Using the appropriate data structure

Vera Sacristán

Computational Geometry
Facultat d'Informàtica de Barcelona
Universitat Politècnica de Catalunya

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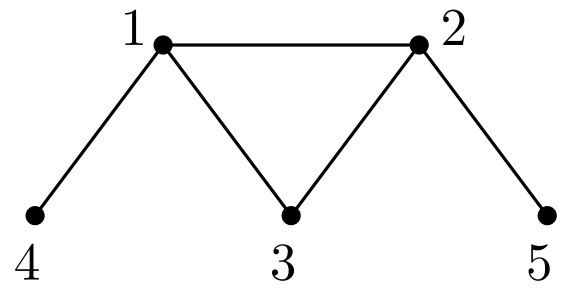

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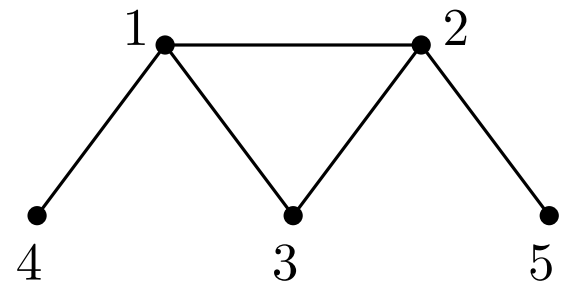

Incidence matrix

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

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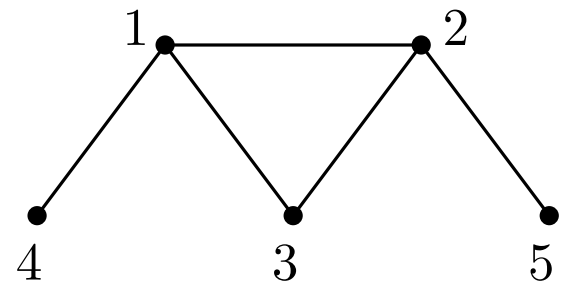

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Adjacency list

$$((2,3,4),(1,3,5),(1,2),(1),(2))$$

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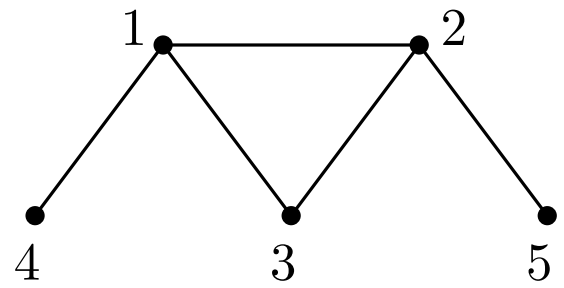

Are v_i and v_j connected?

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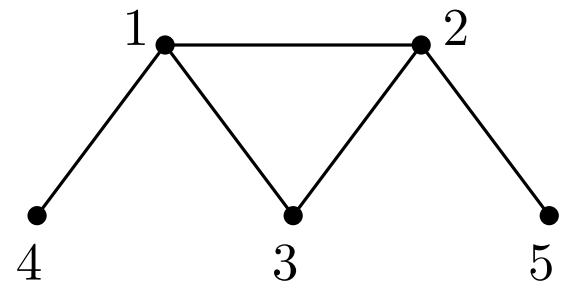

Are v_i and v_j connected?

$O(1)$

$O(n)$

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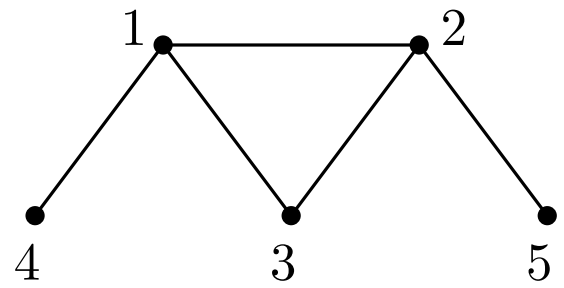

Are v_i and v_j connected?	What is degree(v_i)?
$O(1)$	
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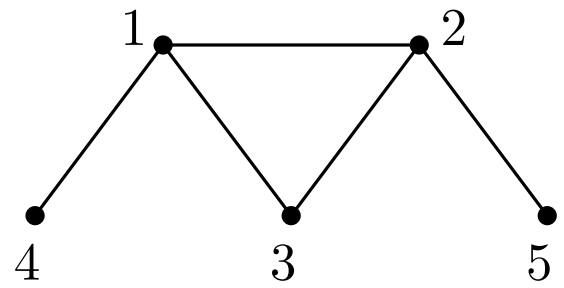

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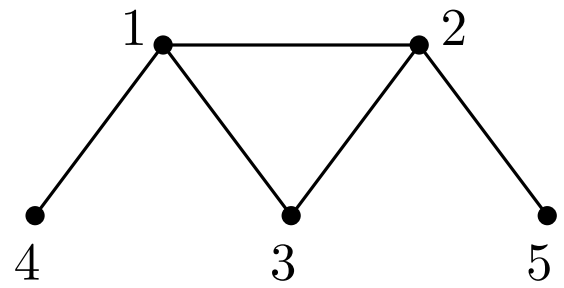

Are v_i and v_j connected?	What is degree(v_i)?	Storage space
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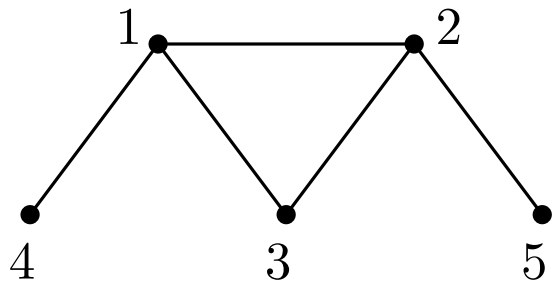

Are v_i and v_j connected?	What is degree(v_i)?	Storage space
$O(1)$	$O(n)$	$O(n^2)$
$O(n)$	$O(degree)$	$O(edges)$

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Let S be a set in a given universe $\mathcal{U}$.

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priority queue

- next
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augmented dictionary

Data structures to implement these operations

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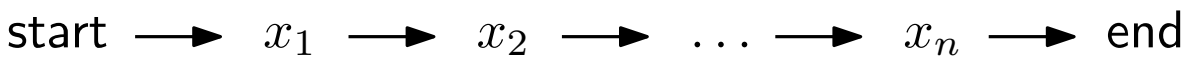
Linked list



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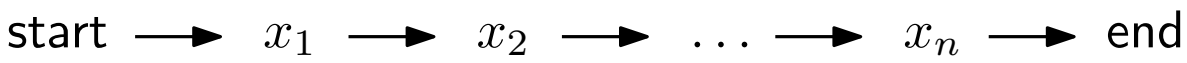
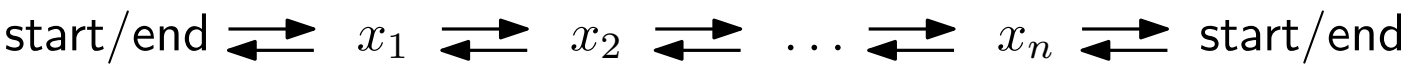


Doubly linked list



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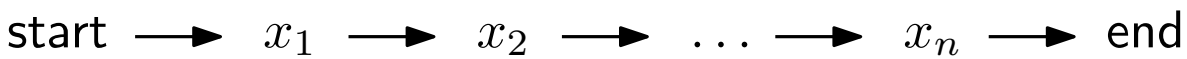

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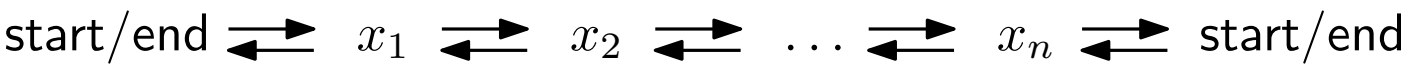

How are the operations done?

- Locate: explore the list in $O(i)$ time
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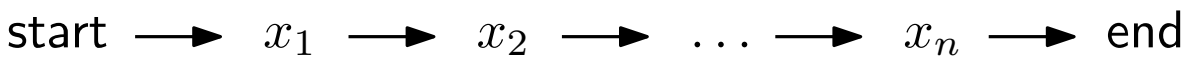

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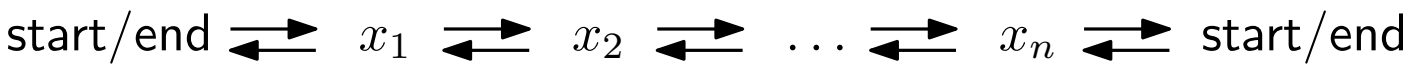

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Two particular lists are:

Stack: LIFO list (insert/delete only at the end)

Queue: LILO list (insert at the end, delete at the beginning)

Data structures to implement these operations

Binary-search trees

Balanced binary trees

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acyclic connected graphs

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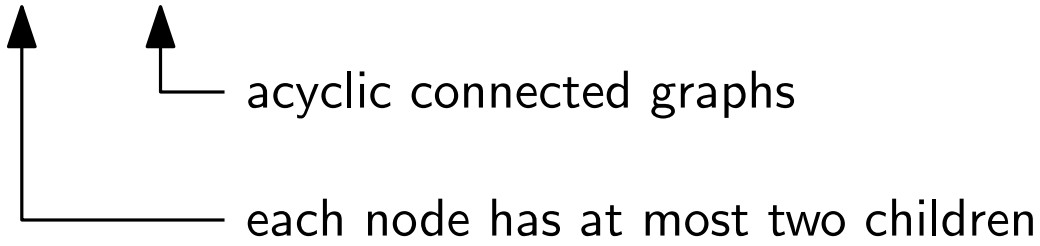

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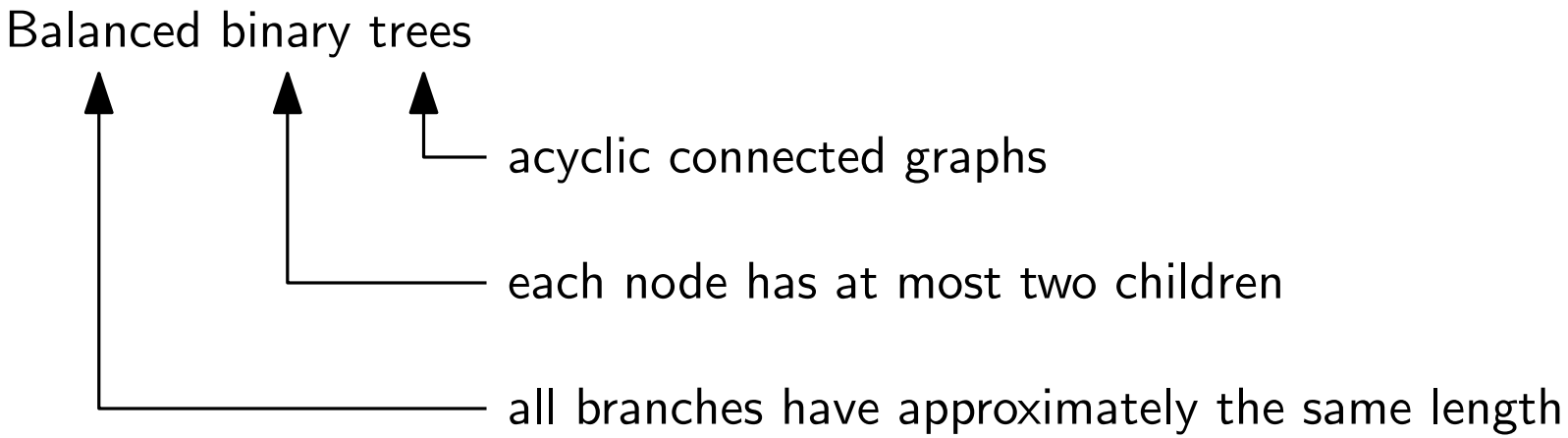

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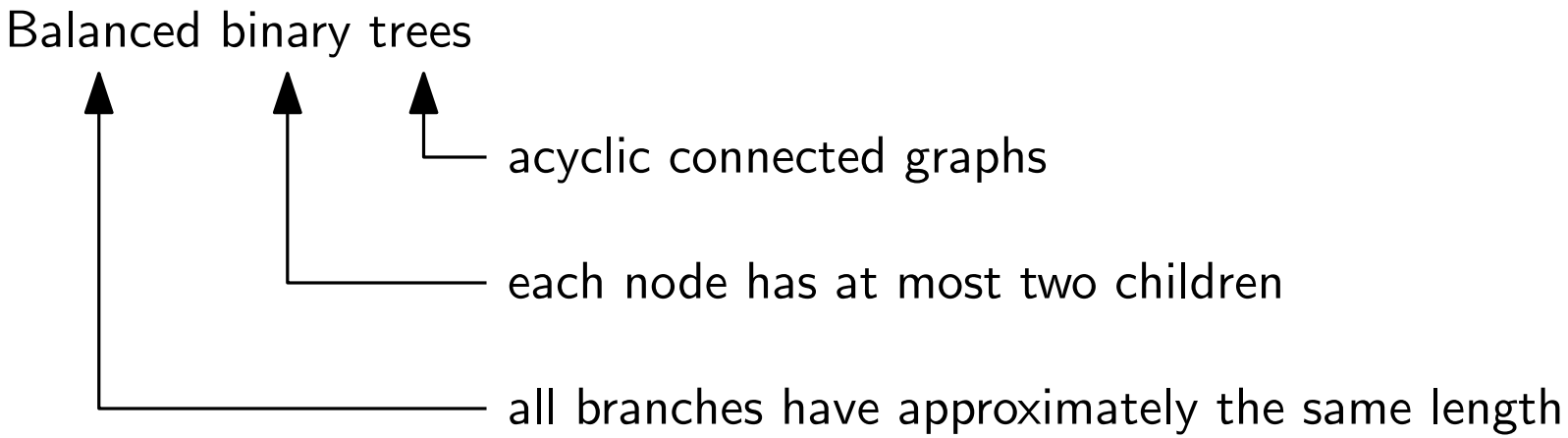

Example

$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ in $\mathcal{U} = (\mathbb{R} \leq)$

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Balanced binary trees

- acyclic connected graphs
- each node has at most two children
- all branches have approximately the same length

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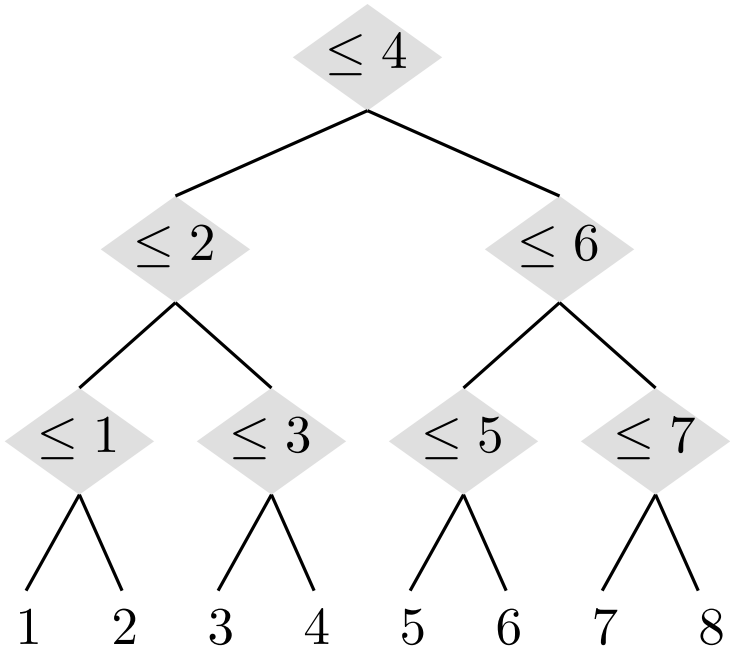

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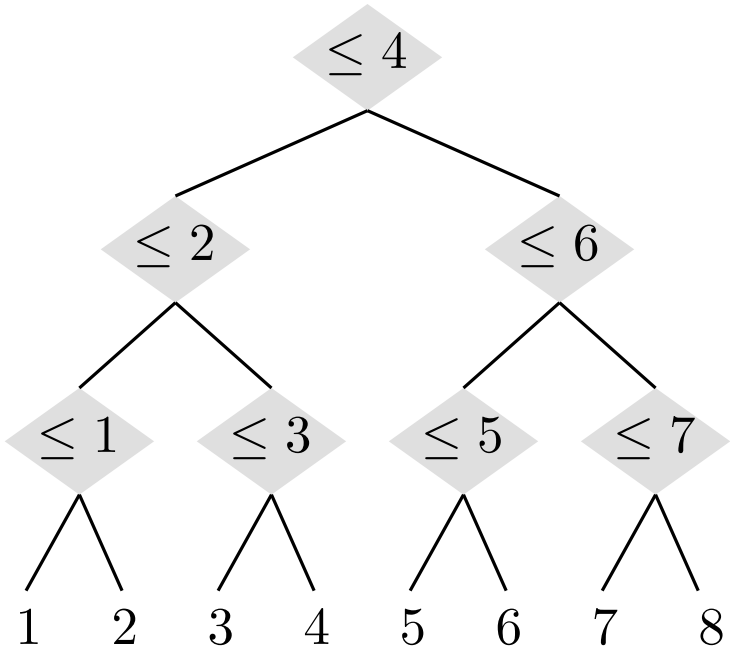
$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ in $\mathcal{U} = (\mathbb{R} \leq)$

- Locate(3)
- Locate(3.5)
- Min(S)
- Insert(3.5)
- Delete(3)



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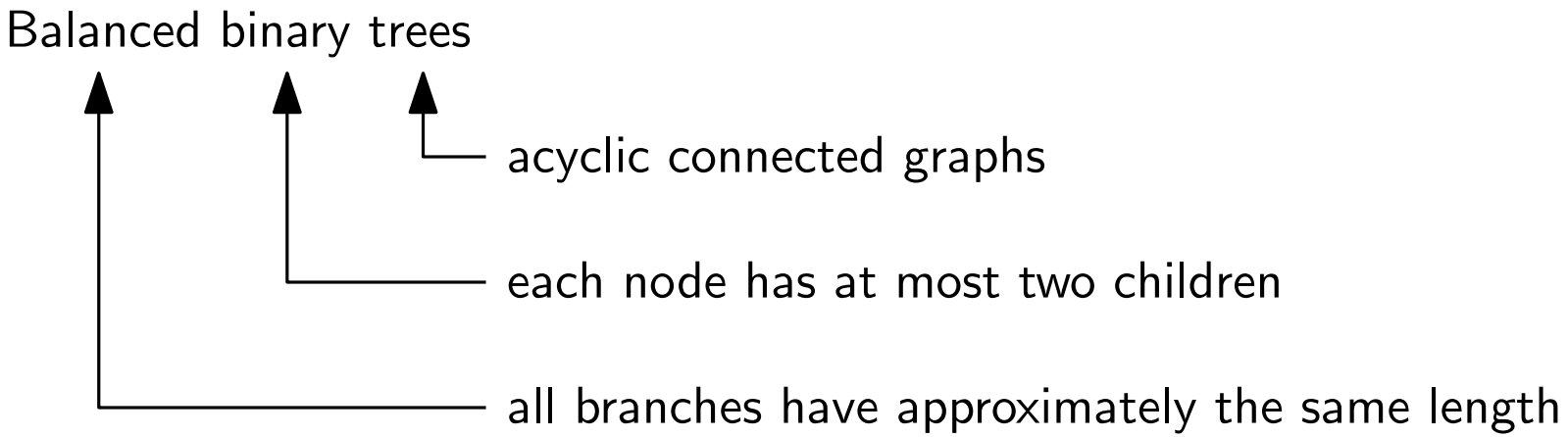

Theorem

Given a set S of n elements ot a totally ordered univers $\mathcal{U}$, it is possible to build a balanced binary search tree representing S using $O(n)$ space and $O(n \log n)$ time.

Then:

- memberQ
- locate
- min/max
- insert/delete

run in $O(\log n)$ time

- next/previous

runs in $O(1)$ time

FURTHER READING

F. P.Preparata and M. I Shamos
Computational Geometry: An Introduction
Springer-Verlag, 1985.

J-D. Boissonnat and M. Yvinec
Algorithmic Geometry
Cambridge University Press, 1997.