Discrete and Algorithmic Geometry
Problems list 1
Year 2016-2017 Q1

1. Propose an algorithm to compute the area of a simple polygon.
2. Let S be a set of n non-ordered segments that are the edges of a convex polygon P. Describe an algorithm to compute in $O(n \log n)$ time the sorted list of vertices of P from S.
3. Given a simple polygon P and a line ℓ which does not intersect P, propose an algorithm for computing the point of P closest to ℓ. Can the previous algorithm be improved for convex polygons? If so, how?
4. Propose an algorithm to compute the the external and the internal common tangent lines (more properly speaking, supporting lines) for disjoint convex polygons. Does this problem make sense when the polygons are simple?
5. Propose an algorithm that, given a point p external to a convex polygon P, finds the point of P closest to p. What happens if instead of finding the closest point we look for the farthest? What if we restrict the search to the vertices of P?
6. Propose an algorithm that, given two disjoint convex polygons, P and Q, finds the closest pair of points $p \in P$ and $q \in Q$.
7. Given a convex polygon P and a point q external to P, find an algorithm to compute the line through q maximizing the distance to P.
8. An *orthogonal polygon* is a polygon all whose angles are right angles. Characterize the orthogonal star-shaped polygons and their kernel. Infer an algorithm to compute the kernel of such a polygon in optimal time.
9. If B is a convex n-gon in the plane, and q is a point external to B, we define $d(q, B) = \min\{d(q, y) \mid y \in B\}$. Give an algorithm to find, given a convex m-gon A disjoint from B,
   (a) $d_M(A, B) = \min\{d(x, B) \mid x \in A\}$ (minimal distance from A to B);
   (b) $d_H(A, B) = \max\{d(x, B) \mid x \in A\}$ (Hausdorff distance from A to B).
10. Let P be a convex n-gon and q a point in the plane. Give an algorithm to compute the longest chord in P whose supporting line contains q. Raise the two cases where q is internal/external to P.
11. Let Q be a convex polygon in the plane.
    (a) Study and describe the locus L of all points in the plane that see Q with a straight angle (see the figure).
    (b) Propose an algorithm to compute L.
    (c) Given a fixed squared angle and a convex polygon Q, give an algorithm to find the position of Q minimizing the lost area (the region labeled R in the figure).