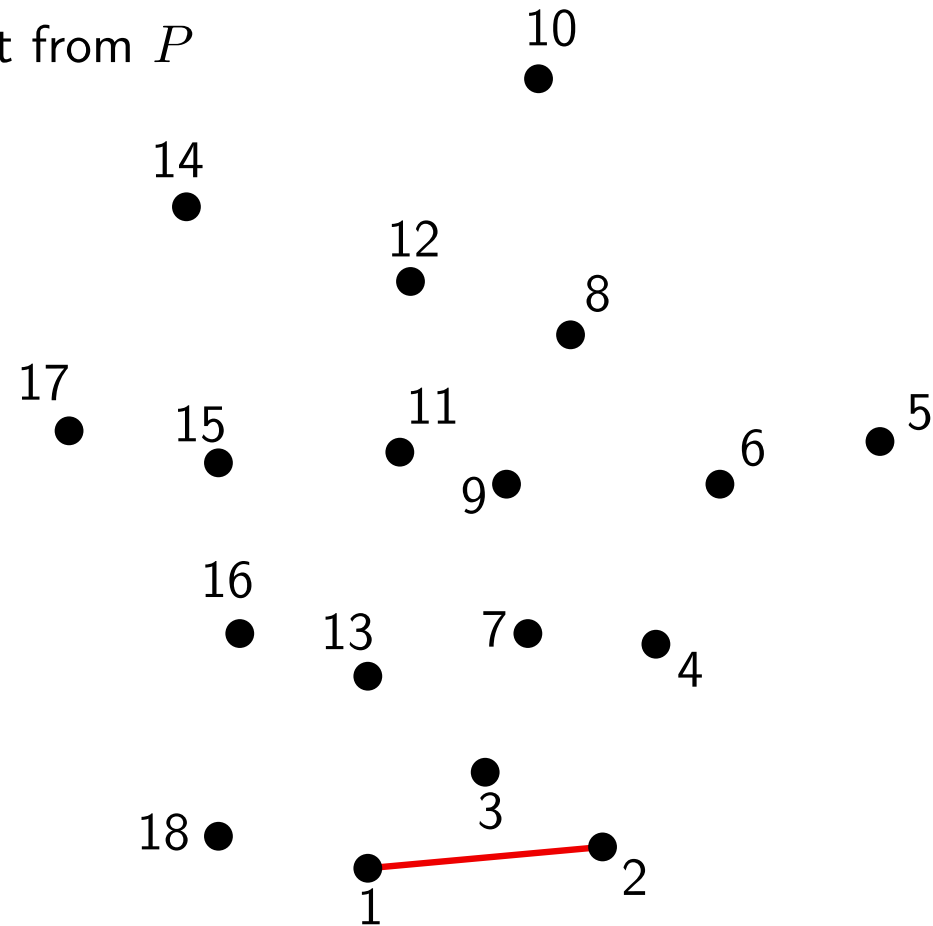
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

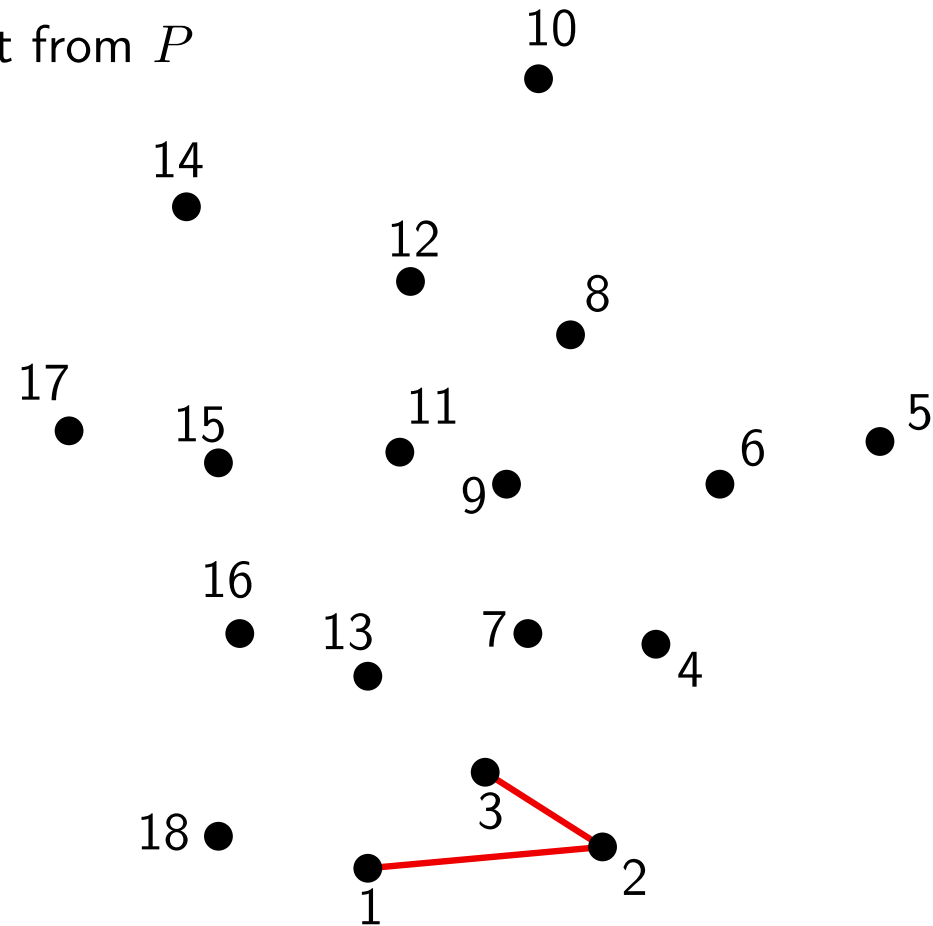
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

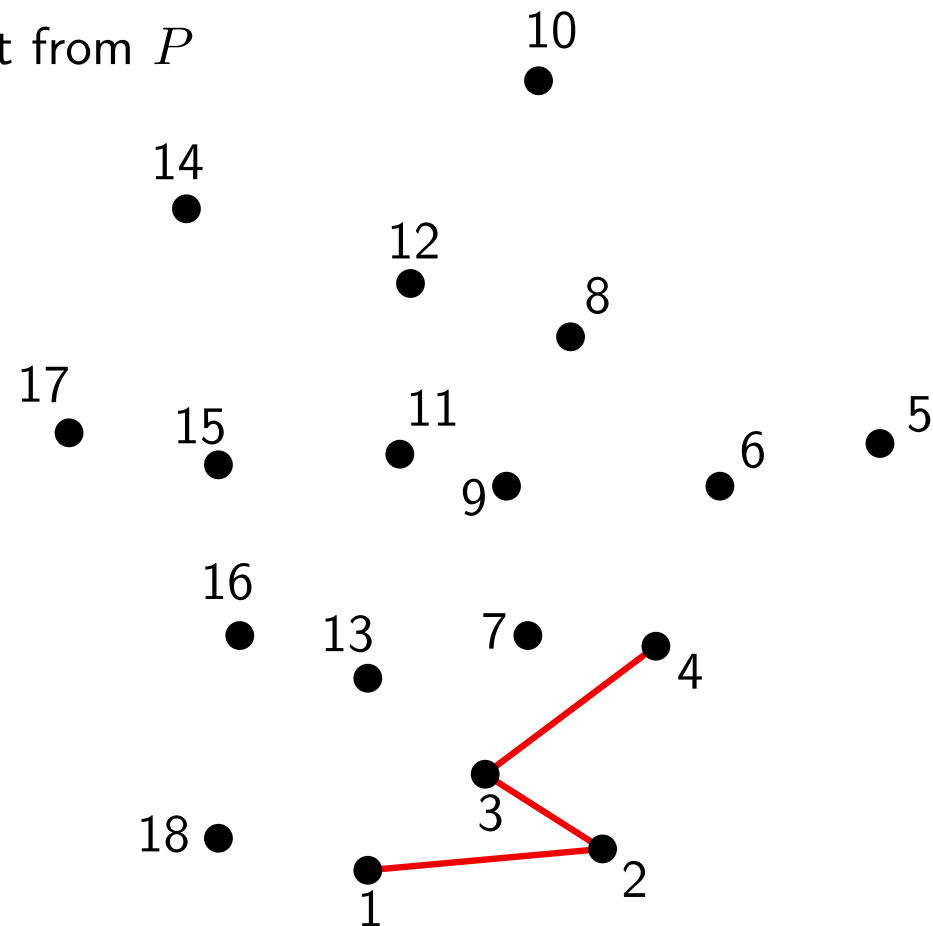
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

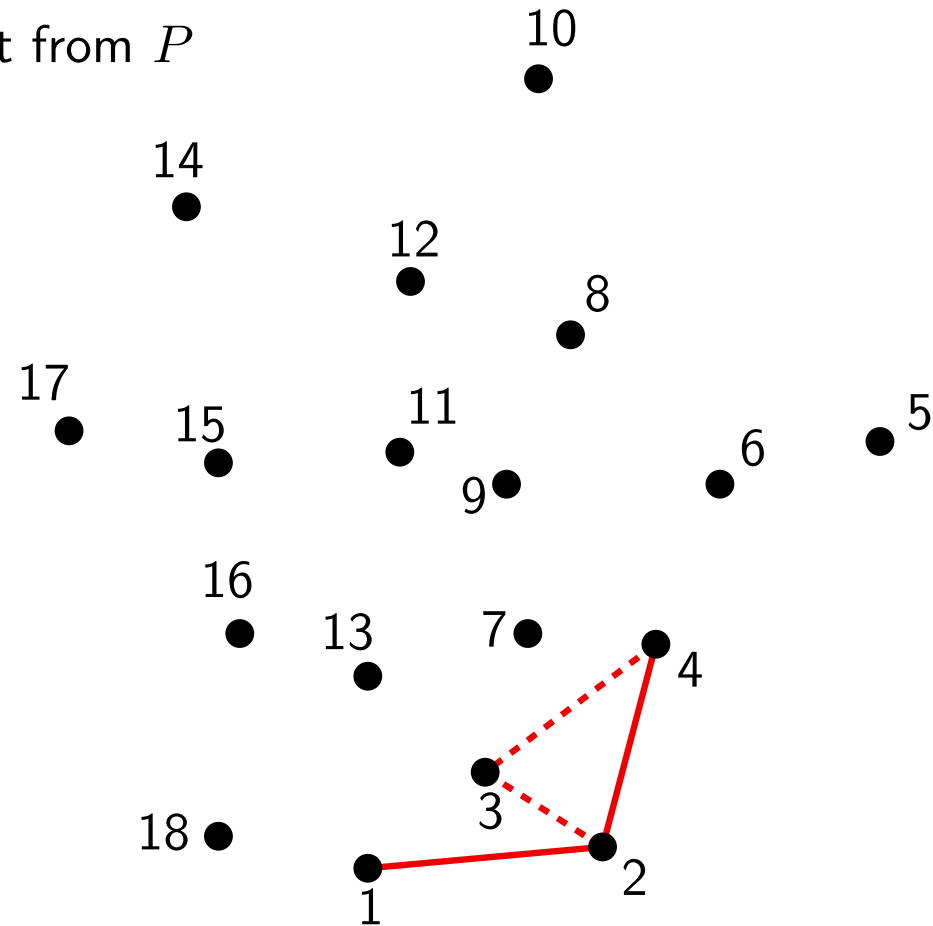
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

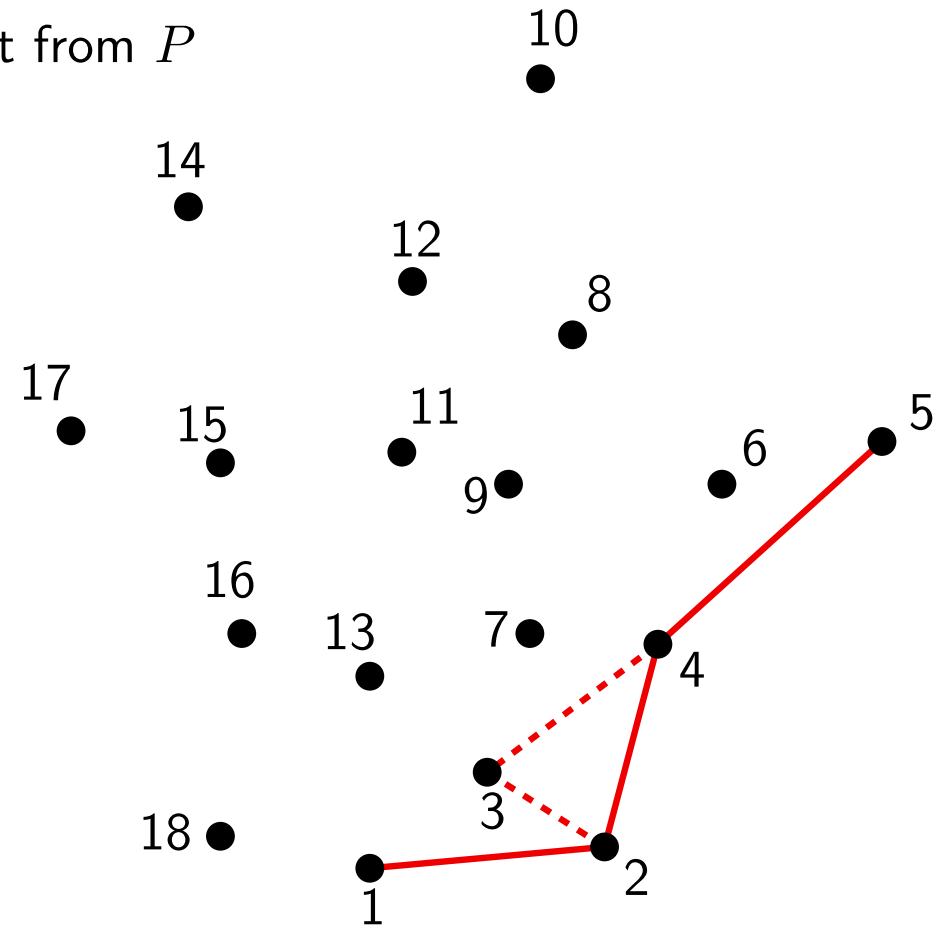
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

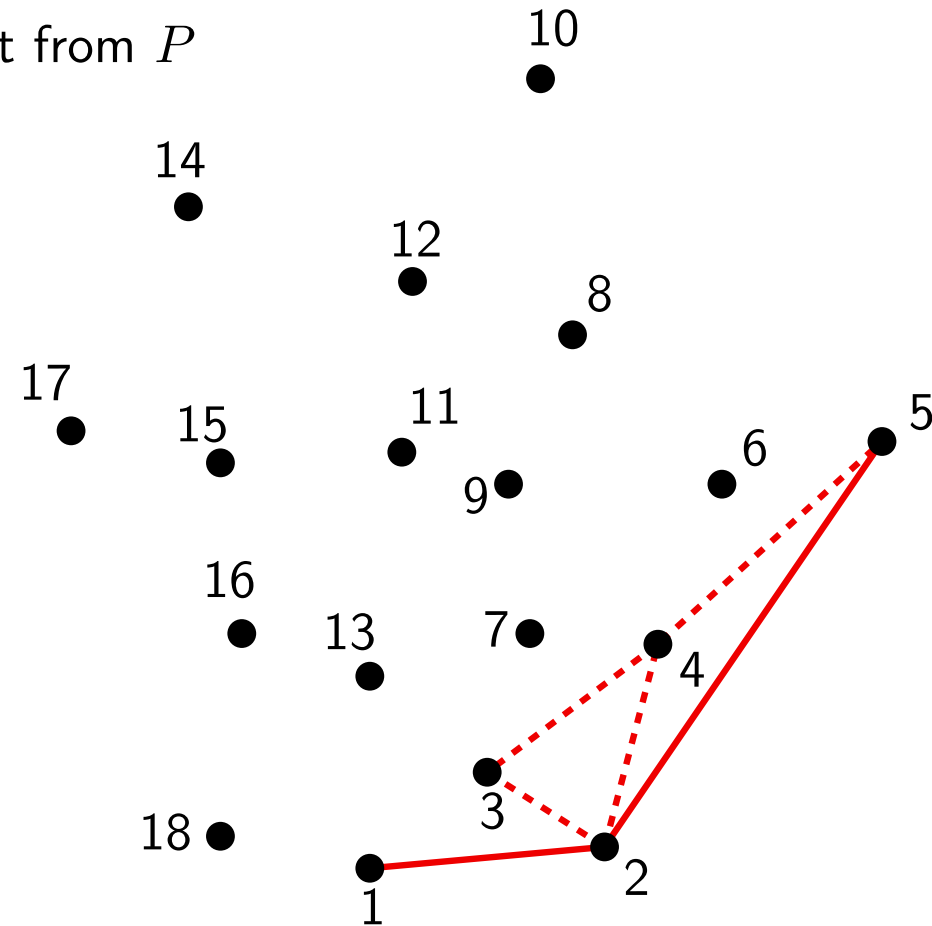
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

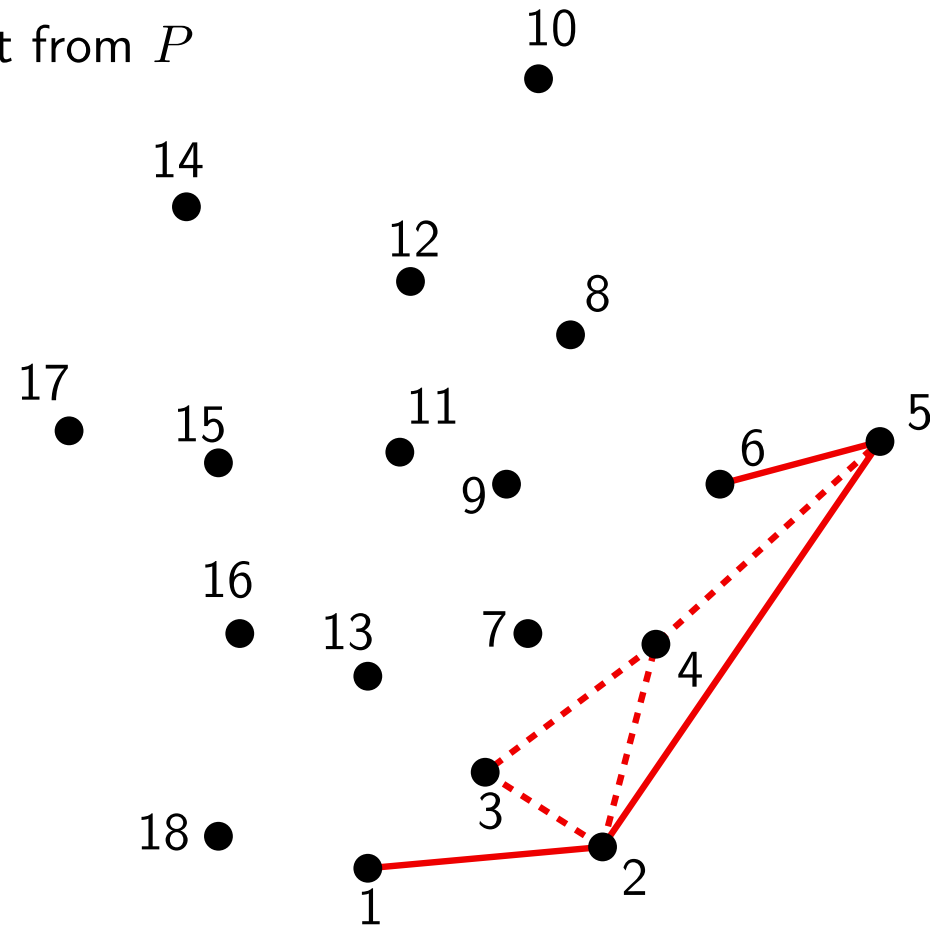
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

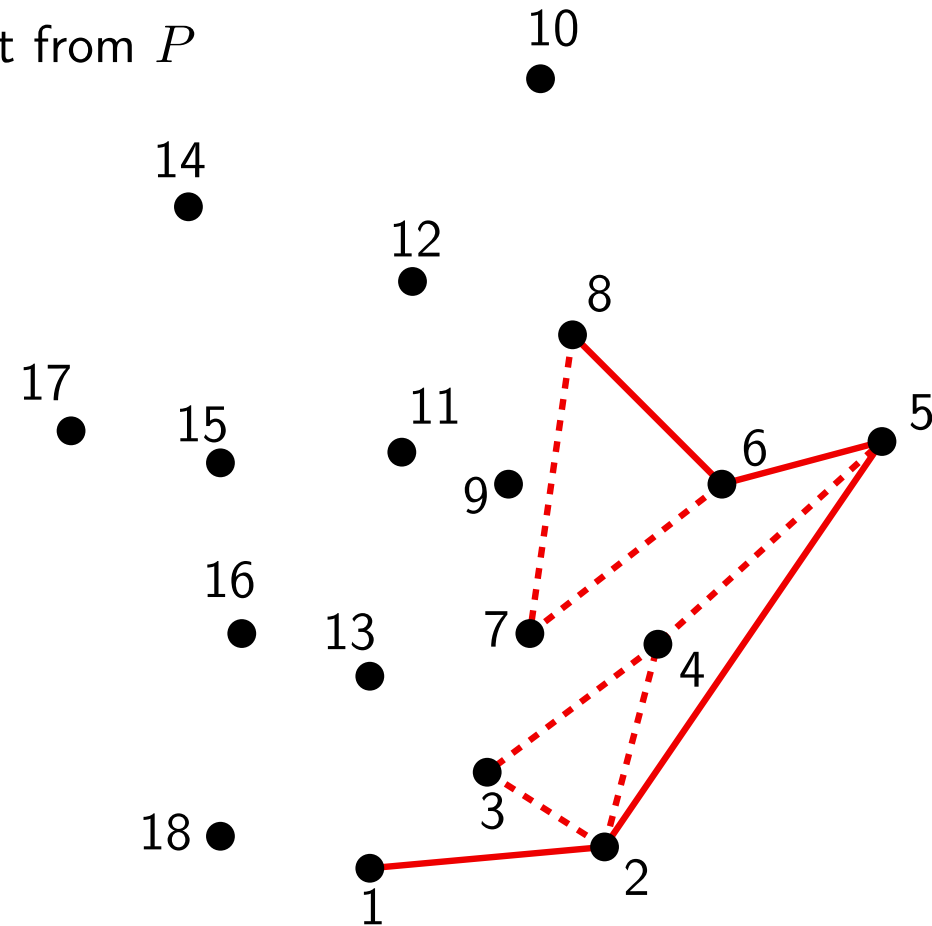
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

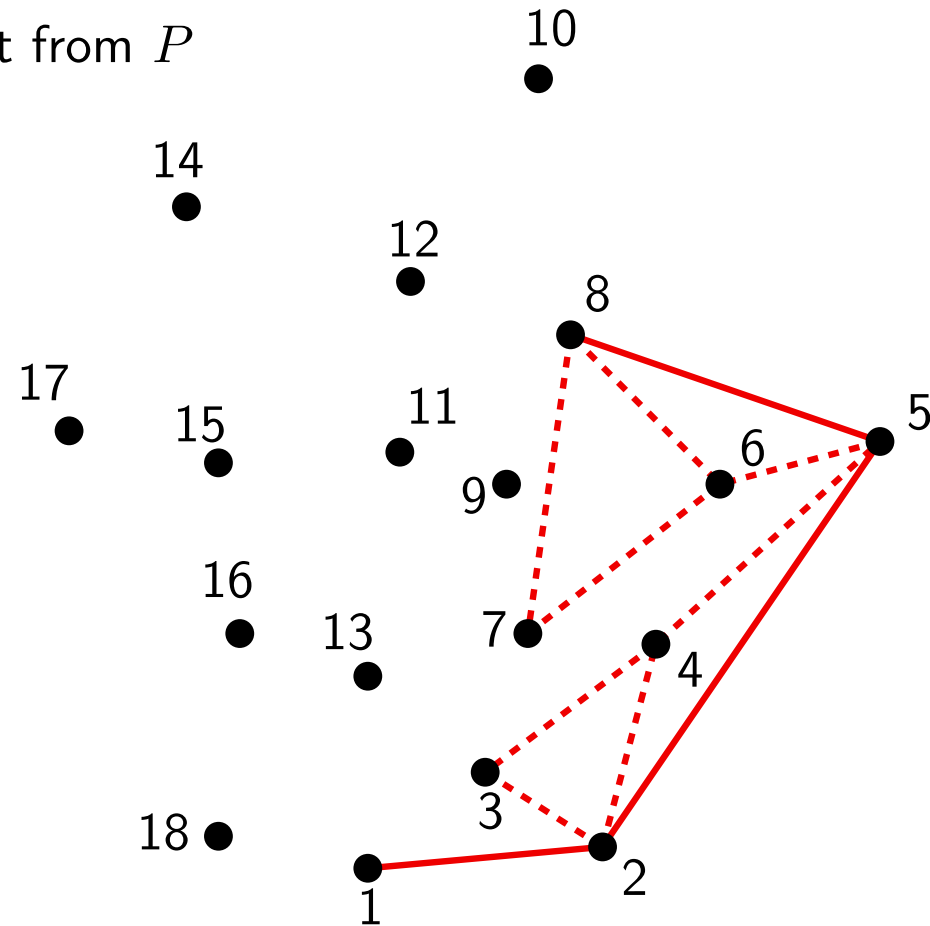
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

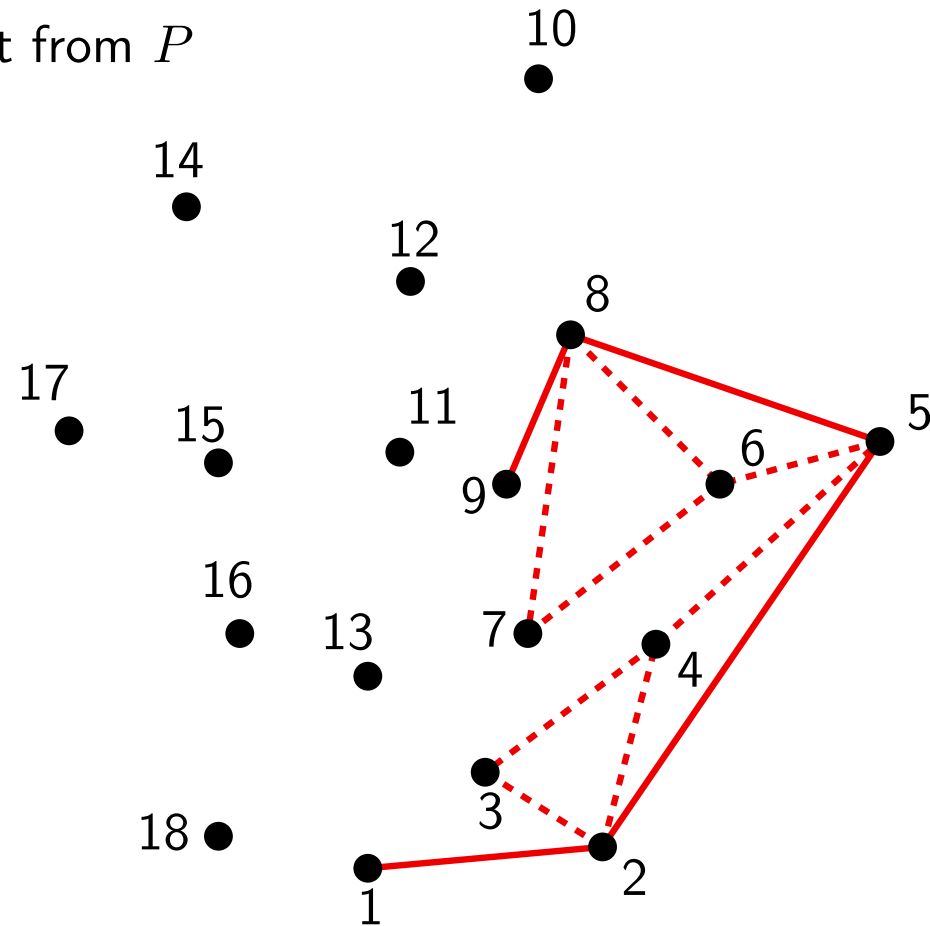
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

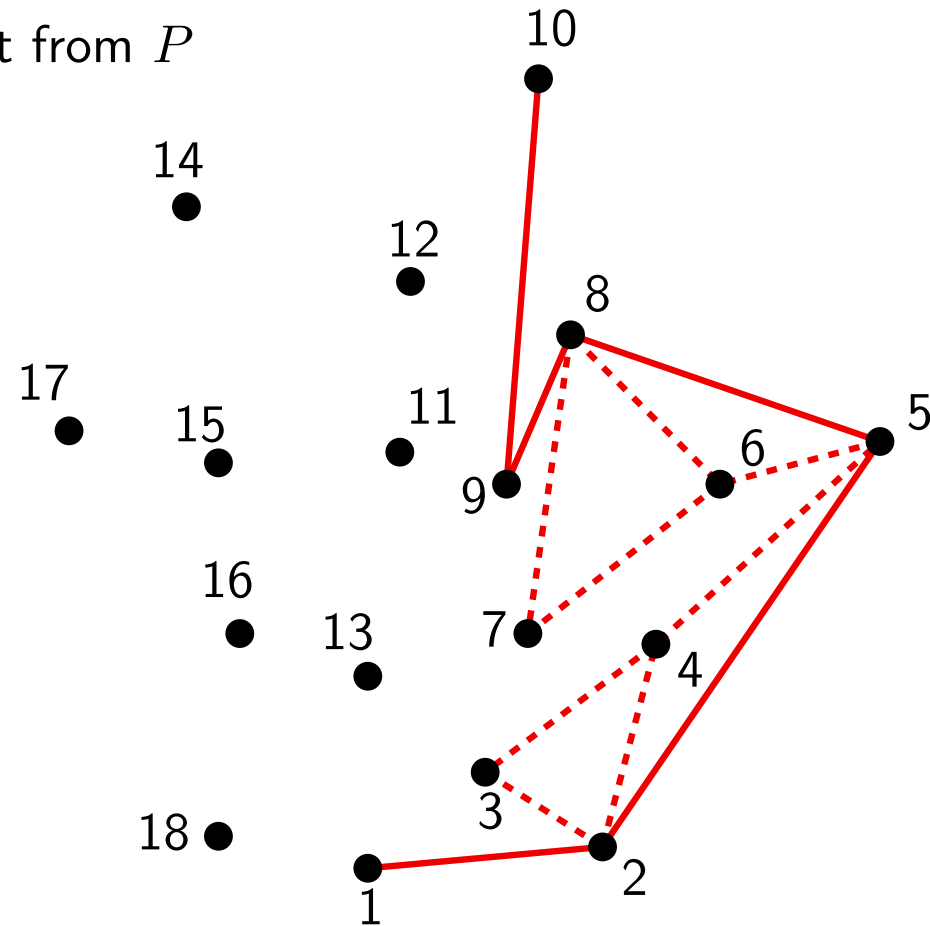
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

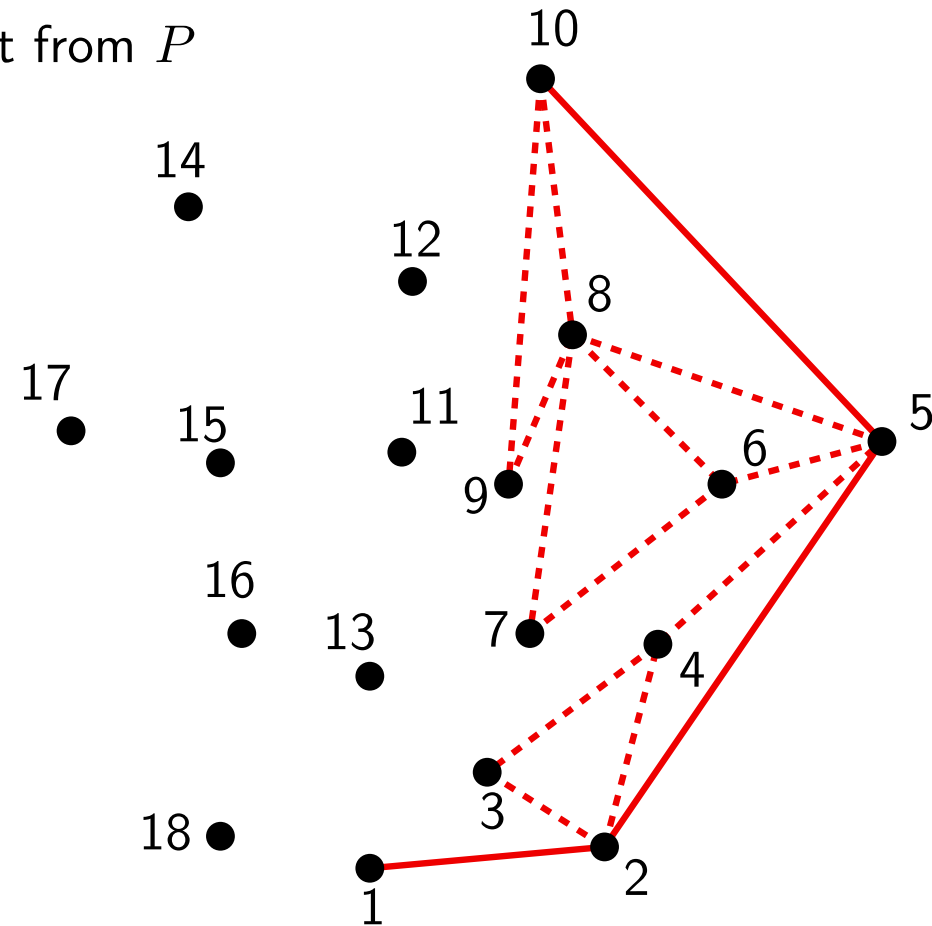
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

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- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

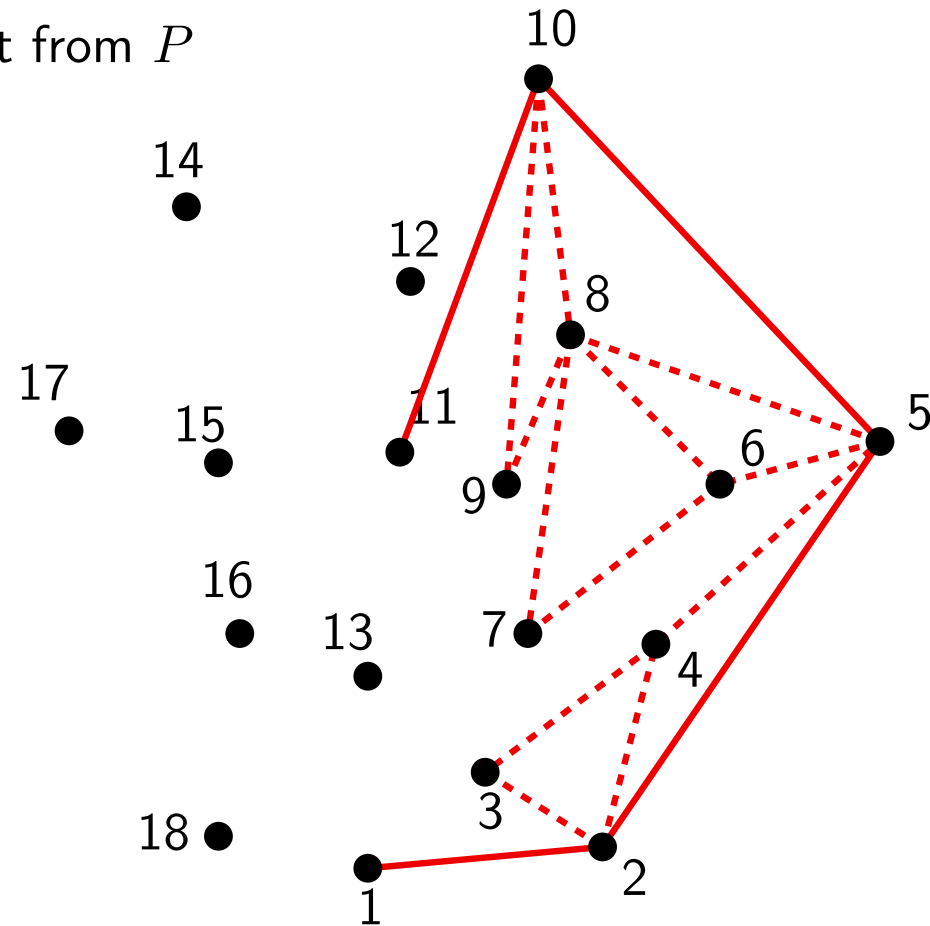
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

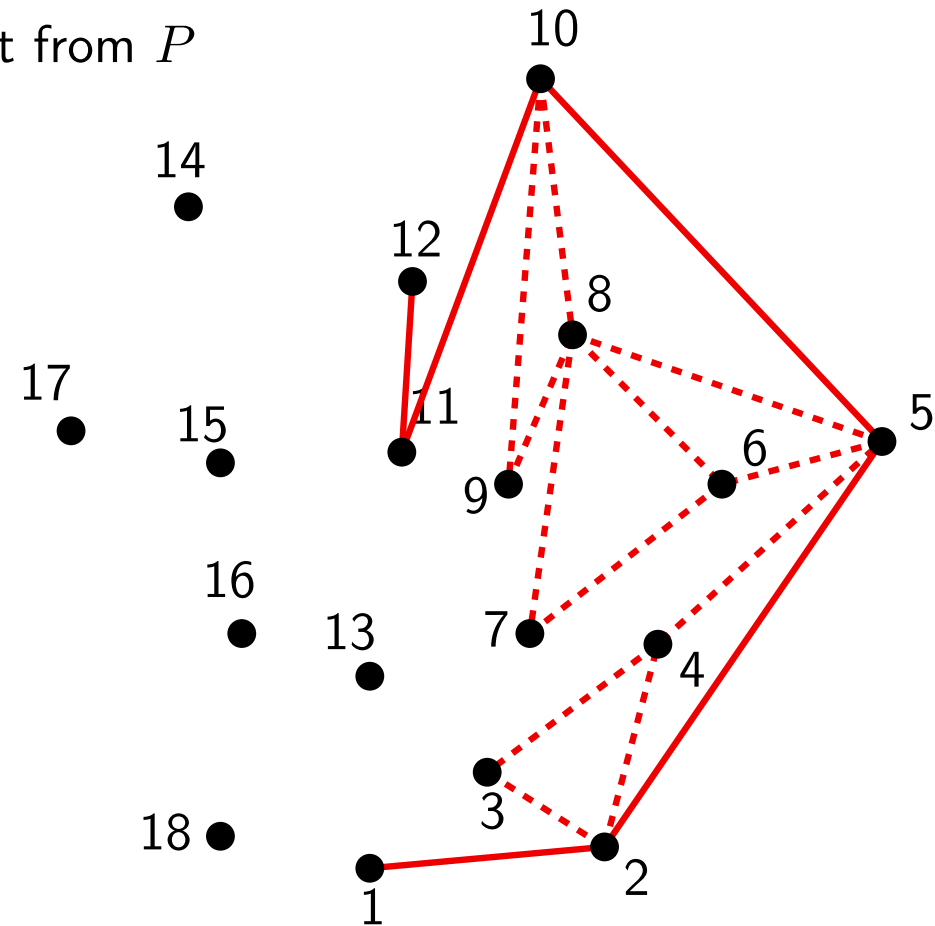
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

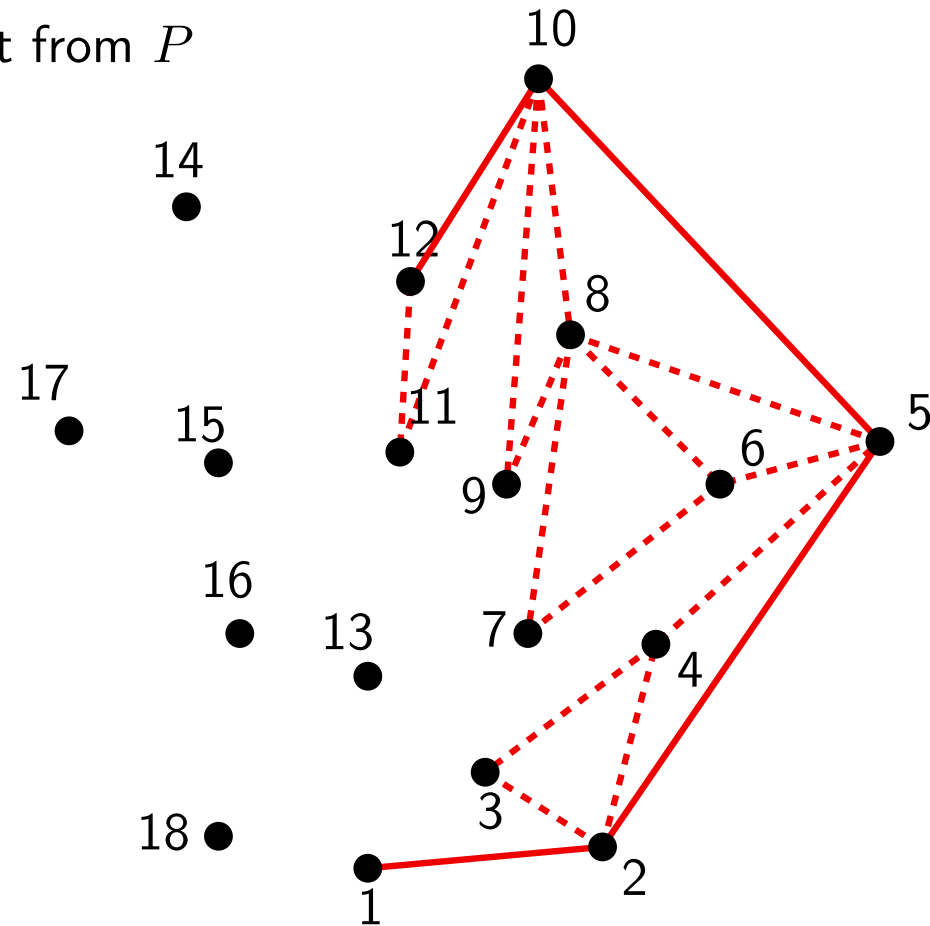
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

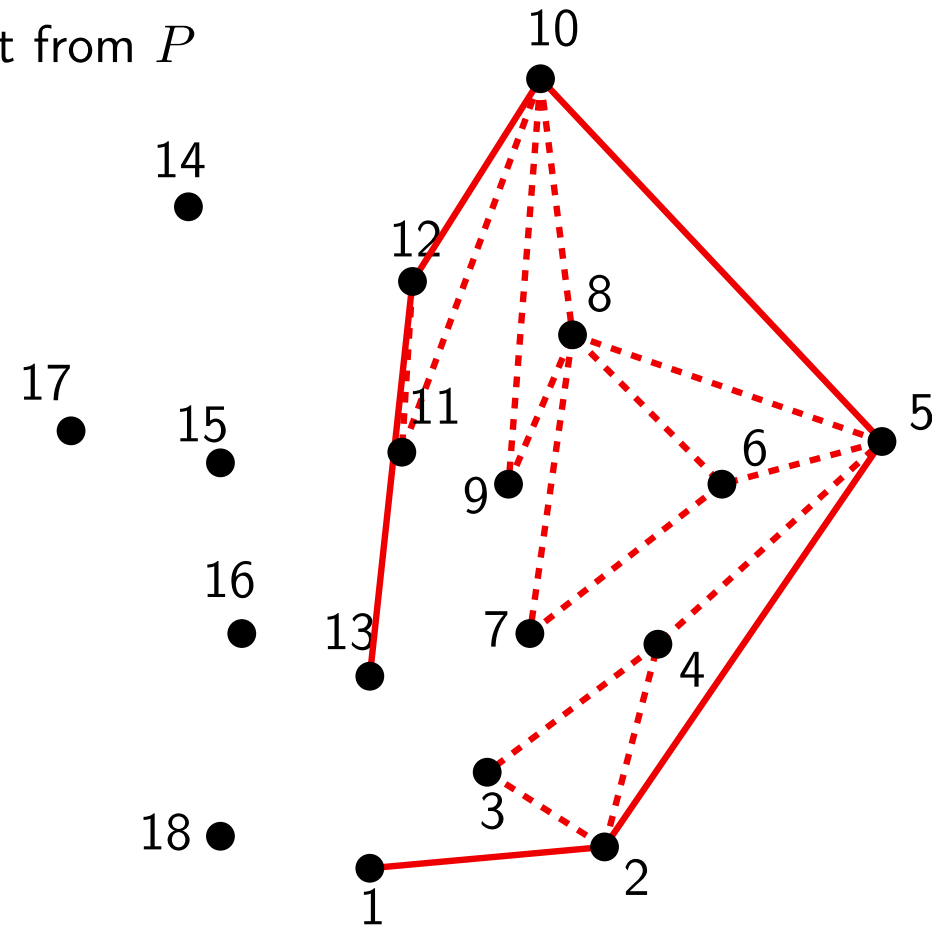
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

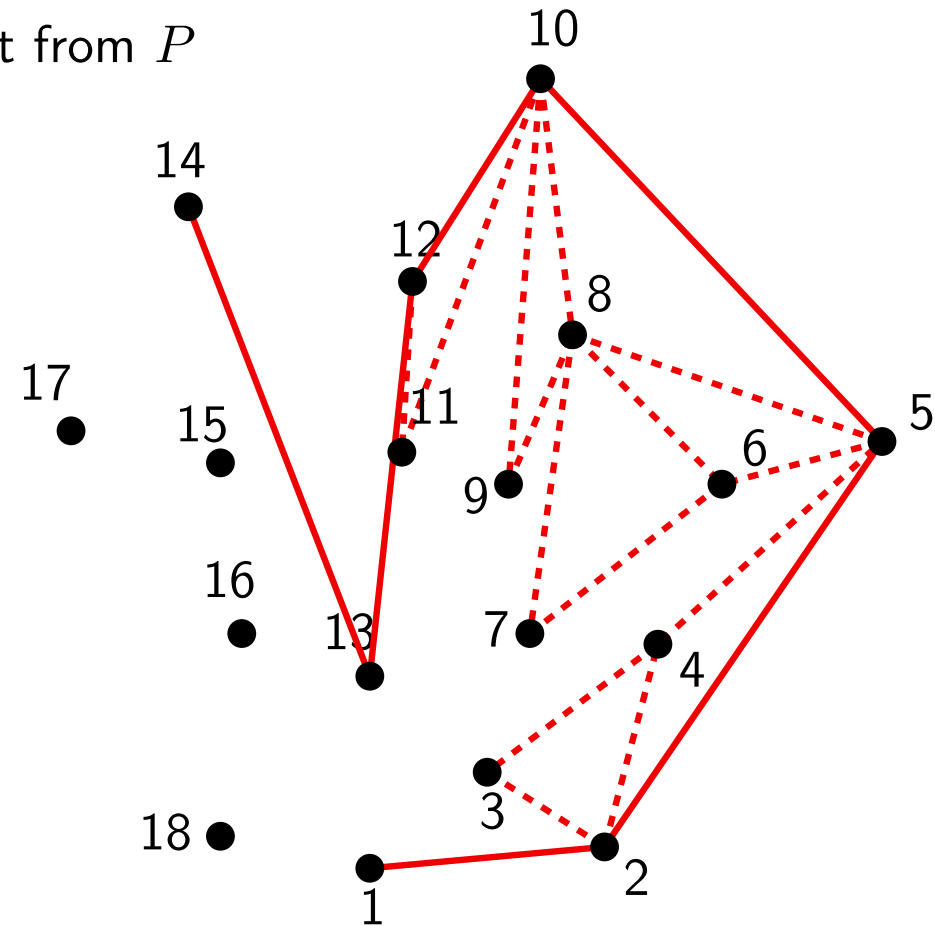
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

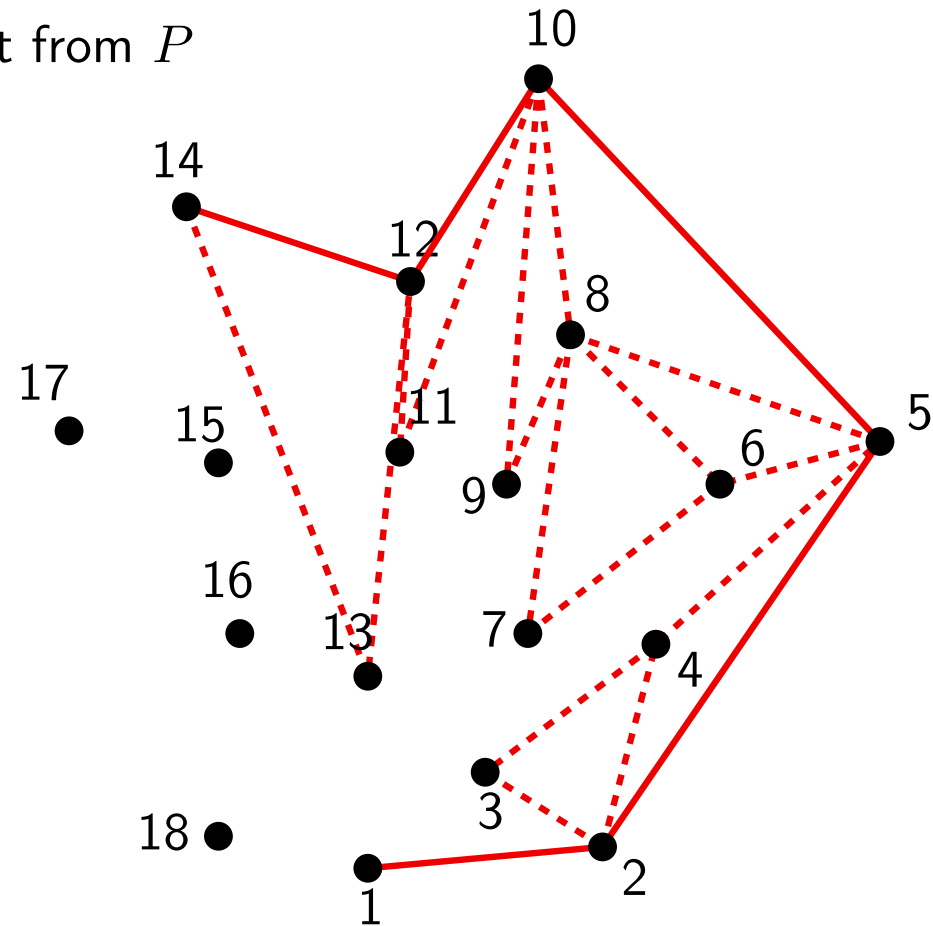
While there exist points $p_i \in P$ to be explored, do:

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- If $p^- p p_i$ is a left turn:
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 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

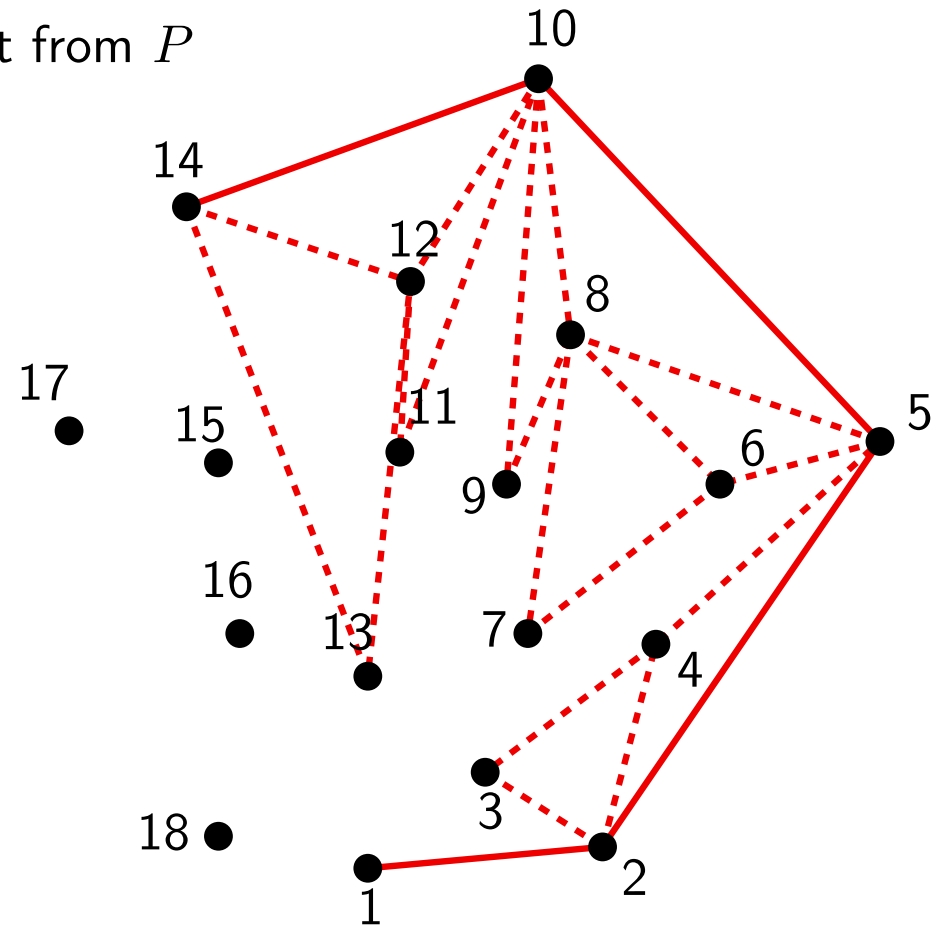
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

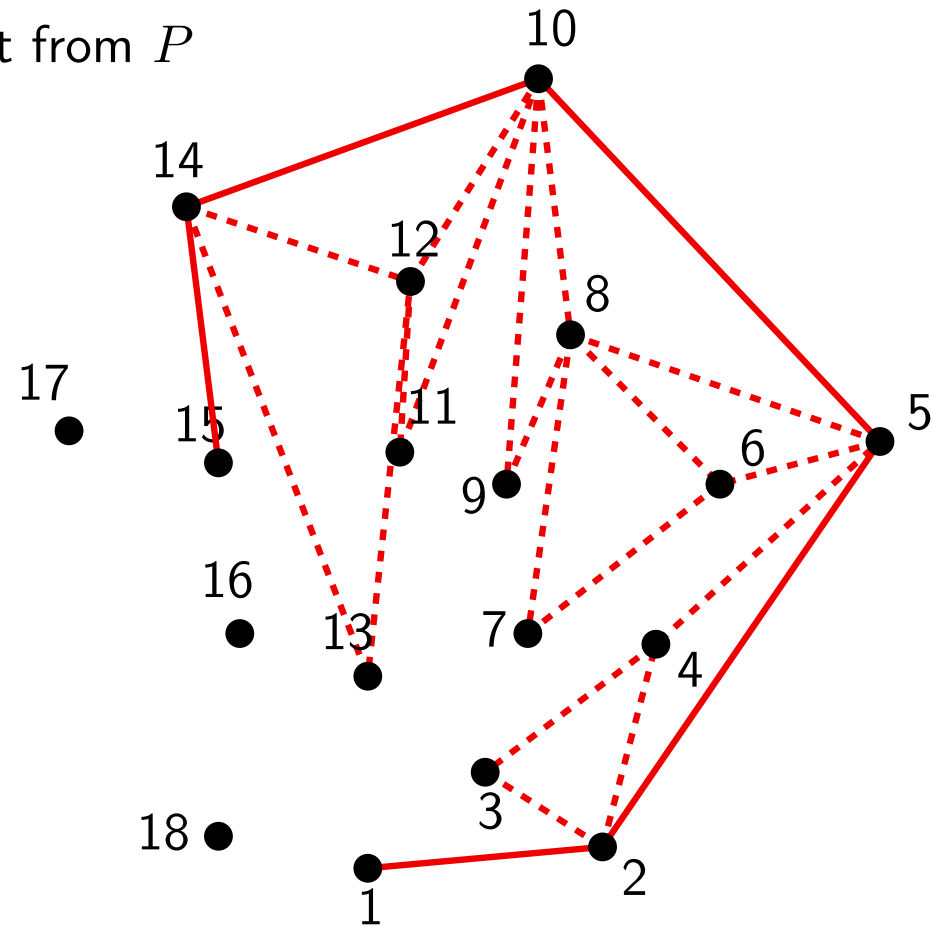
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

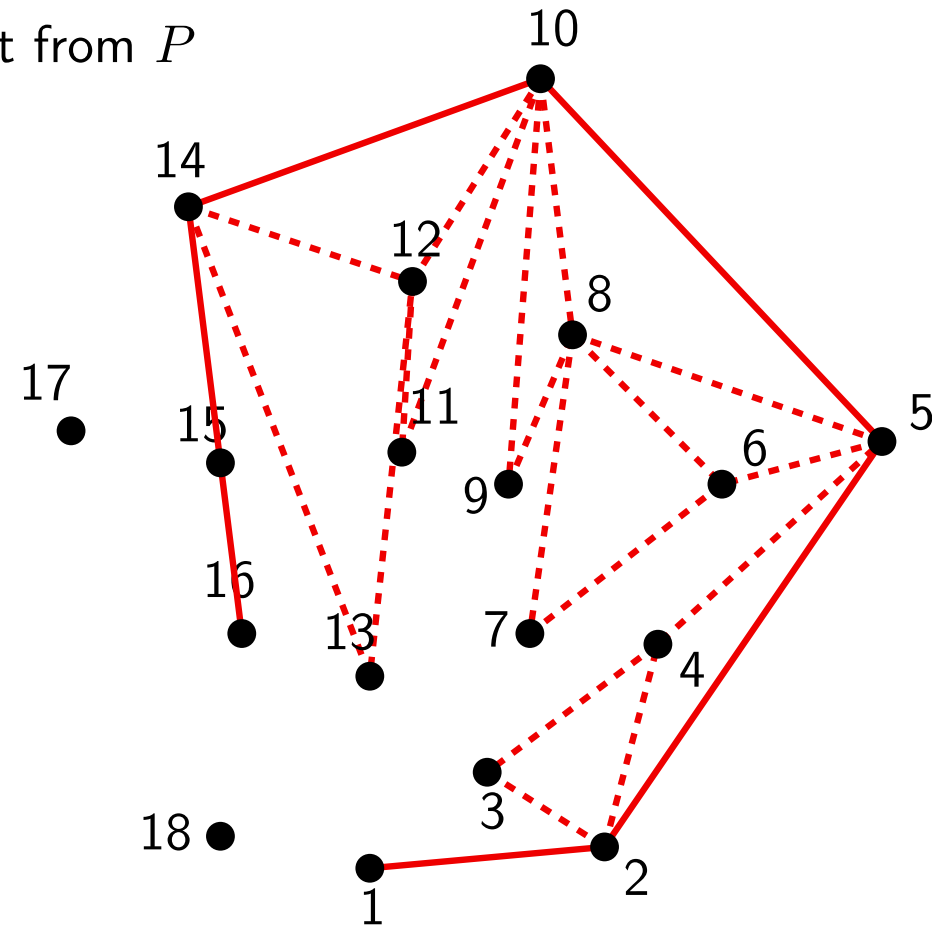
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

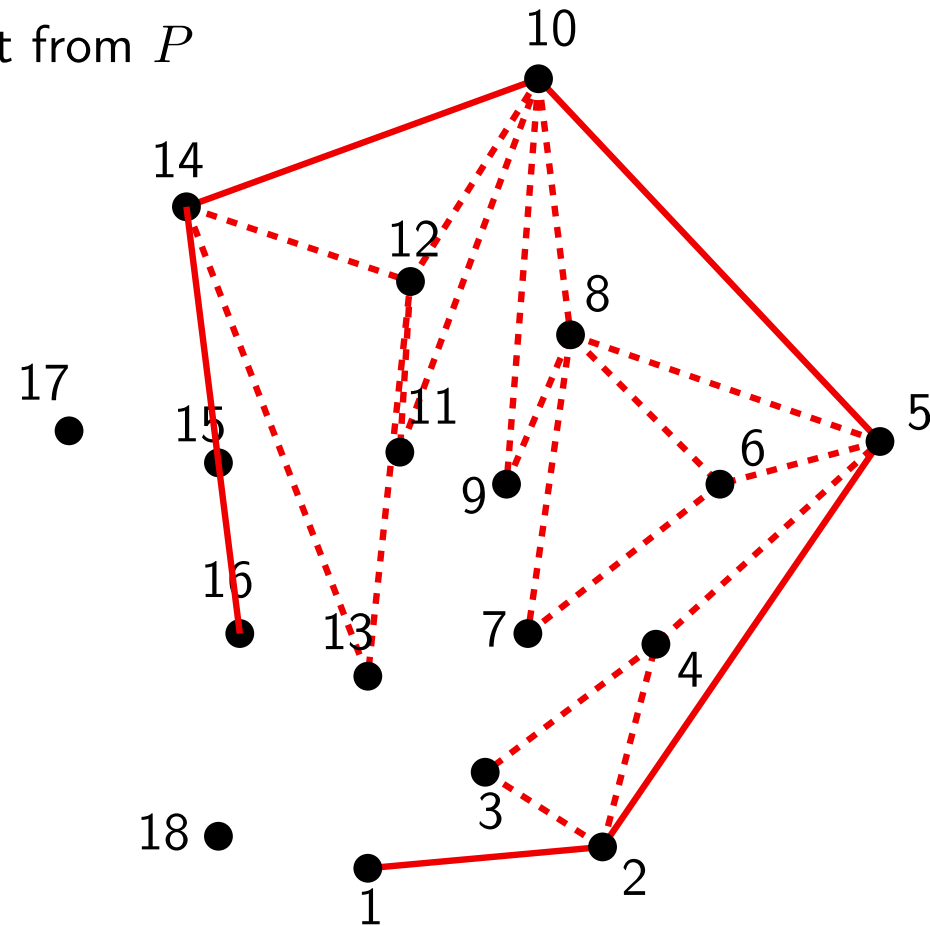
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

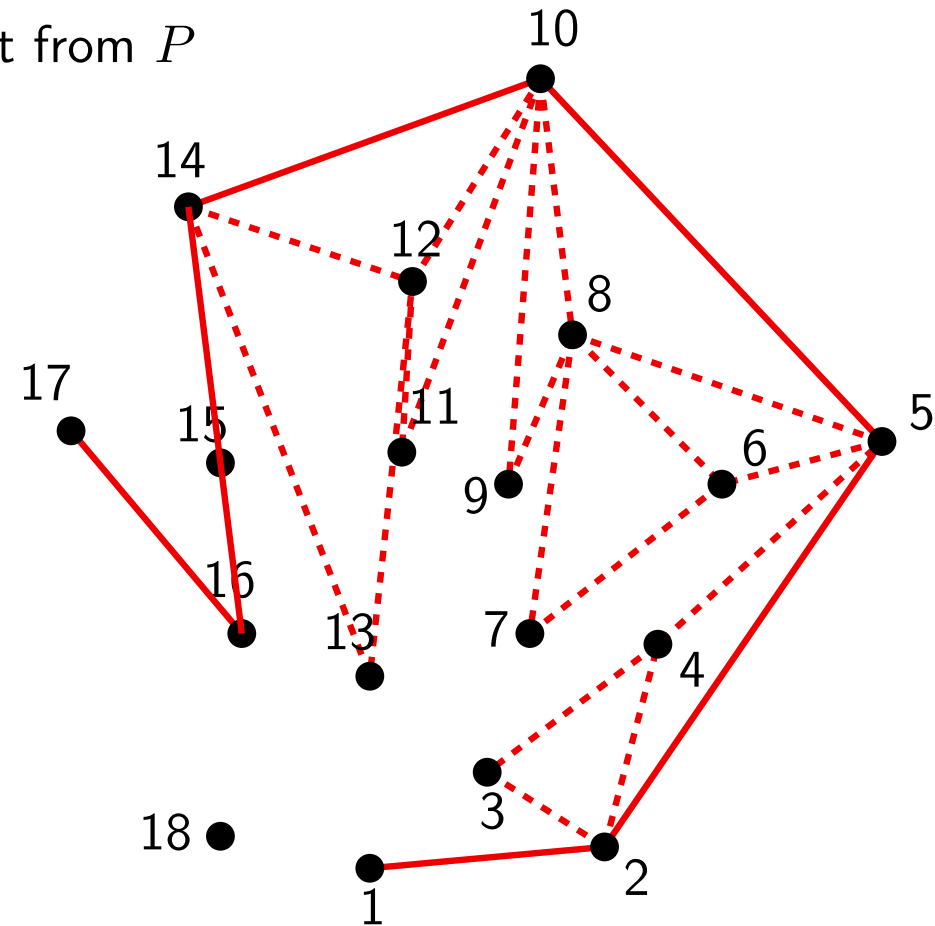
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

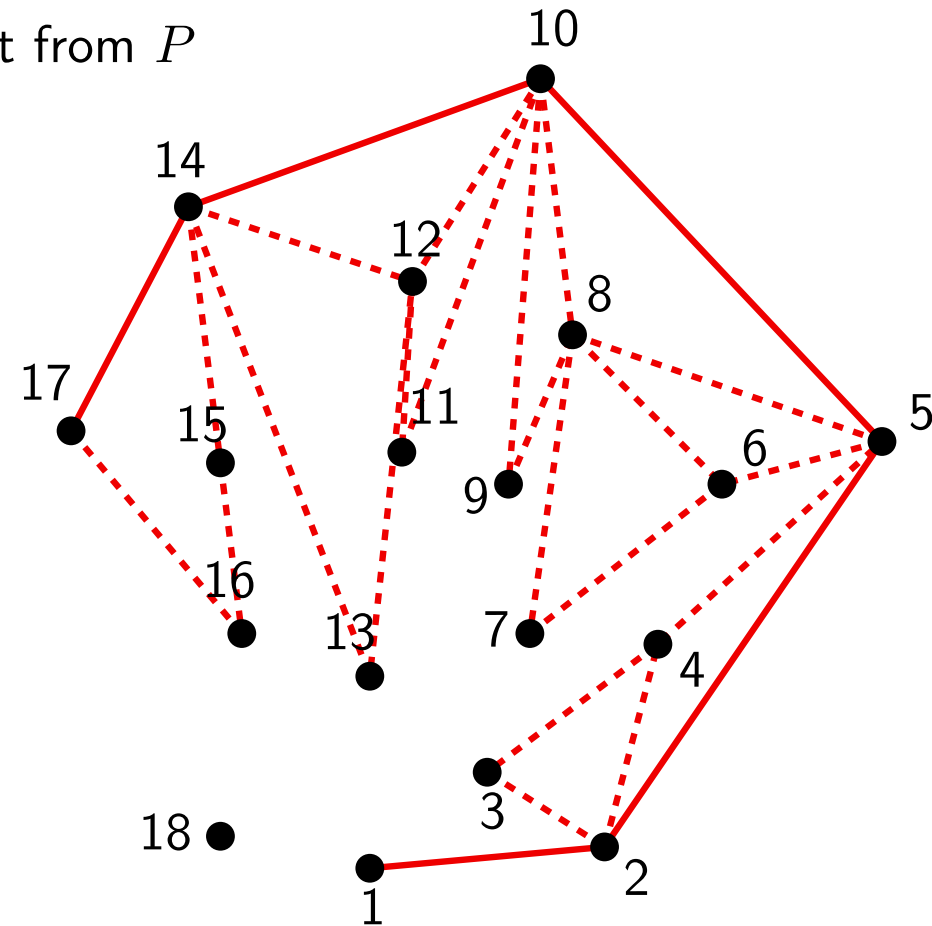
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

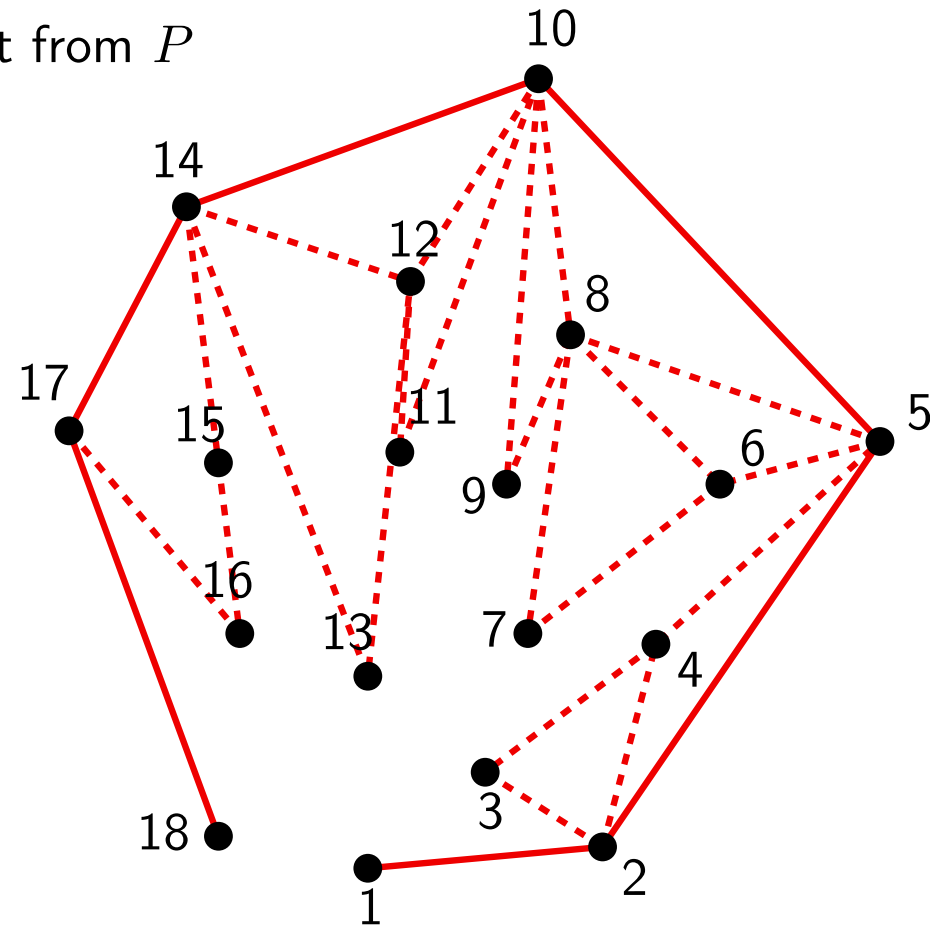
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

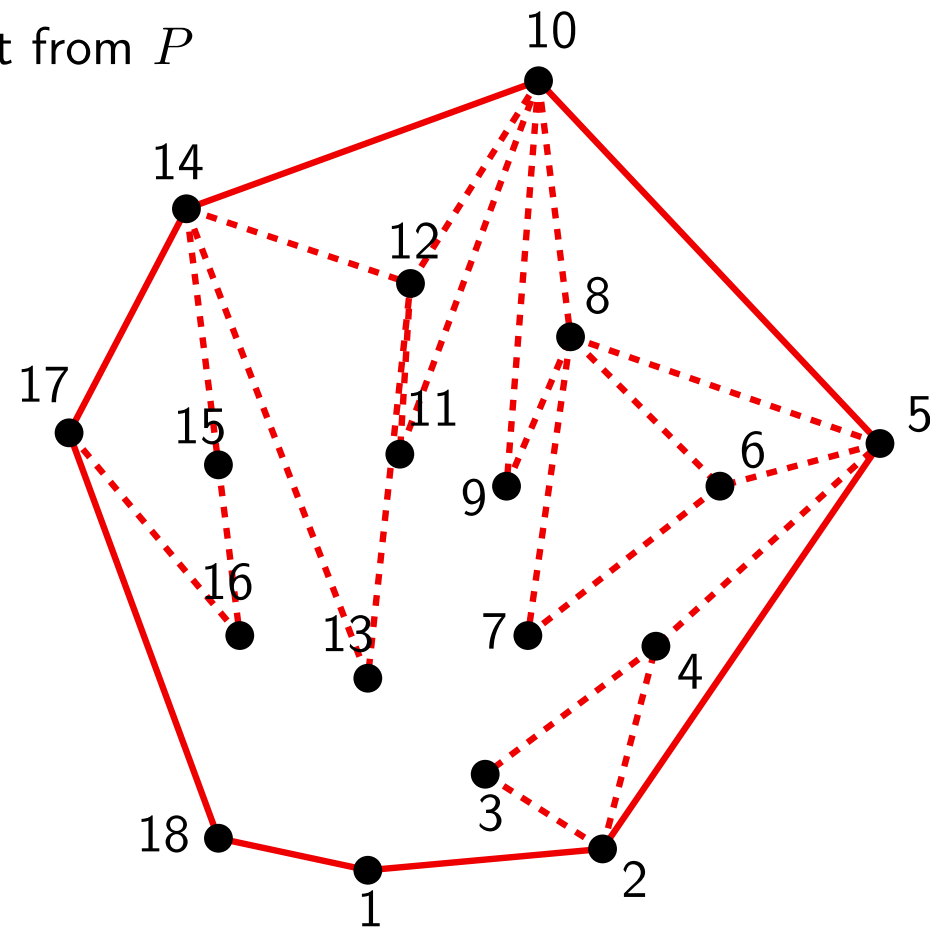
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:

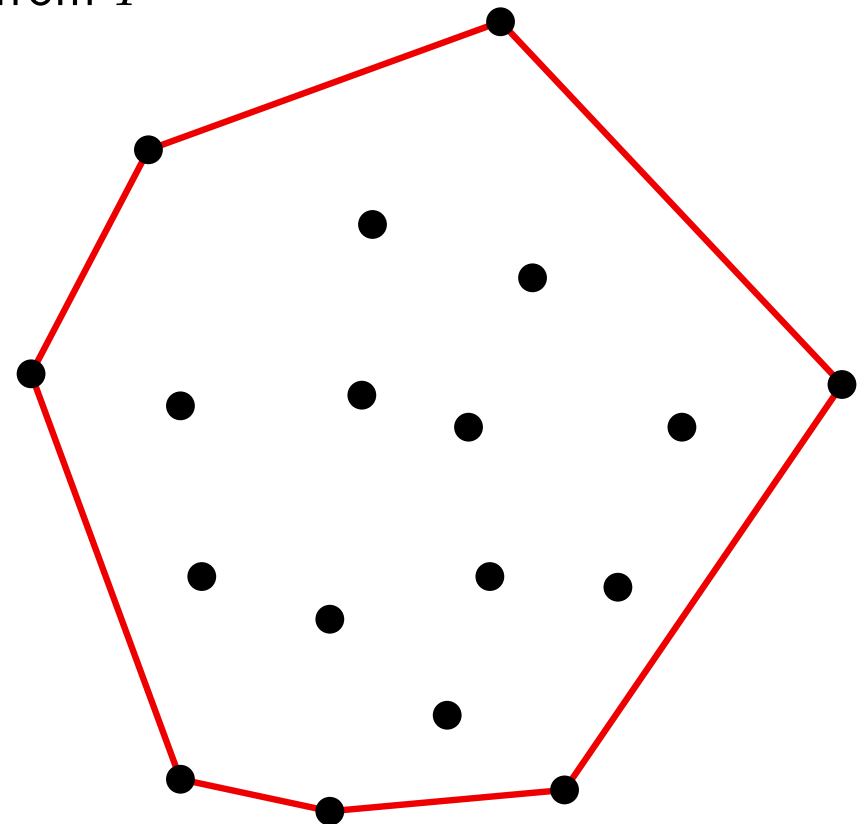
- Push p_i in l

- Advance i

- Else:

- Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:

- Push p_i in l

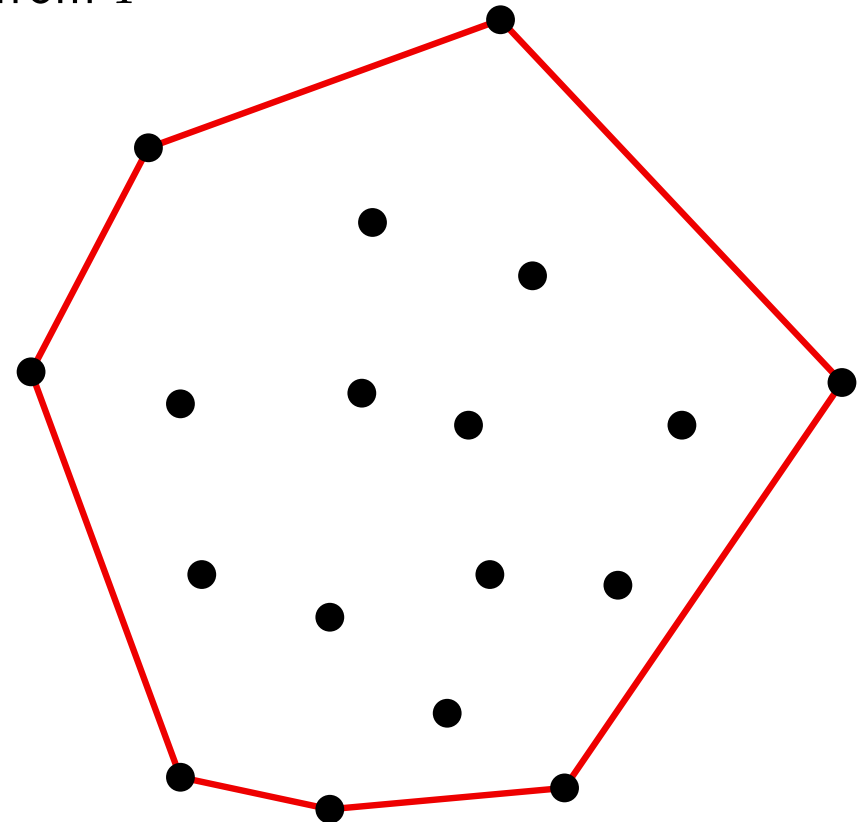
- Advance i

- Else:

- Pop p from l

Return l

Running time: $O(n \log n)$



CONVEX HULL IN 2D

Incremental algorithm

CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l

CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

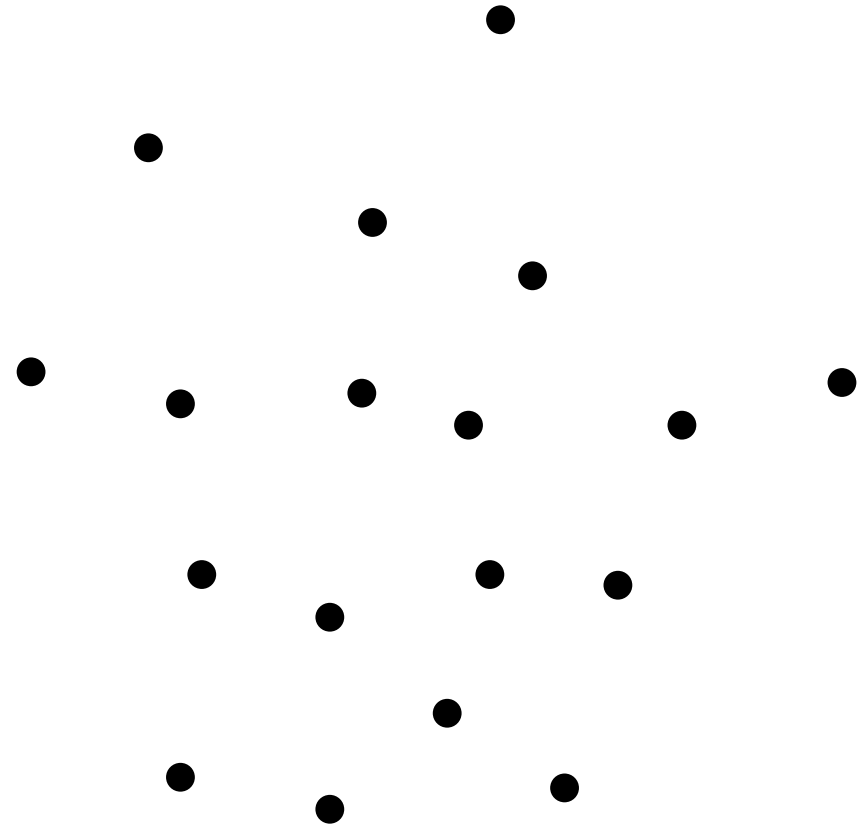
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

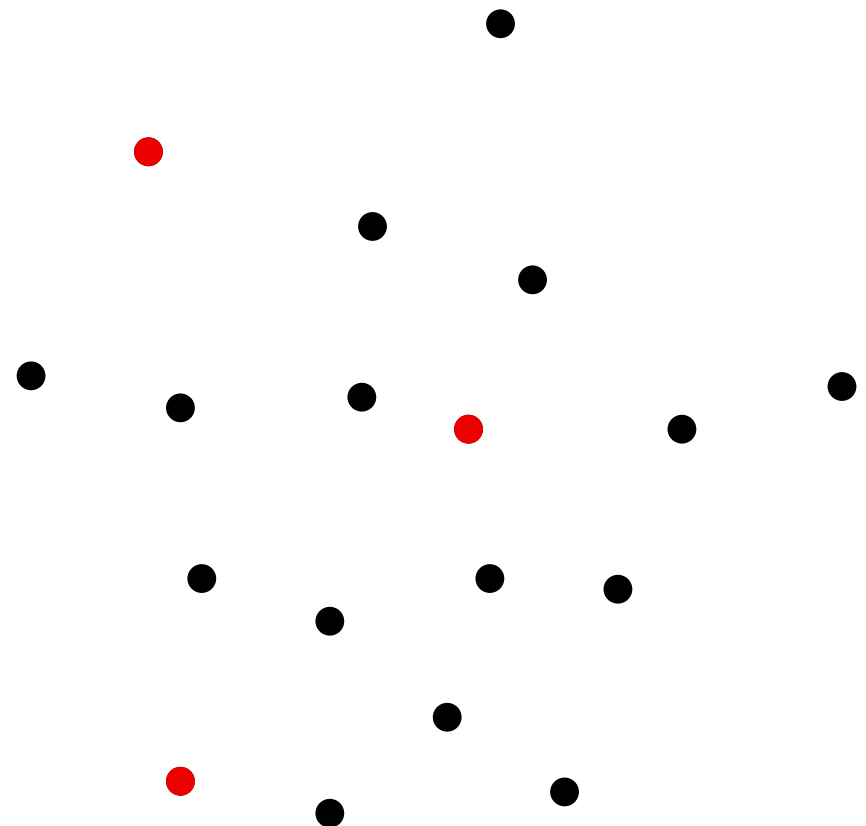
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

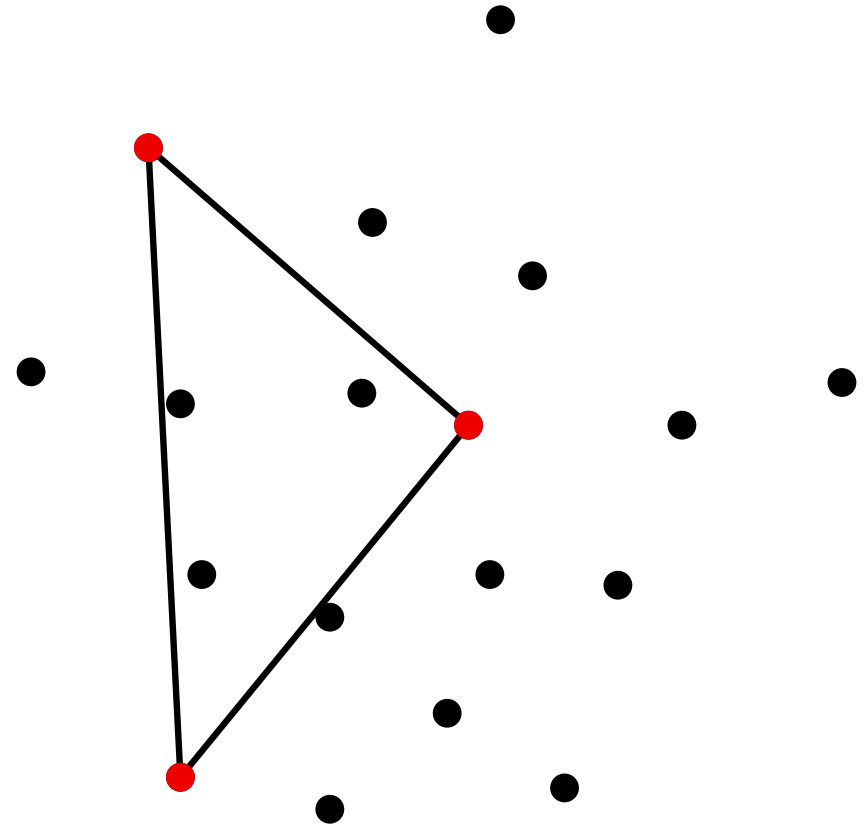
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

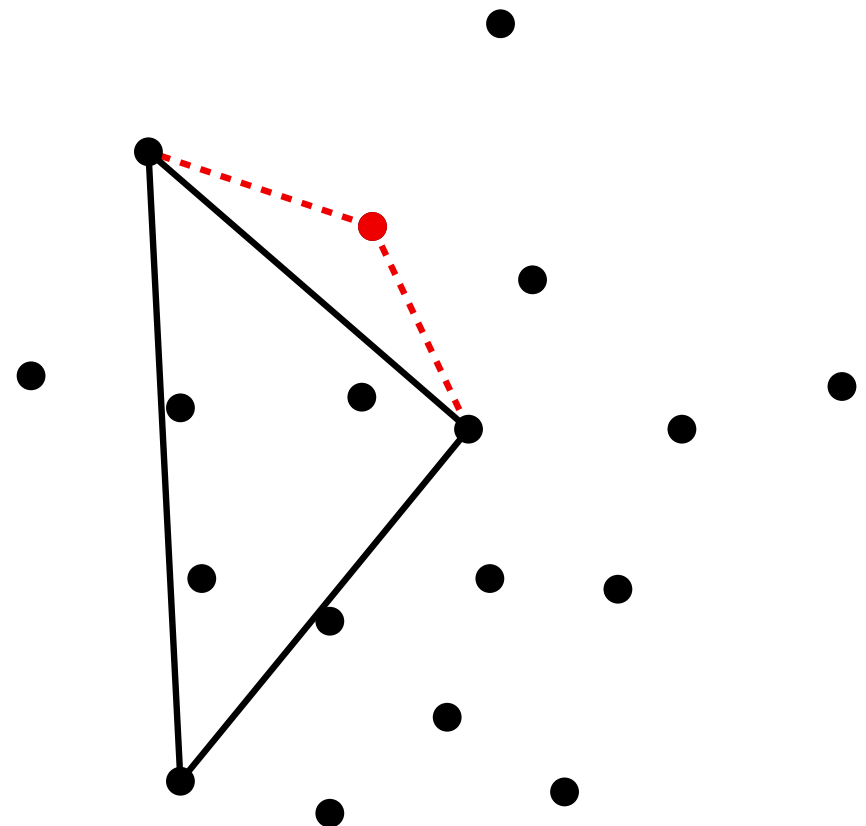
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

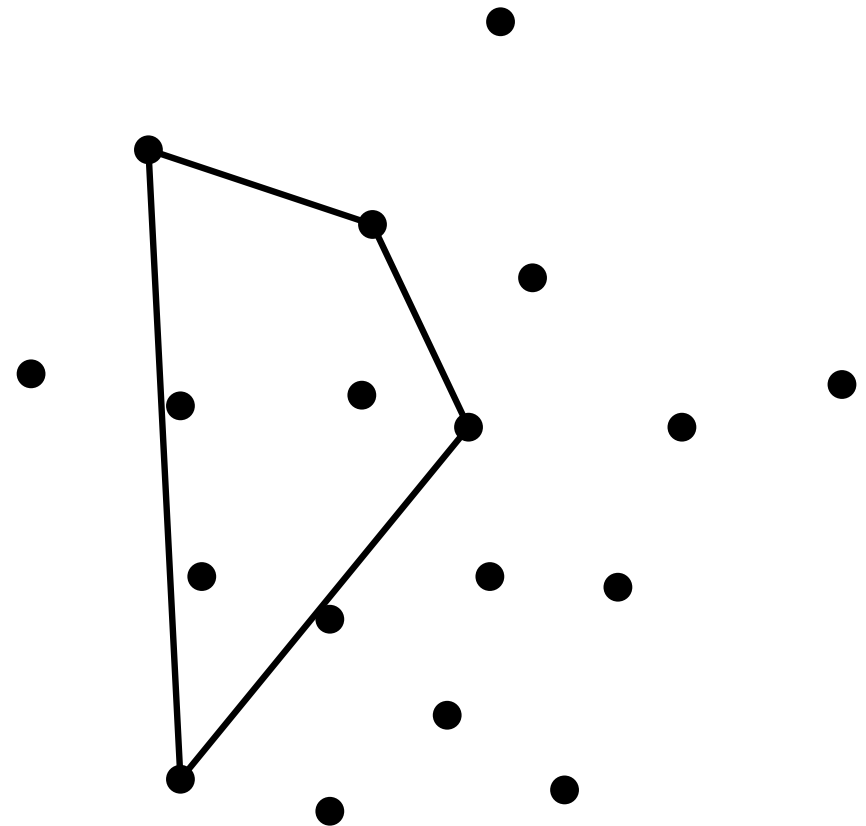
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

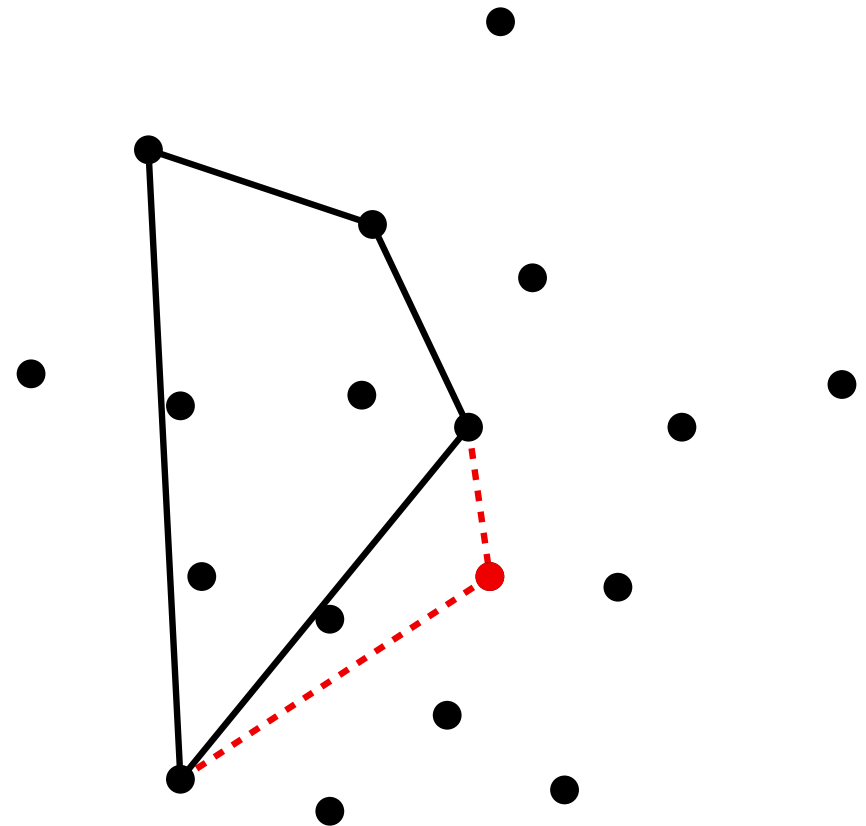
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

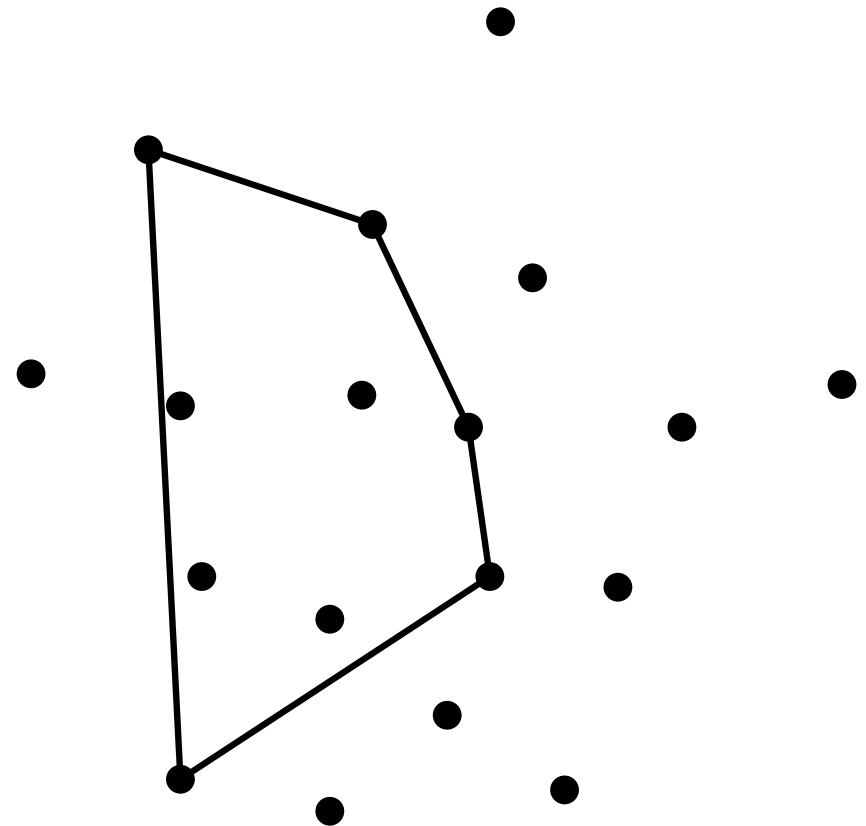
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

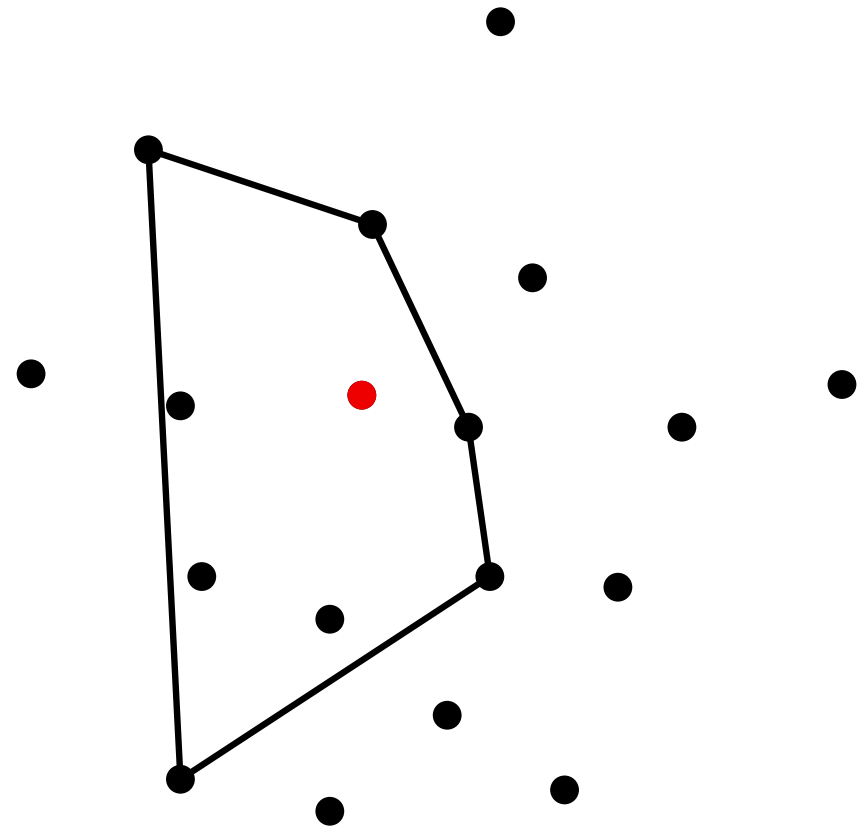
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

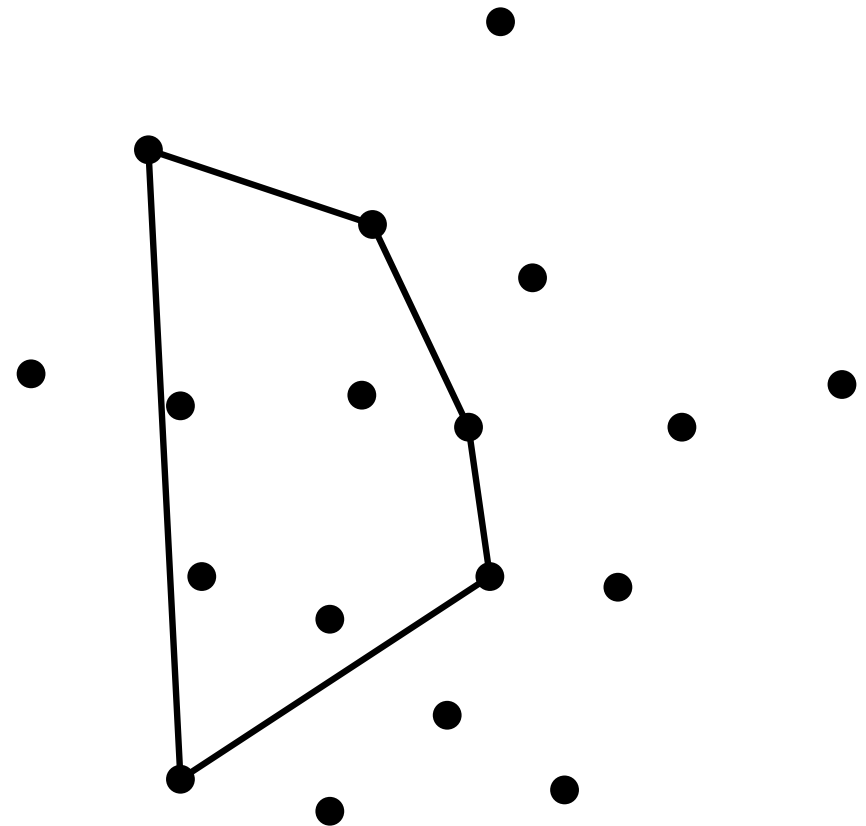
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

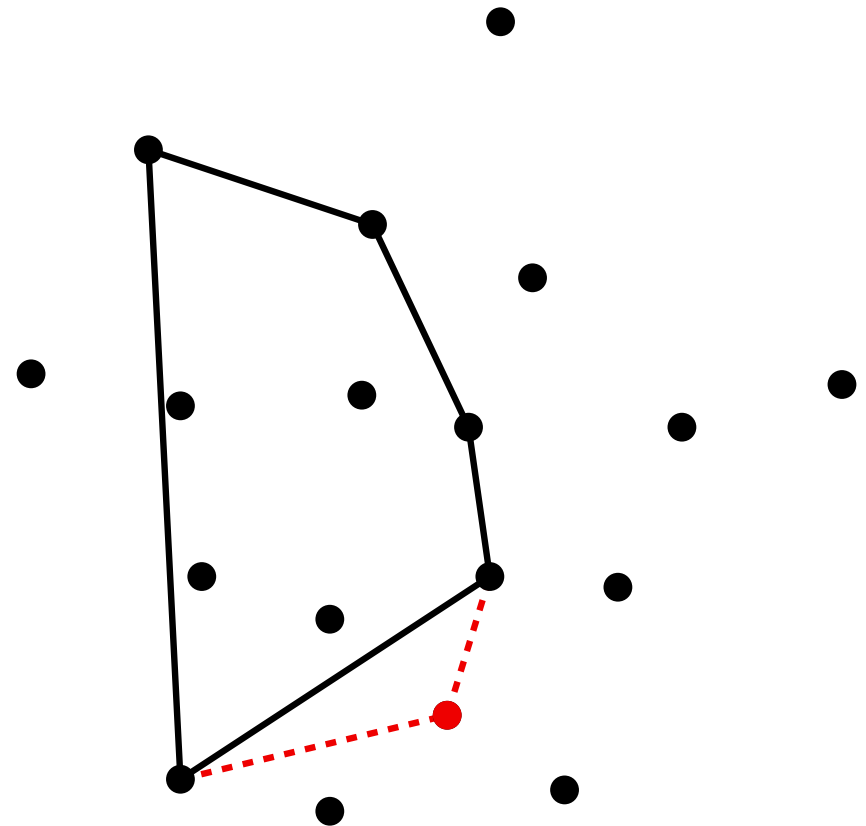
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

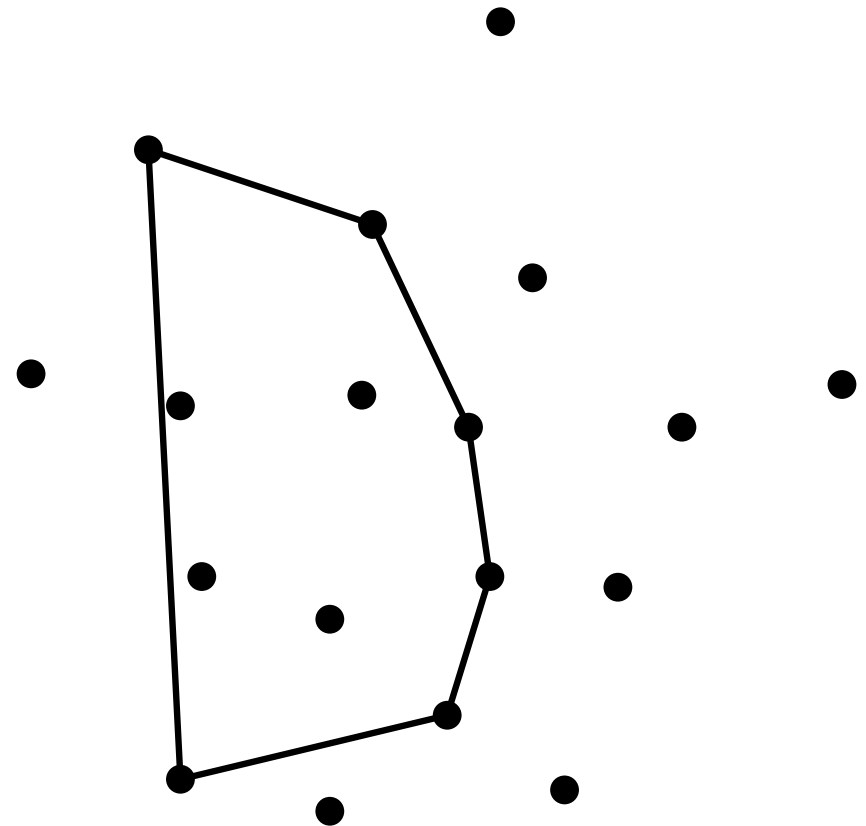
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

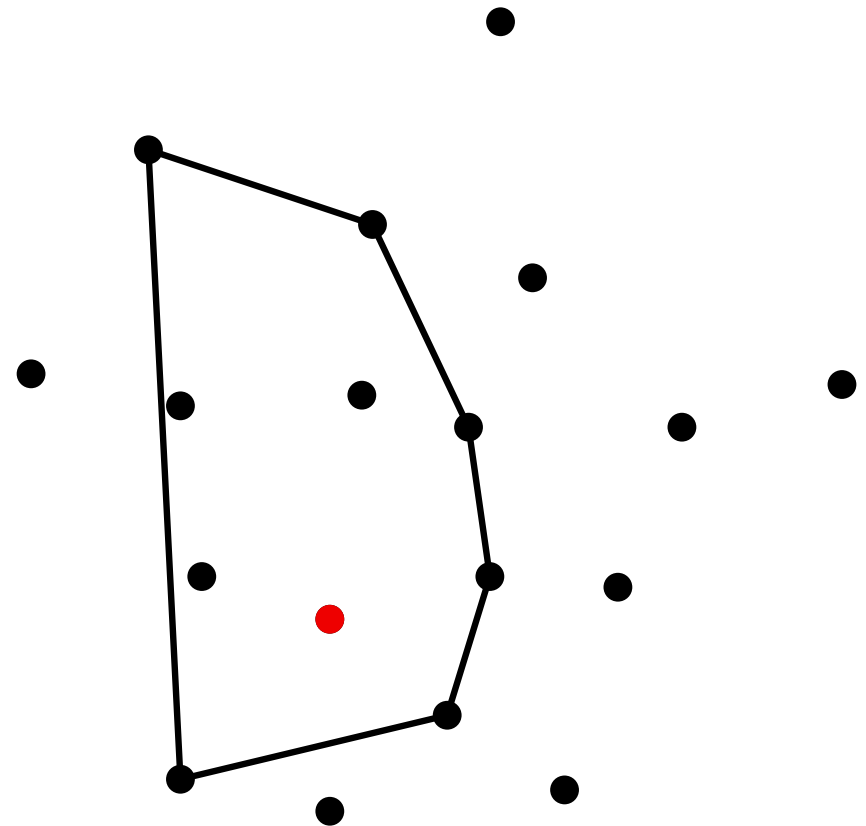
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

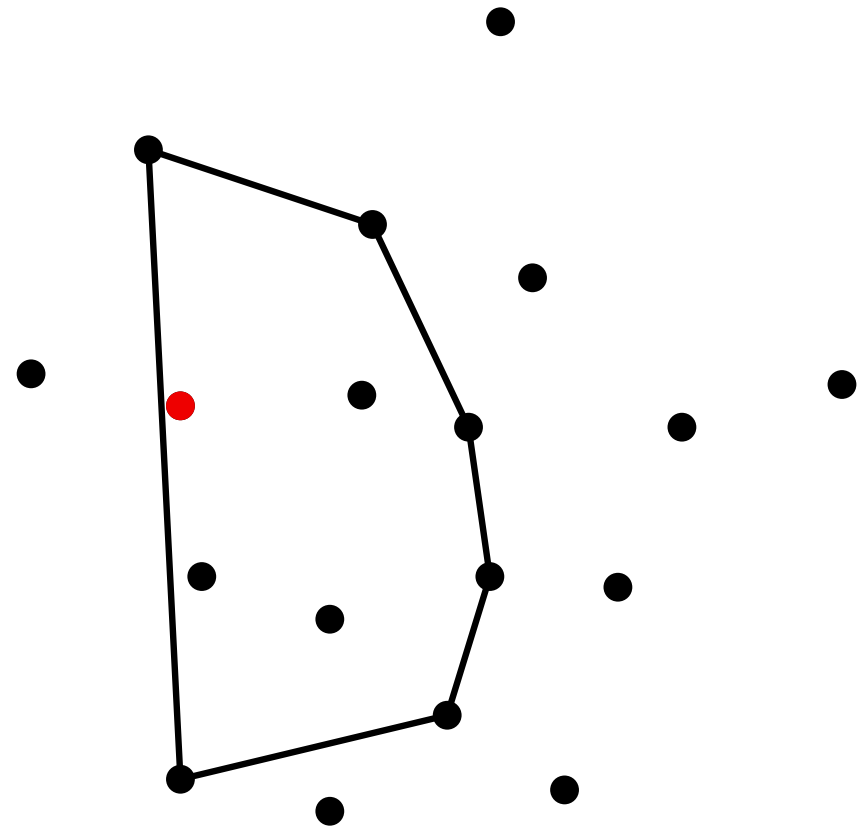
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

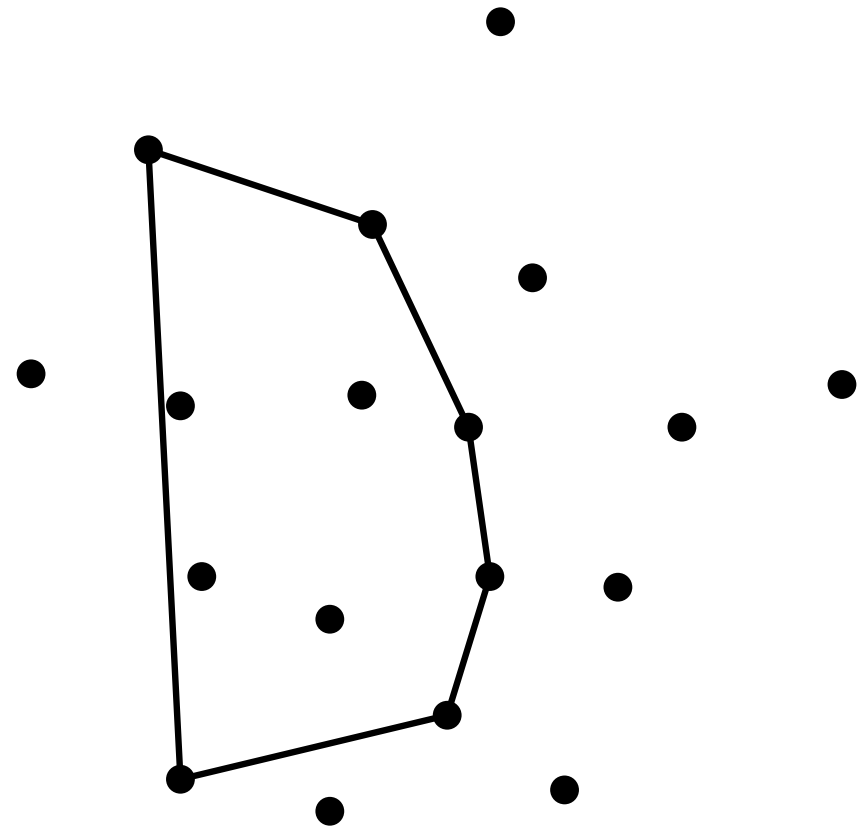
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

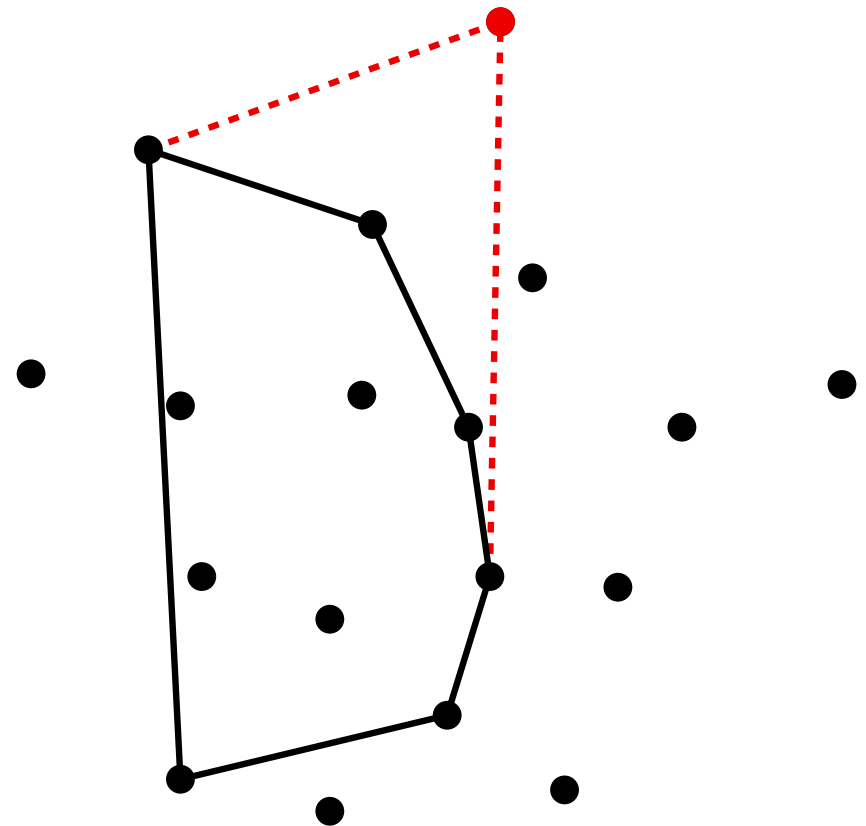
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

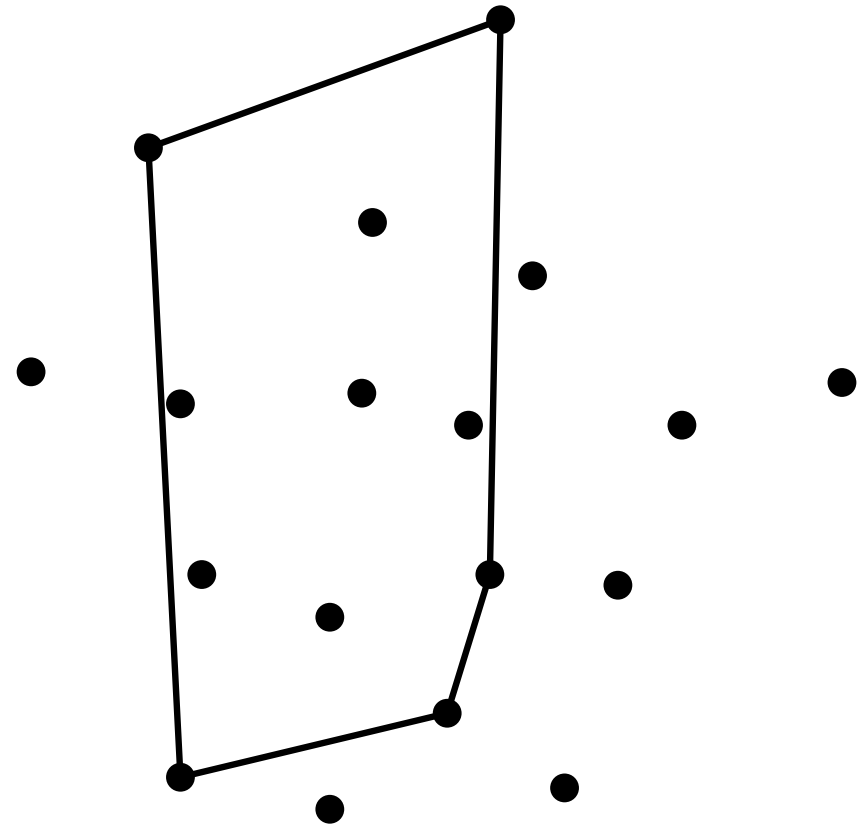
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

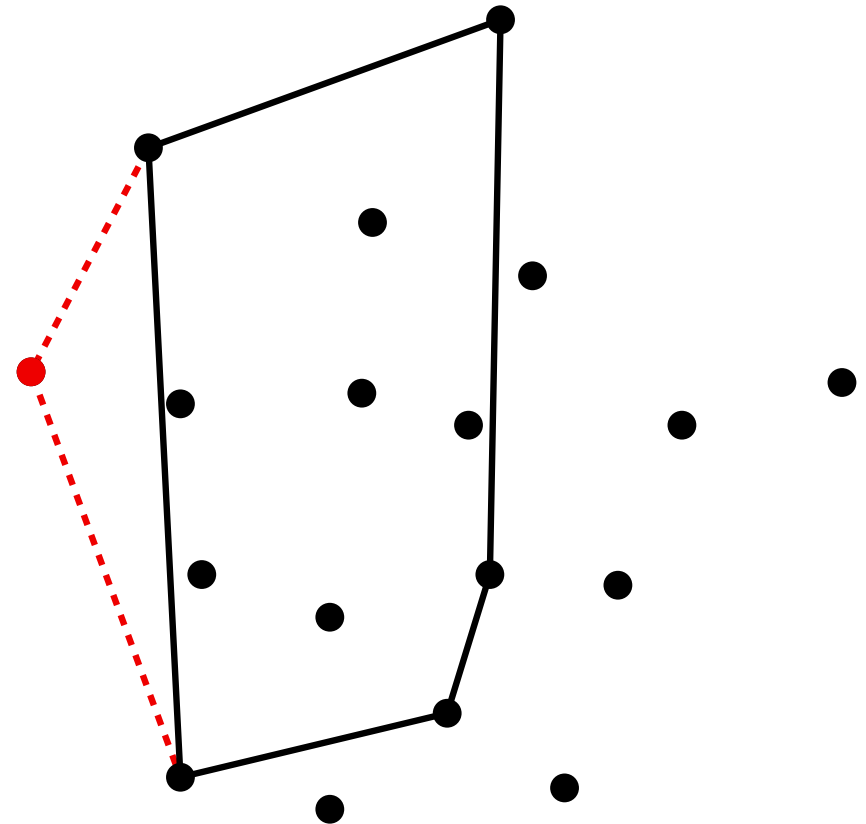
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

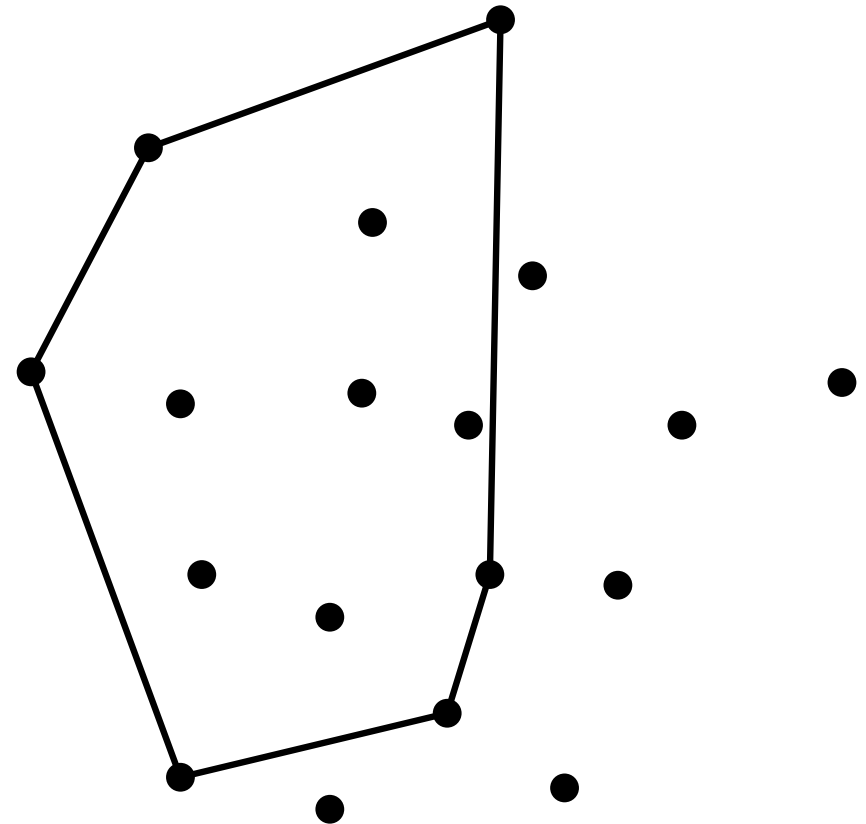
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

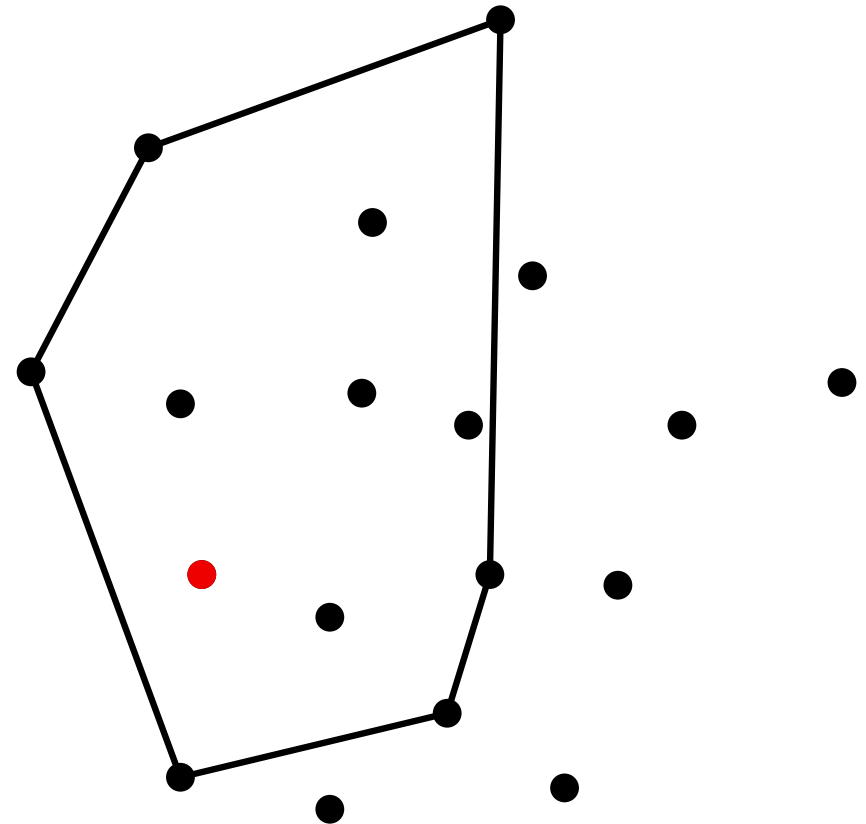
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

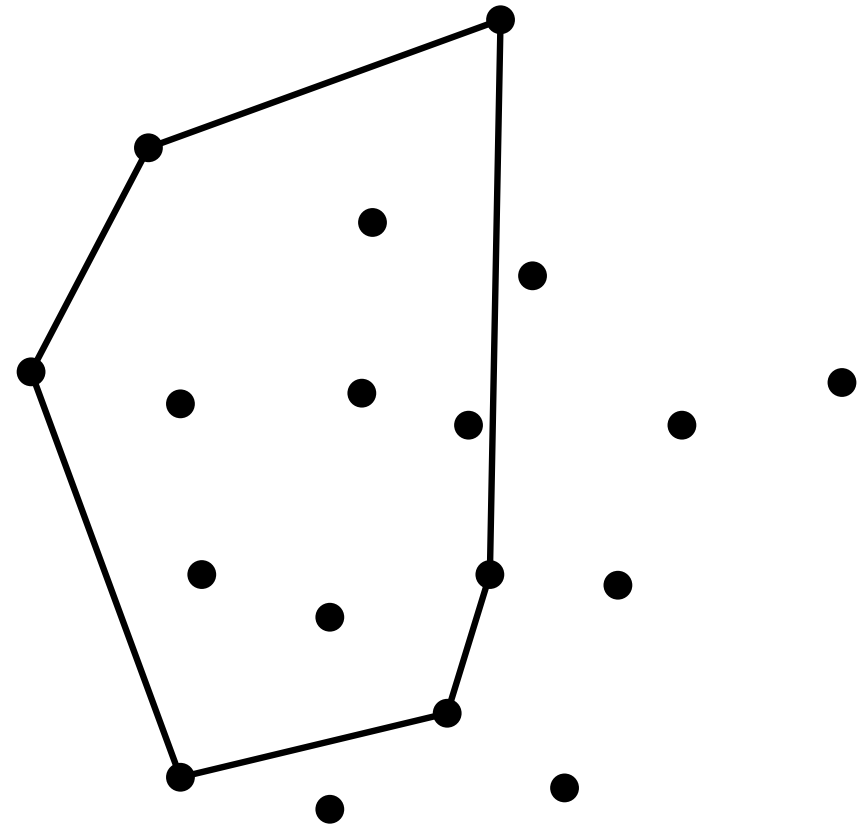
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

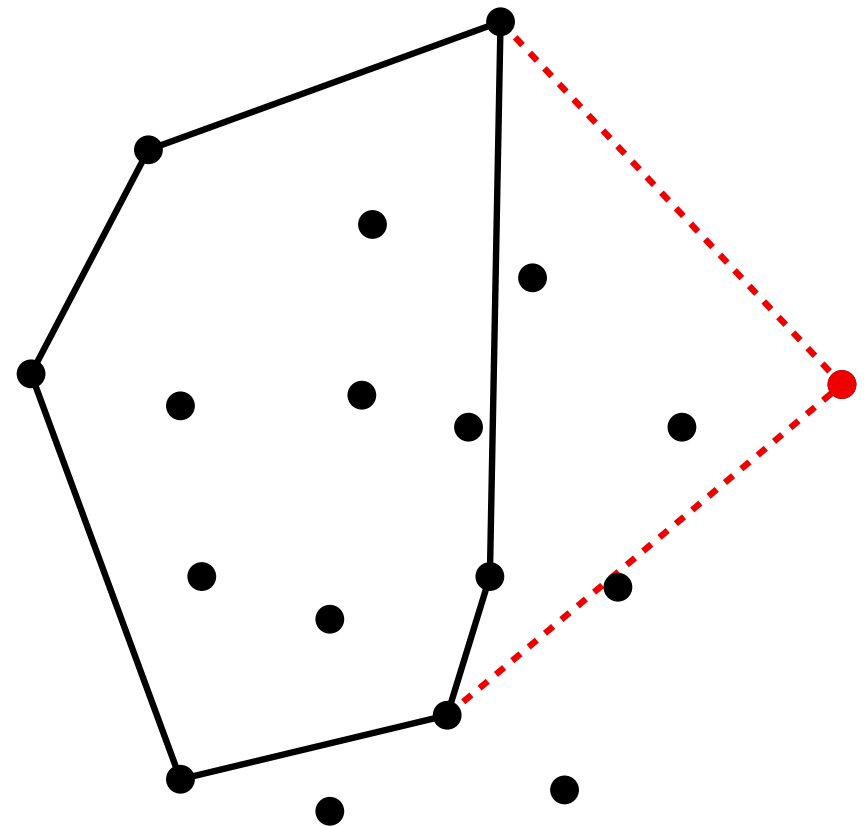
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

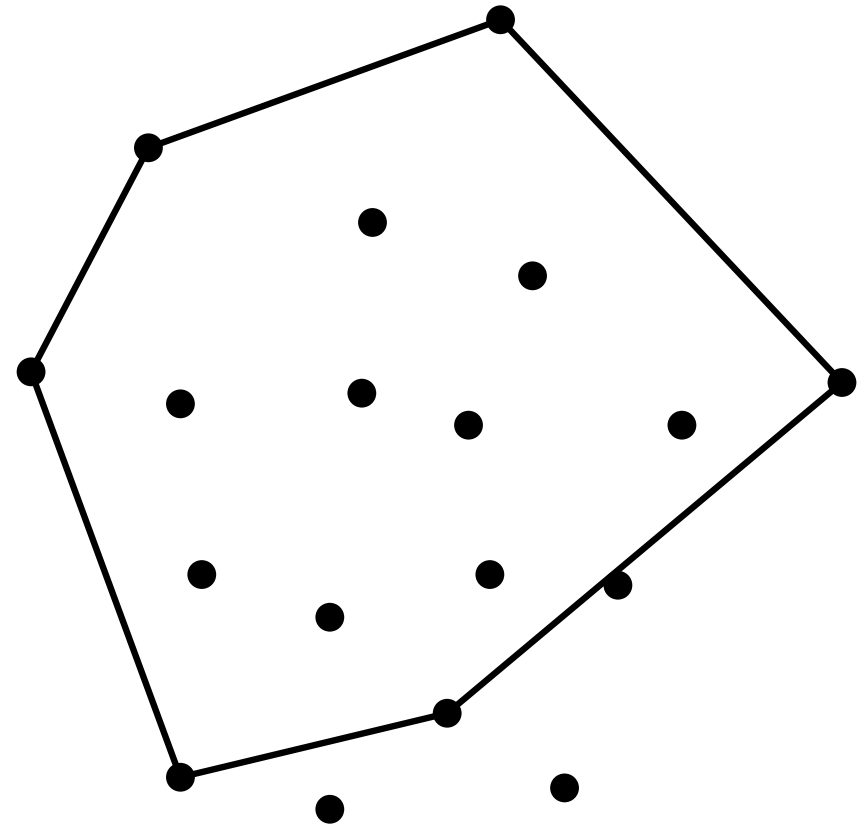
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

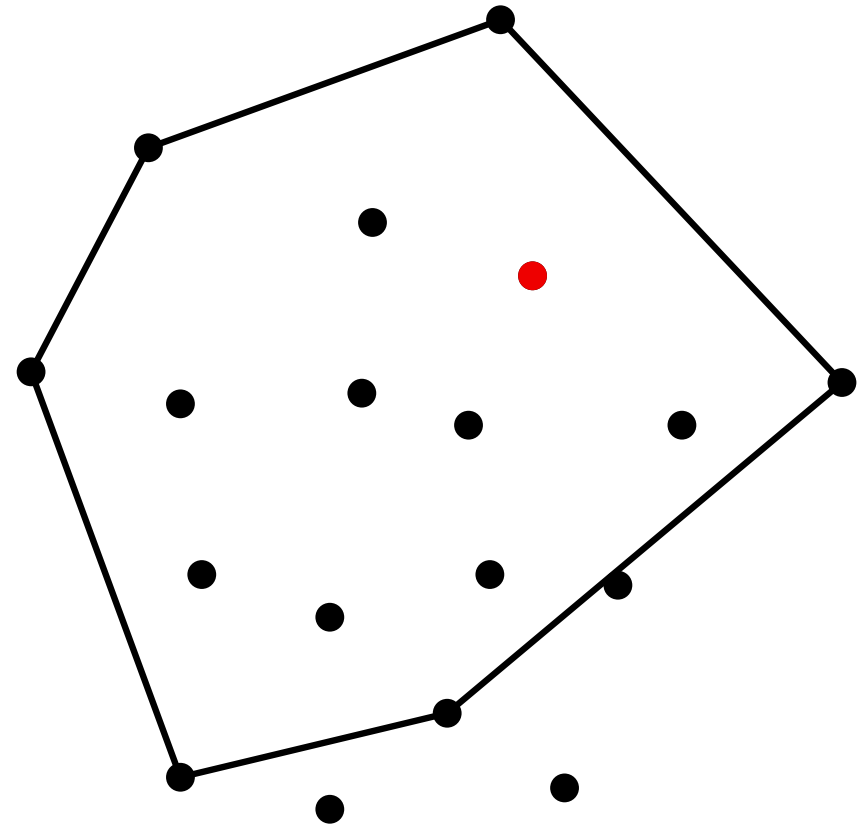
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

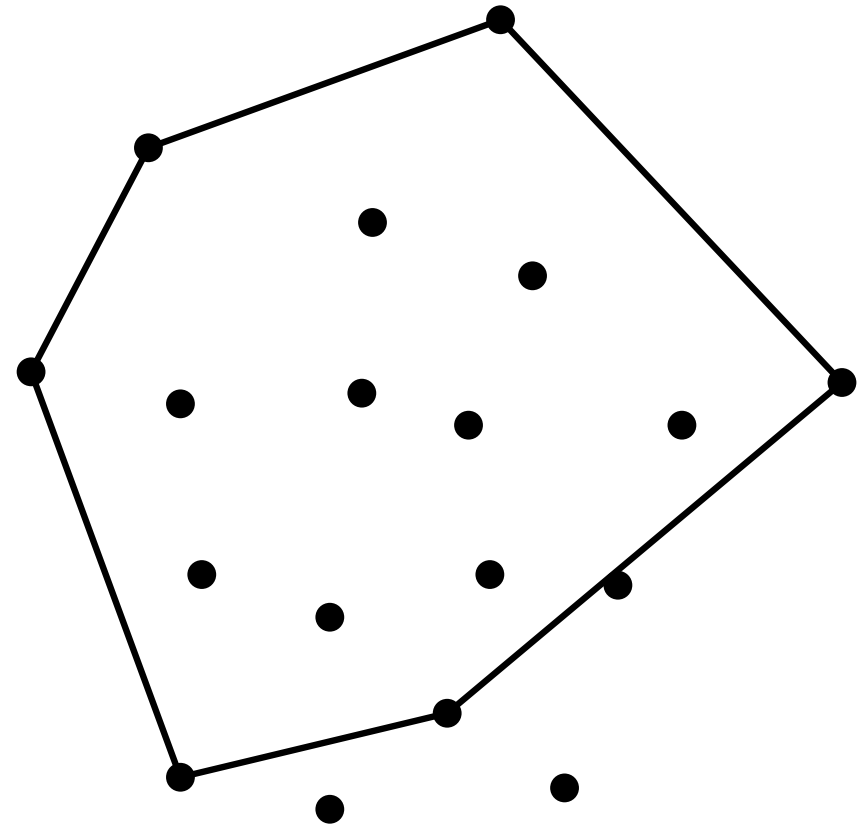
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

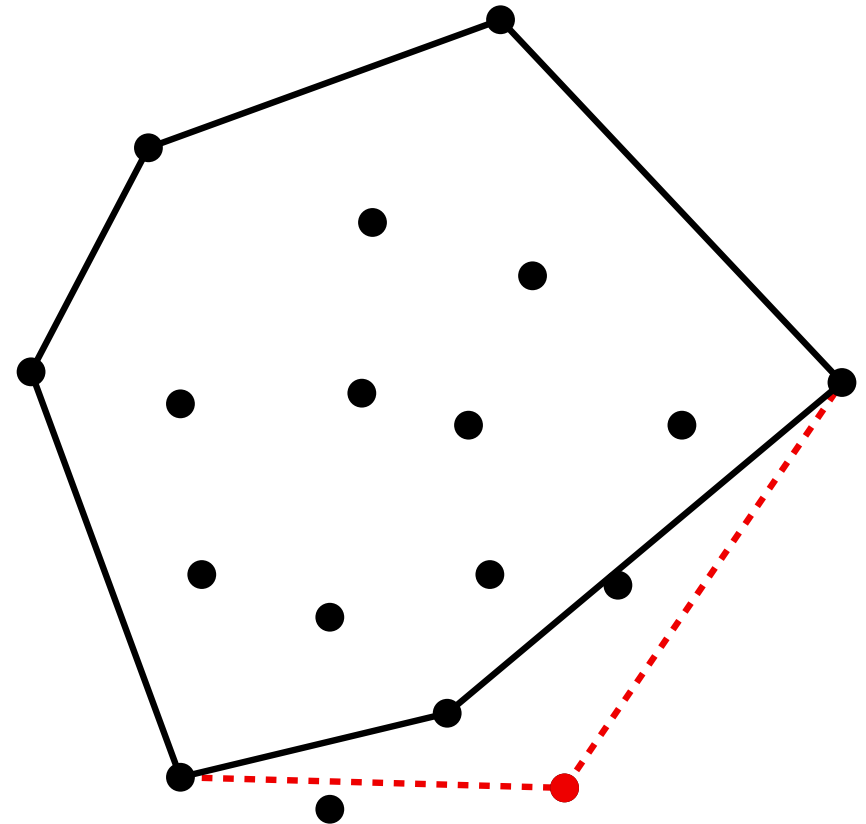
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

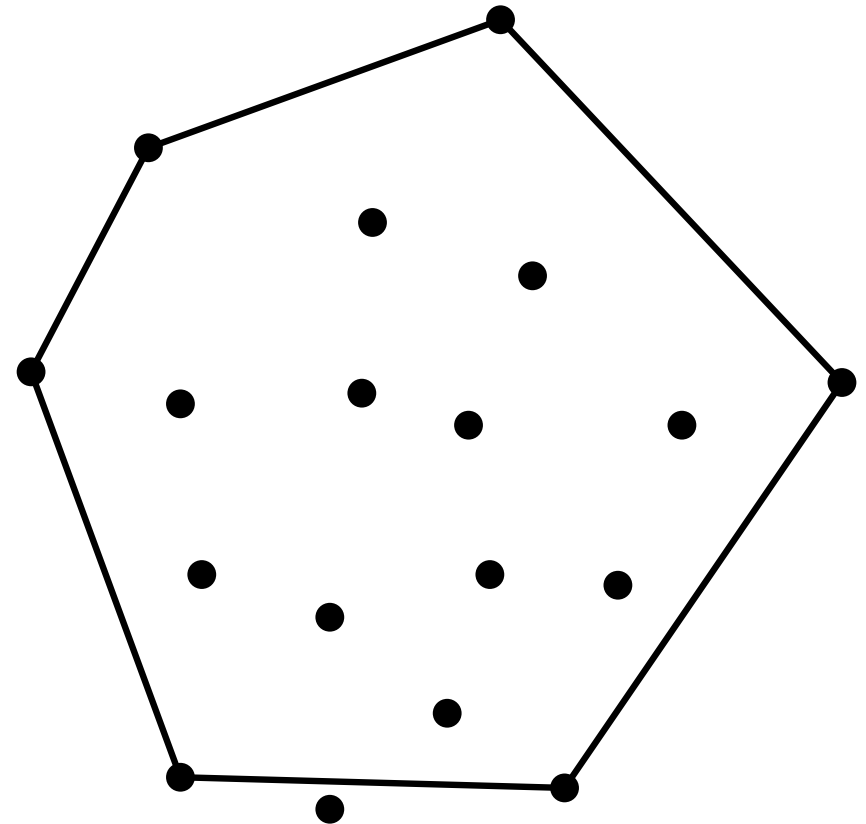
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

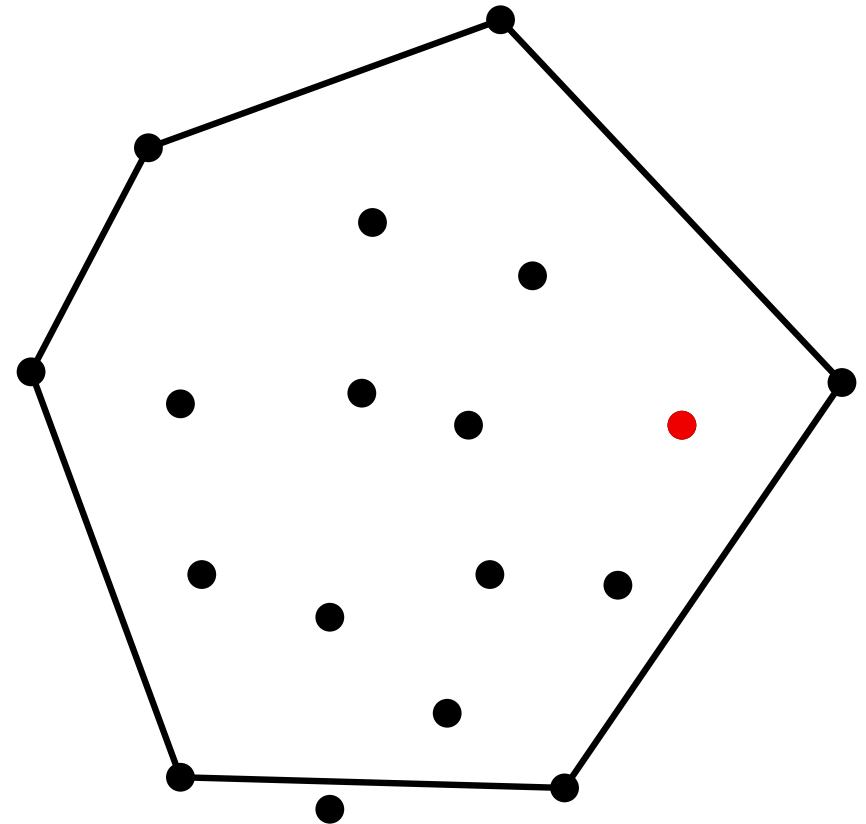
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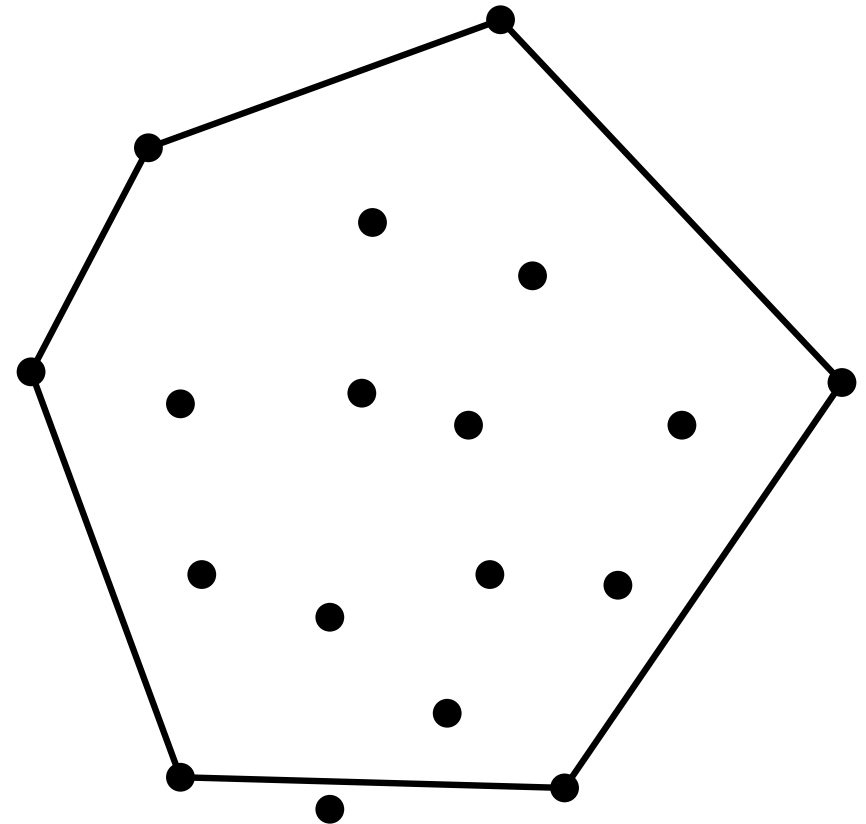
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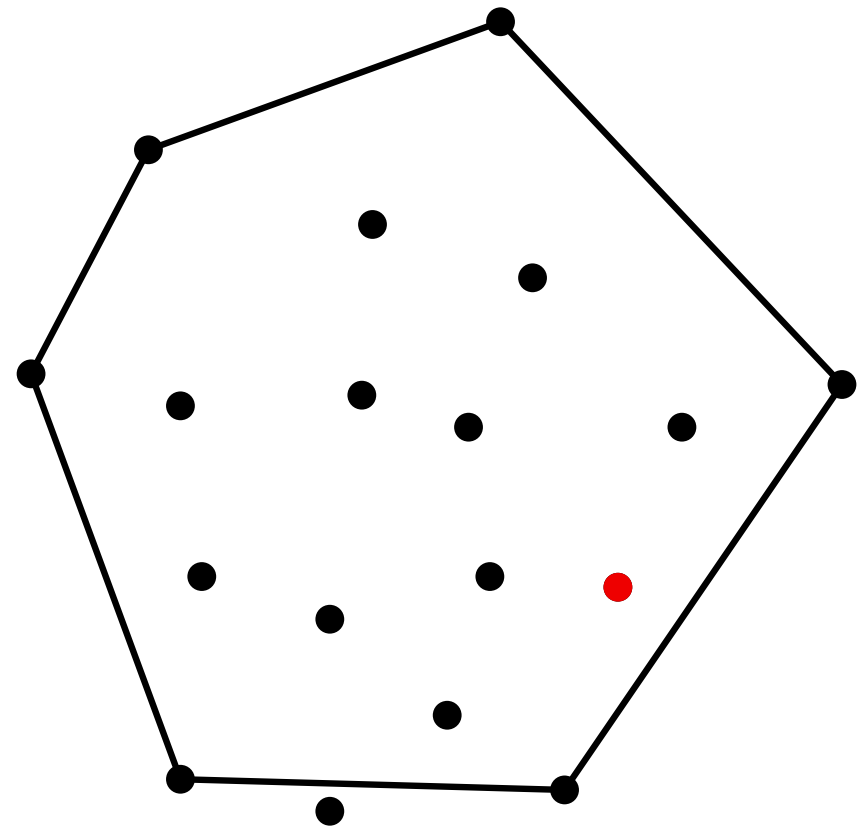
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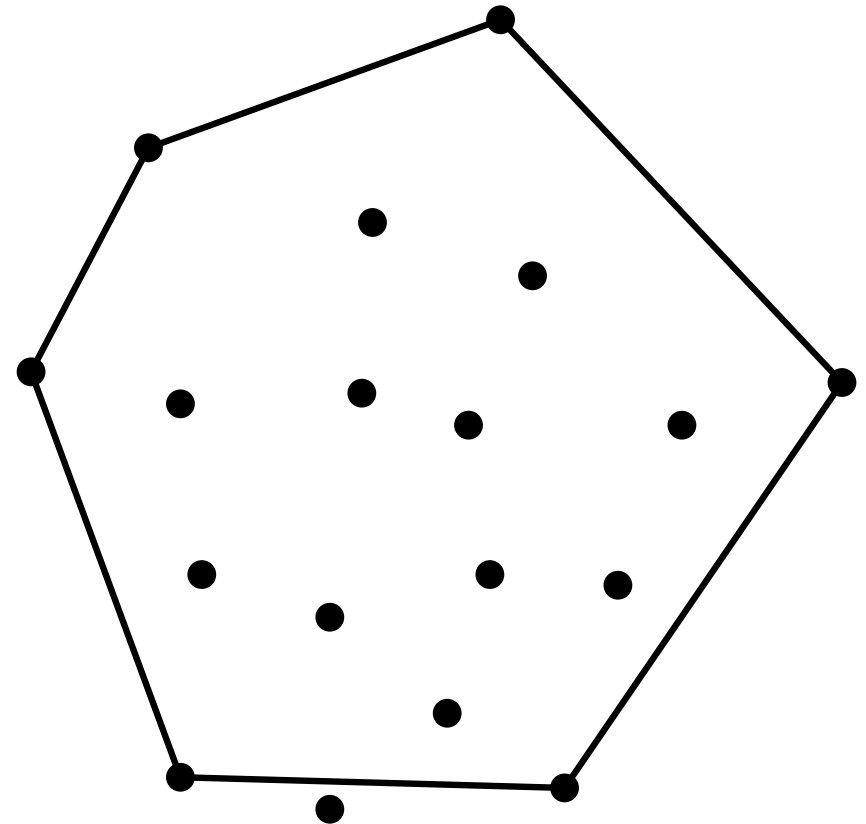
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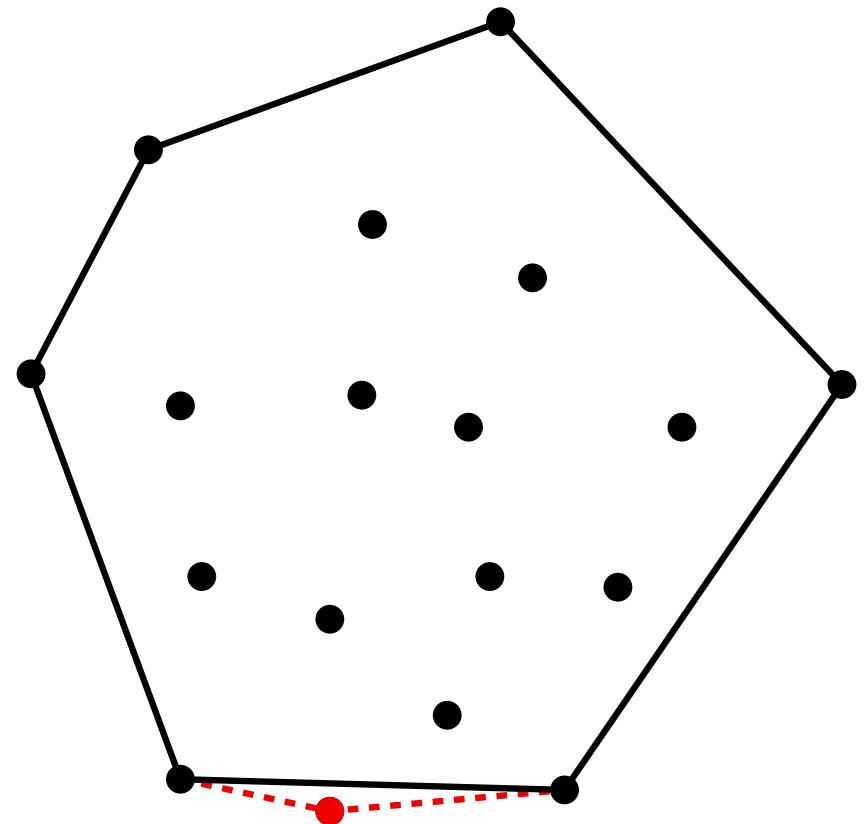
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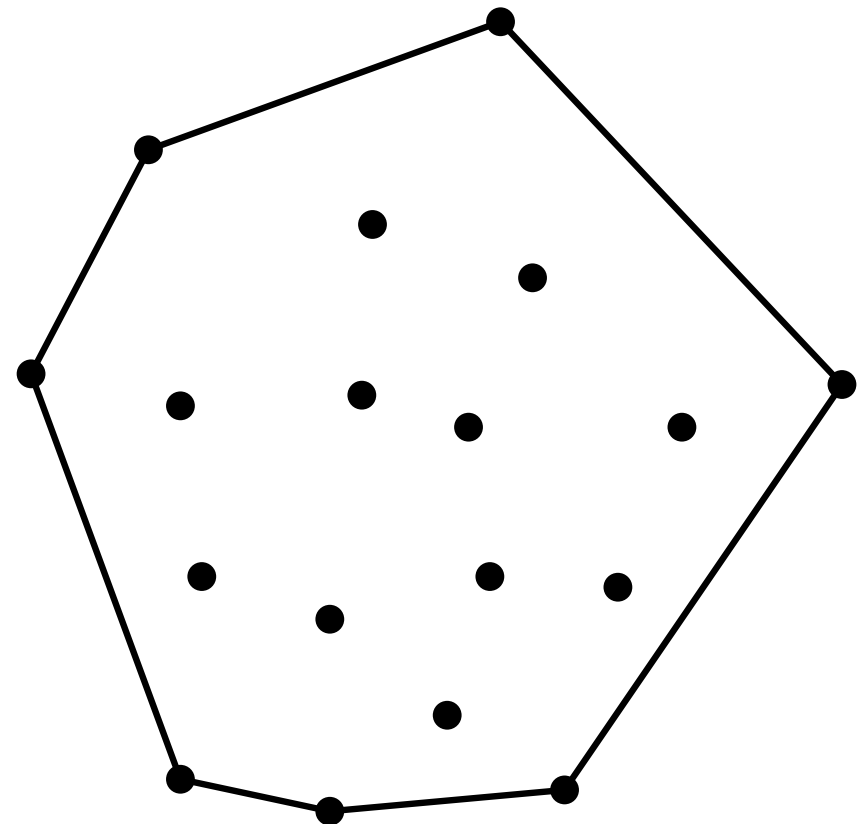
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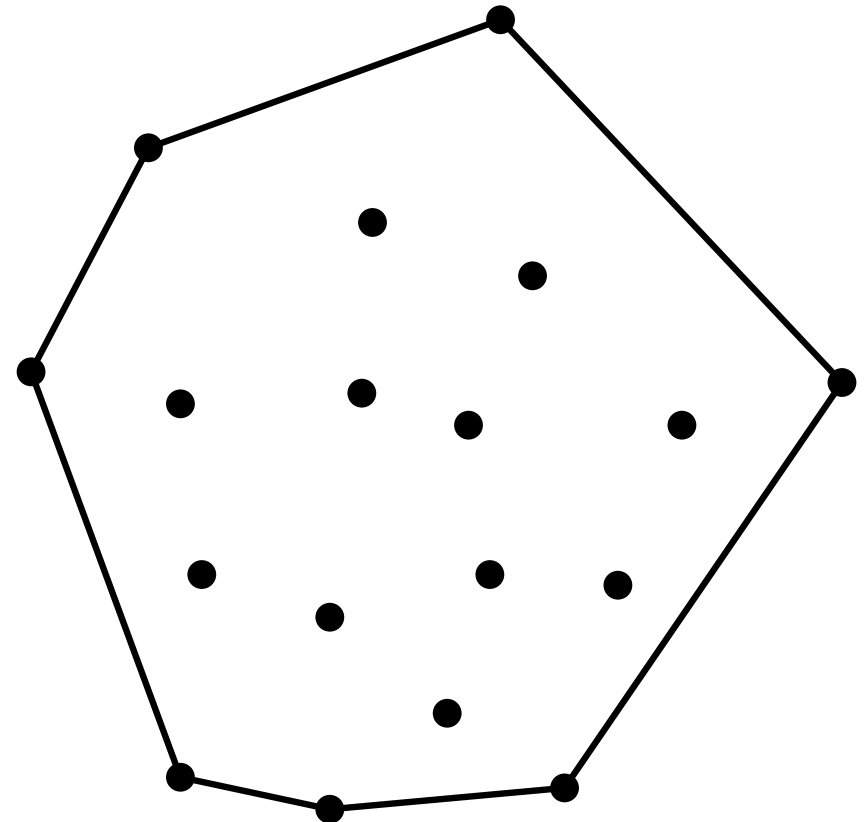
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Running time: $O(n \log n)$



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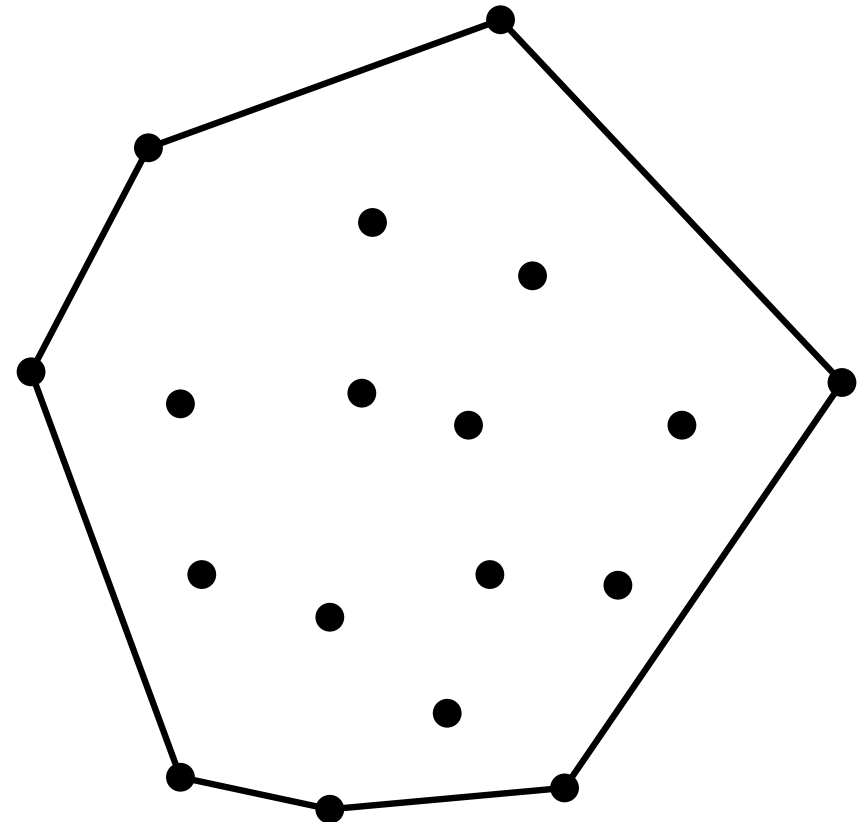
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Return l

Running time: $O(n \log n)$

By storing l in a structure allowing binary search and updatings (insertions and deletions) in $O(\log n)$ time.



CONVEX HULL IN 2D

Divide-and-conquer algorithm

CONVEX HULL IN 2D

Divide-and-conquer algorithm

Initialization

1. Sort the points by abscissae

CONVEX HULL IN 2D

Divide-and-conquer algorithm

Initialization

1. Sort the points by abscissae

Division

1. Divide the points (x_i, y_i) into two subsets, wrt the median value of the abscissae

CONVEX HULL IN 2D

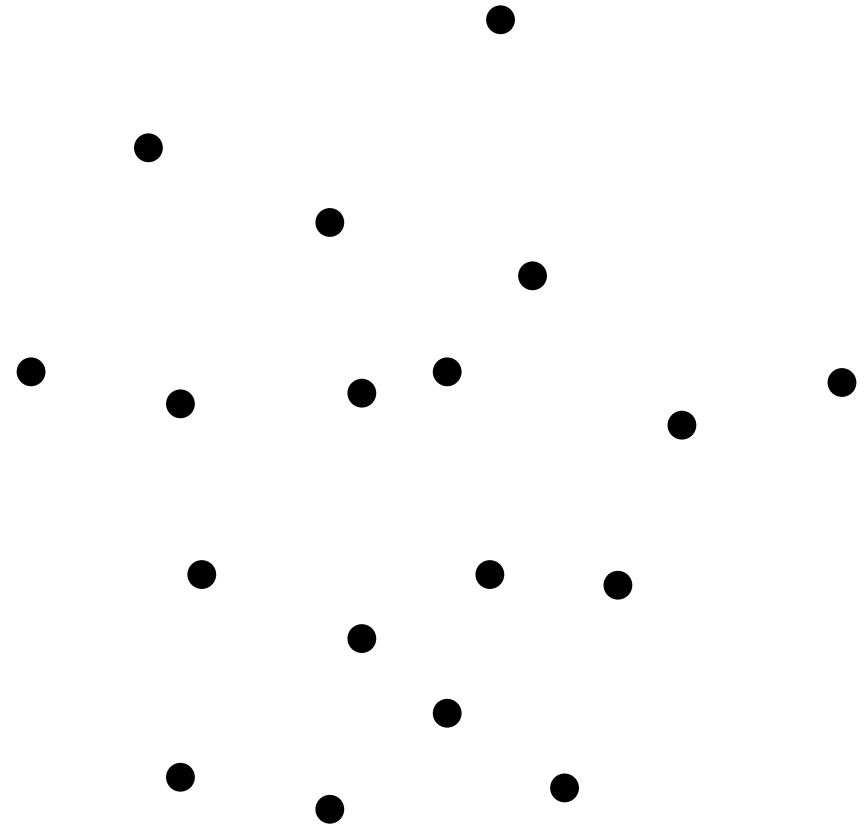
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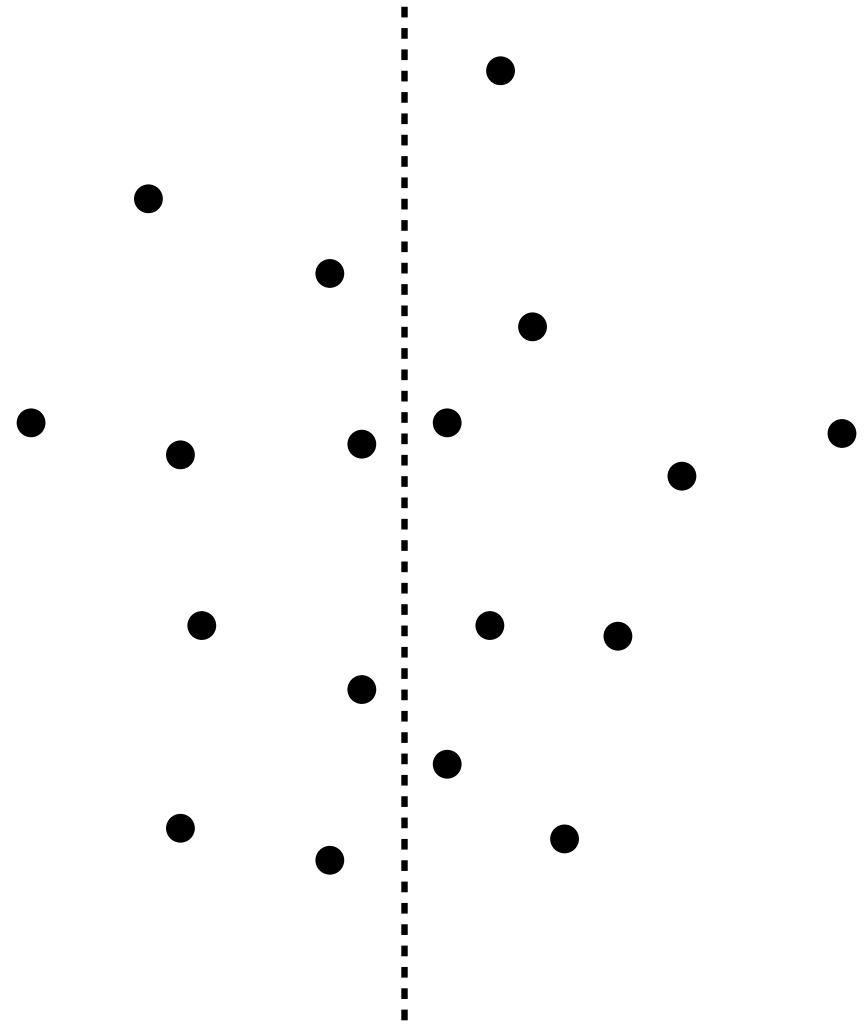
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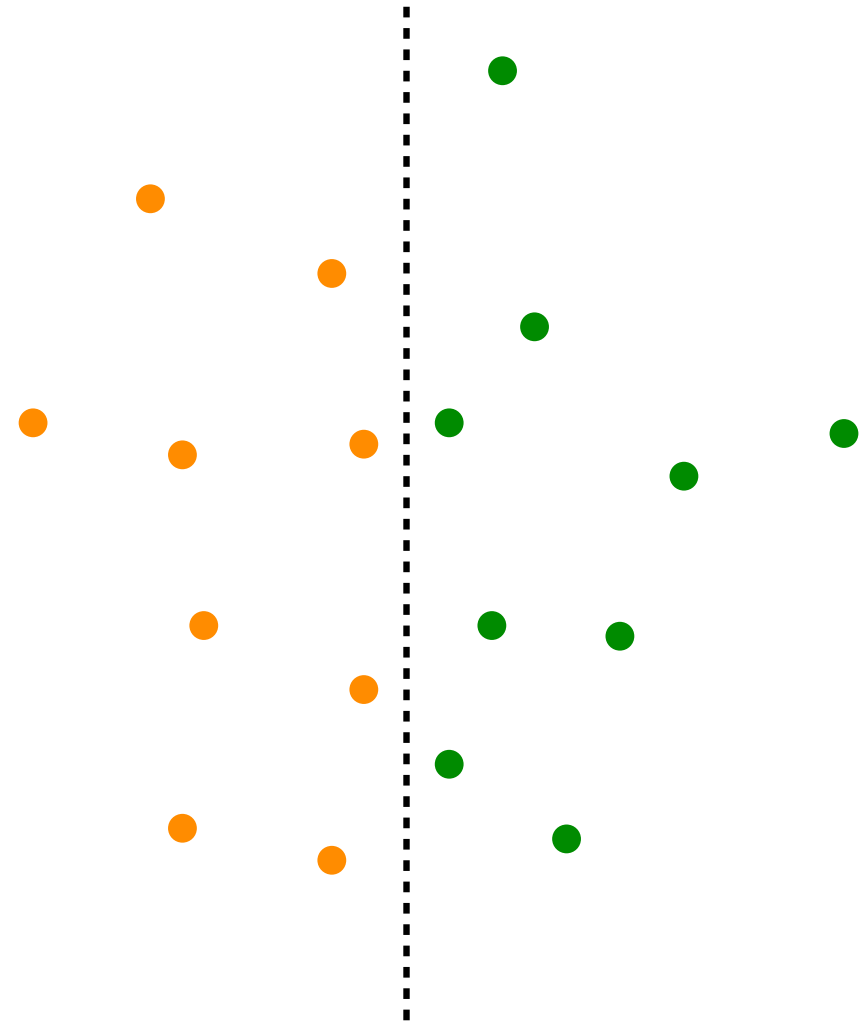
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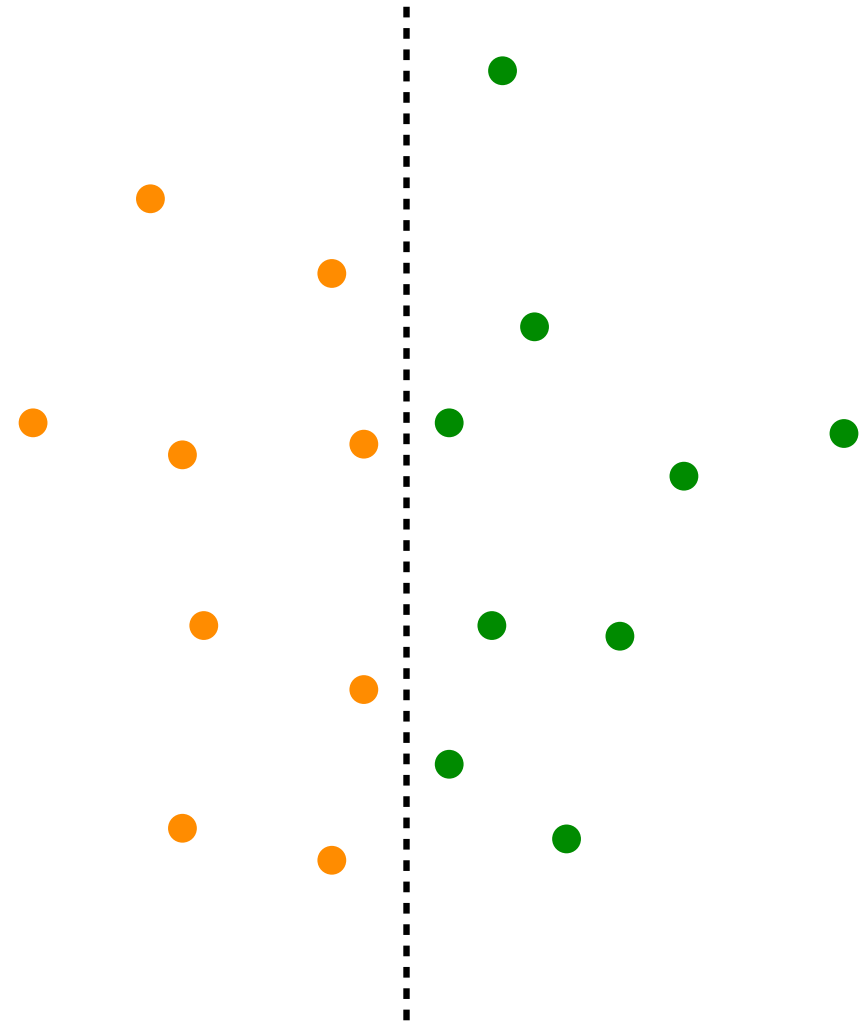
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1. Recursively compute the convex hull of the two subsets



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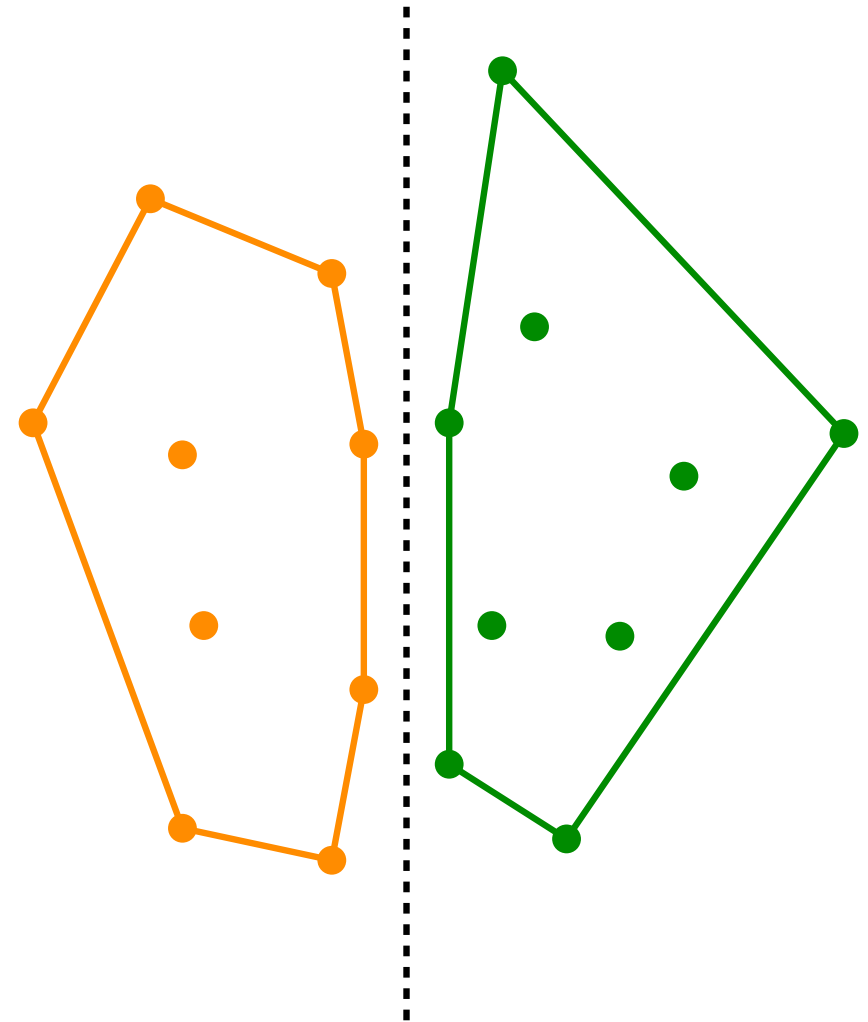
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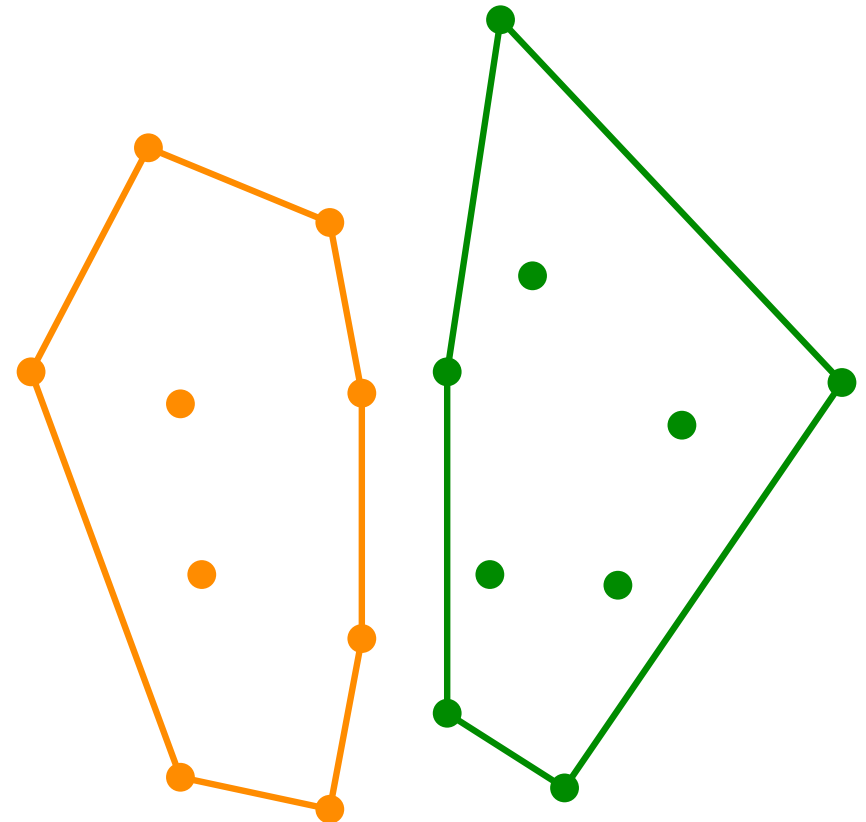
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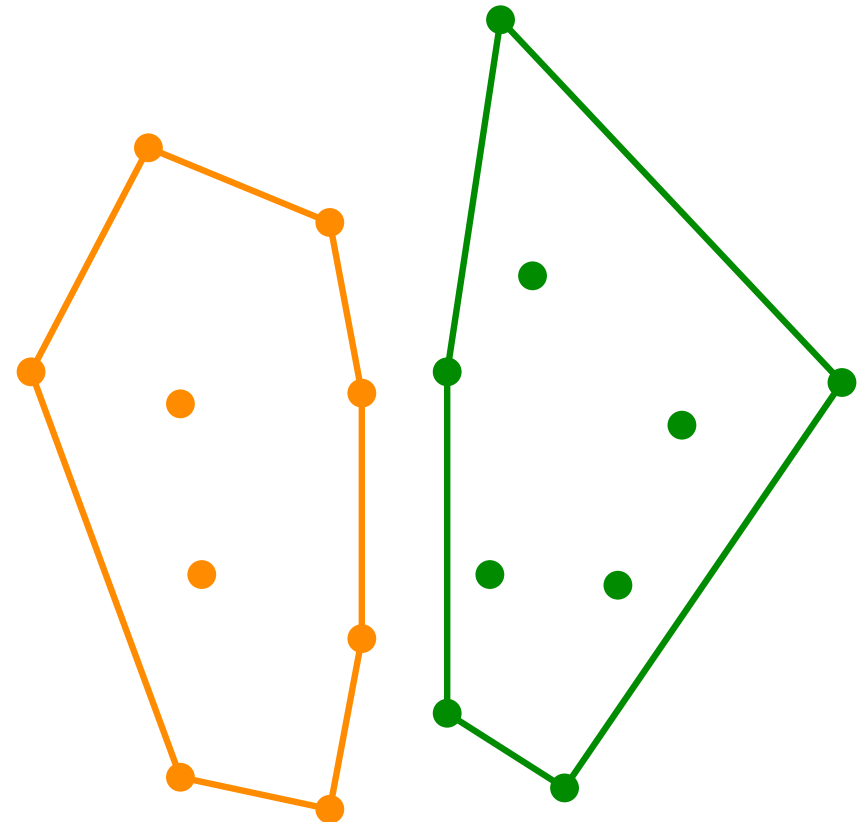
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1. Compute the external common tangents of the two convex polygons
2. Delete the interior chains of the two polygons and join the external chains through the supporting segments



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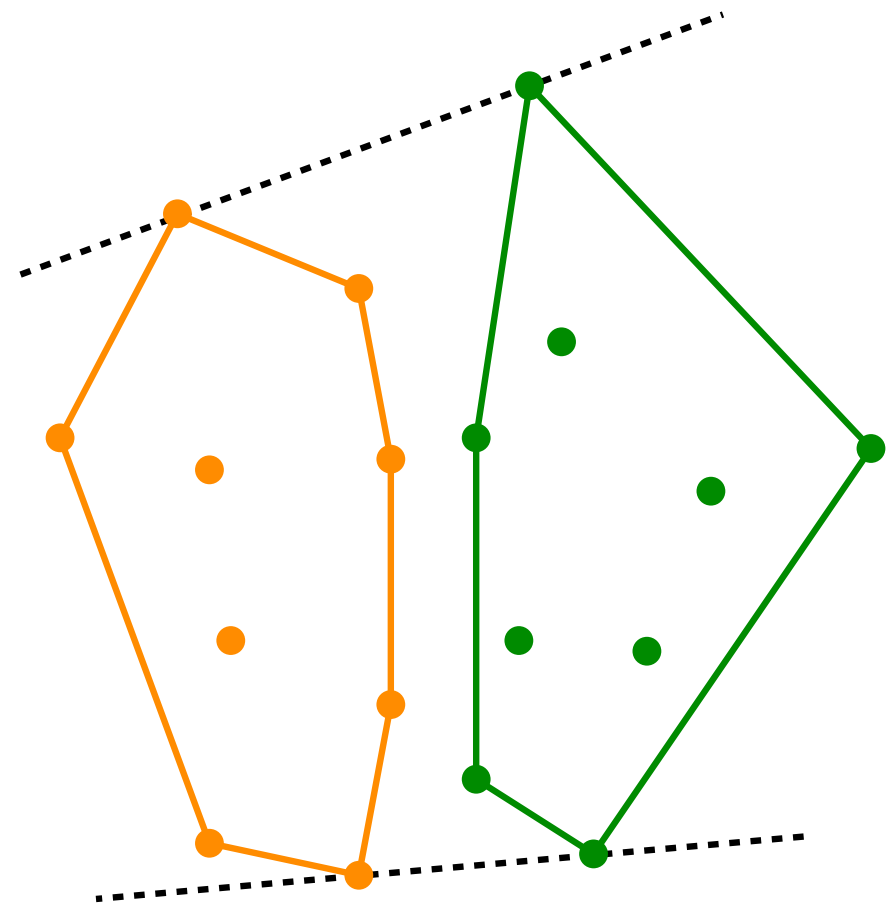
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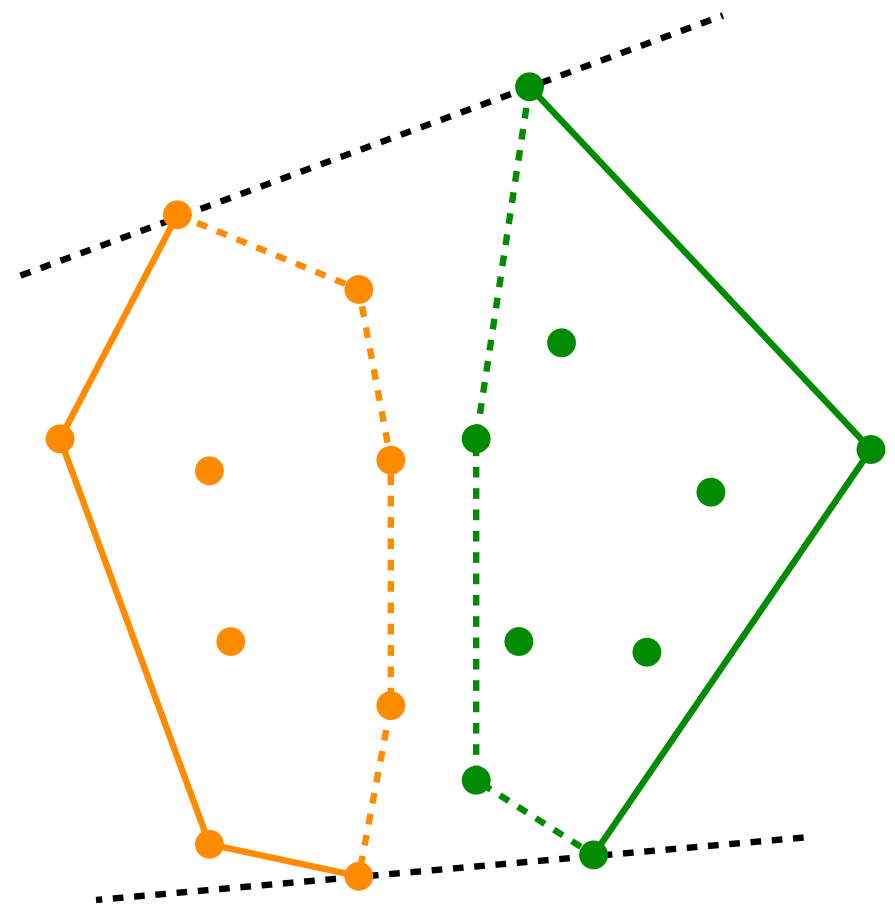
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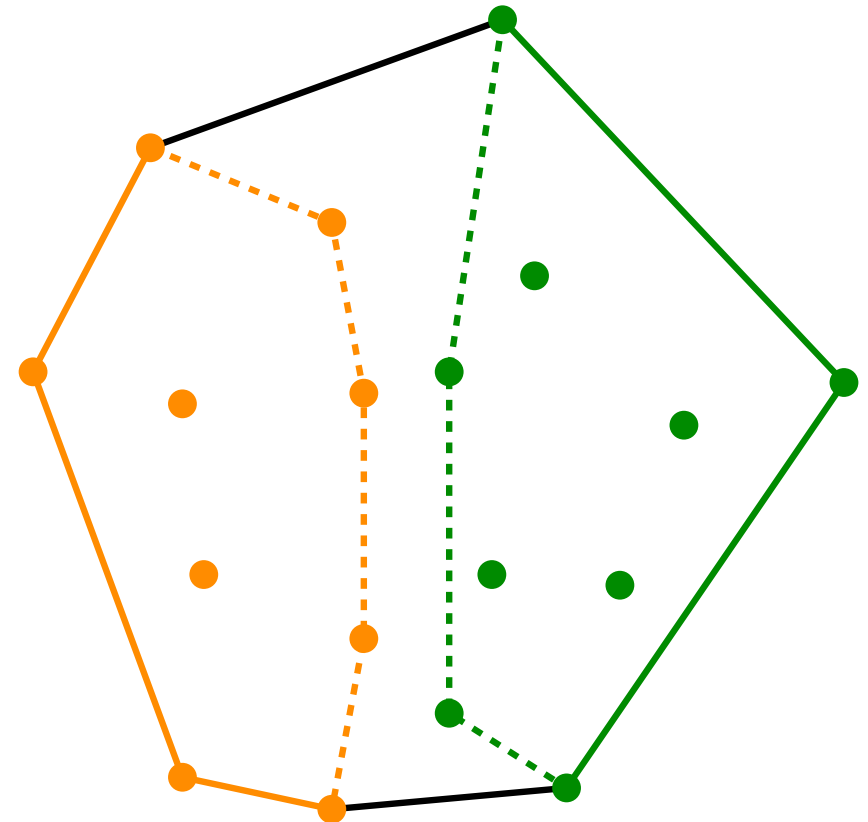
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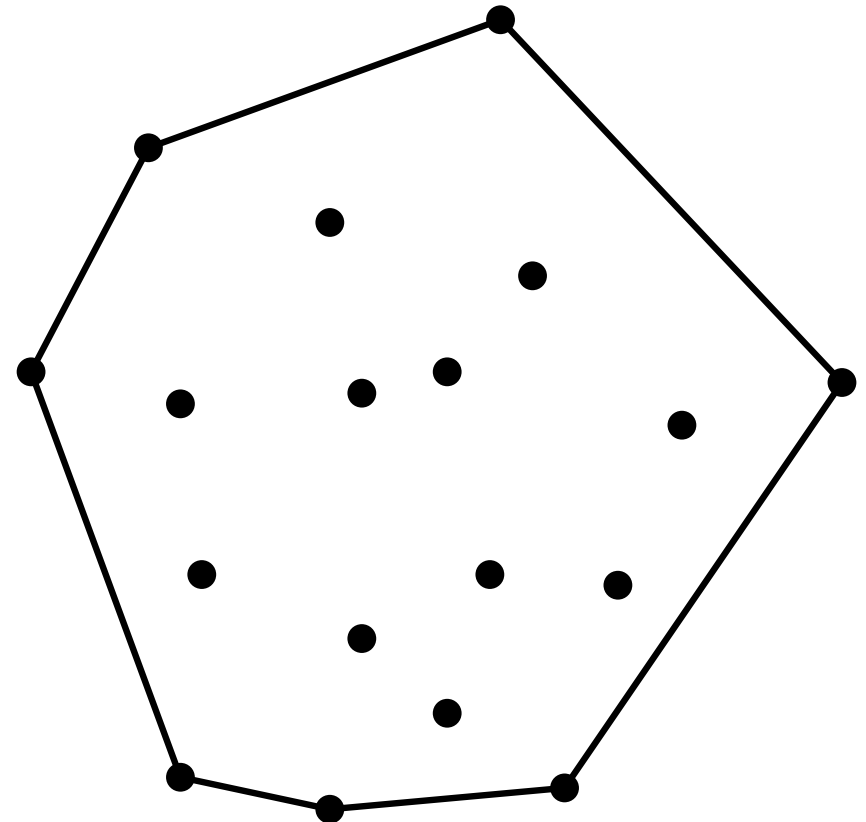
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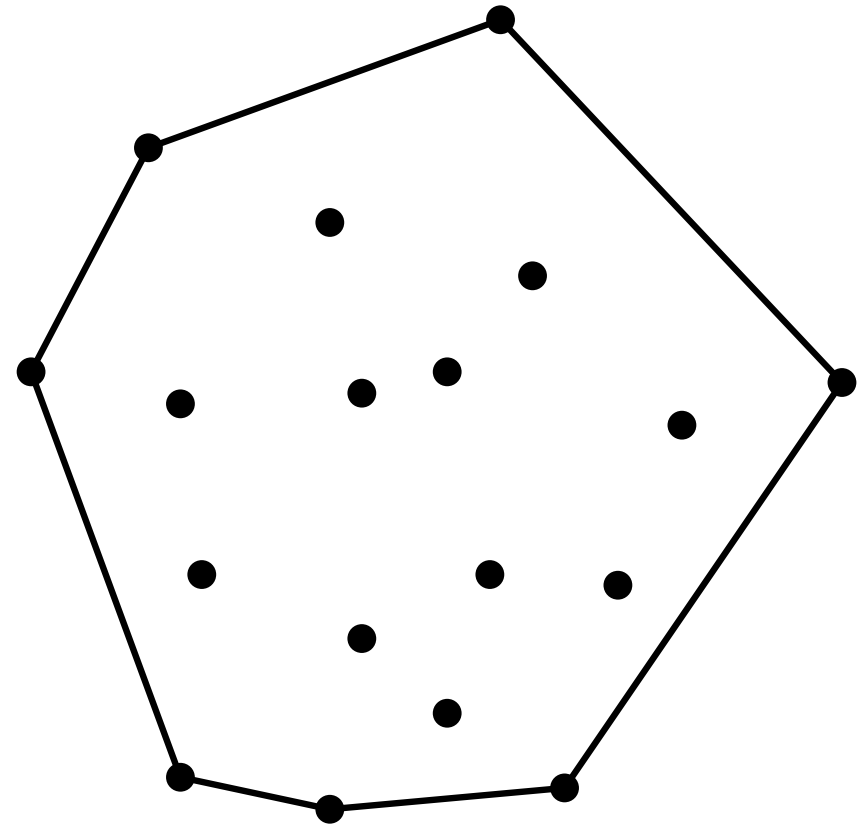


CONVEX HULL IN 2D

Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)



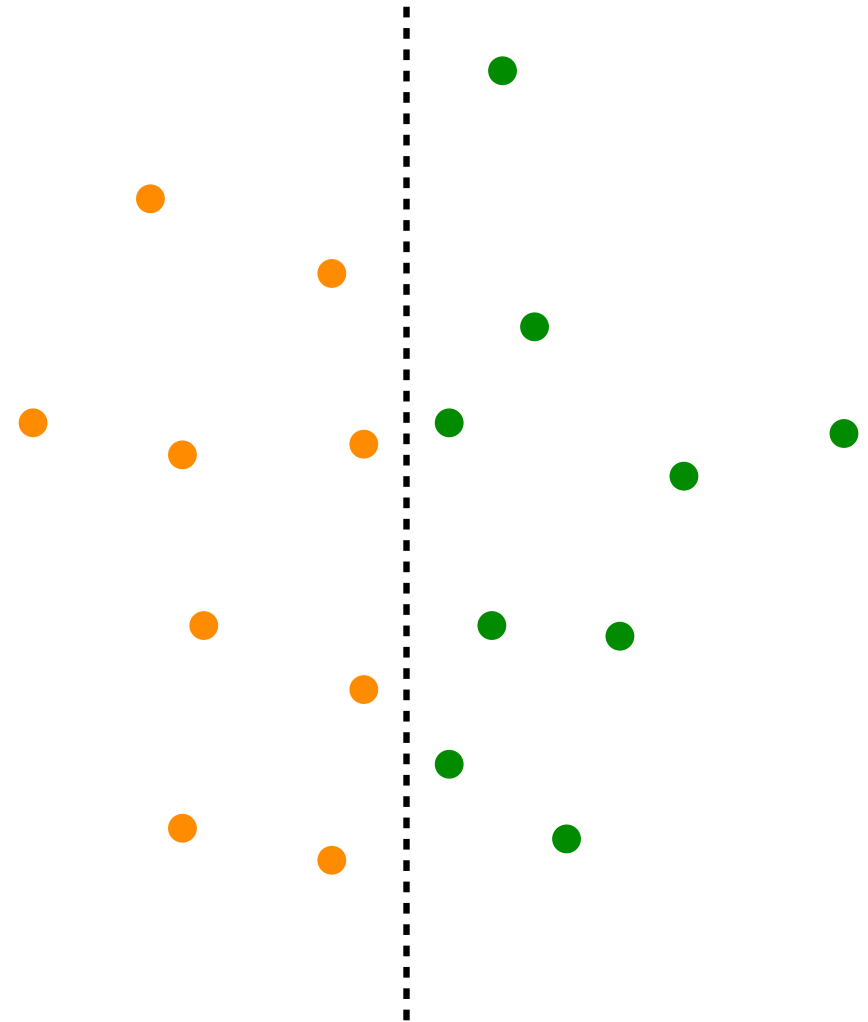
CONVEX HULL IN 2D

Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

Division: $O(n)$



CONVEX HULL IN 2D

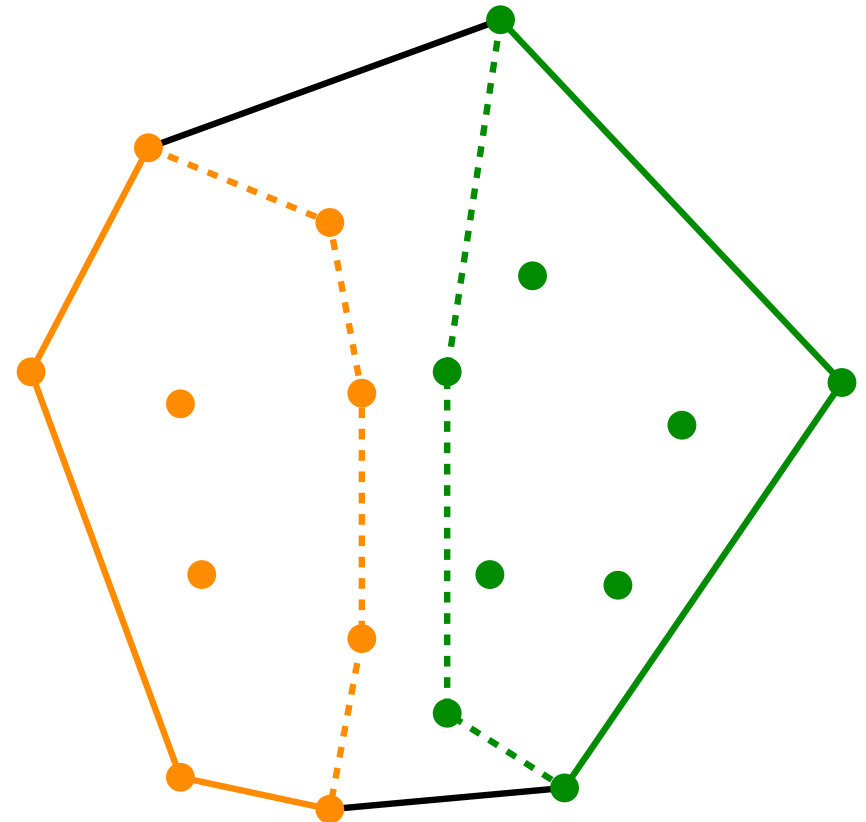
Divide-and-conquer algorithm

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Initialization: $O(n \log n)$ (only once)

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Divide-and-conquer algorithm

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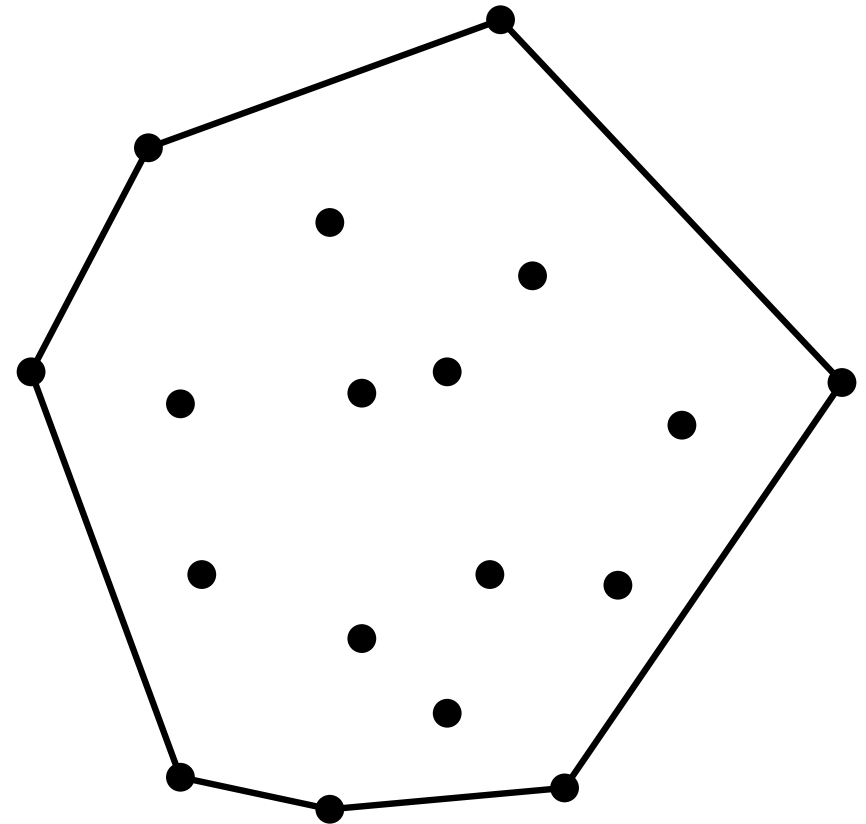
Division: $O(n)$

Merge: $O(n)$

Advance:

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n) = O(n \log n)$$

Overall: $O(n \log n)$



CONVEX HULL IN 2D

Lower bound

CONVEX HULL IN 2D

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Input: n real numbers

x_1, \dots, x_n real numbers

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p_1, \dots, p_n , with $p_i = (x_i, x_i^2)$



CONVEX HULL IN 2D

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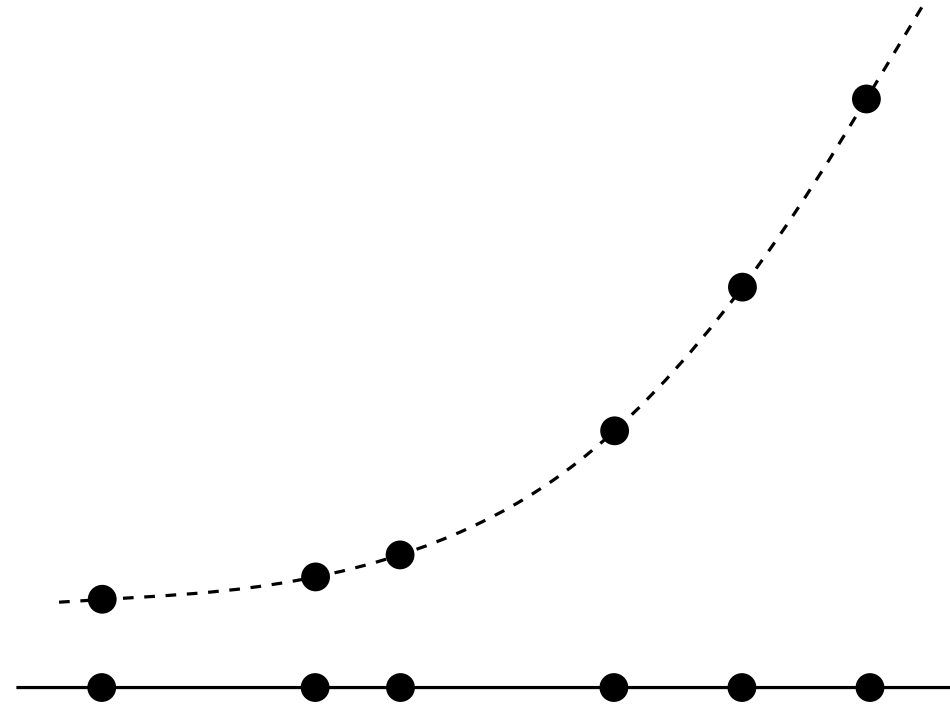
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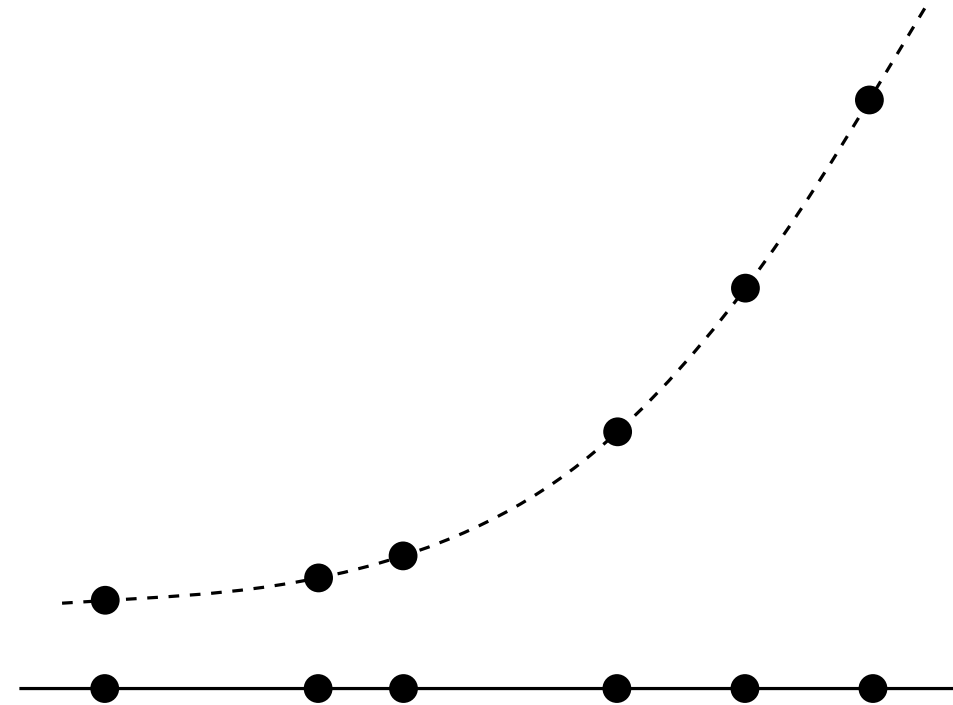
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Output: convex hull of the points

Sorted list of the vertices of the convex hull



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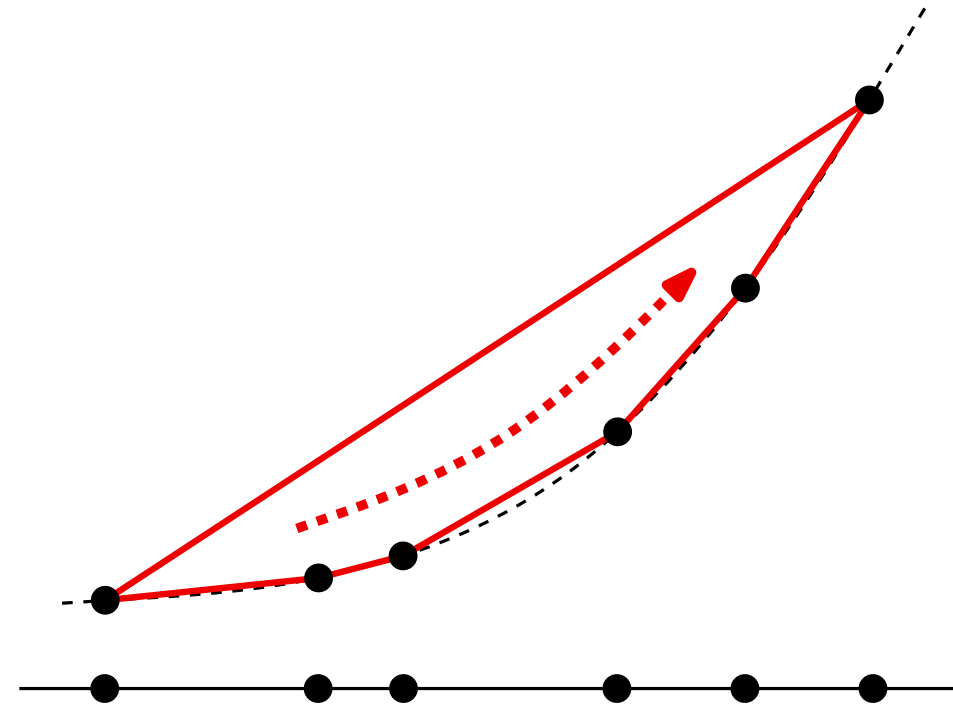
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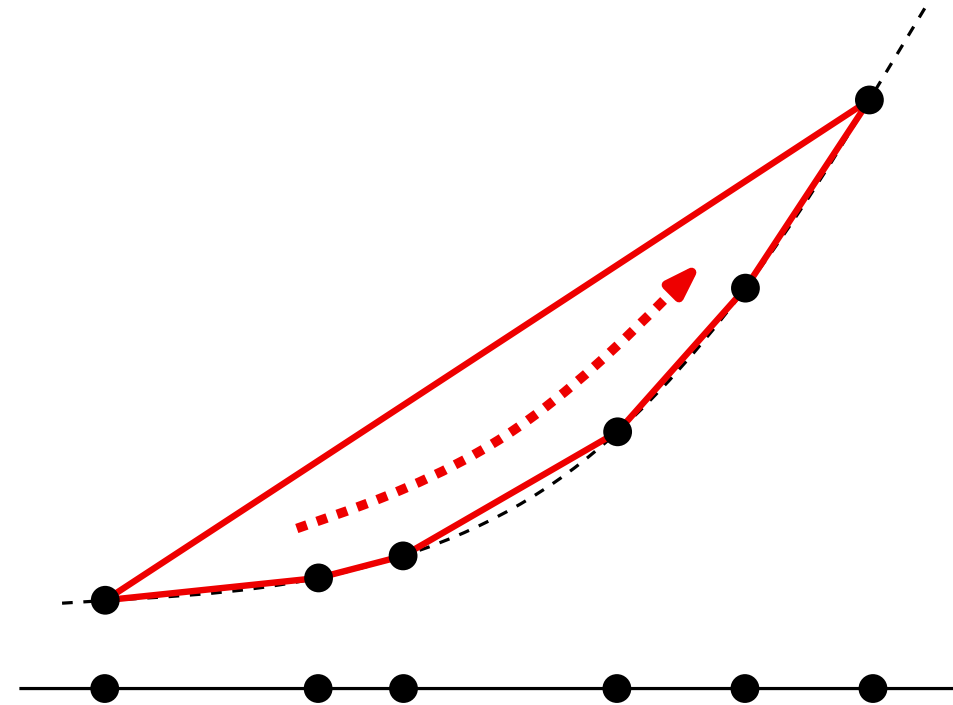
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Output: sorting the numbers

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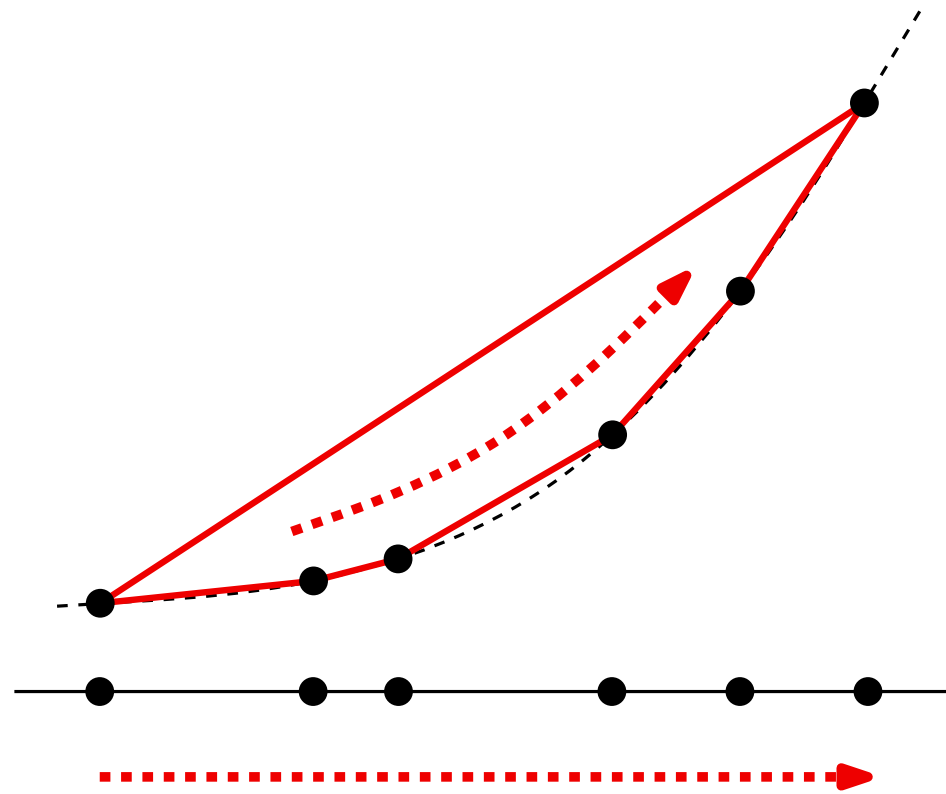
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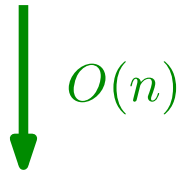


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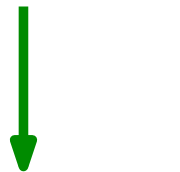
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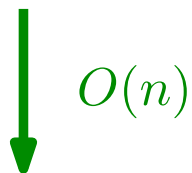
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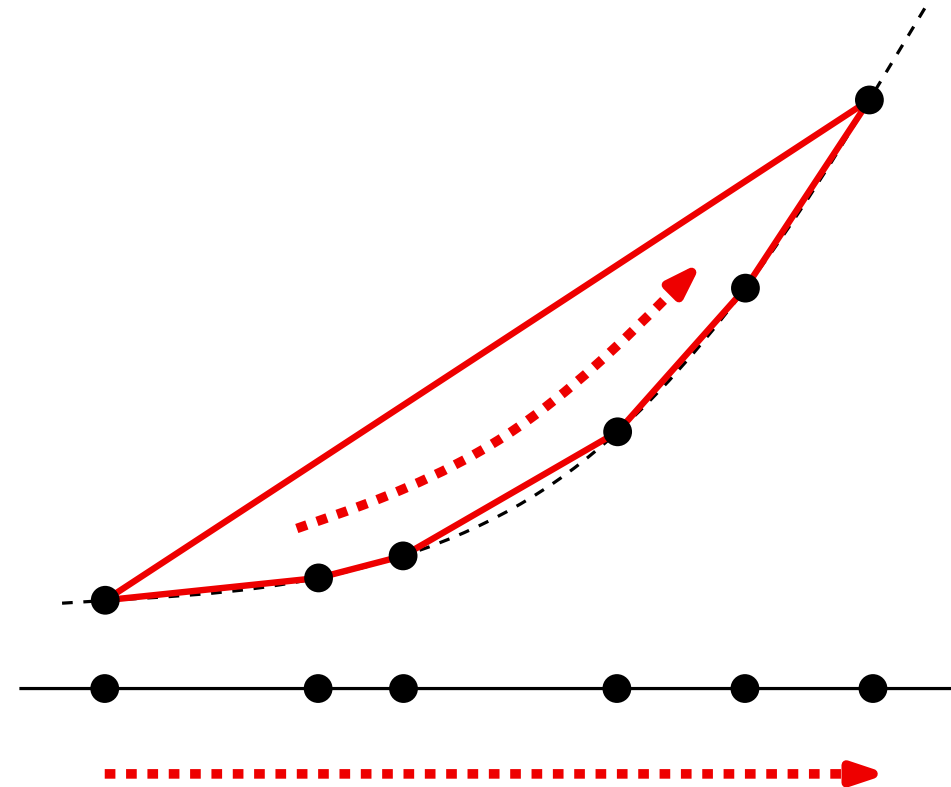
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CONVEX HULL IN 2D

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$O(n)$

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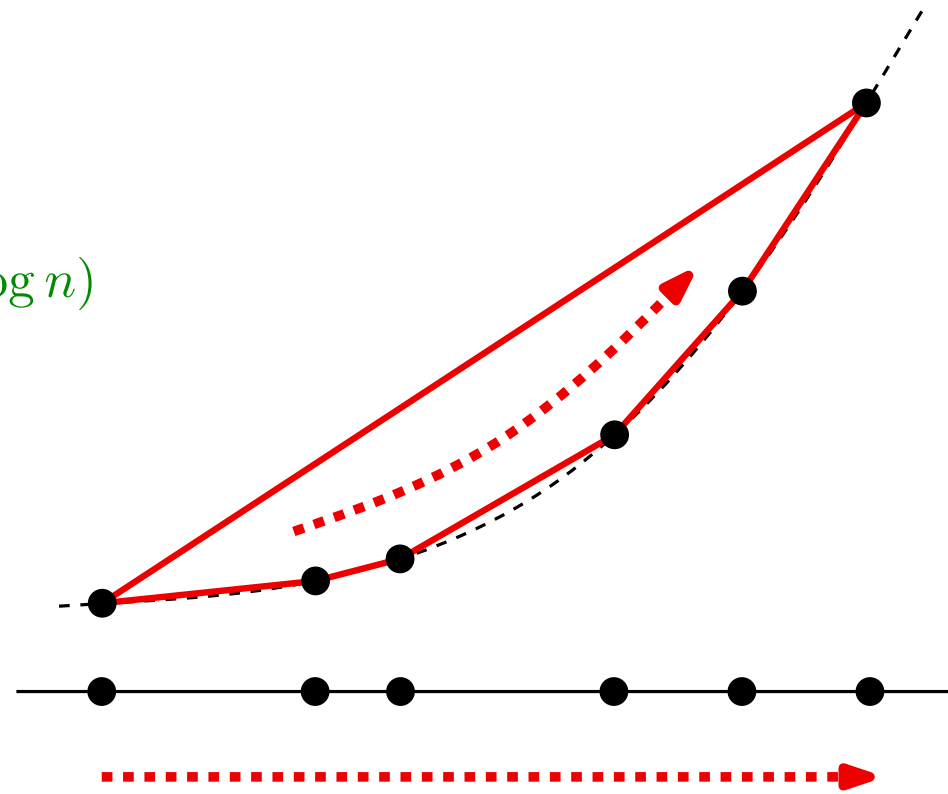
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$\Omega(n \log n)$



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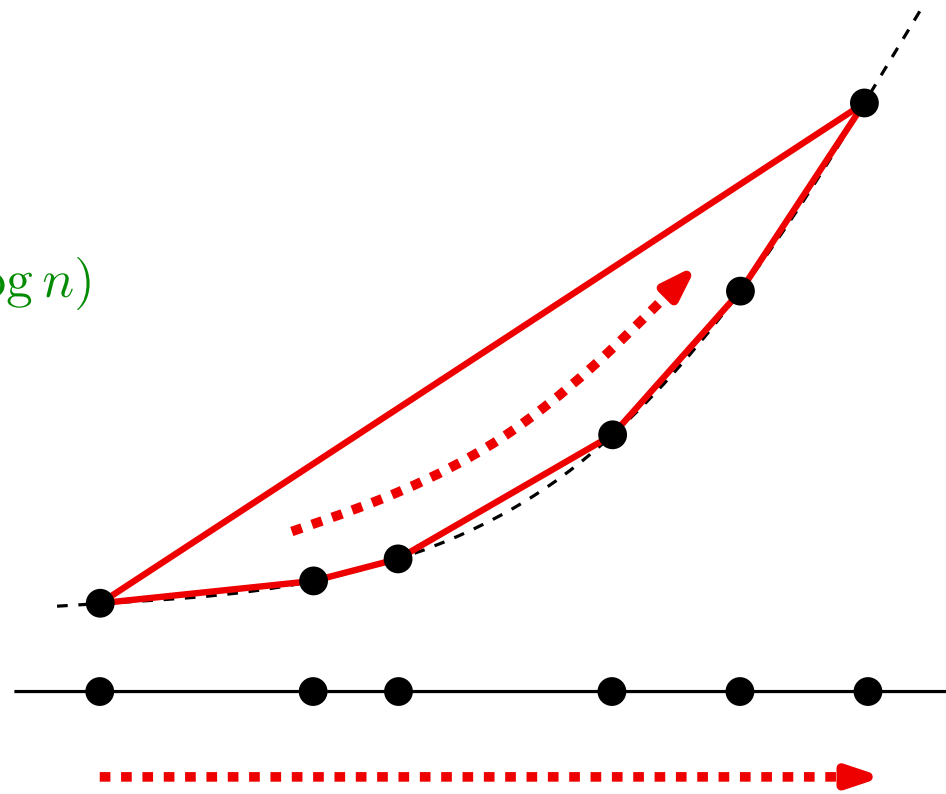
Sorted list of the vertices of the convex hull

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Output: sorting the numbers

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CONVEX HULL IN 2D

Further questions

- Can you do faster than $O(n \log n)$ time if the convex hull vertices don't need to be given in order?

CONVEX HULL IN 2D

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Not much! In fact, the following problem is $\Omega(n \log h)$:

Given a point set P and integer h , does the convex hull of P have h distinct vertices?

However, this running time can be achieved: there are convex hull algorithms that compute the convex hull vertices, in order, in $O(n \log h)$ time!

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Extension: do the previous strategies extend to the 3D case?

- Is it possible to design a 3-dimensional gift wrapping convex hull algorithm?
- Is it possible to design a 3-dimensional incremental convex hull algorithm?
- Is it possible to design a 3-dimensional divide-and-conquer convex hull algorithm?

CONVEX HULL IN 2D

FURTHER READING

- J. O'Rourke, **Computational Geometry in C (2nd ed.)**, Cambridge University Press, 1998.
- F. Preparata, M. Shamos, **Computational Geometry: An introduction (revised ed.)**, Springer, 1993.

...AND PLAYING

See demo in course web page!