Using the appropriate data structure

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**Depending on what we want to do with our data,
it may be convenient to store them in different ways**

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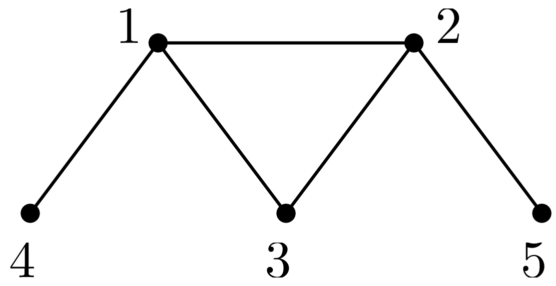
Consider the following graph G:



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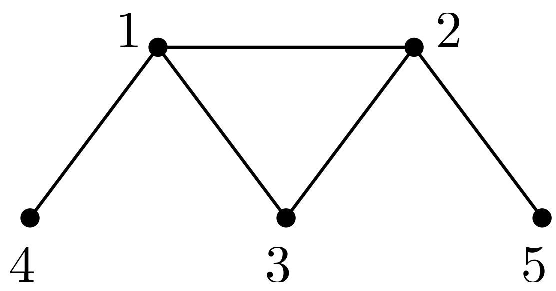

Incidence matrix

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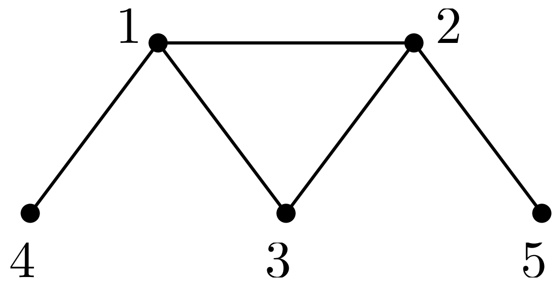

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Adjacency list

$$((2,3,4),(1,3,5),(1,2),(1),(2))$$

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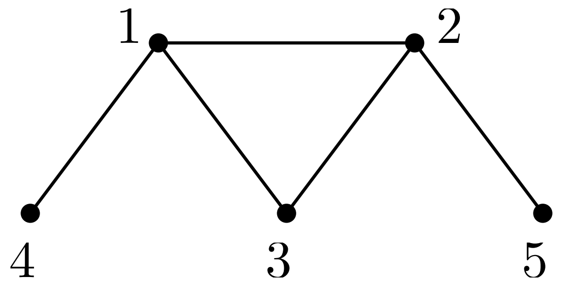

Are v_i and v_j connected?

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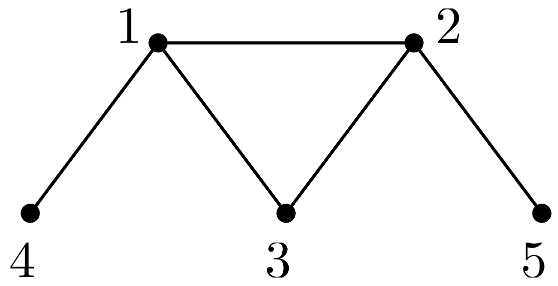

Are v_i and v_j connected?

$O(1)$

$O(n)$

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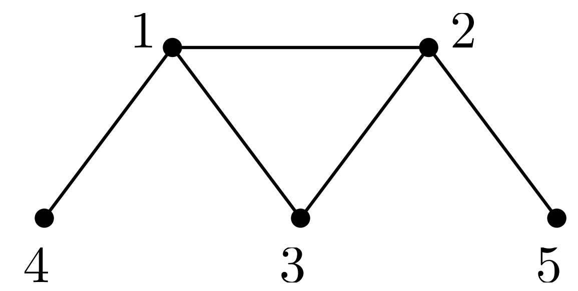

Are v_i and v_j connected?	What is degree(v_i)?
$O(1)$	
$O(n)$	

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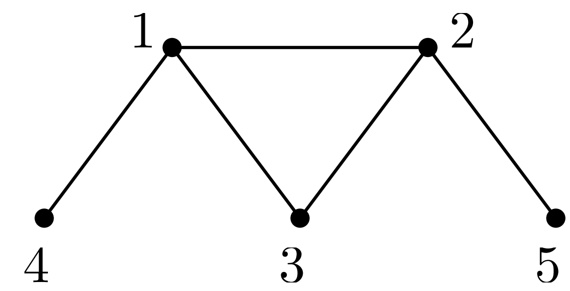

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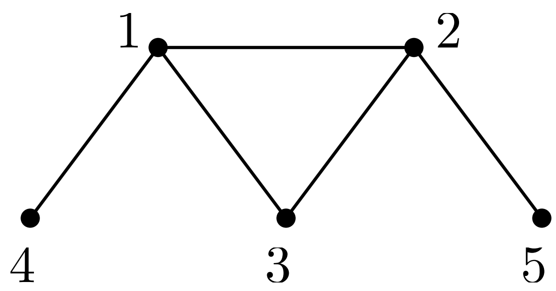

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$O(1)$	$O(n)$	
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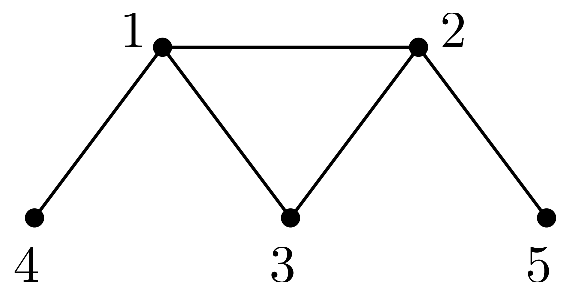

Are v_i and v_j connected?	What is degree(v_i)?	Storage space
$O(1)$	$O(n)$	$O(n^2)$
$O(n)$	$O(degree)$	$O(edges)$

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If the universe $\mathcal{U}$ is totally sorted:

- locate
- insert
- delete
- minimum

locate, insert, delete: dictionary

locate, insert, delete, minimum: priority queue

USING THE APPROPRIATE DATA STRUCTURE

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Let S be a set in a given universe $\mathcal{U}$.

Any data structure representing S and allowing the following operations:

- query
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is called a dictionary

If the universe $\mathcal{U}$ is totally sorted:

Operations			
• locate • insert • delete	dictionary	priority queue	augmented dictionary
• minimum		priority queue	augmented dictionary
• maximum • next • previous			augmented dictionary

Data structures to implement these operations

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Linked list

start $\longrightarrow$ x_1 $\longrightarrow$ x_2 $\longrightarrow$ $\ldots$ $\longrightarrow$ x_n $\longrightarrow$ end

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Doubly linked list

$$\text{start/end} \rightleftarrows x_1 \rightleftarrows x_2 \rightleftarrows \ldots \rightleftarrows x_n \rightleftarrows \text{start/end}$$

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How are the operations done?

- Locate: explore the list in $O(i)$ time
- Insert/ delete: Once located, change pointers in $O(1)$ time
- Min/max: $O(1)$ time
- Next/previous: $O(1)$ time

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Two particular lists are:

Stack: LIFO list (insert/delete only at the end)

Queue: LILO list (insert at the end, delete at the beginning)

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Binary-search trees

Balanced binary trees

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↑ acyclic connected graphs

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acyclic connected graphs

each node has at most two children

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- trees: acyclic connected graphs
- binary: each node has at most two children
- Balanced: all branches have approximately the same length

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Example

$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ in $\mathcal{U} = (\mathbb{R} \leq)$

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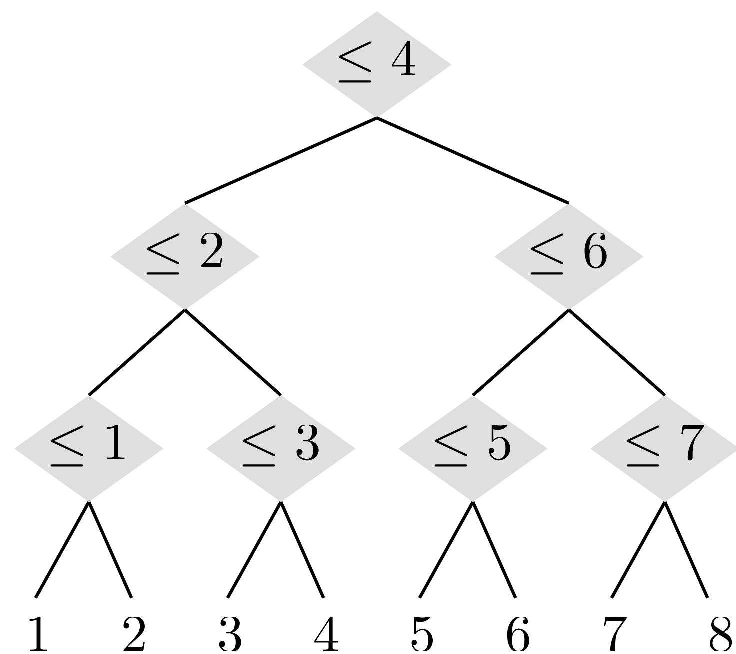


Data structures to implement these operations

Binary-search trees

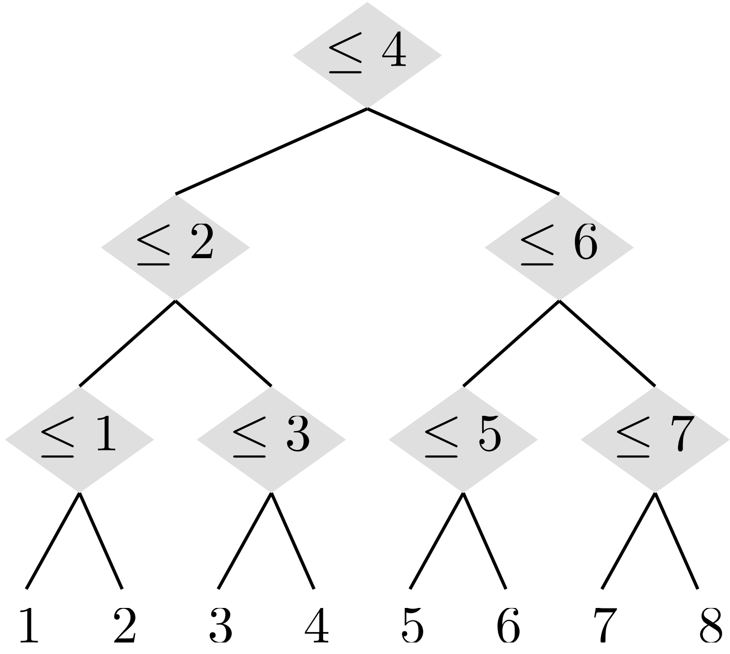
Balanced binary trees

- acyclic connected graphs (trees)
- each node has at most two children (binary)
- all branches have approximately the same length (balanced)

Example

$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ in $\mathcal{U} = (\mathbb{R} \leq)$

- Locate(3)
- Locate(3.5)
- Min(S)
- Insert(3.5)
- Delete(3)



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Binary-search trees

Balanced binary trees

- trees: acyclic connected graphs
- binary: each node has at most two children
- balanced: all branches have approximately the same length

Theorem

Given a set S of n elements ot a totally ordered univers $\mathcal{U}$, it is possible to build a balanced binary search tree representing S using $O(n)$ space and $O(n \log n)$ time.

Then:

- memberQ
- locate
- min/max
- insert/delete

run in $O(\log n)$ time

- next/previous runs in $O(1)$ time

INITIAL READING

F. P.Preparata and M. I Shamos
Computational Geometry: An Introduction
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FURTHER READING

T. H. Cormen, C. E. Leiserson, and R. L. Rivest
Introduction to Algorithms
The MIT Press - McGraw Hill Book Company, 1990 (3rd ed. 2009)

A. V. Aho, J. E. Hopcroft, and J. D. Ullman
Data structures and algorithms
Addison-Wesley, 1983