Computational Geometry – Presentation topics

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NOTICE!

All references are available:

- either they are attached to the link (sometimes only accessible through the UPC library system),
- or they can be found in the university library,
- or you can ask me for a copy.

Please use the references given. Do not replace them by others without telling me first!

1: Rotating calipers

- References:
  - G. Toussaint, Solving Geometric Problems with the Rotating Calipers, *Proc. IEEE Mediterranean Electrotechnical Conference*, 1983.
  - G. Toussaint, Applications of the Rotating Calipers to Geometric Problems in Two and Three Dimensions, *International Journal of Digital Information and Wireless Communications* 4(3): 372-386, 2014.

2: Three dimensional convex hull

- Suggested scheme:
  - Definition and basic properties of polytopes (in particular, three-dimensional polytopes have linear complexity and can be stored in a DCEL). Reference: J. O'Rourke, *Computational Geometry in C*, Cambridge University Press, 2nd edition, 1998.
  - Gift wrapping algorithm. Reference: F. Preparata, M. I. Shamos, *Computational Geometry: an Introduction*, Ed. Springer, 1985.
  - Divide and conquer algorithm. Reference: F. Preparata. M. I. Shamos, *Computational Geometry: an Introduction*, Ed. Springer, 1985.
  - Incremental algorithm. References:
    - J. O'Rourke, *Computational Geometry in C*, Cambridge University Press, 2nd edition, 1998.
    - M. de Berg, O. Cheong, M. van Kreveld, M. Overmars, *Computational Geometry - Algorithms and Applications*, Ed. Springer, 3rd edition, 2008.

3: An output sensitive algorithm to compute convex hulls in 2 and 3 dimensions

- Reference: T. M. Chan, Optimal Output-Sensitive Convex Hull Algorithms in Two and Three Dimensions, *Discrete and Computational Geometry*, 16:361-368, 1996.

4: Convex hull of a simple polygon

- Suggested scheme:
  - Start motivating the problem: why computing the CH of a polygon could be easier or maybe faster than just computing the CH of its vertices?
  - Quickly summarize the story of the many solutions proposed to this problem. Reference: http://cgm.cs.mcgill.ca/~athens/cs601.
  - Explain the solution proposed in F. Preparata. M. I. Shamos, *Computational Geometry: an Introduction*, Ed. Springer, 1985, pages 166-171.

5: Kernel of a polygon, and visibility polygon of a point in the interior of a polygon

- References:
  - Kernel:
    - Reference: F. Preparata. M. I. Shamos, *Computational Geometry: an Introduction*, Ed. Springer, 1985.
    - An applet: http://www.geometrylab.de/applet-5-en
  - Visibility polygon:
    - J. O'Rourke, *Art Gallery Theorems and Algorithms*, Oxford University Press, 1987, pp. 203-206.
      - Some applets: http://www.geometrylab.de/applet-5-en

6: Minimizing the number of guards of a polygon is NP-hard

- Reference: J. O'Rourke, *Art Gallery Theorems and Algorithms*, Oxford University Press, 1987, Chapter 9 (particularly Section 9.3).

7: Dynamic maintenance of geometric structures

- Suggested scheme:
  - Describe and analyze an algorithm for dynamic maintenance of planar convex hulls.
  - Make a short reference to other geometric structures than can be maintained with similar techniques.
  - Make a short reference to the latest results on this subject.
- Reference: M. Overmars and J. van Leeuwen, Maintenance of configurations in the plane, *Journal of Computer and System Sciences* 23 (2): 166–204, 1981. (Free access from UPC).

8: Kinetic data structures

- Suggested scheme:
  - Describe the problem in general and all the general ideas about how to solve it.
  - Describe and analyze a kinetic data structure for planar convex hulls.
  - Make a short reference to other geometric structures than can be kinetic data structures can be applied to.
- References:
  - L. Guibas, Kinetic Data Structures: A State of the Art Report, in P. Agarwal, L. Kavraki, and M. Mason (eds.), *Robotics: The Algorithmic Perspective (Proceedings of the 3rd Workshop on the Algorithmic Foundations of Robotics)*, A K Peters/CRC Press, pp. 191–209, 1998.
  - L. Guibas, Leonidas, Kinetic Data Structures, in D. Mehta and S. Sahni (eds.), *Handbook of Data Structures and Applications*, Chapman and Hall/CRC, chapter 23, 2001.

9: Constructing the Voronoi diagram by sweeping (Fortune's algorithm)

- References:
  - To get a general vision of the algorithm: J. O'Rourke, *Computational Geometry in C*, Cambridge University Press, 2nd edition, 1998.
  - To understand the algorithm in full detail: M. de Berg, O. Cheong, M. van Kreveld, M. Overmars, *Computational Geometry - Algorithms and Applications*, Ed. Springer, 3rd edition, 2008.

10: Crust, anticrust, and medial axis

- Suggested scheme:
  - Definition
  - Applications
  - Algorithms
- References:
  - N. Amenta, M. Bern, D. Eppstein, The Crust and the Beta-Skeleton: Combinatorial Curve Reconstruction, *Graphical Models and Image Processing* 60(2): 125-135, 1998. (Free access from UPC).
  - http://sarielhp.org/research/CG/applets/Crust/Crust.html
  - Search the internet to find out about the applications

11: Alpha shapes

- I suggest sticking to the case alpha < 0 only.
- References:
  - H. Edelsbrunner, D. G. Kirkpatrick, R. Seidel, On the Shape of a Set of Points in the Plane, *IEEE Transactions on Information Theory* IT-D(4): 551-559, 1983. (Free access from UPC).
  - http://cgm.cs.mcgill.ca/~godfried/teaching/projects97/belair/alpha.html

12: Proximity graphs

- References:
  - G. T. Toussaint, The relative neighborhood graph of a finite planar set, *Pattern Recognition* 12: 261-268, 1980.
  - J.W. Jaromczyk, G.T. Toussaint, Relative Neighborhood Graphs and Their Relatives, *Proc. IEEE* 80(9): 1502-1517, 1992.

13: Delaunay triangulation of imprecise points

- Reference: O. Devillers, Delaunay triangulation of imprecise points: preprocess and actually get a fast query time, *Journal of Computational Geometry* 2(1): 30-45, 2011.

14: Motion planning for a point/polygonal robot, shortest paths, and visibility graphs

- References:
  - Chapters 13 and 15 of M. de Berg, O. Cheong, M. van Kreveld, M. Overmars, *Computational Geometry - Algorithms and Applications*, Ed. Springer, 3rd edition, 2008.
  - Sections 8.1 to 8.5 of J. O'Rourke, *Computational Geometry in C*, Cambridge University Press, 2nd edition, 1998.

15: Planning the movement of a robot arm

- References:
  - Section 8.6 of J. O'Rourke, *Computational Geometry in C*, Cambridge University Press, 2nd edition, 1998.
  - http://weblogs.asp.net/frank_hileman/vg-net-5-1-released

16: Digital elevation models of terrains

- Suggested scheme:
  - How terrain data is obtained
  - Models to store terrain data:
    - Regular square grid
    - Contour lines
    - Delaunay triangulation (TIN)
  - Pros and cons of the different models
  - Conversion between models
  - Operations and computations on terrains
- Reference: M. van Kreveld: Digital elevation models and TIN algorithms, in *Algorithmic Foundations of Geographic Information Systems*, Lecture Notes in Computer Science 1340: 37-78, 1997. (Free access from UPC).

17: Arrangements: complexity and construction

- Reference: J. Pach (ed.): *New Trends in Discrete and Computational Geometry*, Springer-Verlag, 1993. Chapter 1.

18: Topological sweep

- I suggest to restrict to the topological sweep of an arrangement of lines.
- References:
  - H. Edelsbrunner, L. Guibas, Topologically Sweeping an Arrangement, *Journal of Computer and System Sciences* 38(1): 165-194, 1989. (Free access from UPC).
  - E. Rafalin, D. Souvaine, I. Streinu, Topological Seep in Degenerate Cases, *Revised Papers from the 4th International Workshop on Algorithm Engineering and Experiments*, Lecture Notes In Computer Science 2409: 155 - 165, 2002. (Free access from UPC).

19: Lower bounds for some geometric problems

- Reference: V. Sacristán, *Lower bounds for some geometric problems*, Tech. Rep. MA-IR-98-0034, UPC, Dept. MA II, 1998.

20: Graph drawing

- Suggested scheme:
  - Goals and applications of graph drawing
  - Criteria and parameters for drawing a graph
  - Most used paradigms in graph drawing algorithms
  - Applications
- References:
  - G. Di Battista, P. Eades, R. Tamassia, I. G. Tollis. *Graph Drawing: Algorithms for the Visualization of Graphs*, Prentice Hall, 1999.
  - T. Nishizeki, Md. S. Rahman. *Planar Graph Drawing*, Word Scientific, 2004.
  - R. Tamassia, G. Liotta. Graph Drawing. Chapter 52 of the book: J. E. Goodman, J. O'Rourke (eds.), *Handbook of Discrete and Computational Geometry*, Chapman & Hall / CRC Press, 2nd edition, 2004, pp. 1163-1185.
  - I.F. Cruz, R. Tamassia, Graph Drawing Tutorial

21: Randomized algorithms in Computational Geometry

- Suggested scheme:
  - Deterministic algorithms versus randomized algorithms
  - Typologies of geometric randomized algorithms
  - Some examples (need to be chosen among the many that appear in the literature)
- References:
  - K. Mulmuley, *Computational Geometry. An Introduction through Randomized Algorithms*, Prentice Hall, 1994. (General framework and a large number of examples).
  - R. Motwani, P. Raghavan, *Randomized Algorithms*, Cambridge University Press, 1995. (Only as a general reference).

22. Distributed algorithms in Computational Geometry

- Reference: S. Rajsbaum, J. Urrutia, Some problems in distributed computational geometry, *Theoretical Computer Science* (412) 41: 5760-5770, 2011.

23: Data structures in Computational Geometry

- This subject may be approached in several different ways, depending on the interest of the student. For example:
  - One possible way may be to give a general vision, not going deeply into each structure. In this case, it is convenient to cover the four chapters, since the goal of the Presentation would be to show a good range of structures, their differences and their applications.
  - Another possible way would be to explain a shorter number of structures, but in more detail. In this case, it is convenient to cover at least two of the chapters.
- References:
  - M. de Berg, O. Cheong, M. van Kreveld, M. Overmars, *Computational Geometry - Algorithms and Applications*, Ed. Springer, 3rd edition, 2008. (Chapters 5, 10, 12 and 14).
  - A nice applet: http://symbolcraft.com/graphics/bsp/

24: Computational Geometry on surfaces

- Suggested scheme:
  - Basic objects of geometric nature (point, line, segment, etc.) and working tools (how to represent the objets).
  - Choose one/some of the following problems: convex hull, Voronoi diagram, triangulations.
- Reference: C.I. Grima, A. Márquez: *Computational Geometry on Surfaces*, Kluwer Academic Publishers, 2001.

25: Computational Geometry Algorithms Library (CGAL)

- Reference: http://www.cgal.org/

26: Open problems in Computational Geometry

- Reference: http://cs.smith.edu/~orourke/TOPP/