

CONVEX HULL IN 2D

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Universitat Politècnica de Catalunya

CONVEX HULL IN 2D

Computing the extreme points

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Characterization

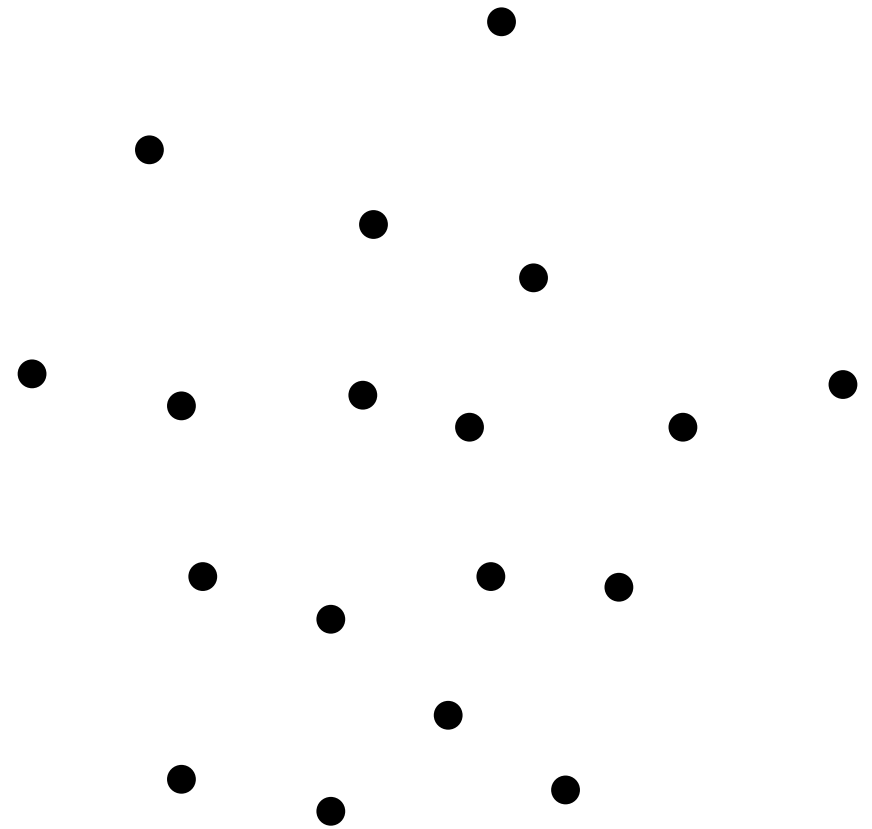
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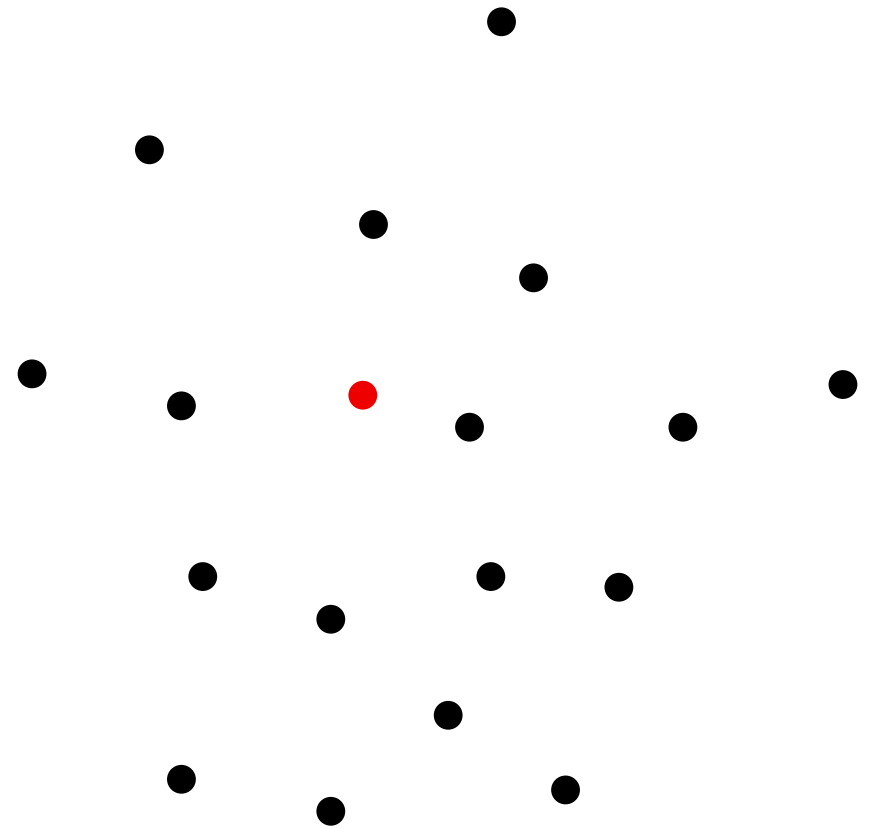


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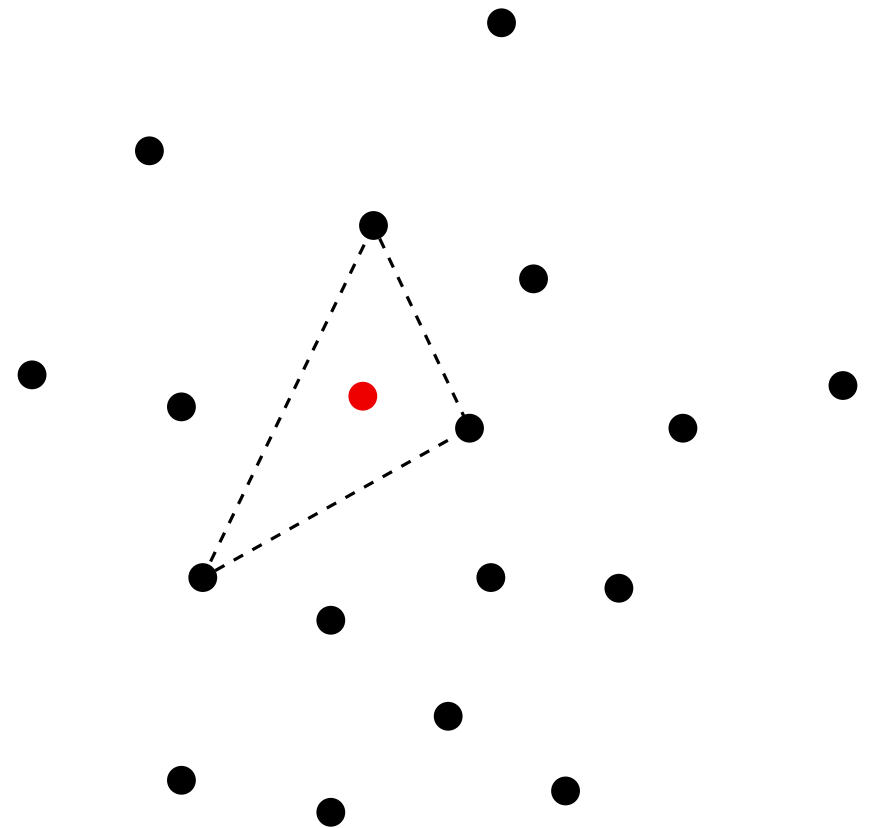


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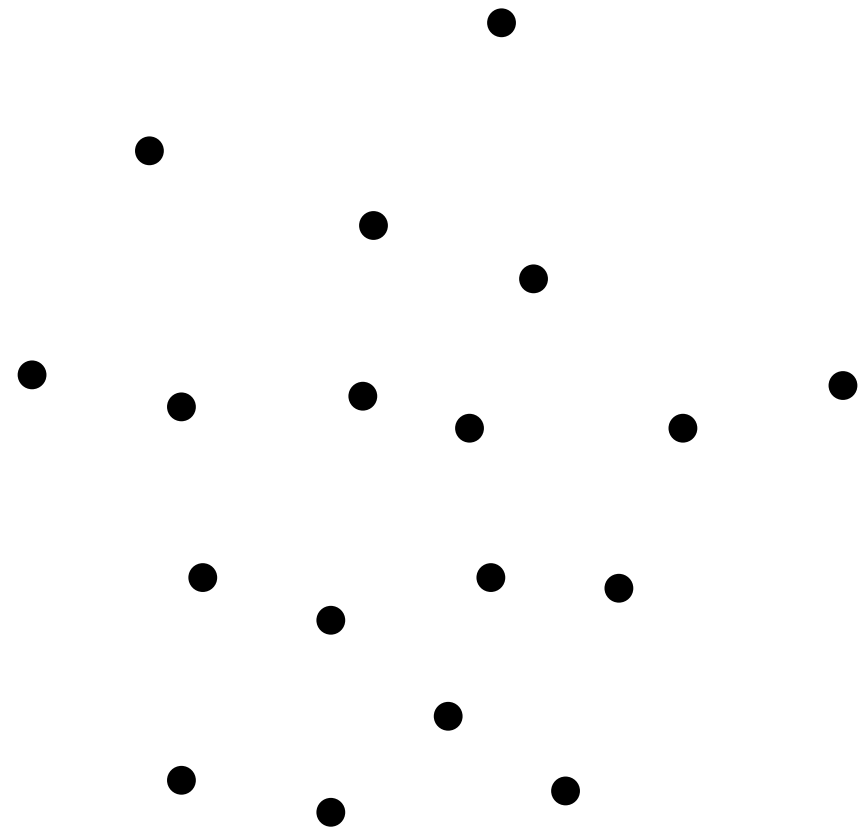
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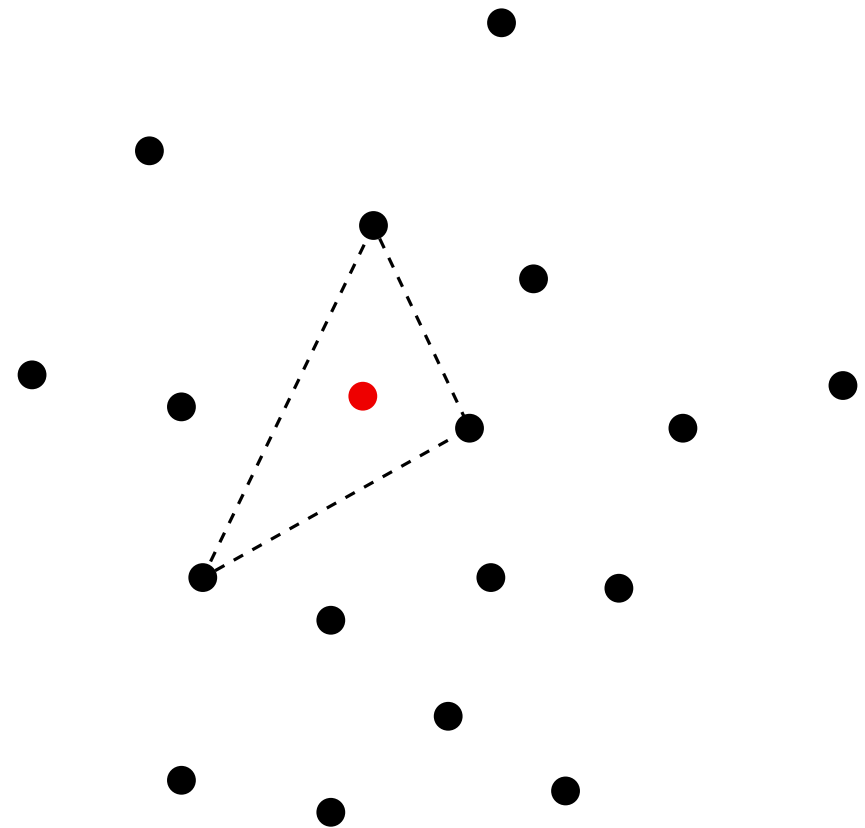
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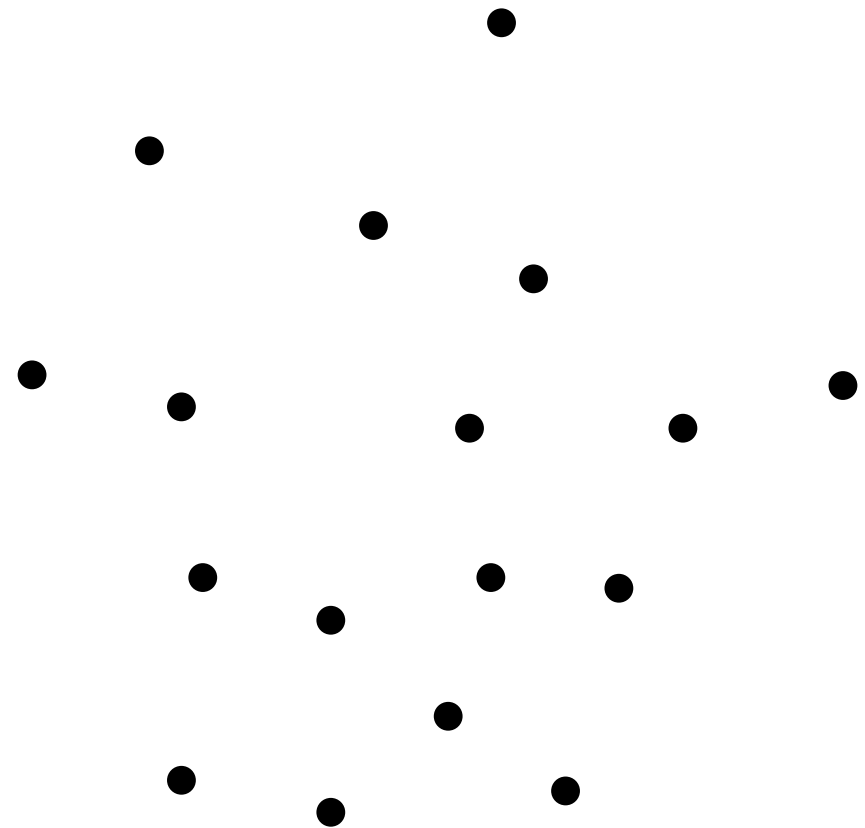
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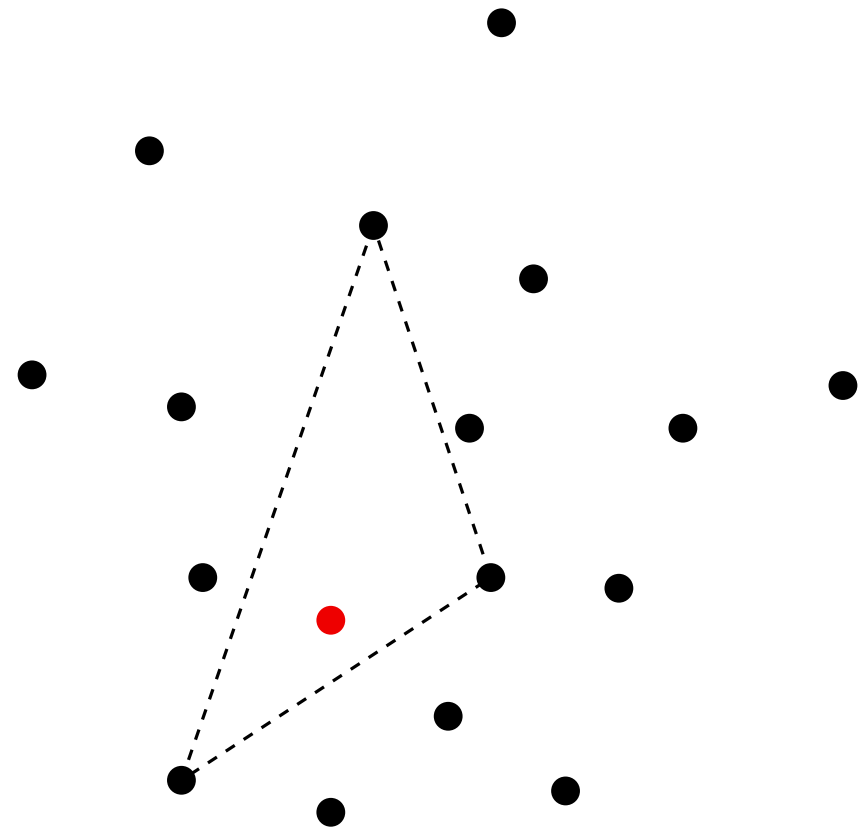
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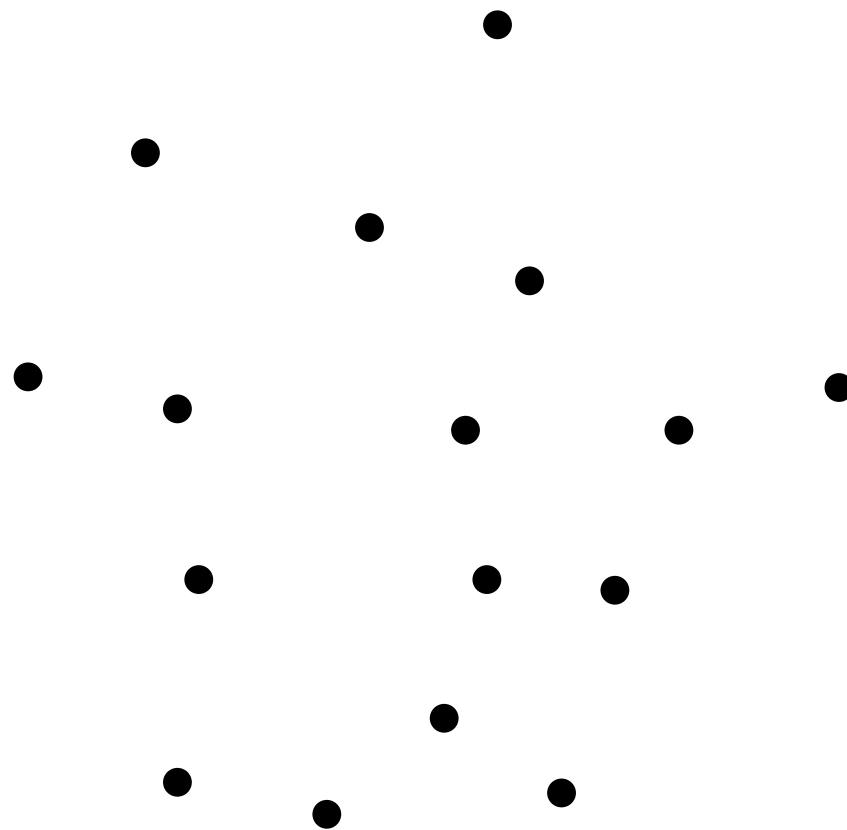
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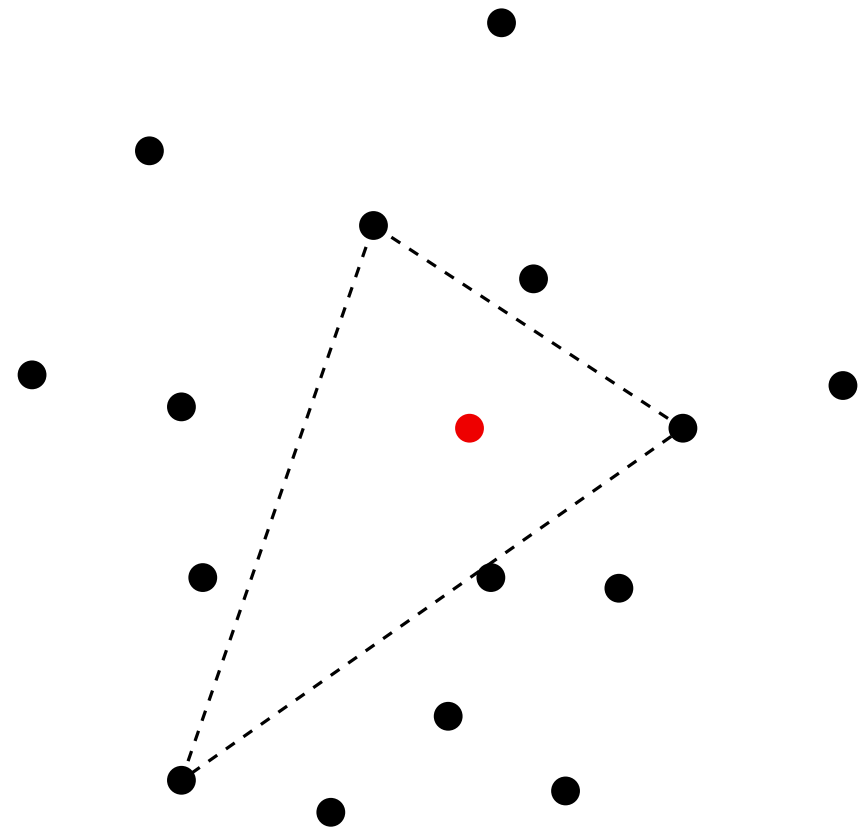
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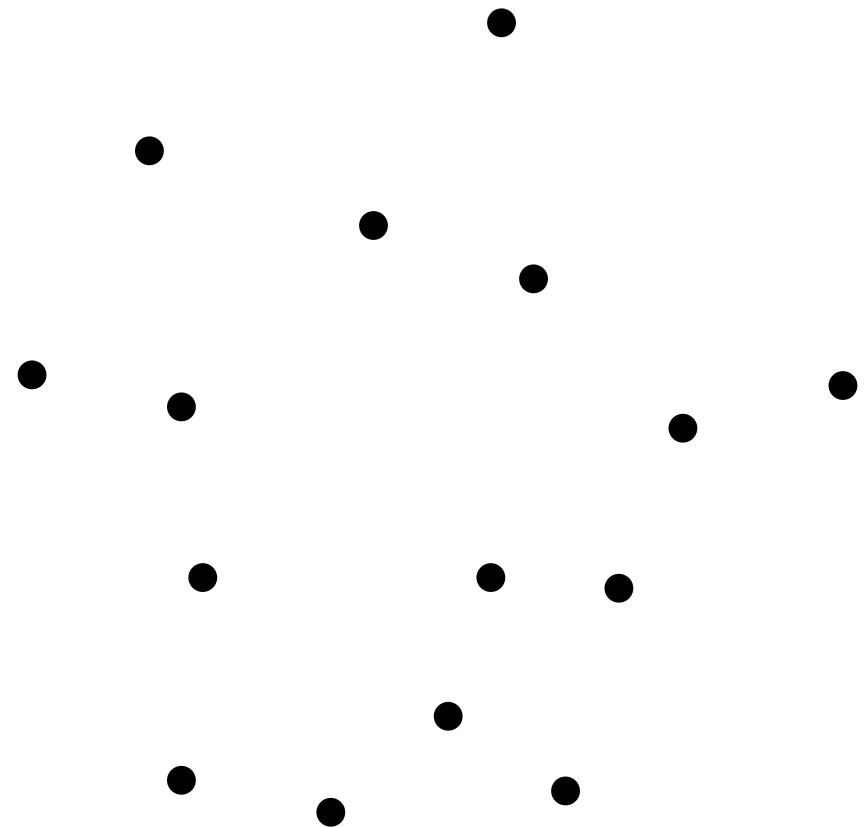
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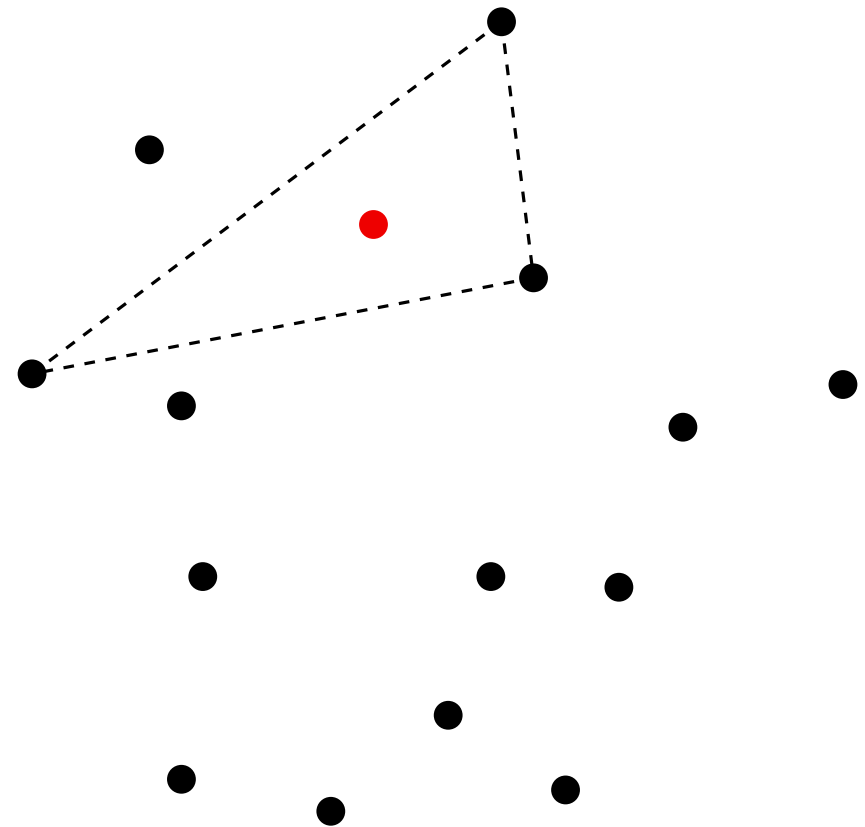
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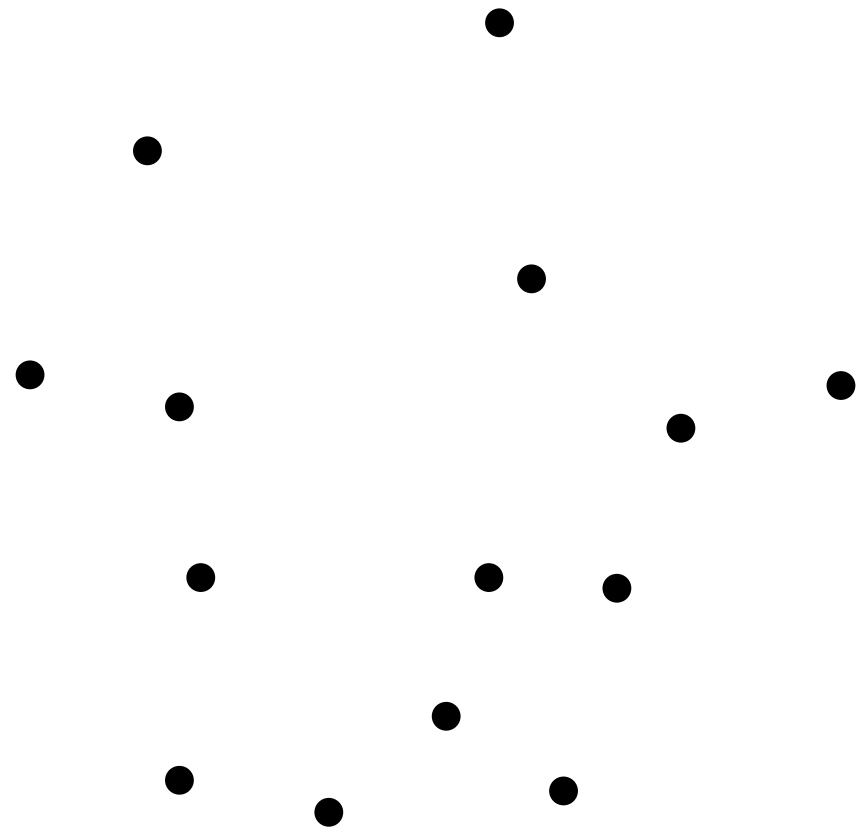
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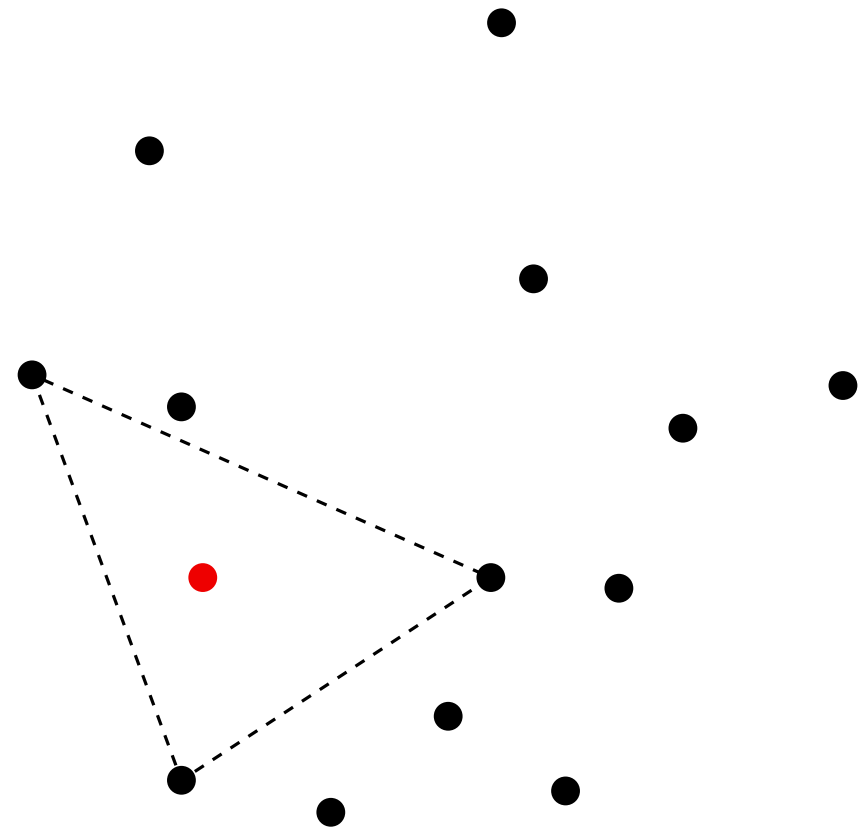
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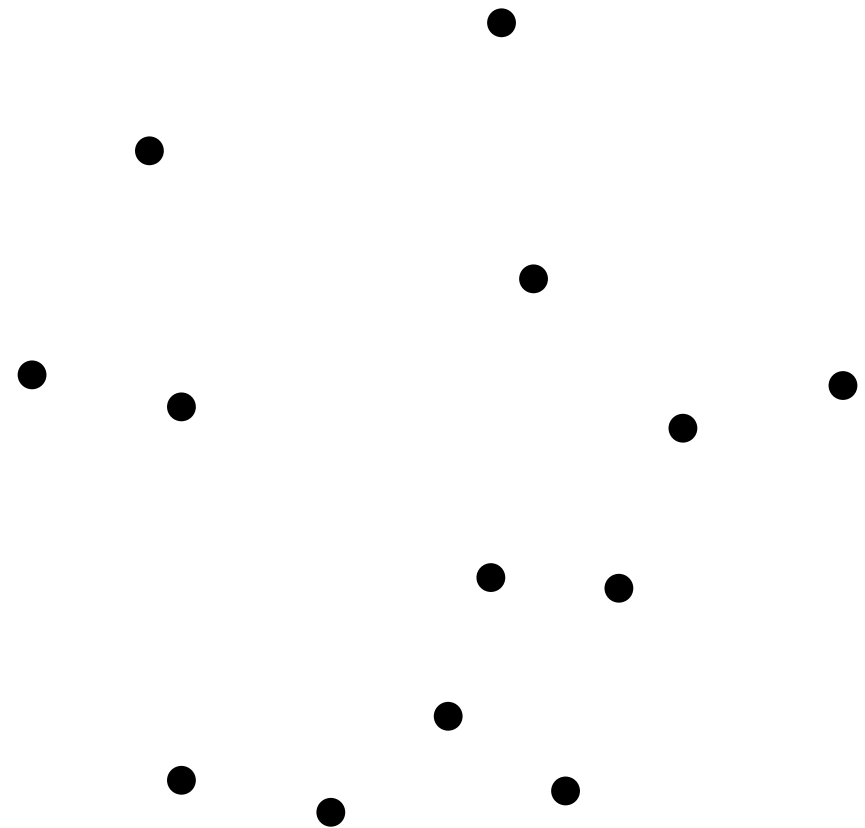
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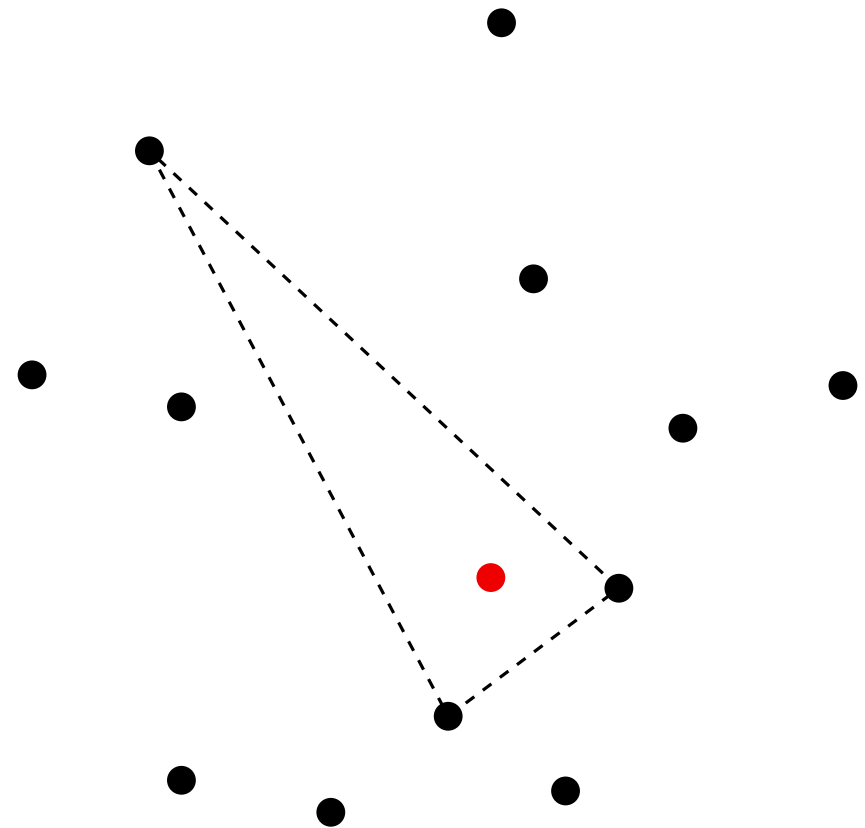
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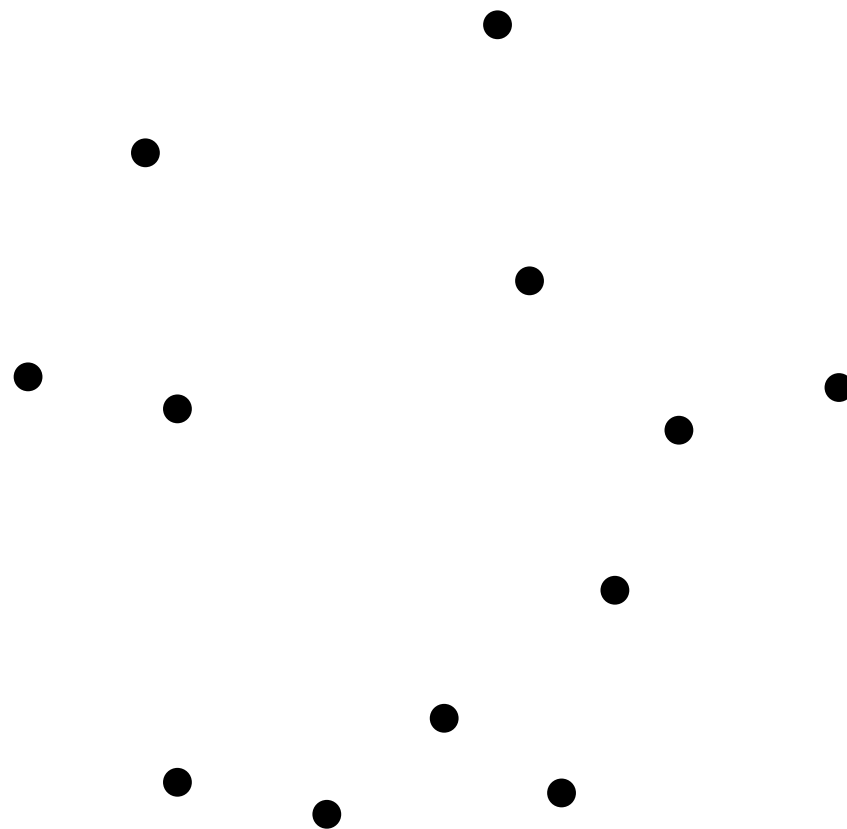
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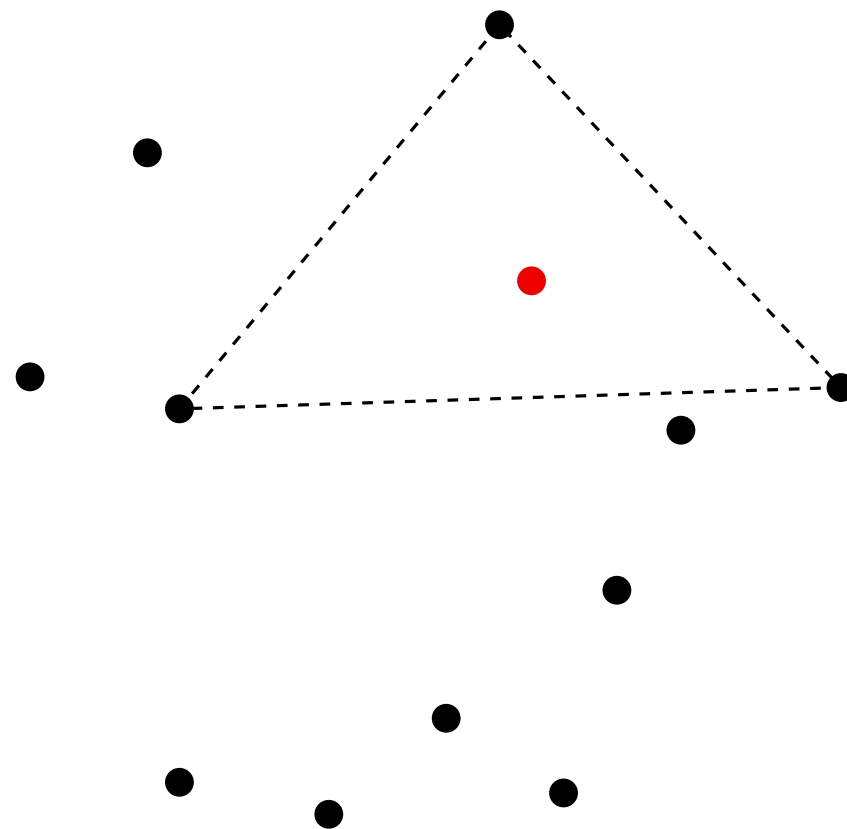
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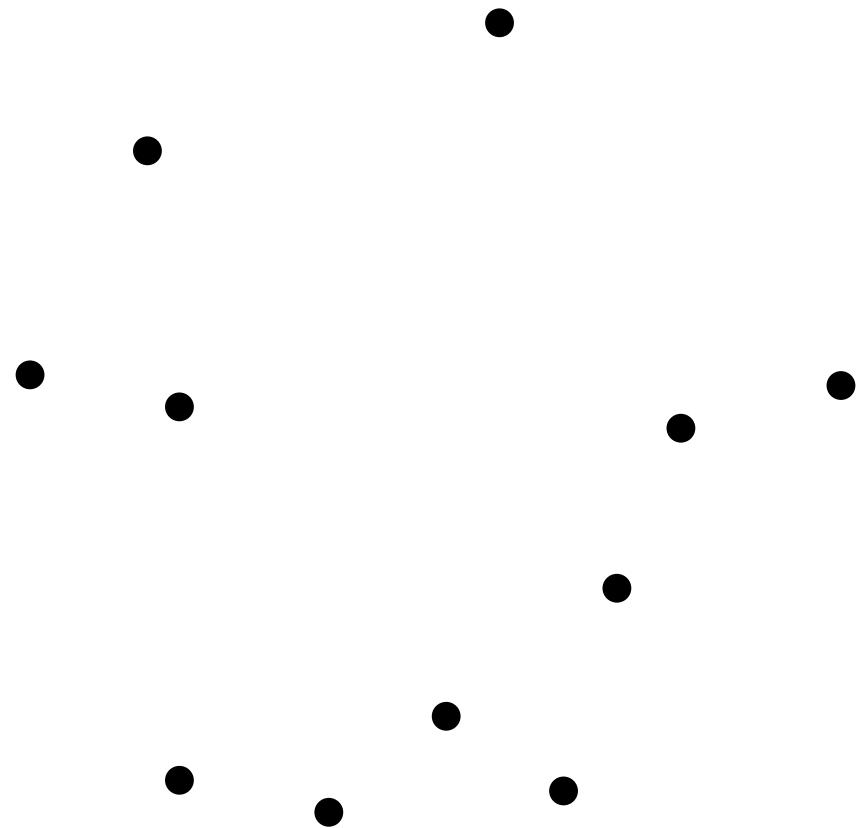
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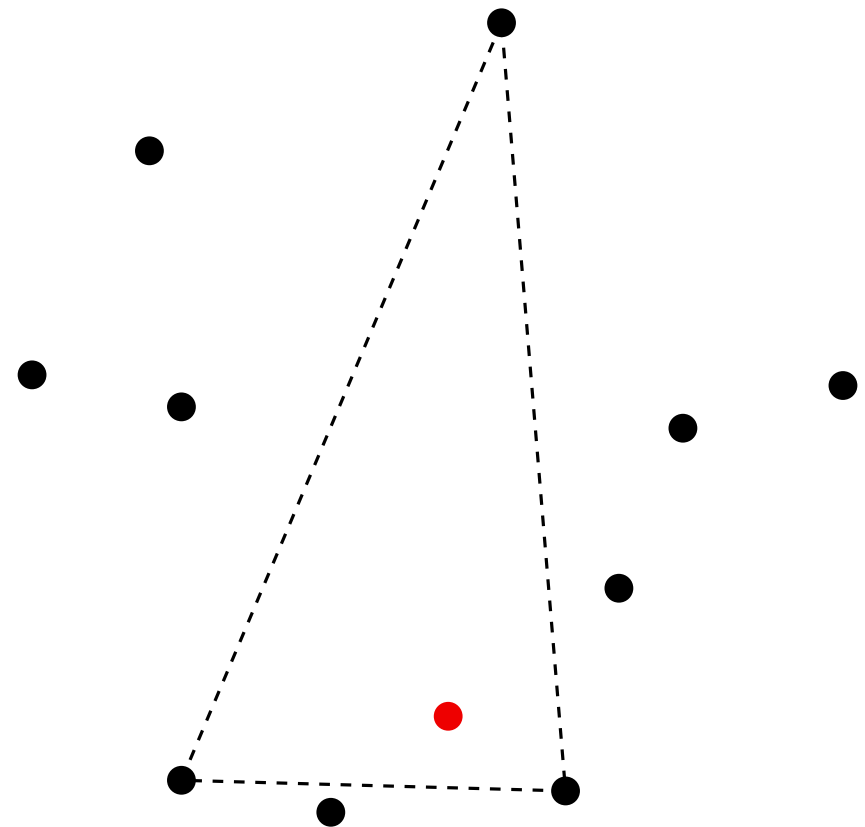
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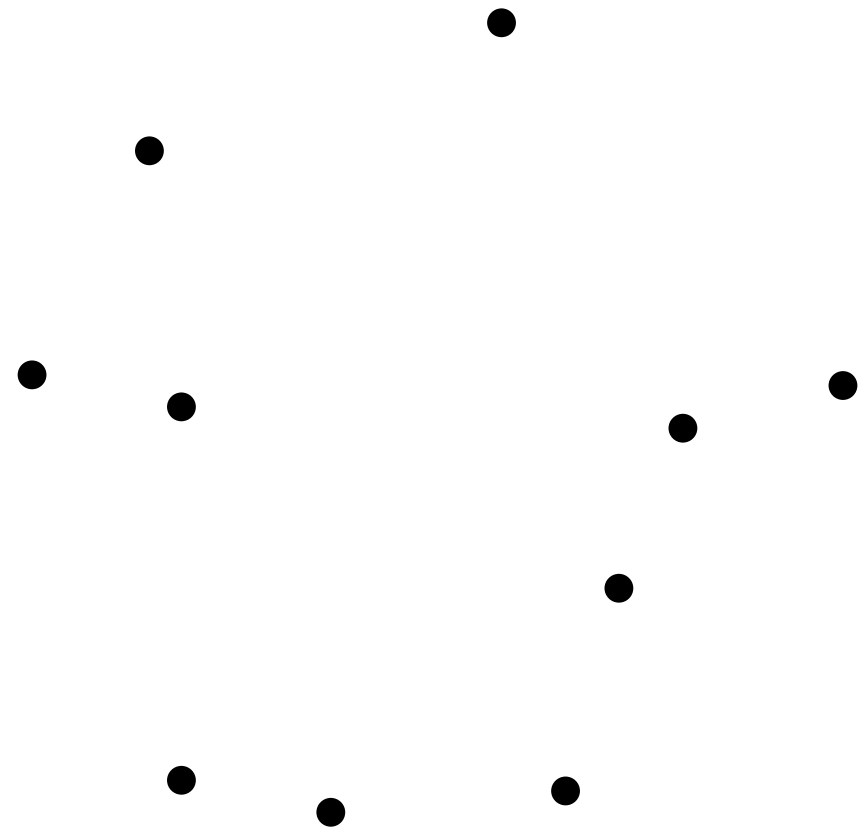
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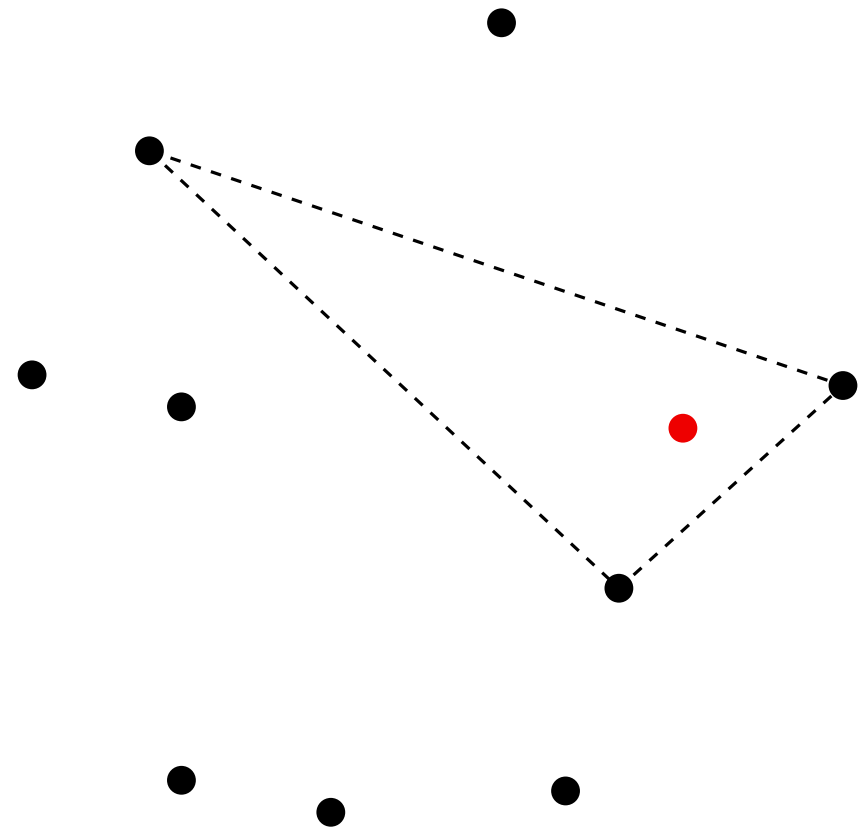
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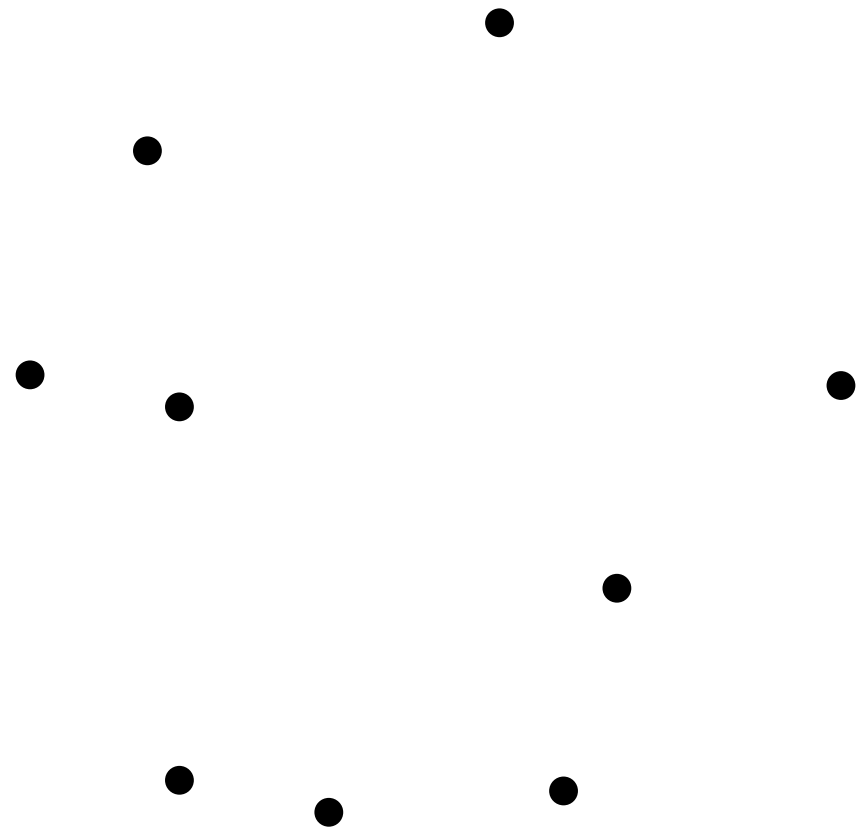
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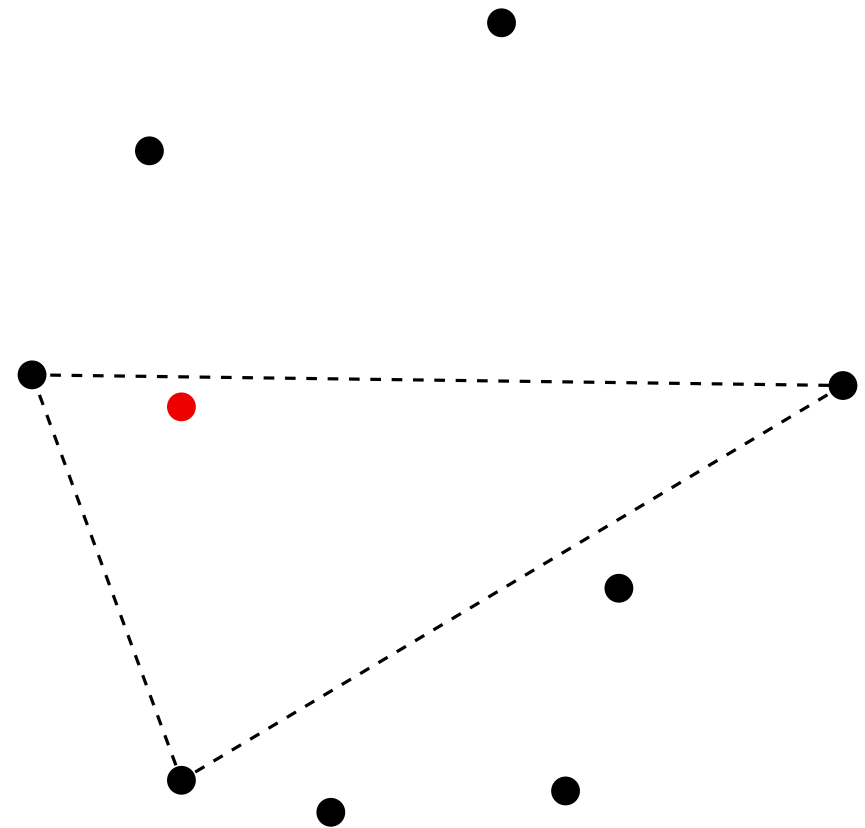
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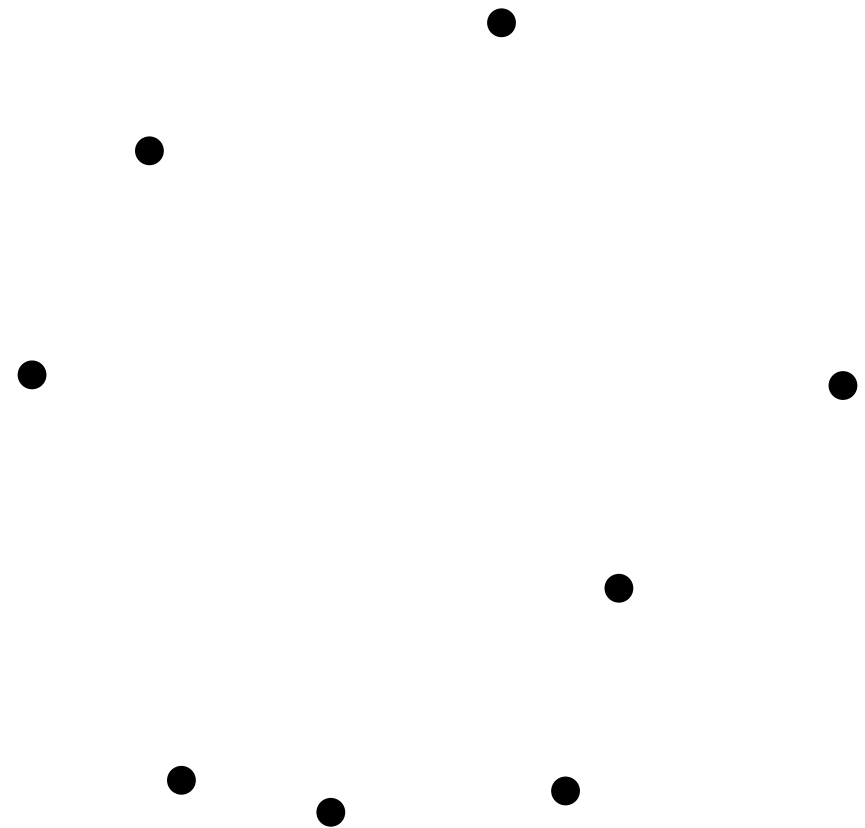
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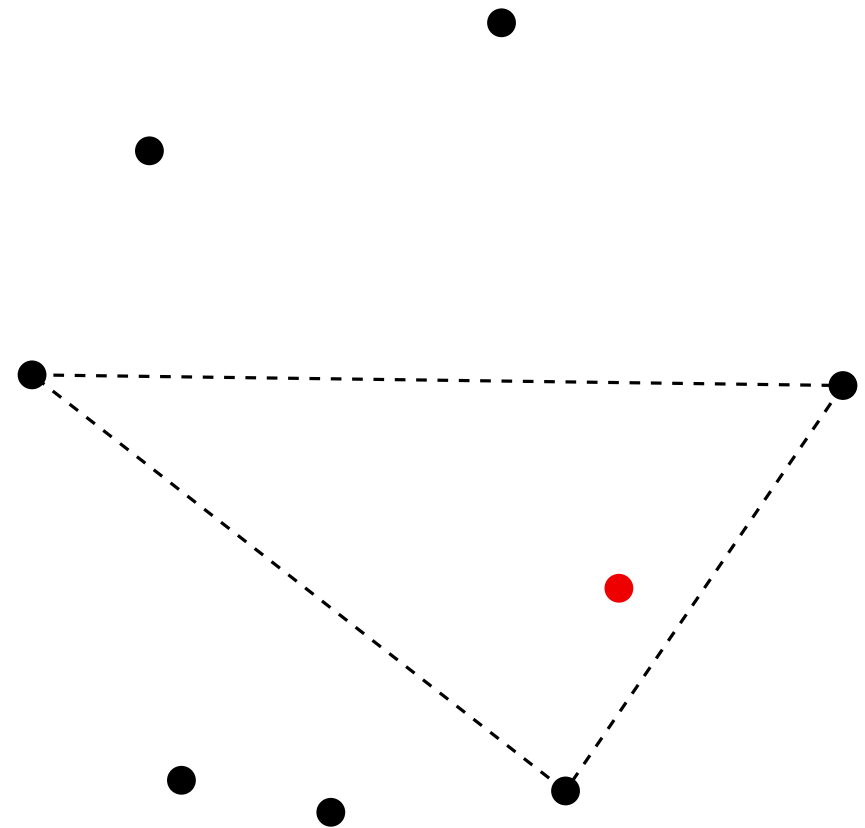
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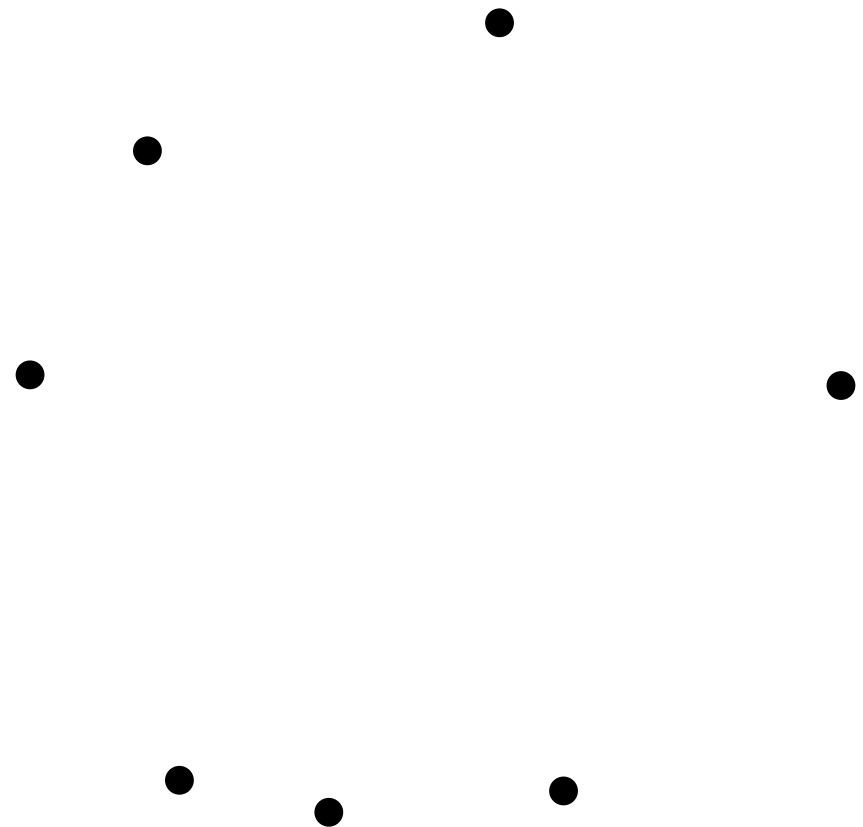
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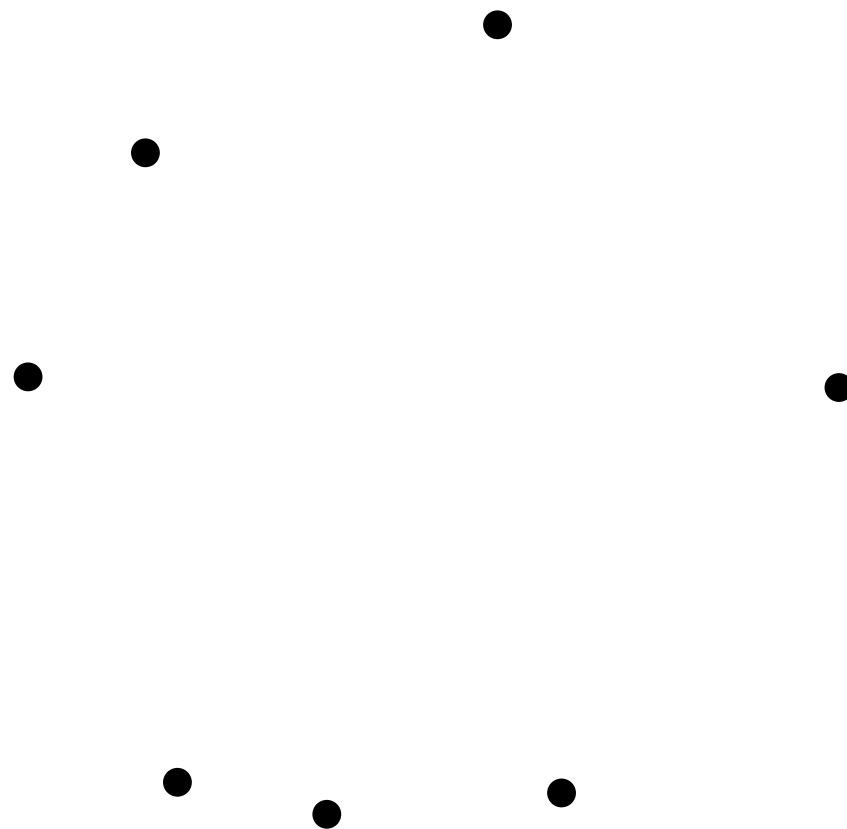
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Running time: $\Theta(n^4)$



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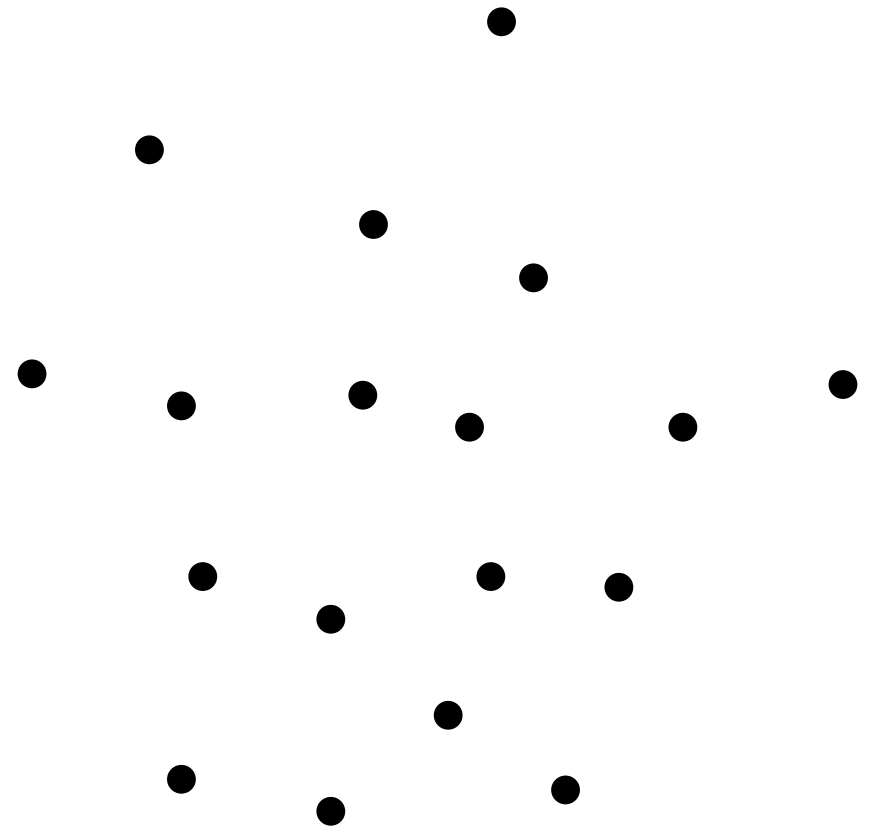
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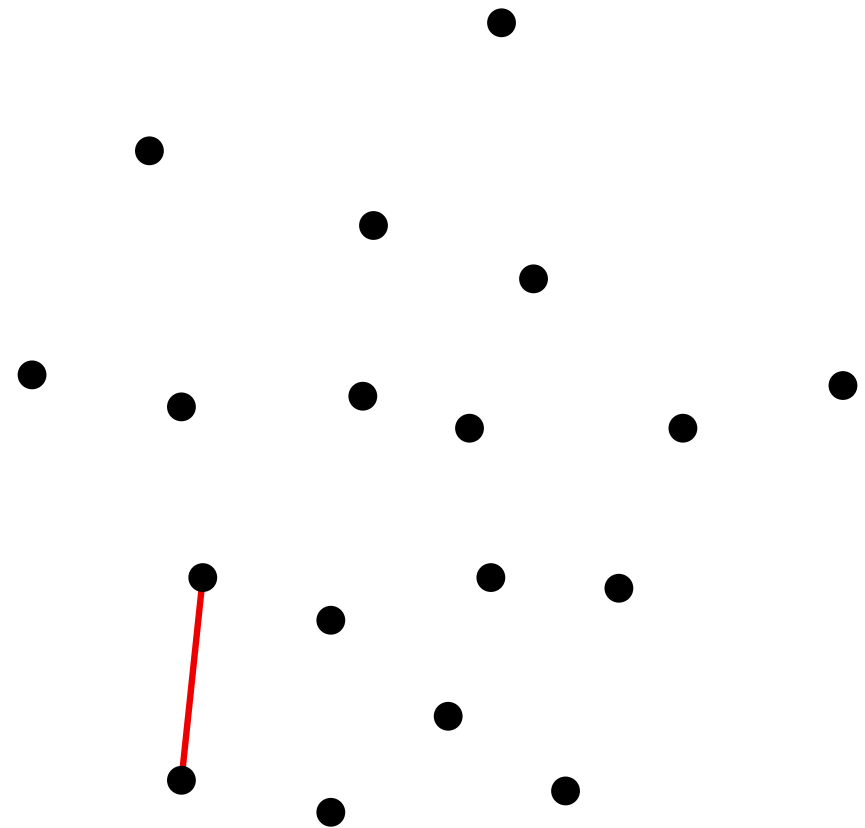


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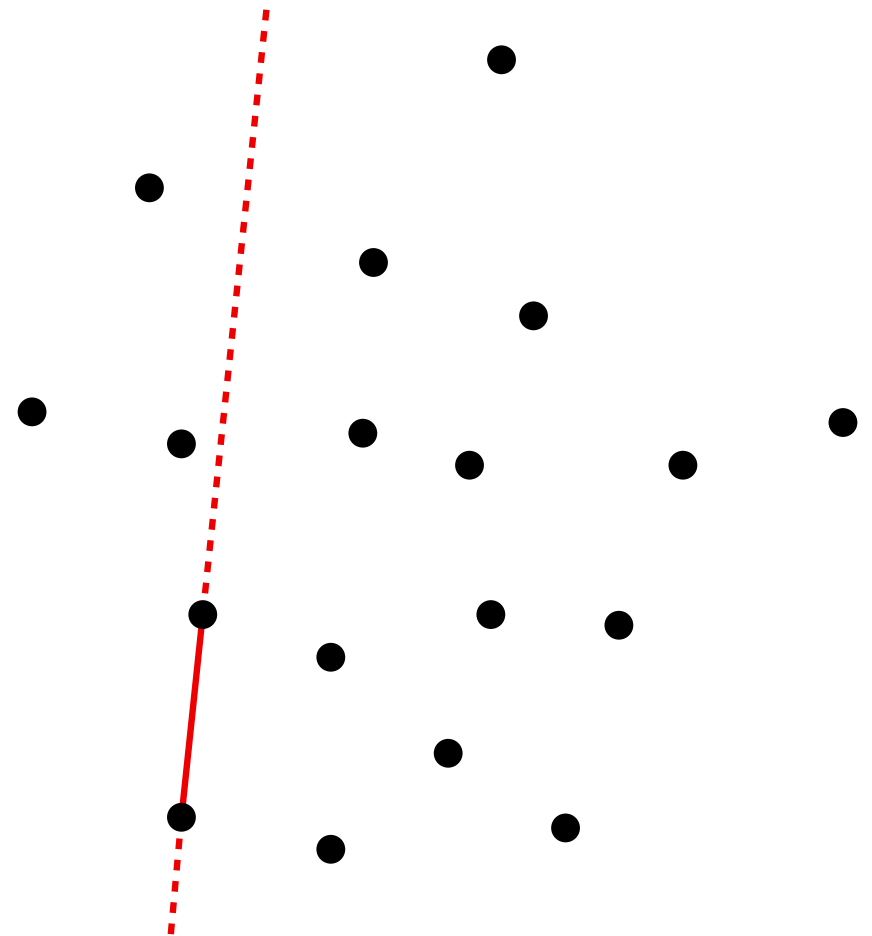


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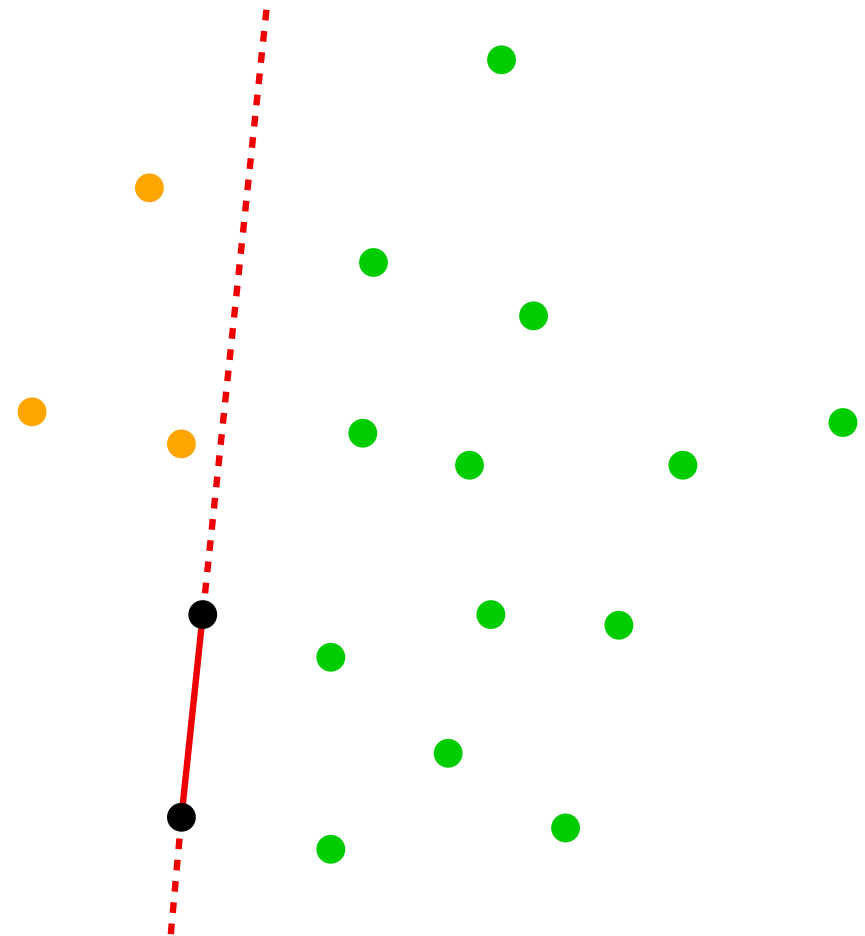


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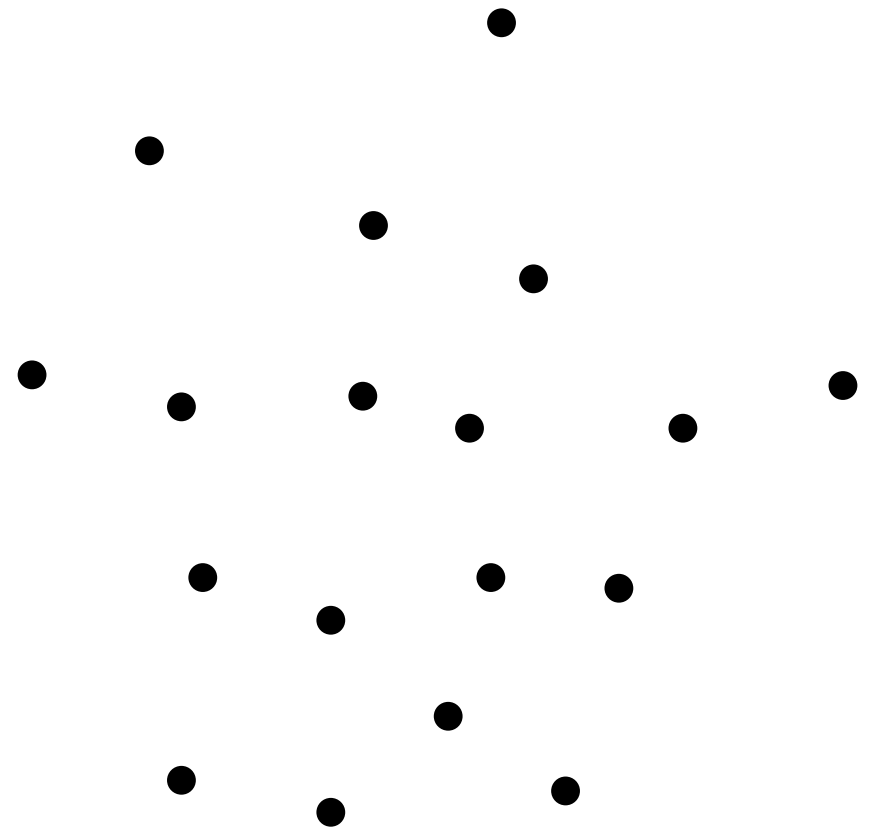


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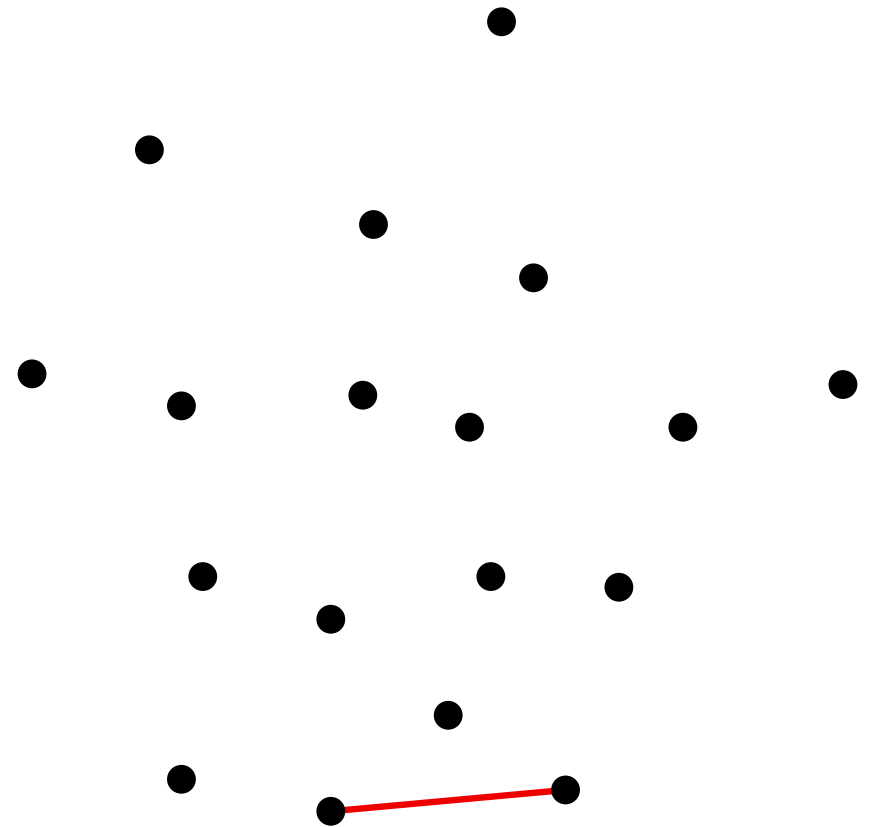


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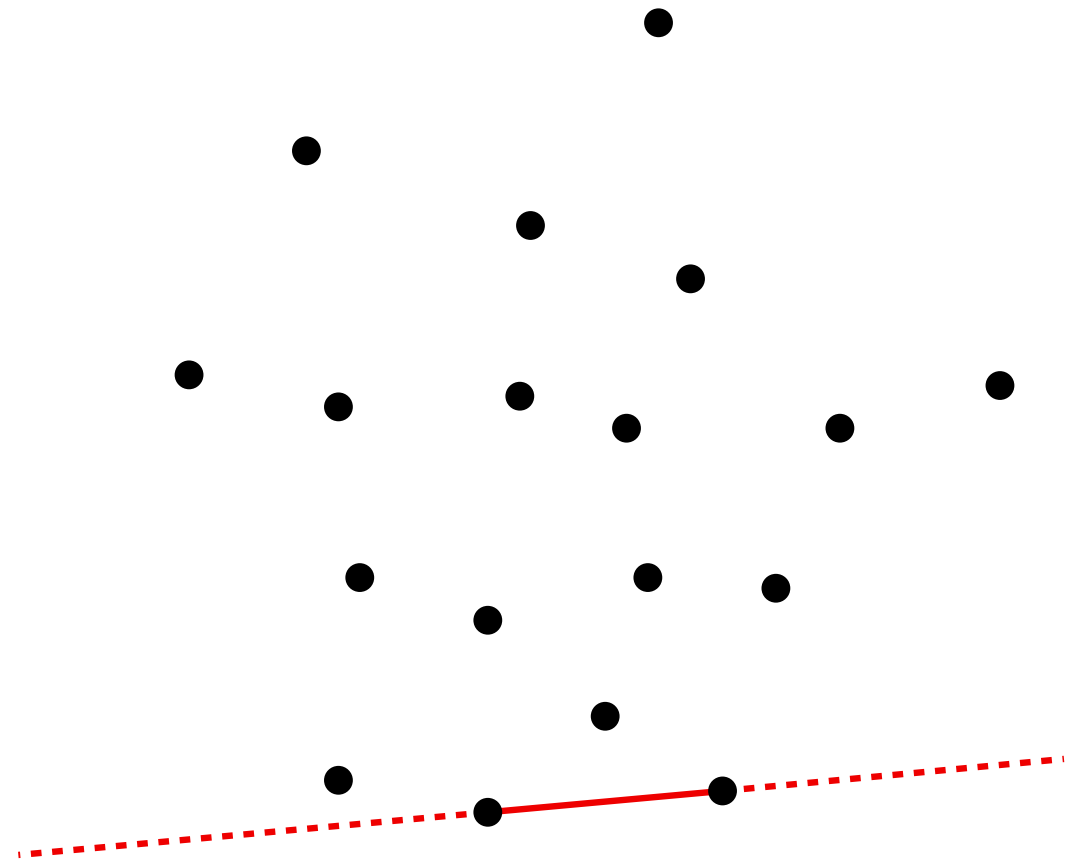


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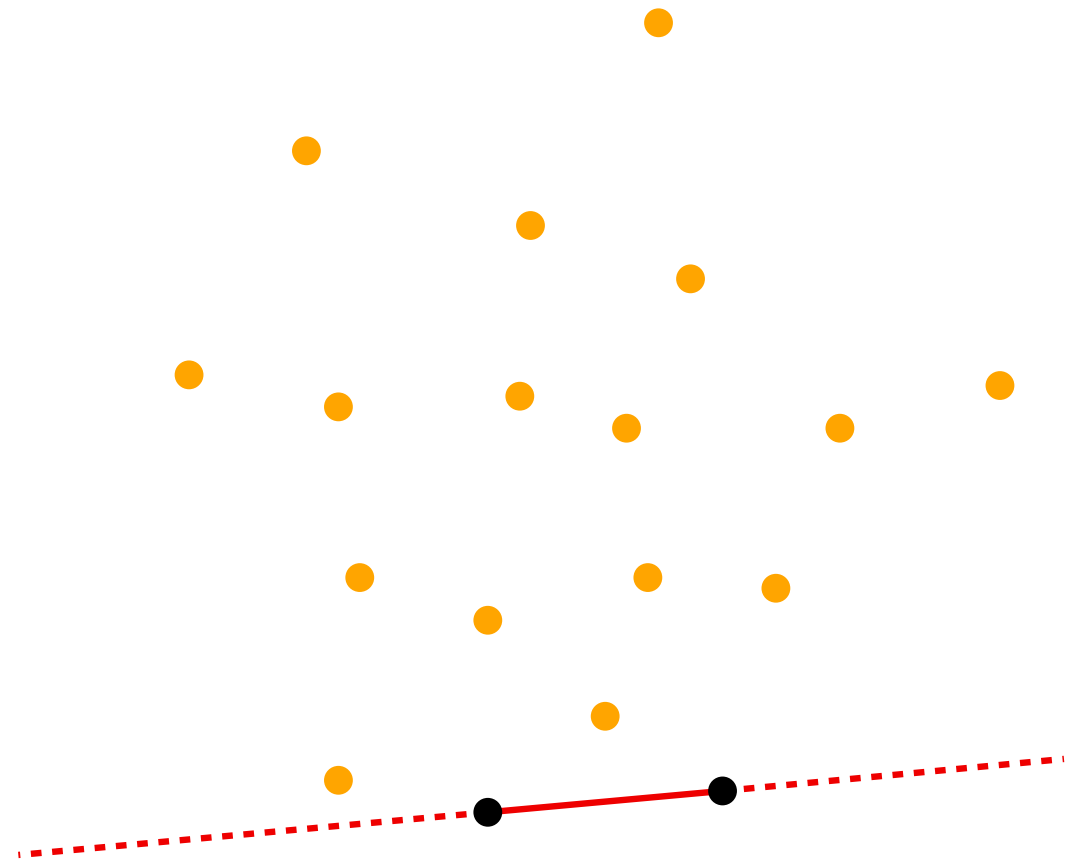


CONVEX HULL IN 2D

Computing the extreme segments

Characterization

Given $X = \{p_1, \dots, p_n\}$, the segment $p_i p_j$ is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line $p_i p_j$.



CONVEX HULL IN 2D

Computing the extreme segments

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Algorithm

Input: p_1, \dots, p_n

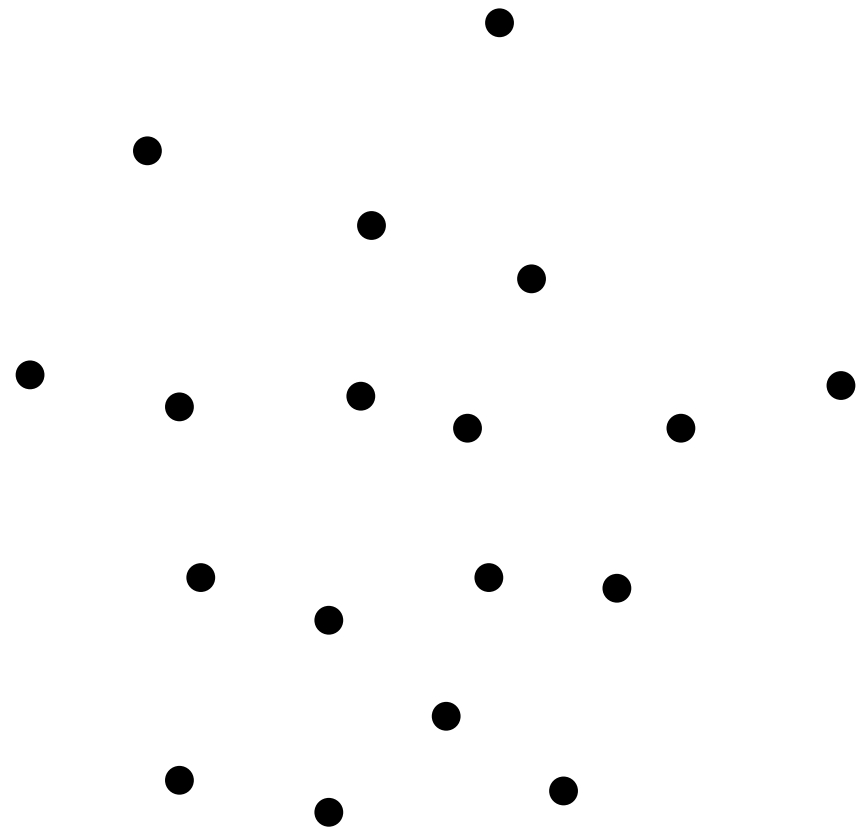
Output: set of the extreme segments

Procedure:

For each i, j ,

Check whether all p_k with $k \neq i, j$
lie in the same halfplane defined by $p_i p_j$.

In the affirmative, return the segment $p_i p_j$.



CONVEX HULL IN 2D

Computing the extreme segments

Characterization

Given $X = \{p_1, \dots, p_n\}$, the segment $p_i p_j$ is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line $p_i p_j$.

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Input: p_1, \dots, p_n

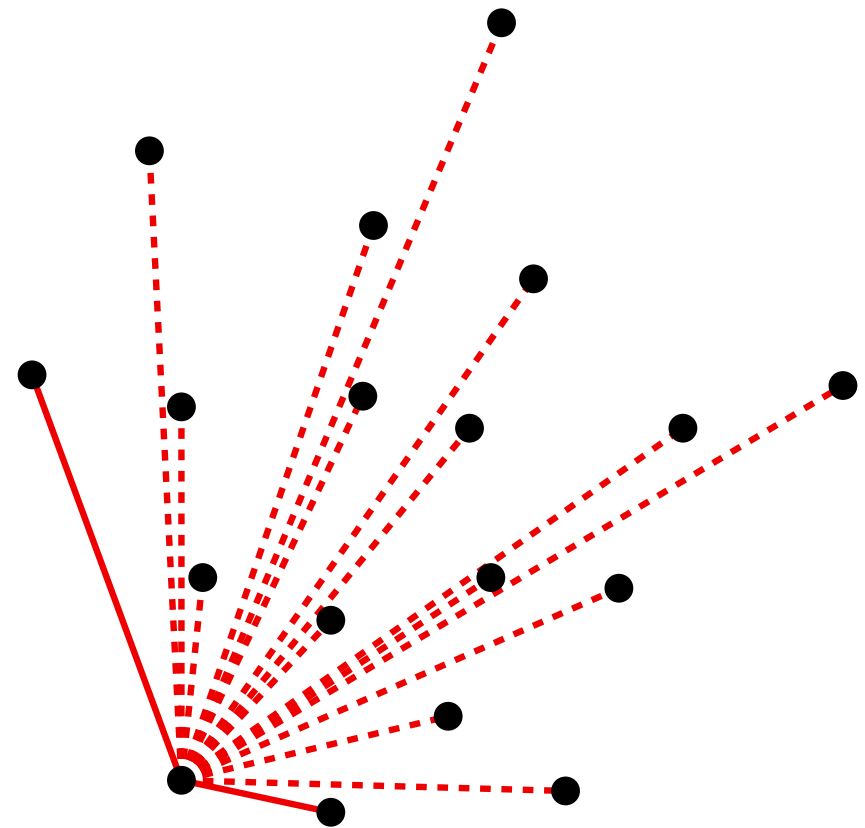
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CONVEX HULL IN 2D

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Algorithm

Input: p_1, \dots, p_n

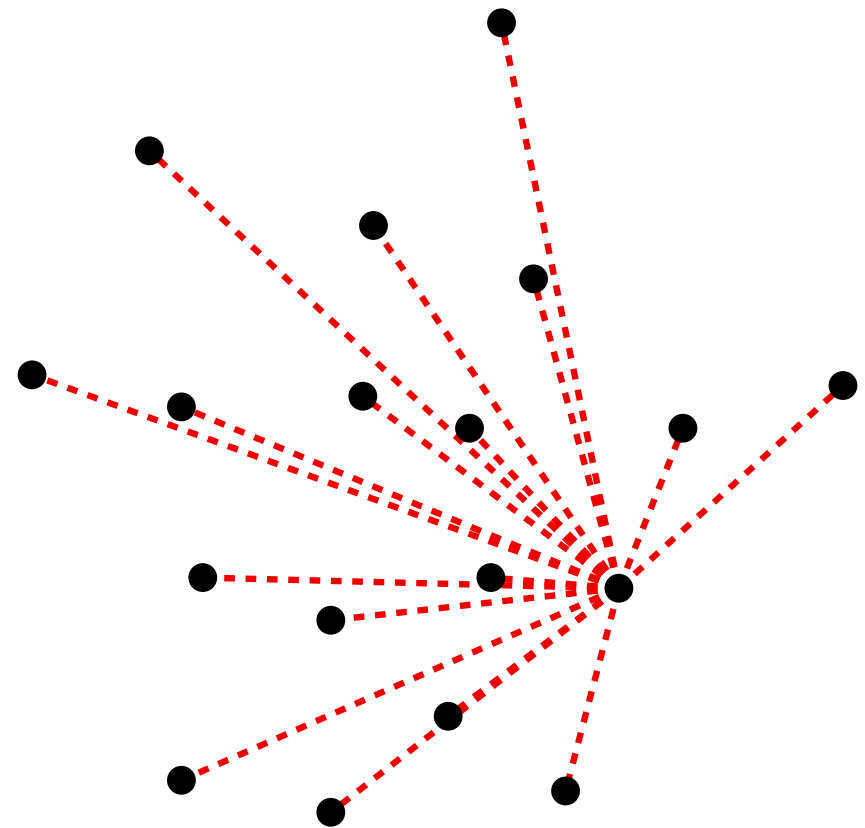
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CONVEX HULL IN 2D

Computing the extreme segments

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Algorithm

Input: p_1, \dots, p_n

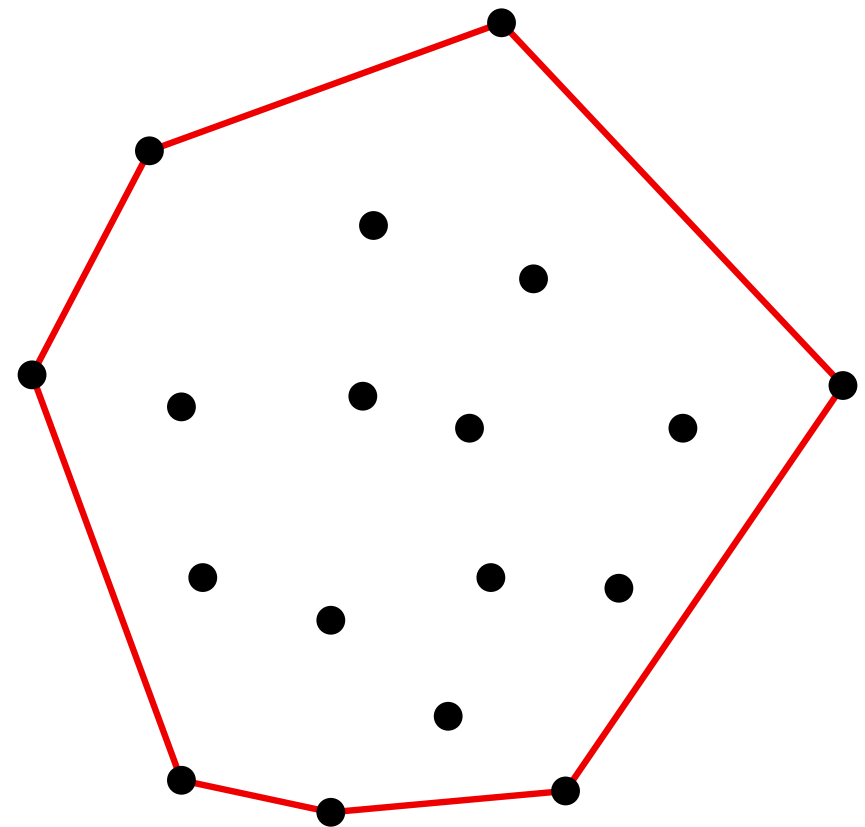
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CONVEX HULL IN 2D

Computing the extreme segments

Characterization

Given $X = \{p_1, \dots, p_n\}$, the segment $p_i p_j$ is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line $p_i p_j$.

Algorithm

Input: p_1, \dots, p_n

Output: set of the extreme segments

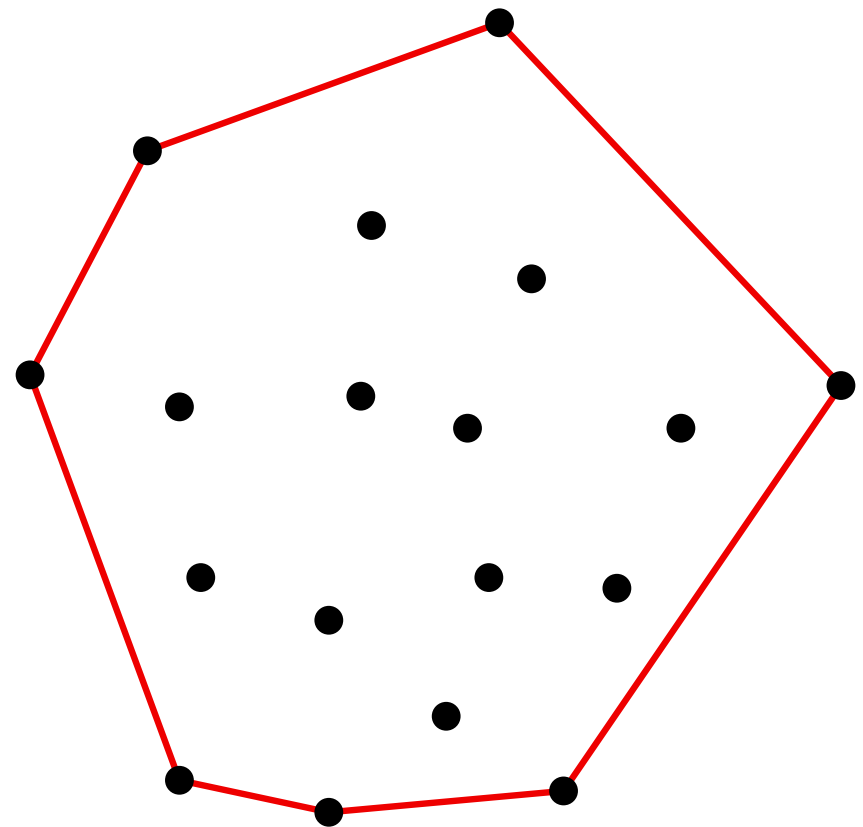
Procedure:

For each i, j ,

Check whether all p_k with $k \neq i, j$
lie in the same halfplane defined by $p_i p_j$.

In the affirmative, return the segment $p_i p_j$.

Running time: $\Theta(n^3)$



CONVEX HULL IN 2D

Computing the convex hull

CONVEX HULL IN 2D

Computing the convex hull (sorted list of its vertices)

CONVEX HULL IN 2D

Computing the convex hull

Input:

$P = \{p_1, \dots, p_n\} \subset \mathbb{R}^2$ a set of n points in the plane

Output:

l , the list of the vertices of $ch(P)$ sorted in counterclockwise order

CONVEX HULL IN 2D

Computing the convex hull

Input:

$P = \{p_1, \dots, p_n\} \subset \mathbb{R}^2$ a set of n points in the plane

Output:

l , the list of the vertices of $ch(P)$ sorted in counterclockwise order

Characterization

Given $X = \{p_1, \dots, p_n\}$, the segment $p_i p_j$ is an edge of the convex hull of X if and only if all the points p_k with $k \neq i, j$ lie to the left of the oriented line $p_i p_j$.

CONVEX HULL IN 2D

Jarvis march

CONVEX HULL IN 2D

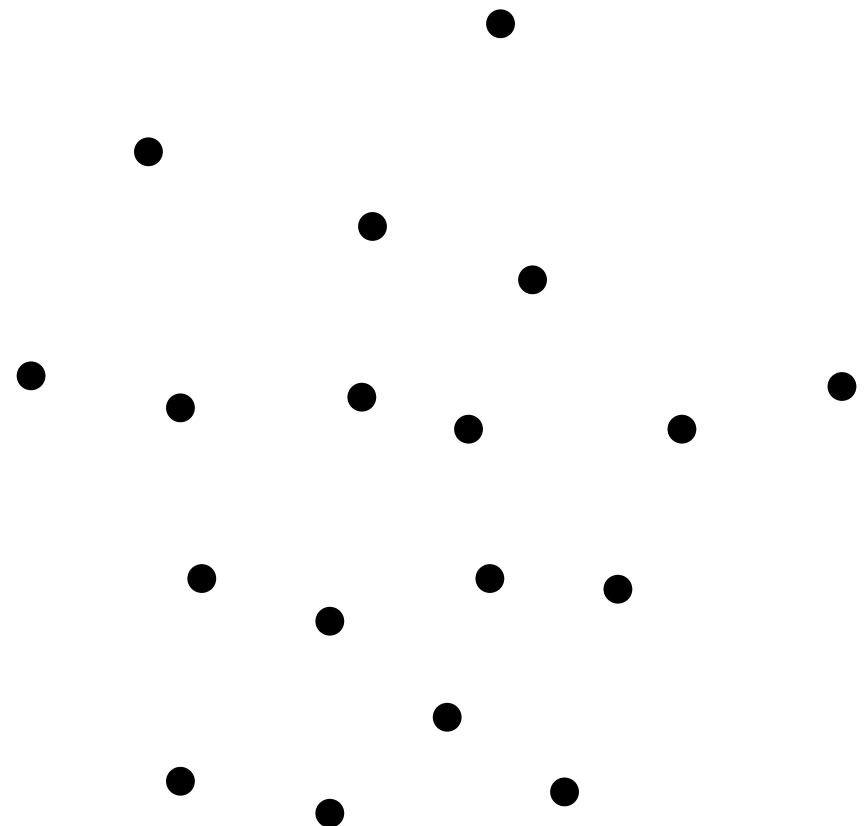
Jarvis march

1. Find a vertex of $ch(P)$ (for example, the lexicographically smaller point $p_i \in P$) and add it to l
2. While $v = \text{Last}(l) \neq \text{First}(l)$, do:
 - (a) Detect the angularly rightmost point $p_j \in P$ with respect to v .
 - (b) Add p_j to l
3. Return l

CONVEX HULL IN 2D

Jarvis march

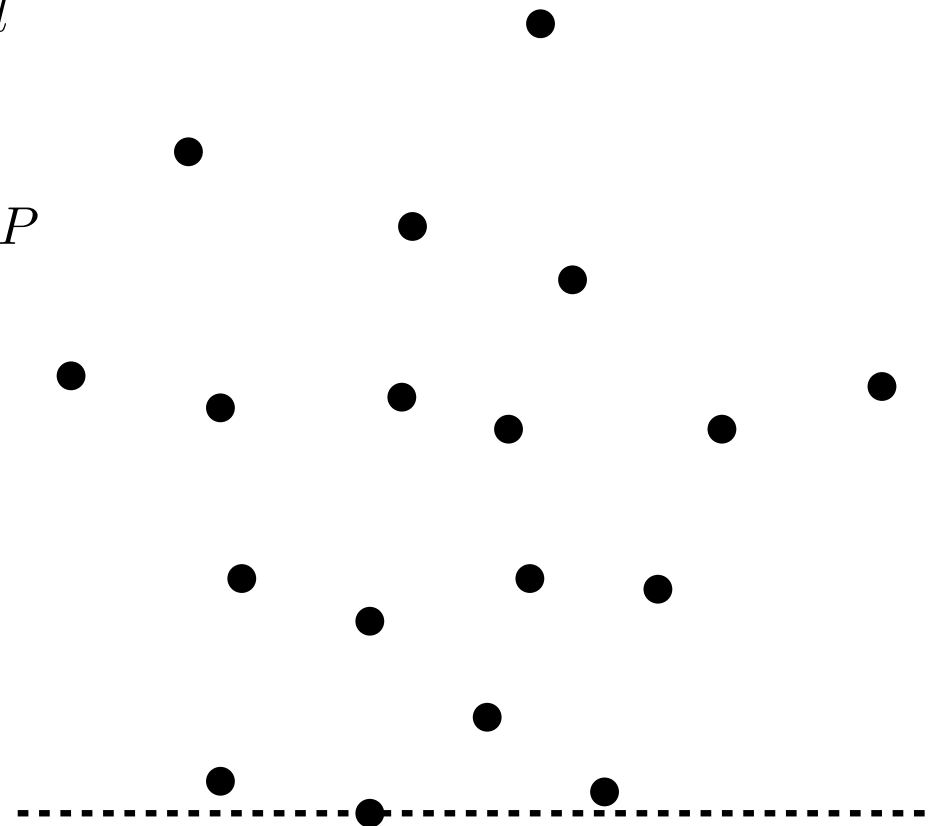
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CONVEX HULL IN 2D

Jarvis march

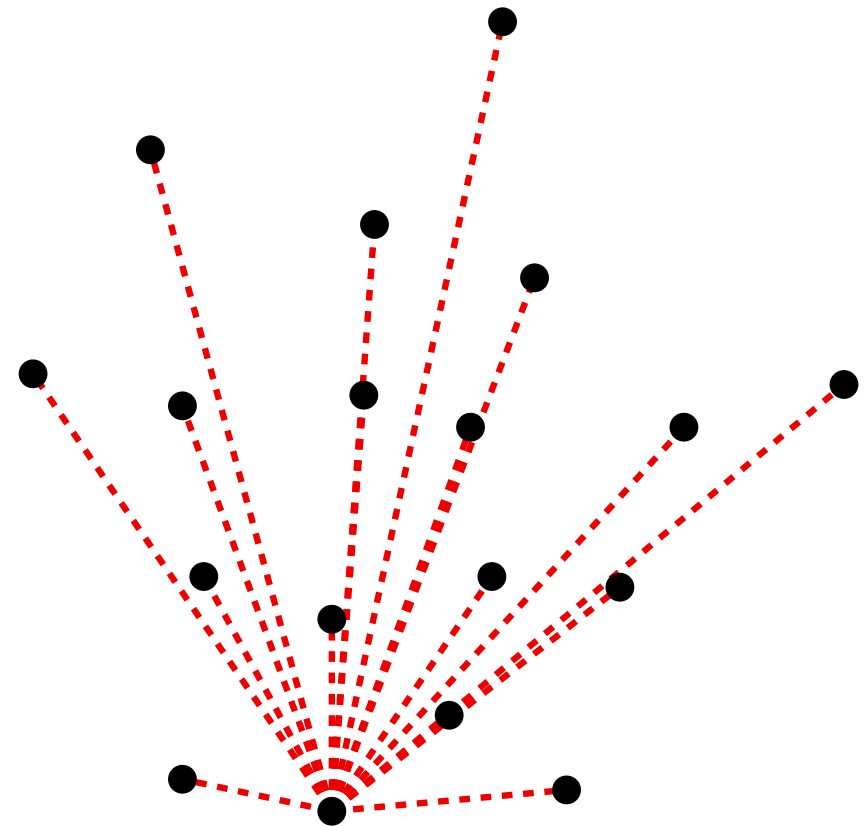
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CONVEX HULL IN 2D

Jarvis march

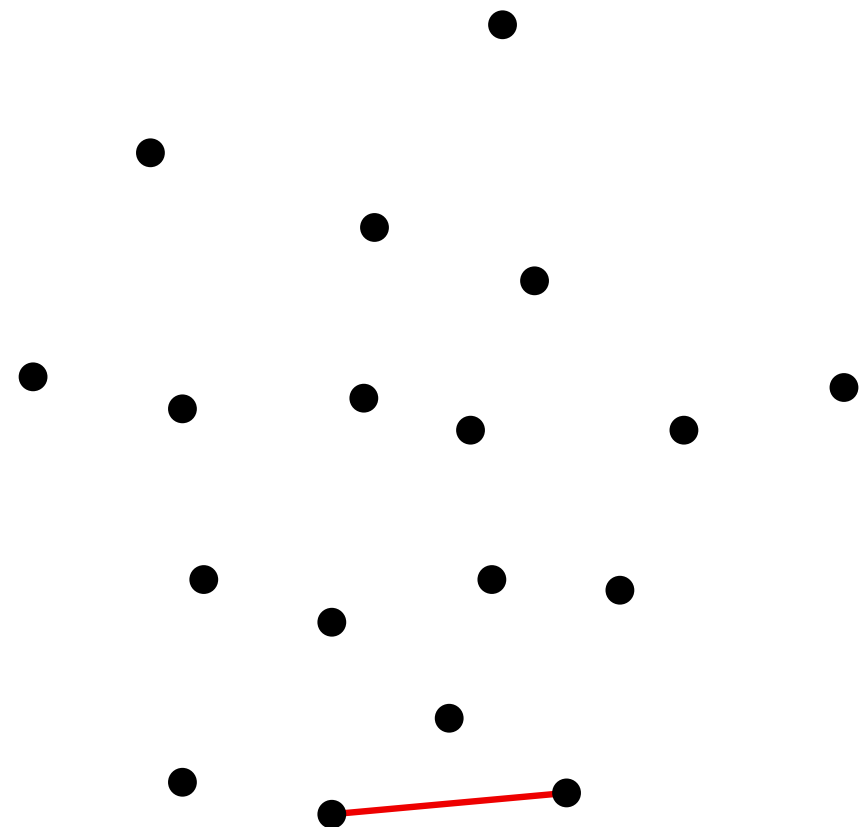
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CONVEX HULL IN 2D

Jarvis march

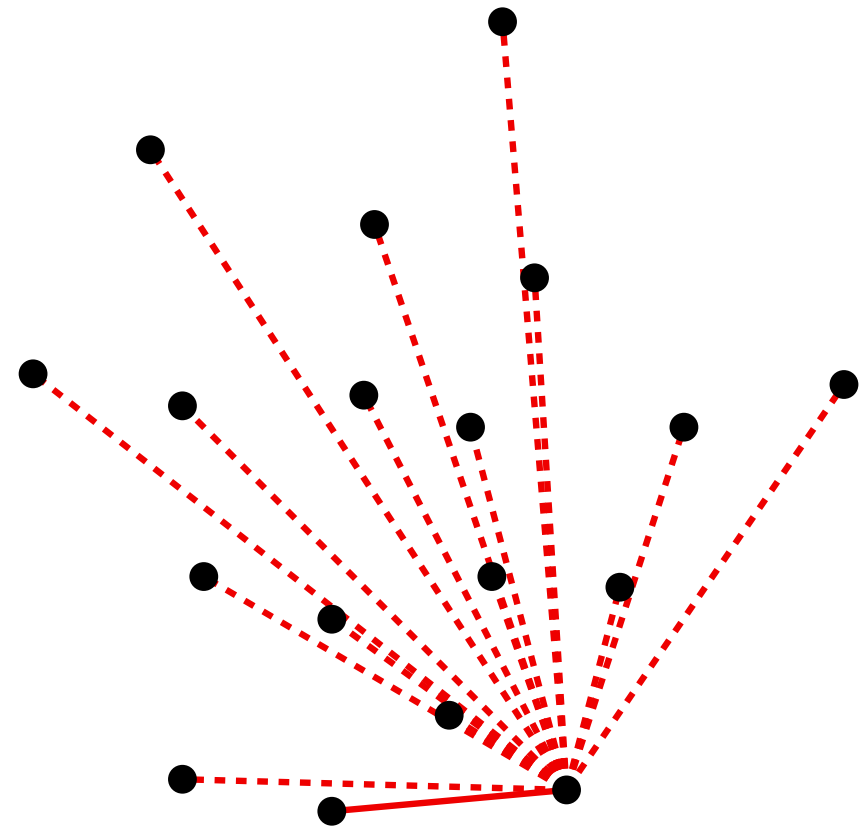
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CONVEX HULL IN 2D

Jarvis march

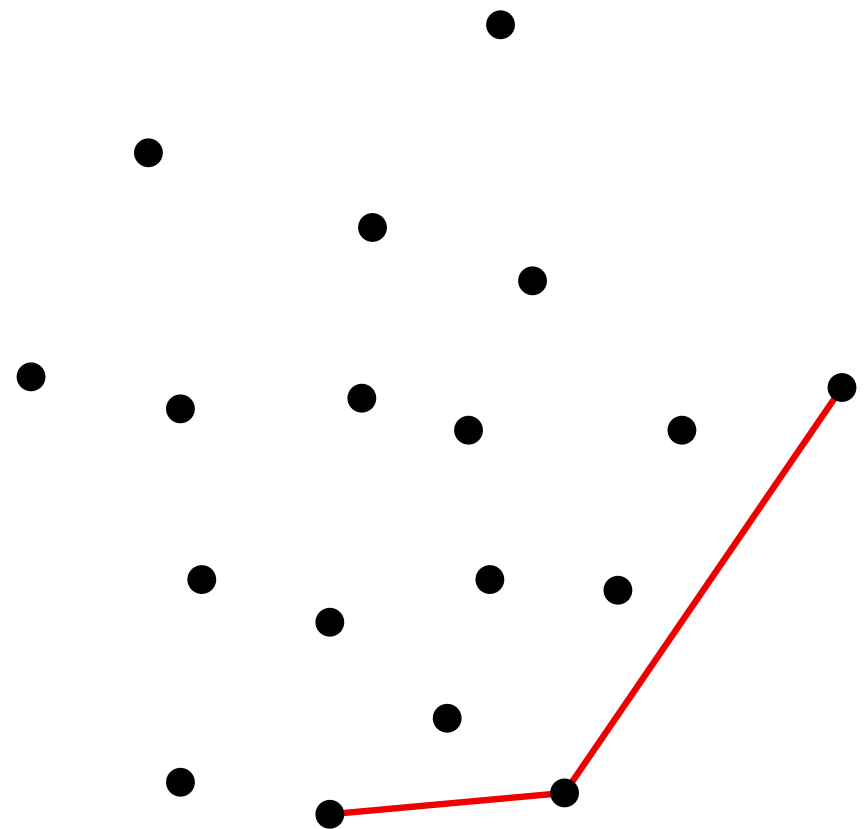
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CONVEX HULL IN 2D

Jarvis march

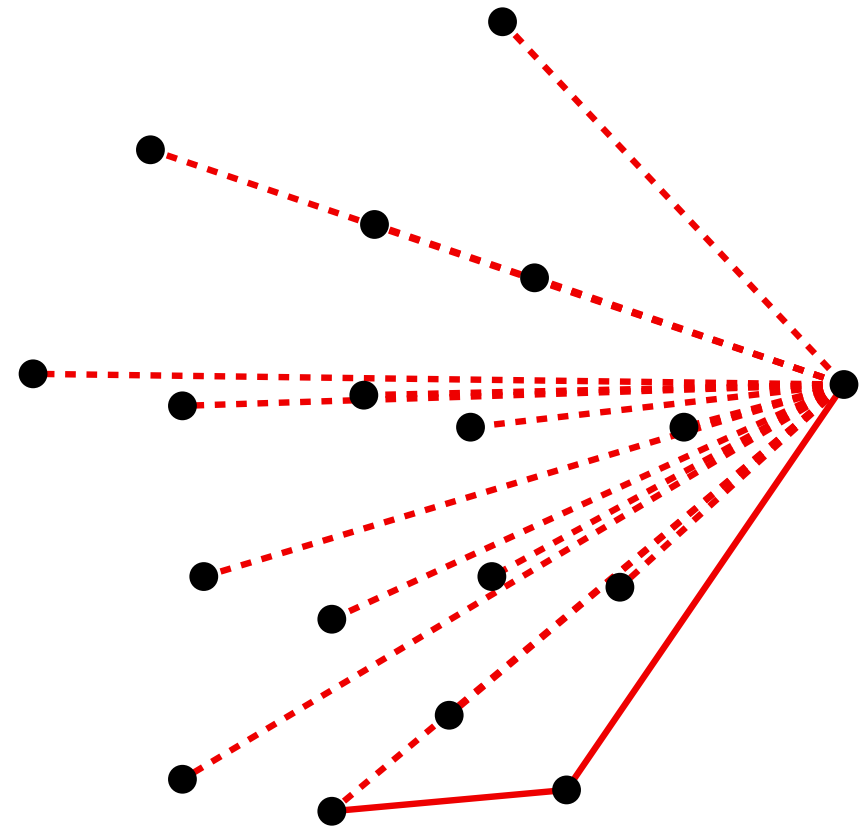
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CONVEX HULL IN 2D

Jarvis march

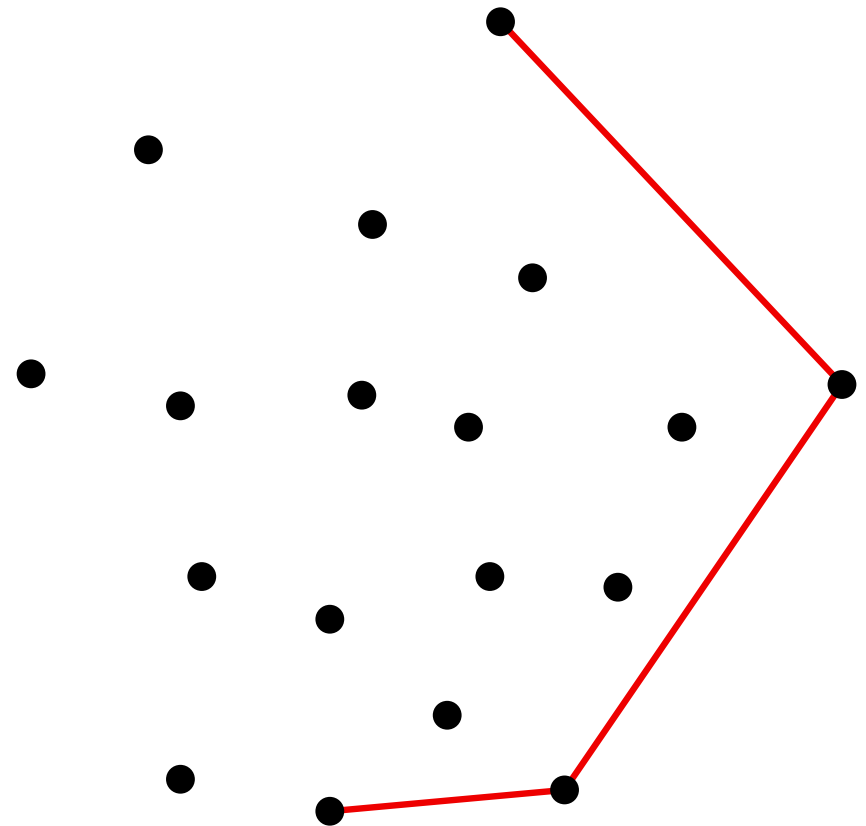
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CONVEX HULL IN 2D

Jarvis march

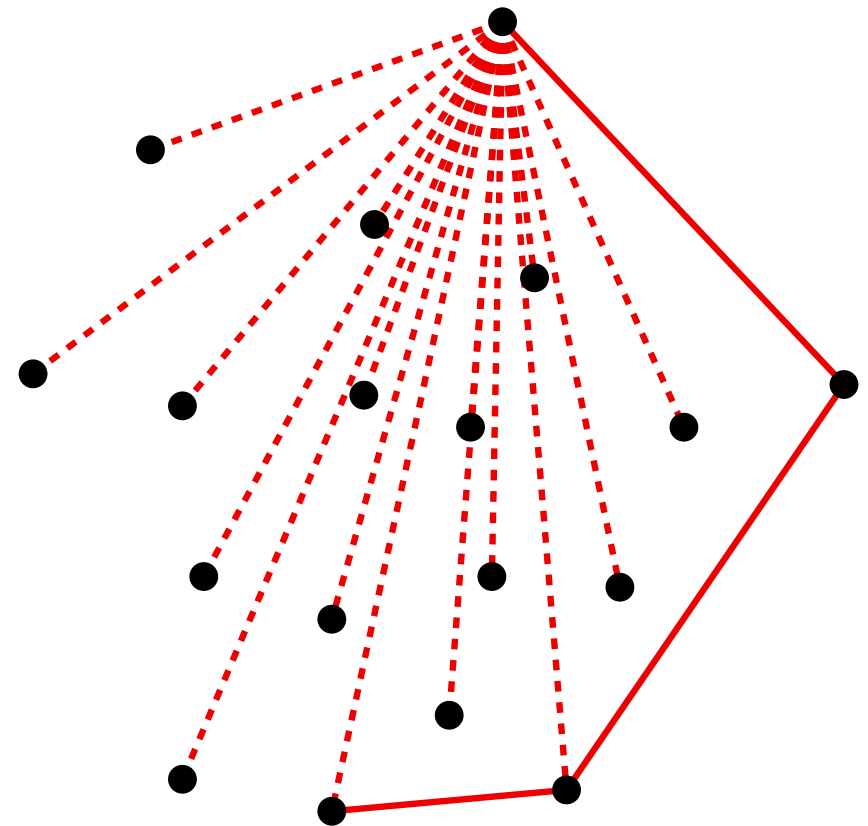
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Jarvis march

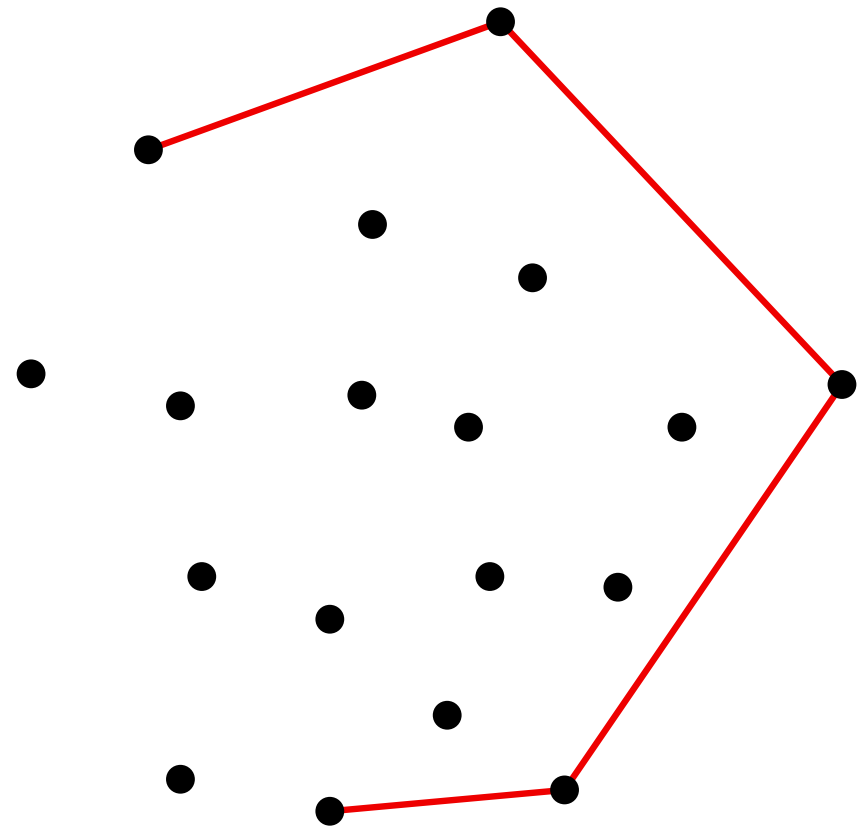
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CONVEX HULL IN 2D

Jarvis march

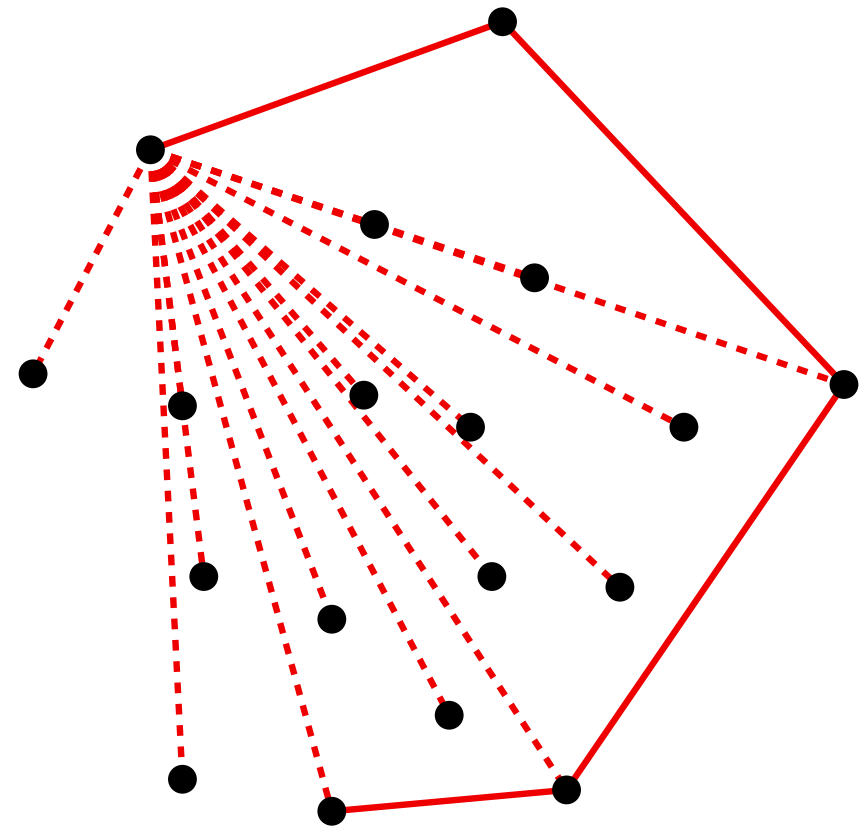
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Jarvis march

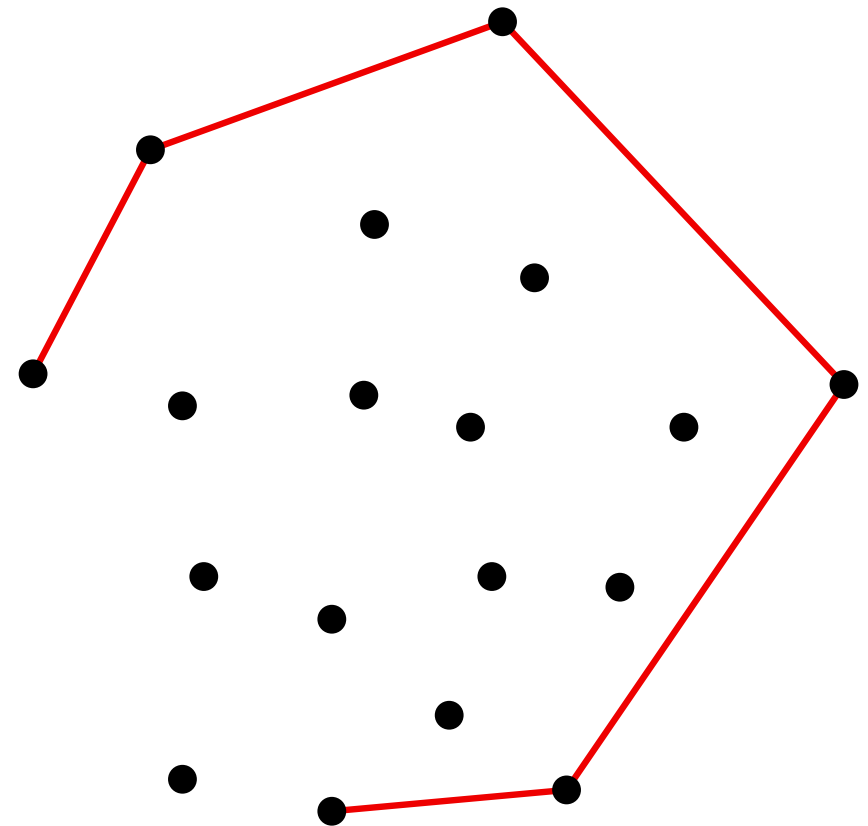
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Jarvis march

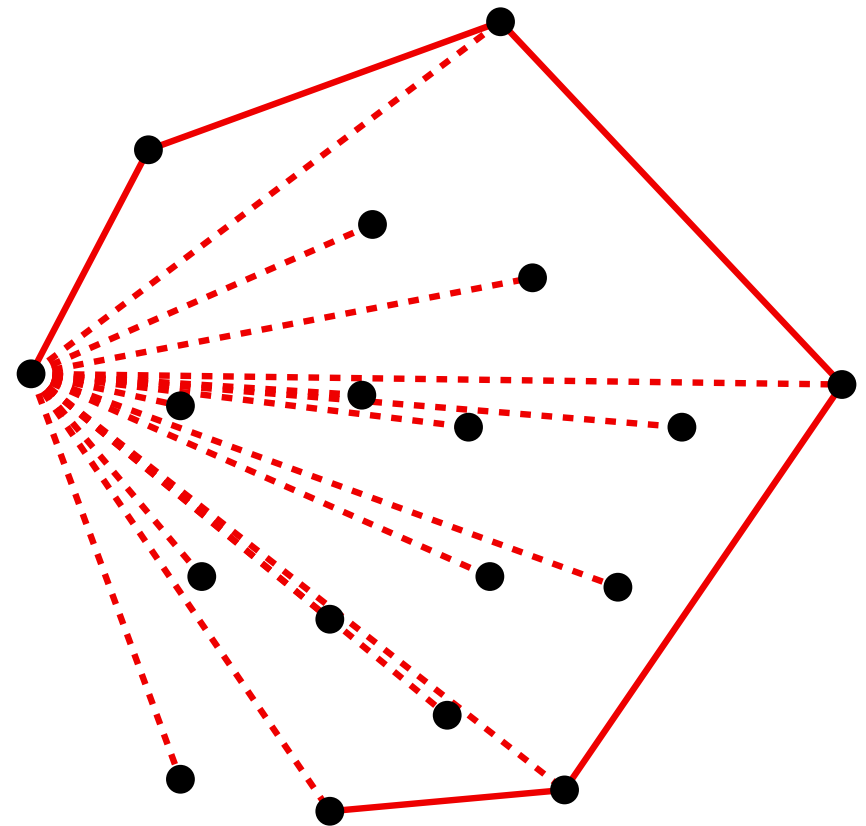
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Jarvis march

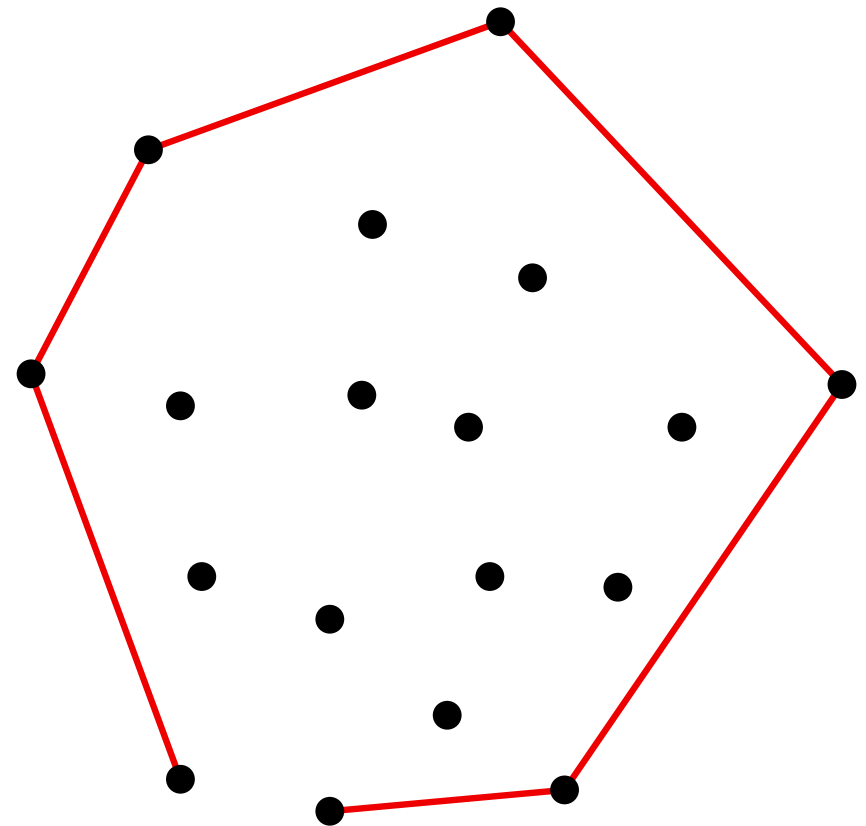
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CONVEX HULL IN 2D

Jarvis march

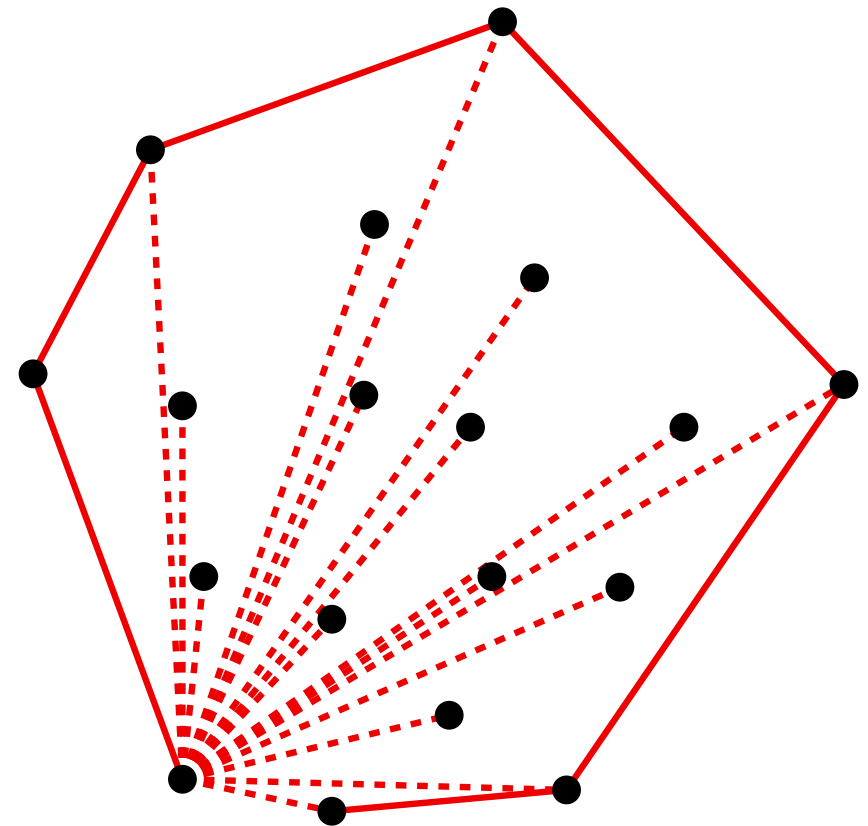
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Jarvis march

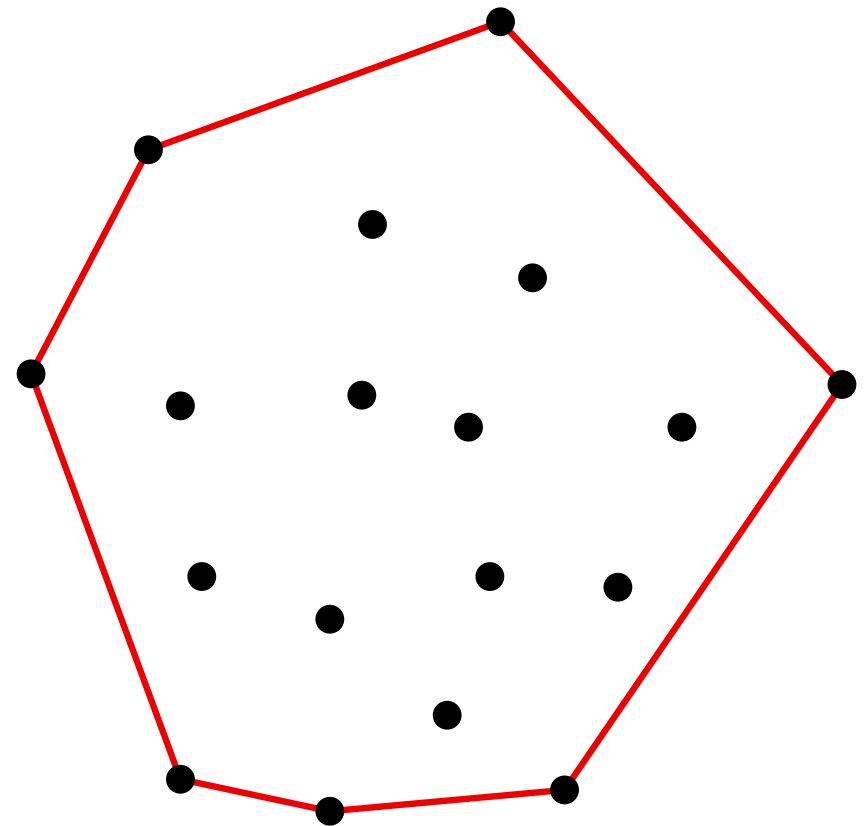
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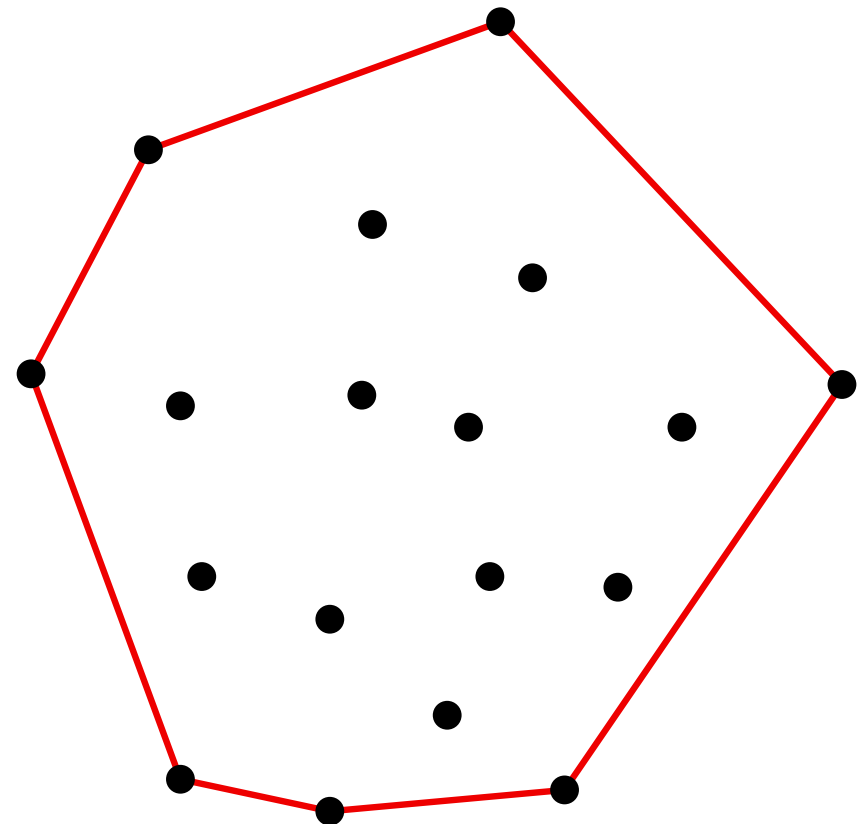


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Time cost: $\Theta(hn) = O(n^2)$



CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

Initialization

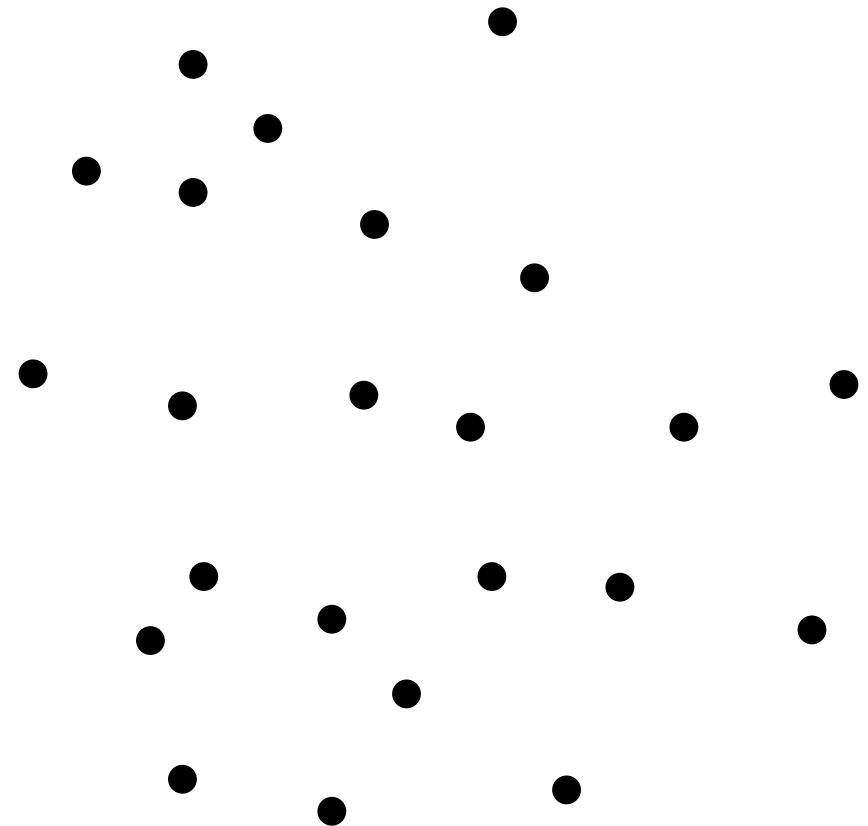
1. Find the extreme points in the horizontal and vertical directions.
2. Compute the convex hull of these (between 2 and 8) points.
3. Test all the remaining points, and classify them according to their position (NE, SE, SW, NW) or eliminate them if they lie in the interior.

CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

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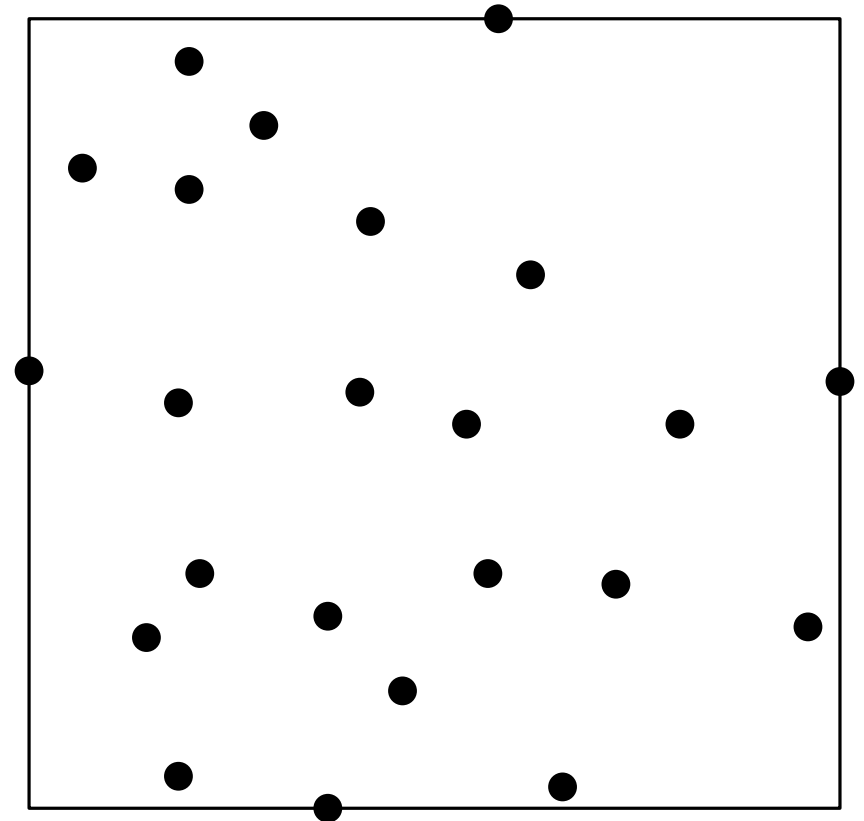


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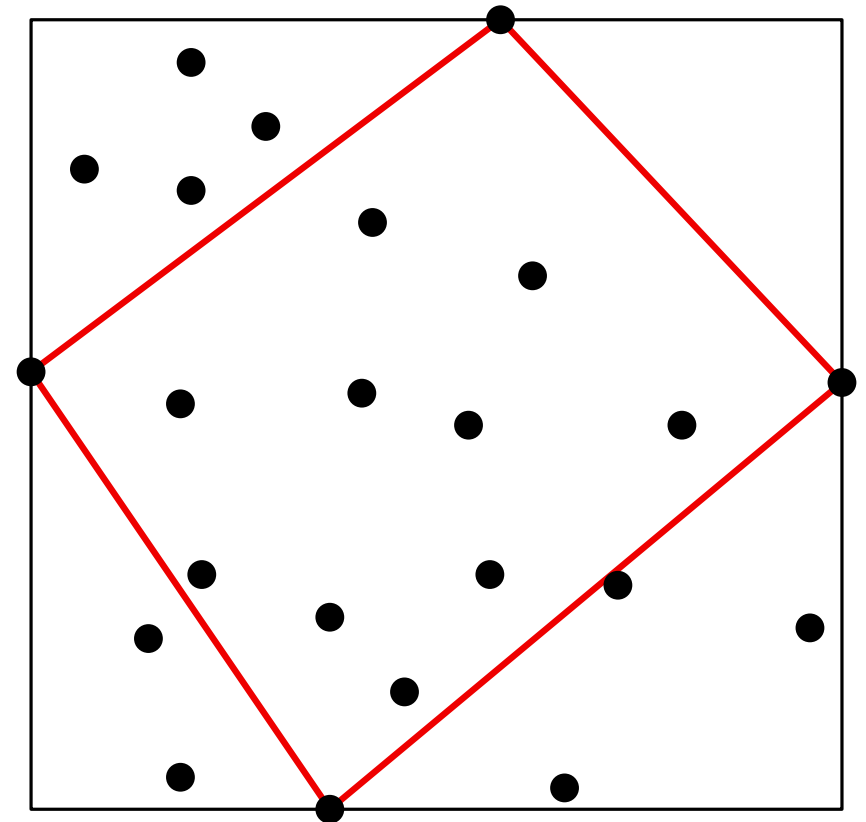


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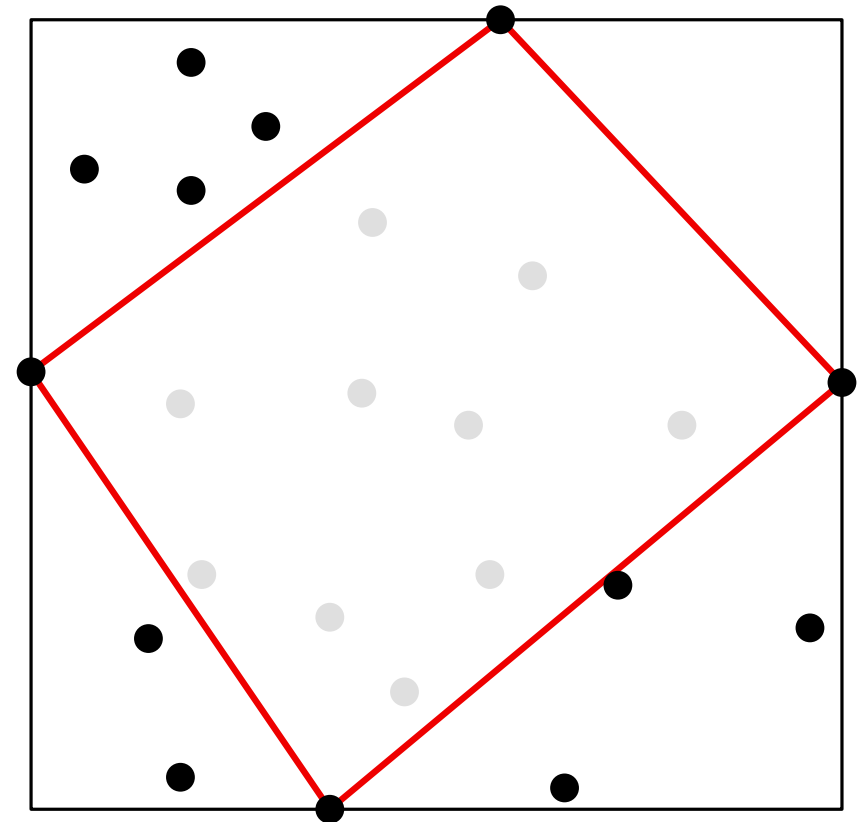


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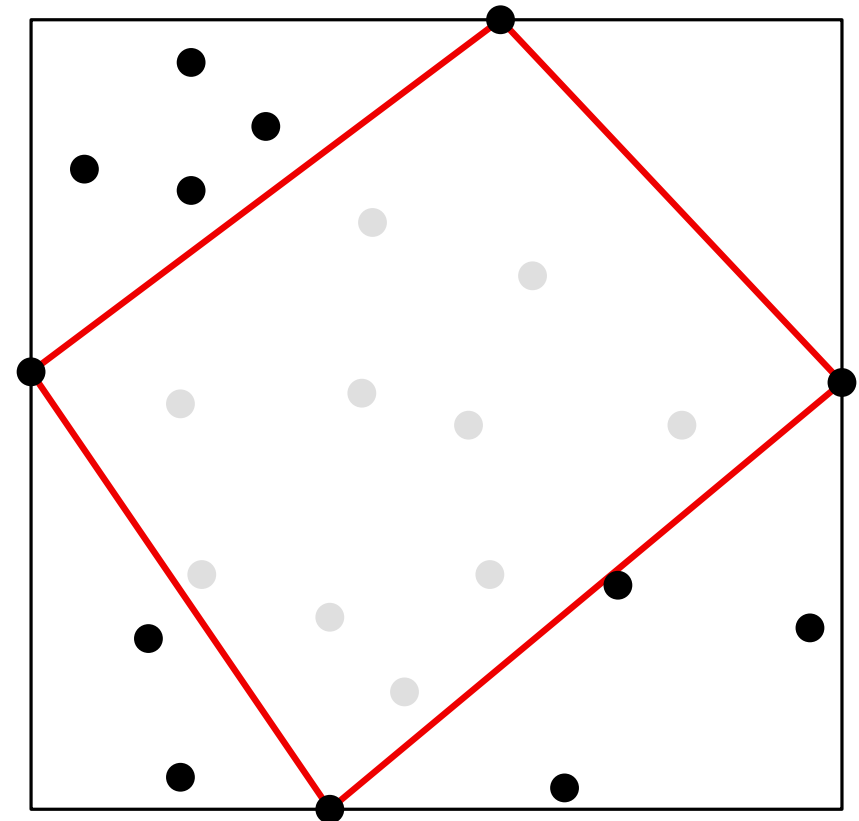
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Running time of this step: $O(n)$



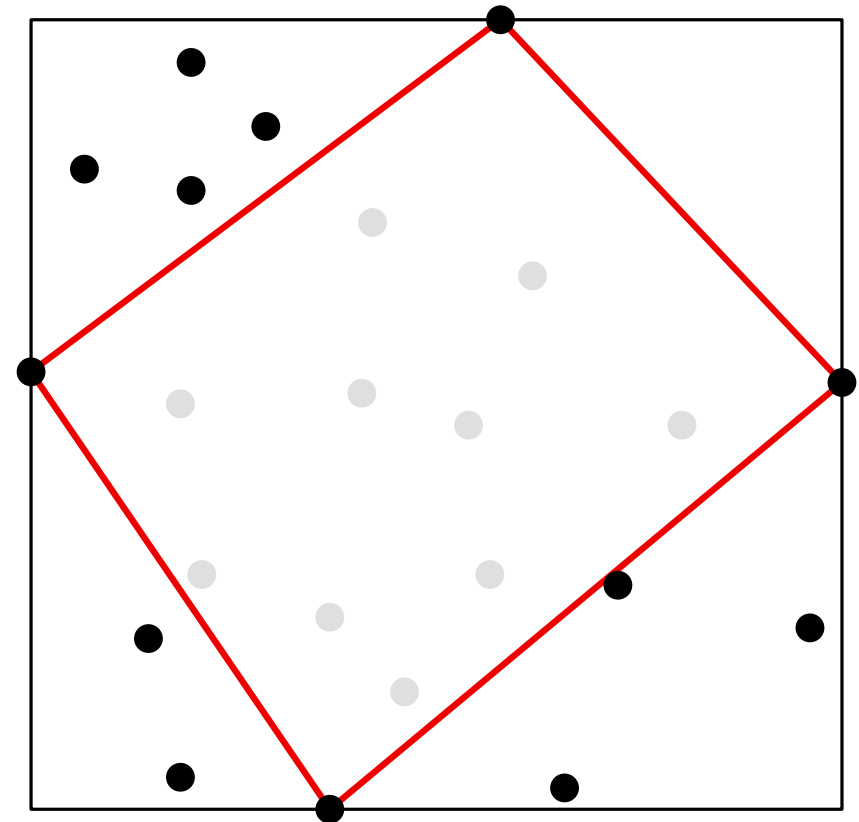
CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

Advance

Recursively, do:

1. Among all points lying in each region, find the extreme point in the direction orthogonal to the edge that determines the region.
2. Connect the extreme point with the endpoints of the edge, and update the convex hull.
3. Test all the remaining points of each region, and classify them according to their position (left or right) or eliminate them if they lie in the interior of the newly created triangle.



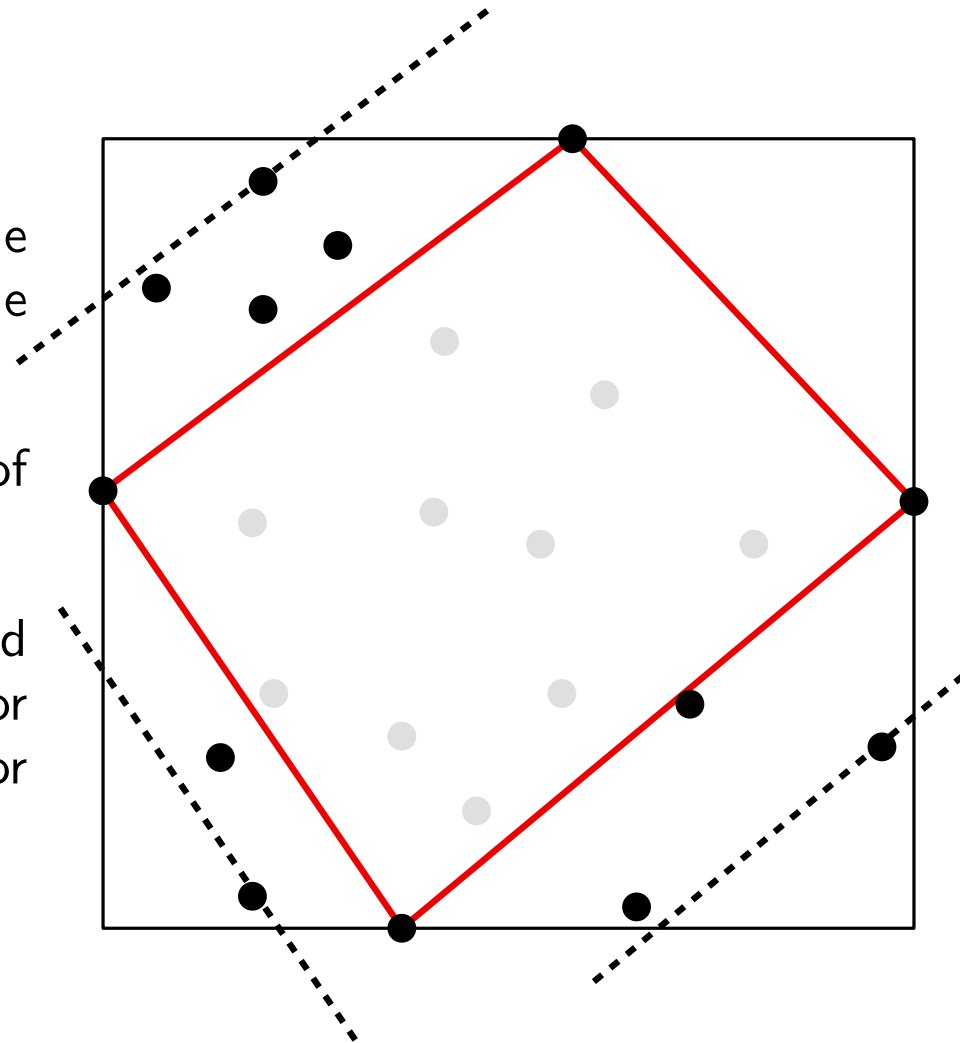
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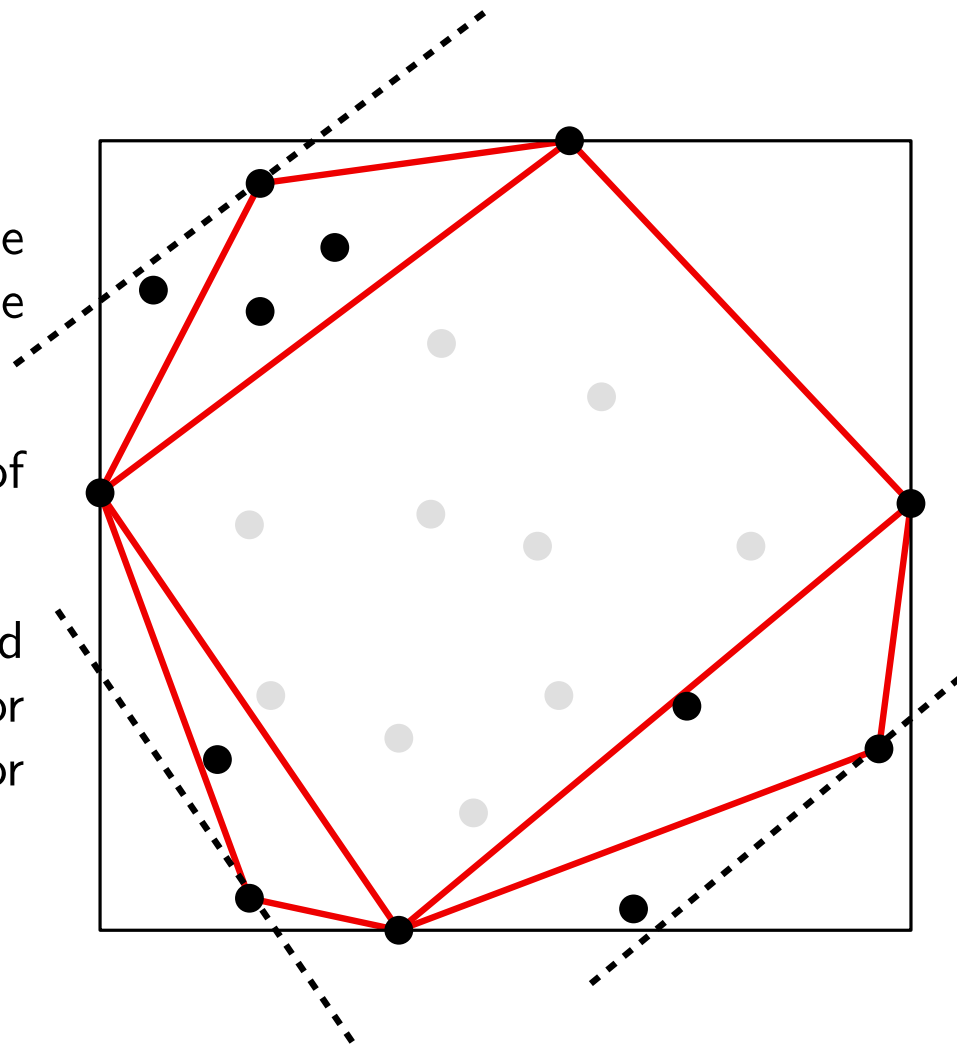
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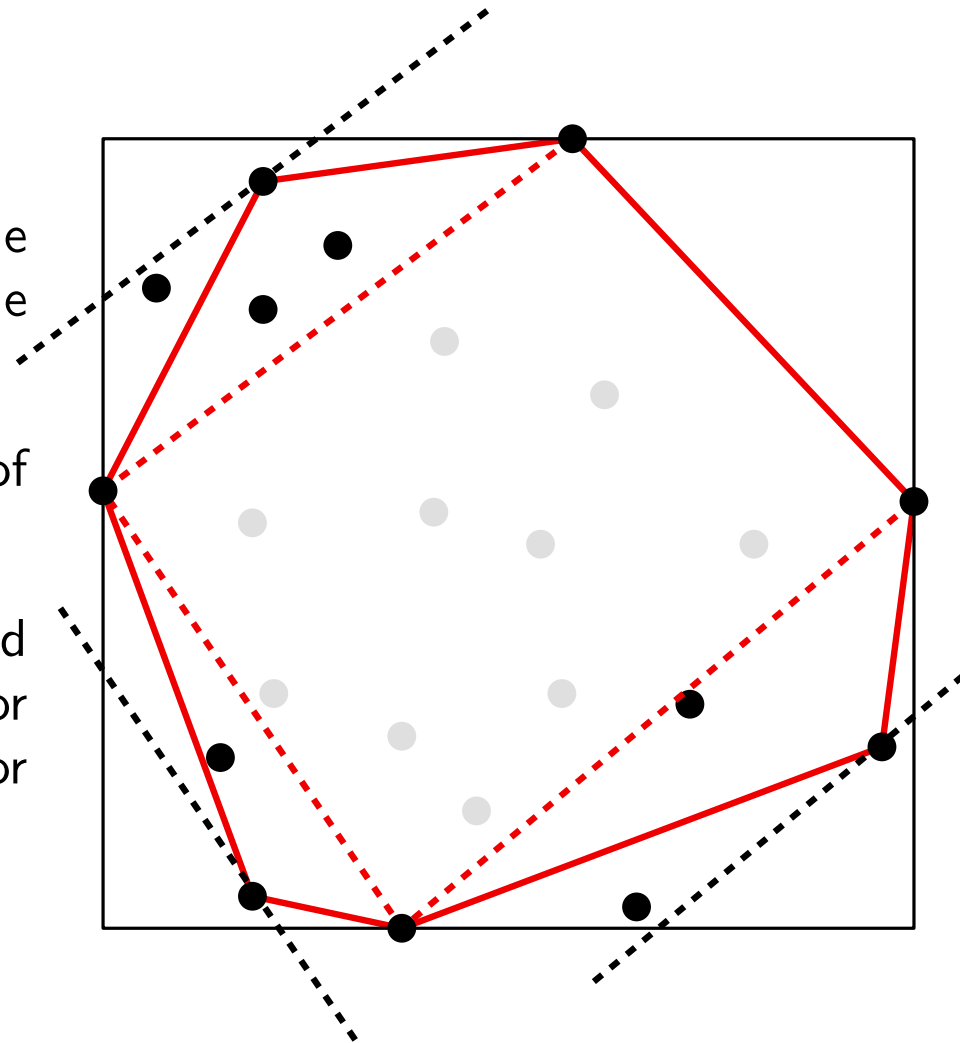
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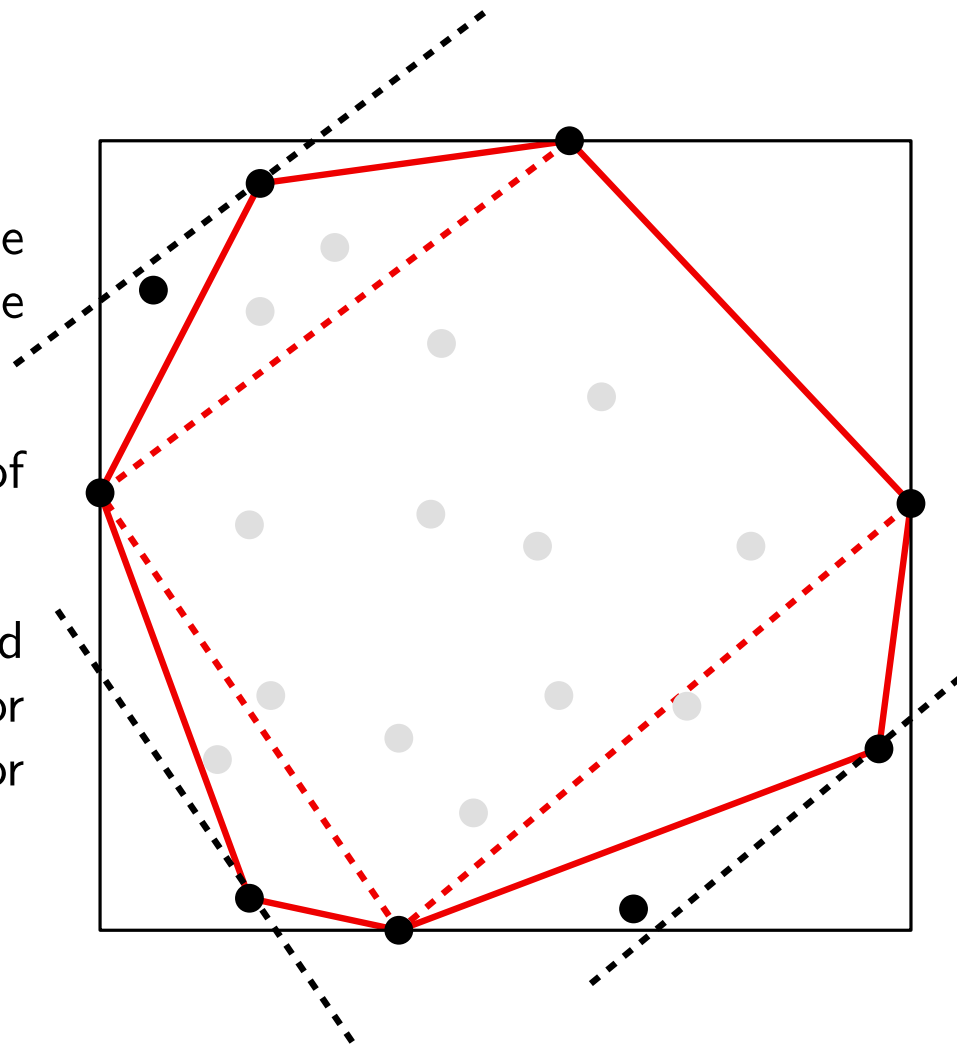
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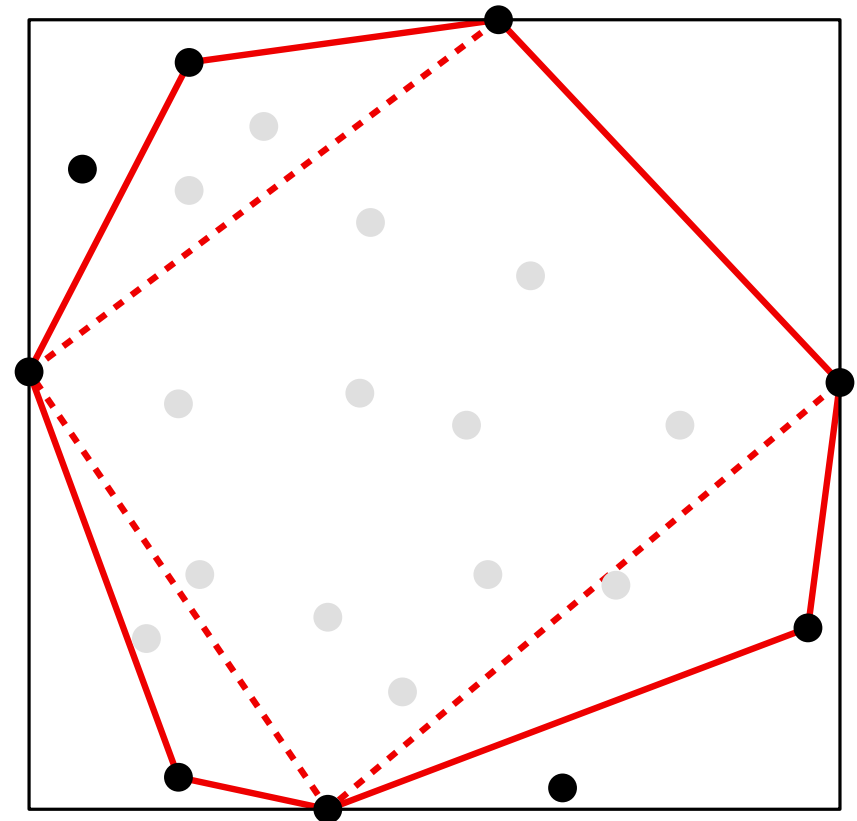
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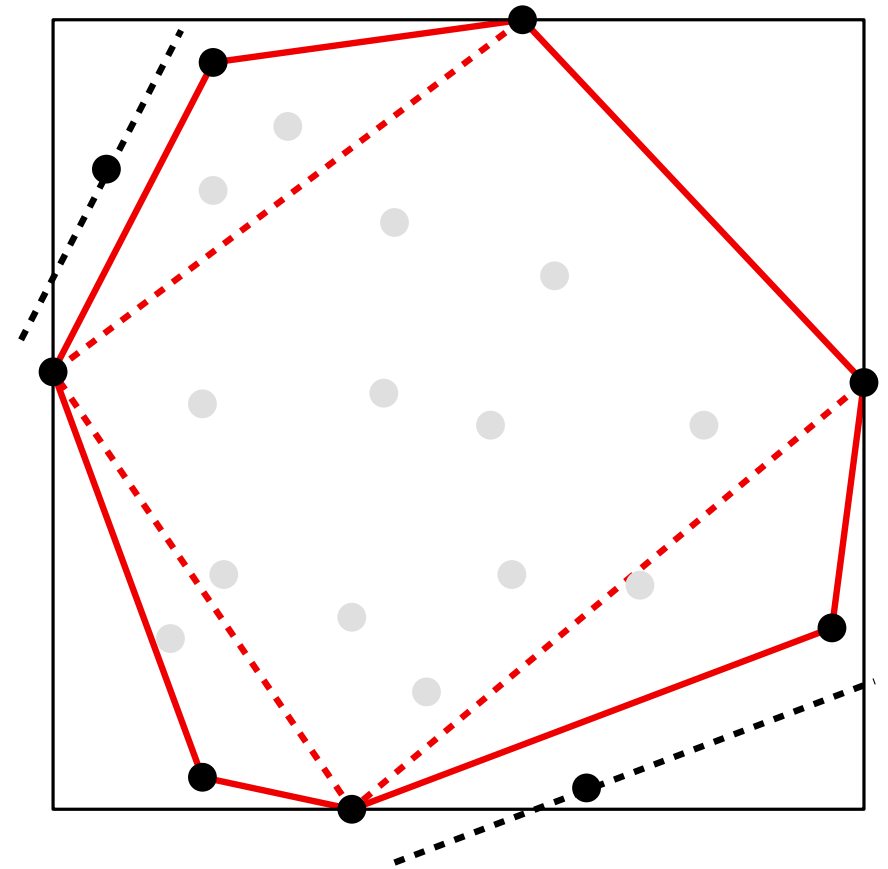
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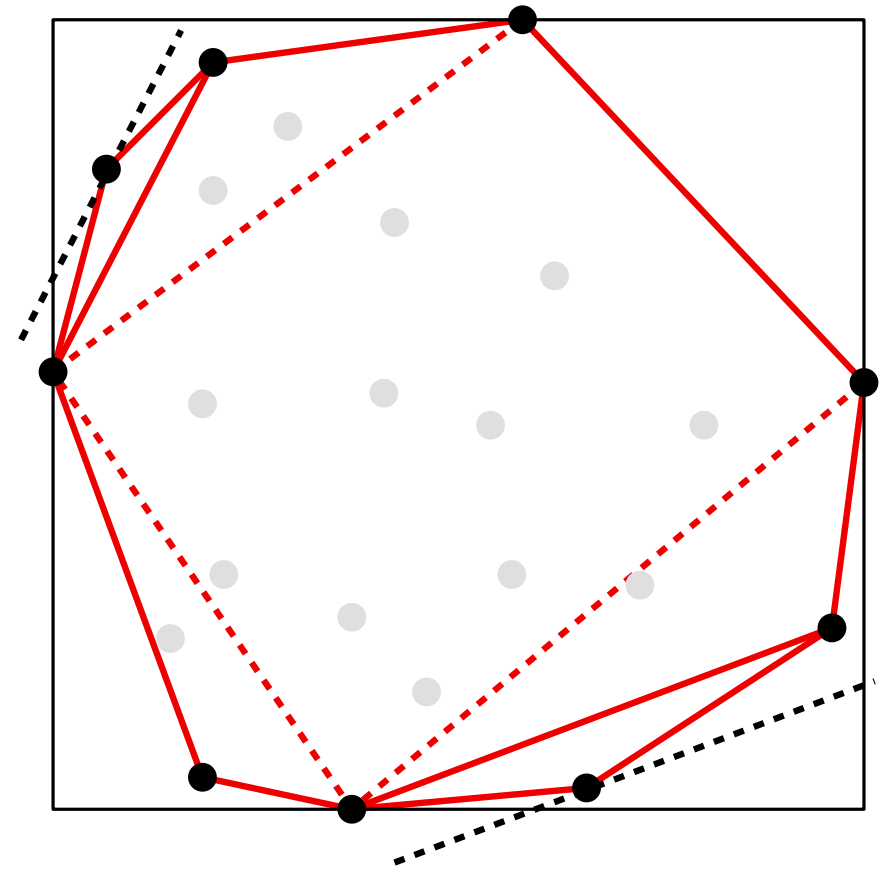
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QuickHull algorithm (by prune-and-search)

Advance

Recursively, do:

1. Among all points lying in each region, find the extreme point in the direction orthogonal to the edge that determines the region.
2. Connect the extreme point with the endpoints of the edge, and update the convex hull.
3. Test all the remaining points of each region, and classify them according to their position (left or right) or eliminate them if they lie in the interior of the newly created triangle.



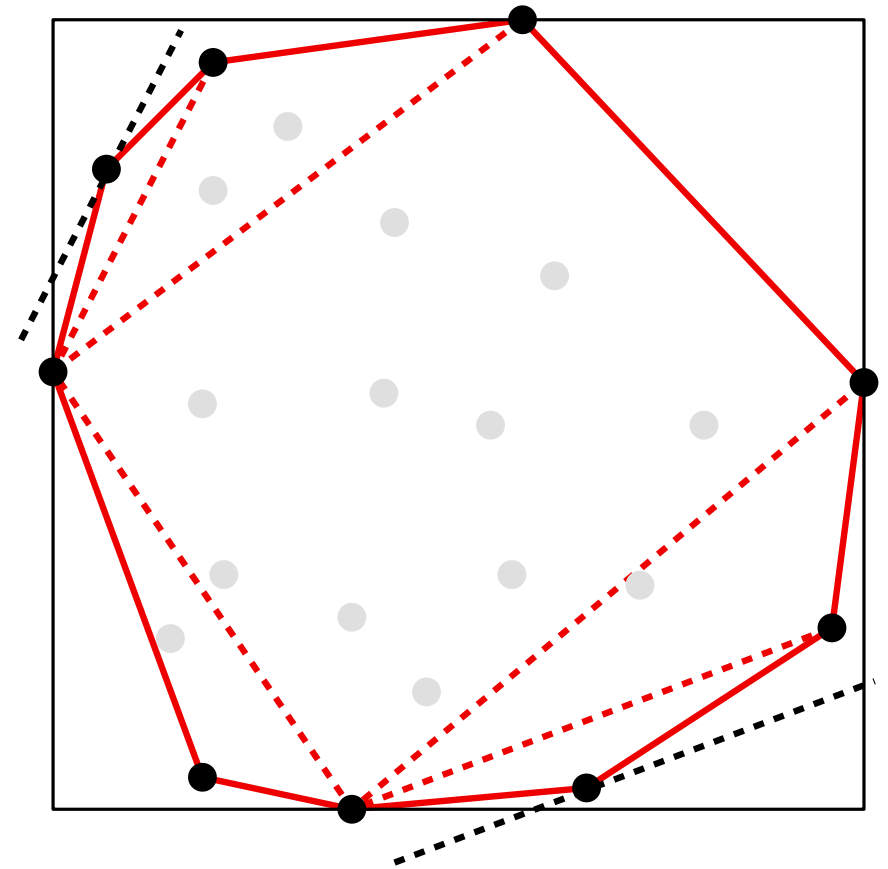
CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

Advance

Recursively, do:

1. Among all points lying in each region, find the extreme point in the direction orthogonal to the edge that determines the region.
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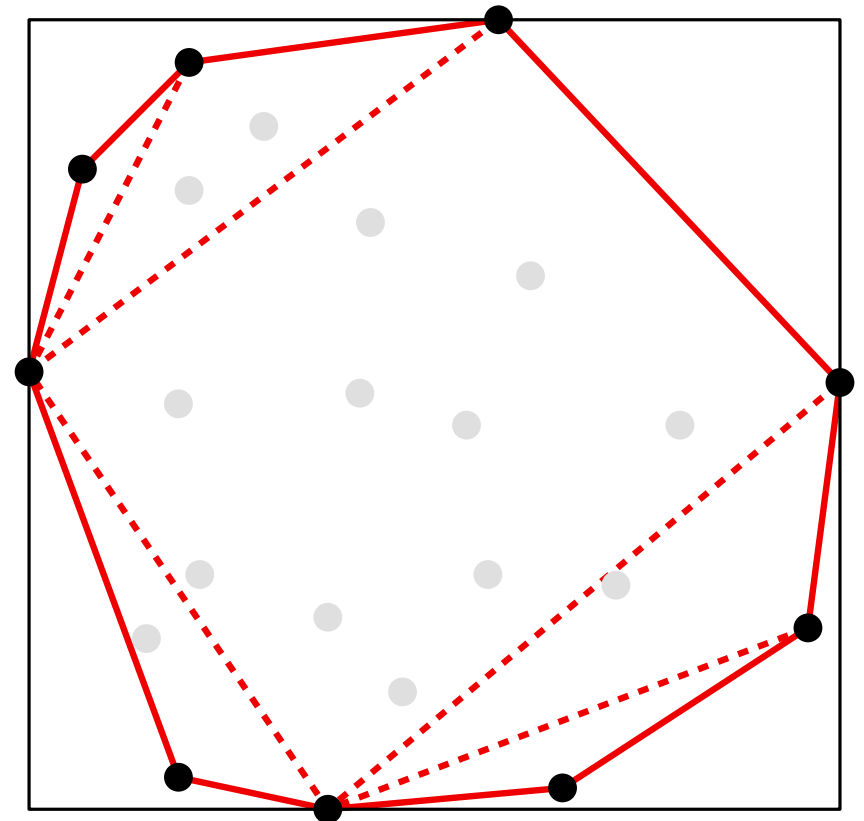
CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

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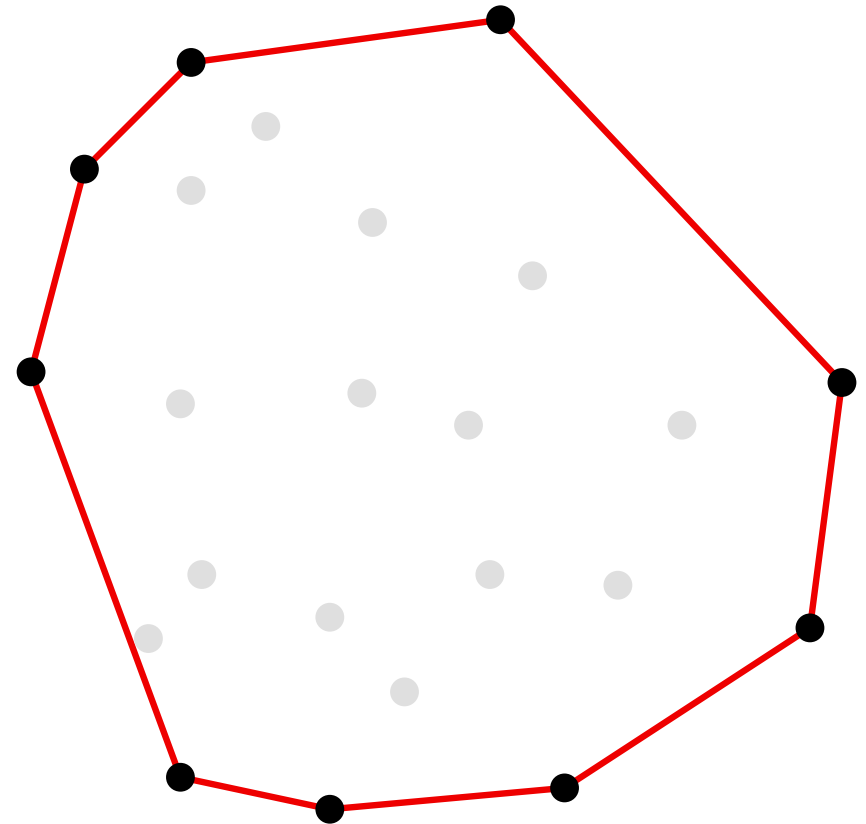
CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

Advance

Recursively, do:

1. Among all points lying in each region, find the extreme point in the direction orthogonal to the edge that determines the region.
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CONVEX HULL IN 2D

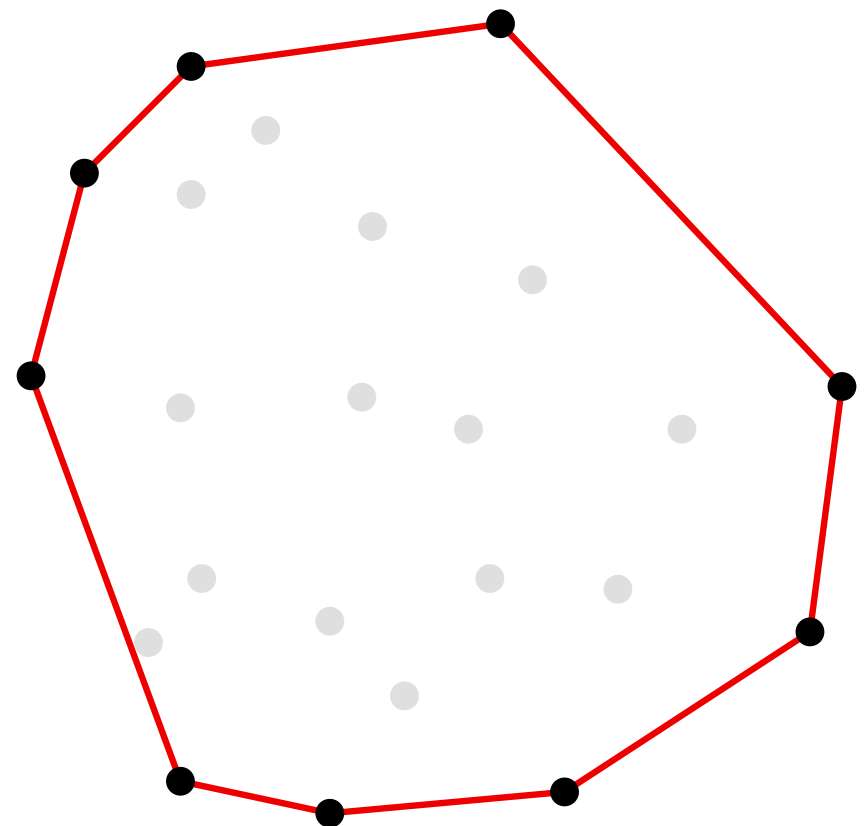
QuickHull algorithm (by prune-and-search)

Advance

Recursively, do:

1. Among all points lying in each region, find the extreme point in the direction orthogonal to the edge that determines the region.
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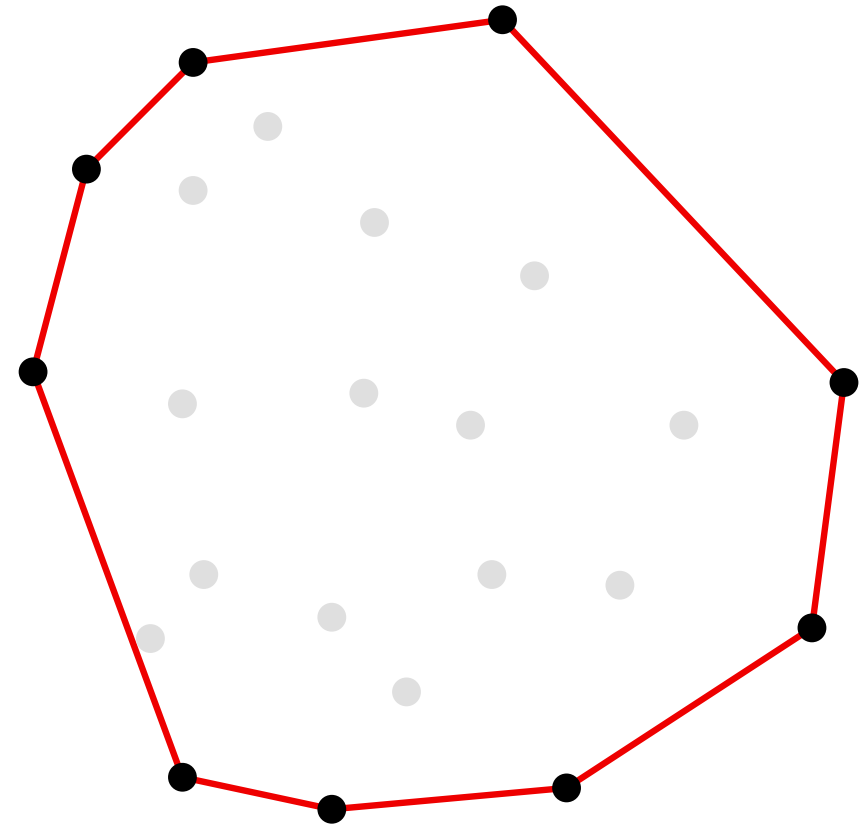
Running time of this step: $O(n^2)$



CONVEX HULL IN 2D

QuickHull algorithm (by prune-and-search)

Overall running time: $O(n^2)$



CONVEX HULL IN 2D

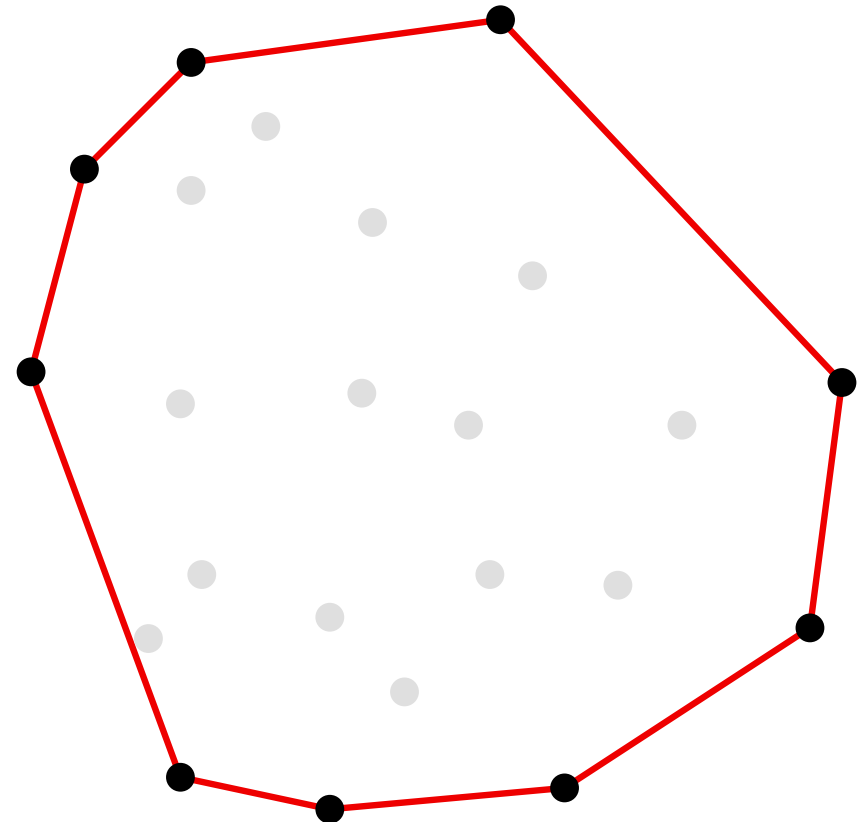
QuickHull algorithm (by prune-and-search)

Overall running time: $O(n^2)$

Nevertheless, the running time of this algorithm depends on the position of the input points.

For example:

- If the input points are in convex position, the running time is $\Theta(n^2)$.
- If the points are such that each prune step eliminates half of the current points, then the algorithm runs in $\Theta(n \log n)$ time.
- If the convex hull is triangular, the algorithm runs in $\Theta(n)$ time.



CONVEX HULL IN 2D

Graham's algorithm

CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:

- Push p_i in l

- Advance i

- Else:

- Pop p from l

Return l

CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

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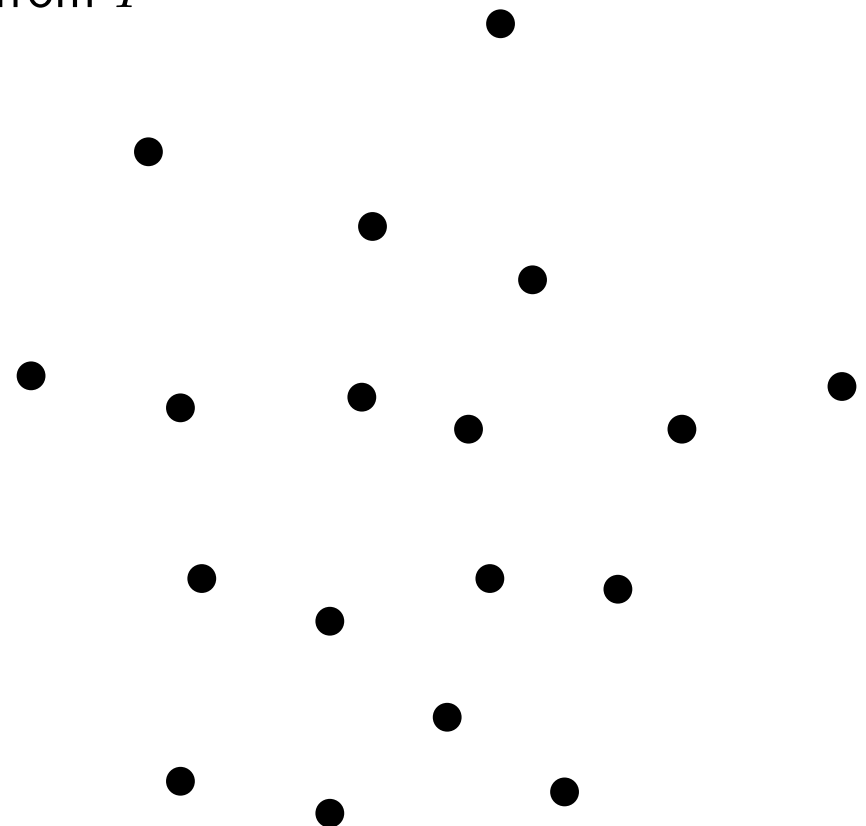
- Push p_i in l

- Advance i

- Else:

- Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:

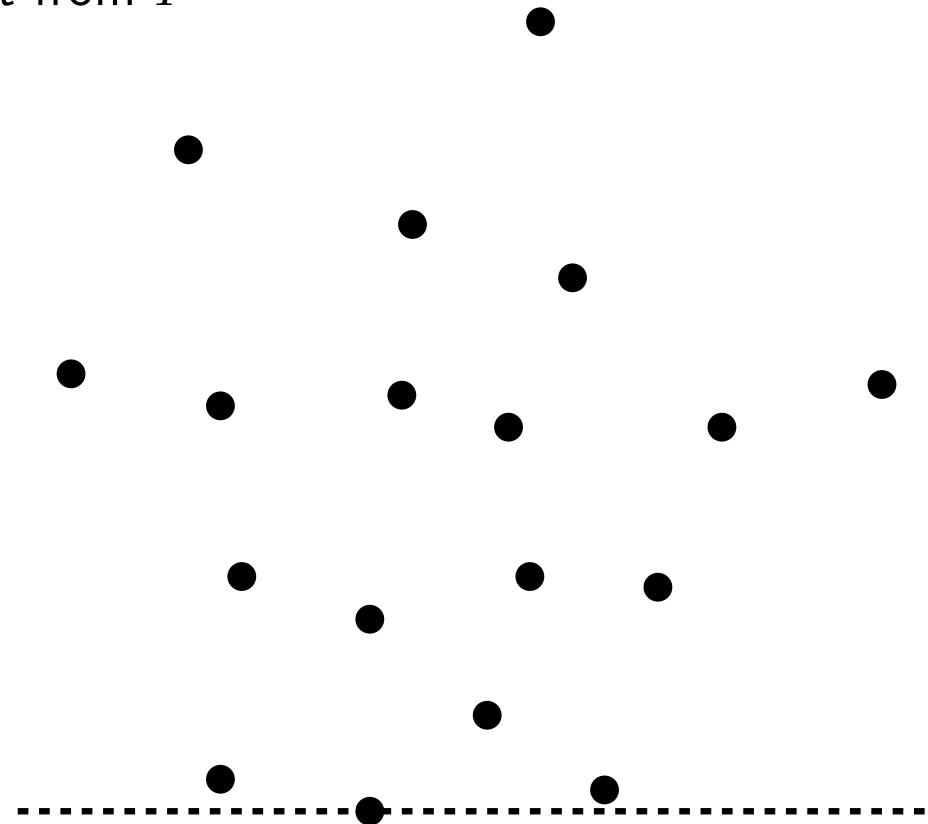
- Push p_i in l

- Advance i

- Else:

- Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

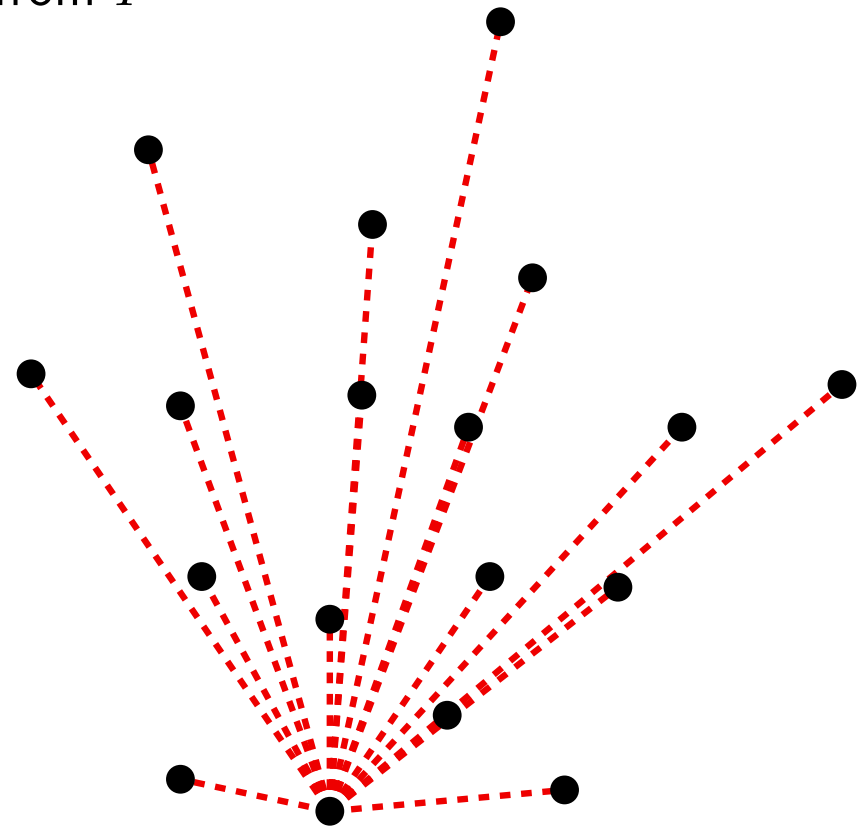
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

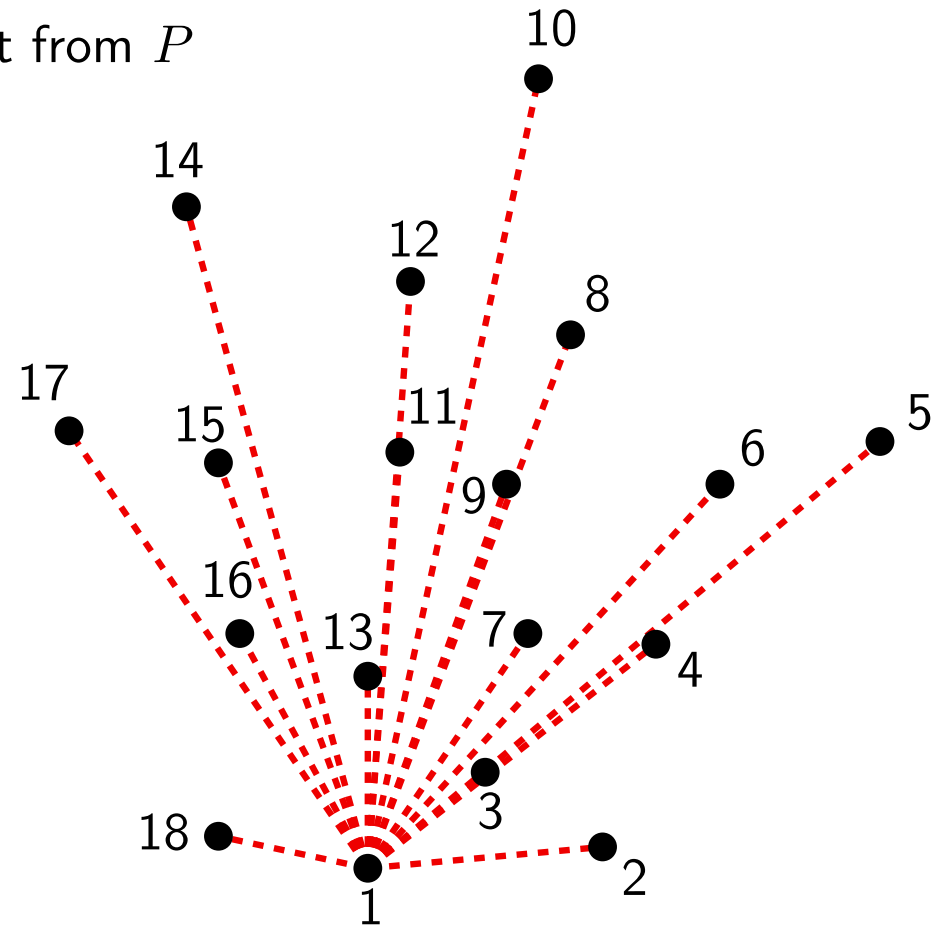
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

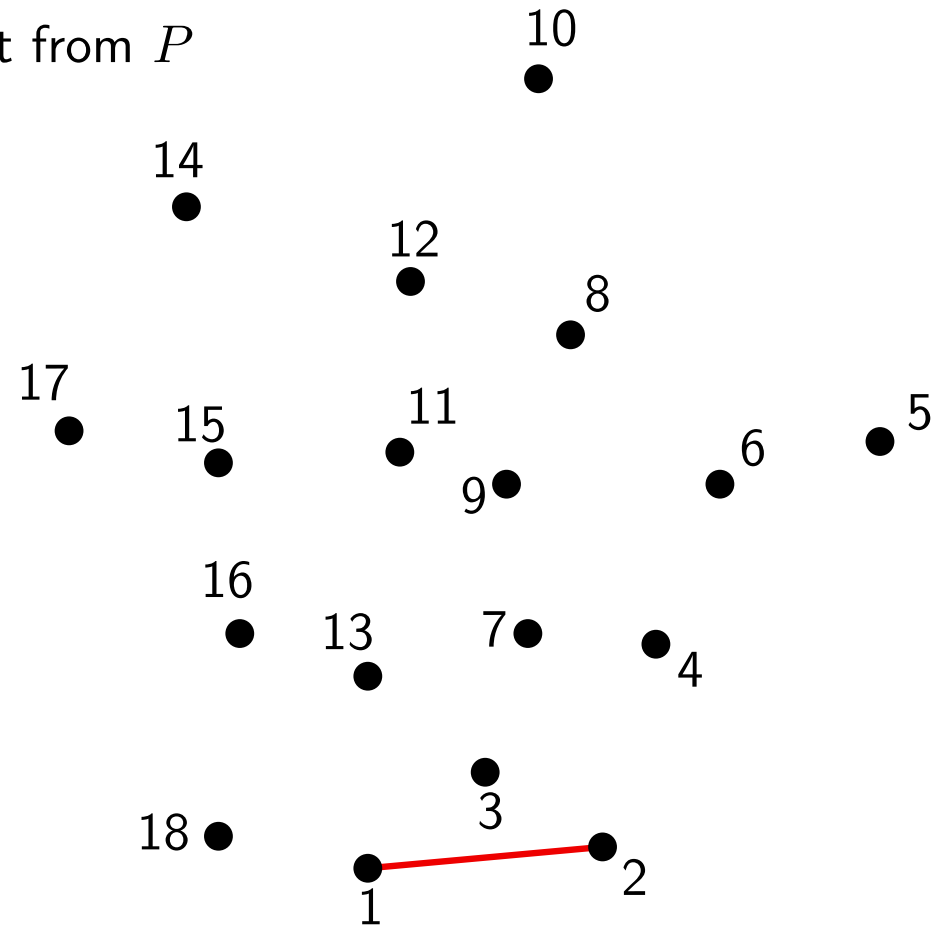
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

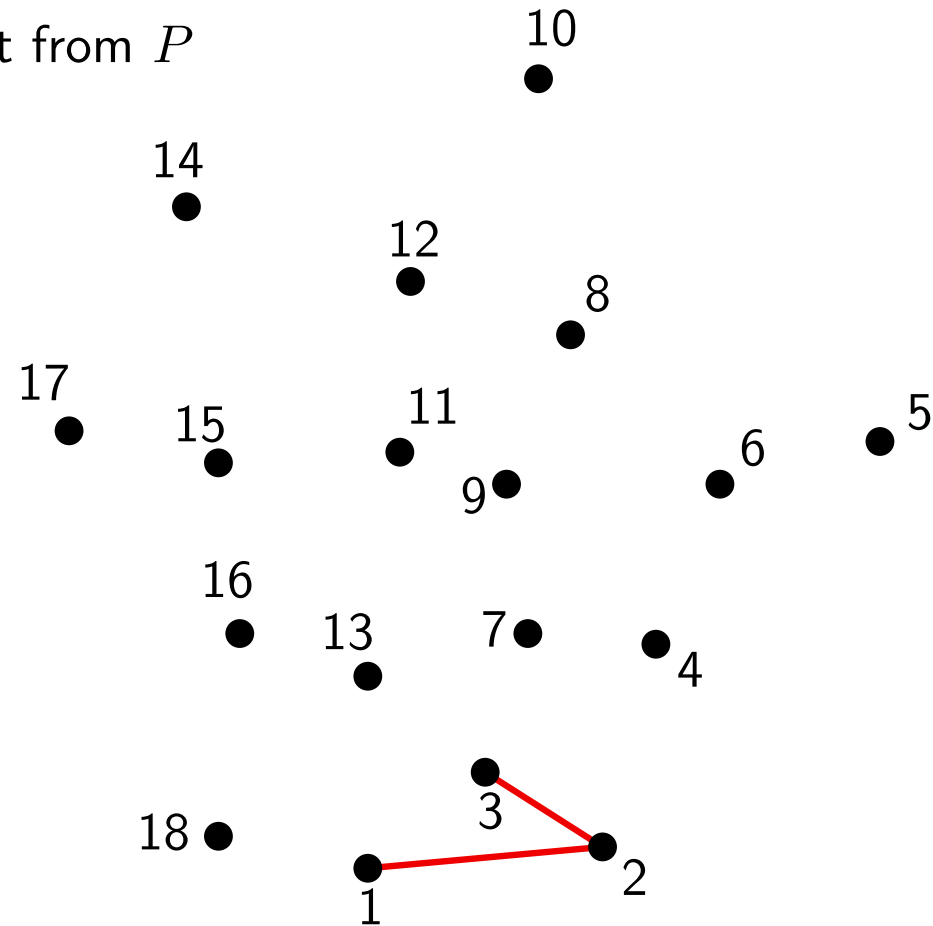
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

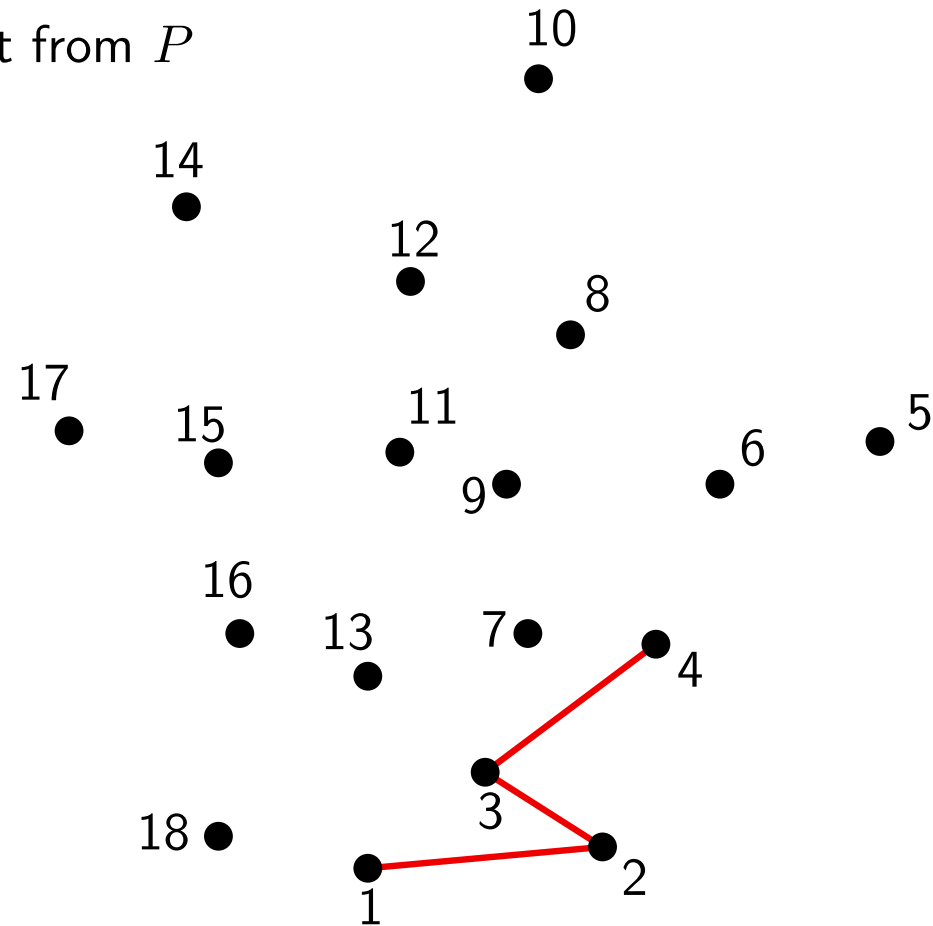
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

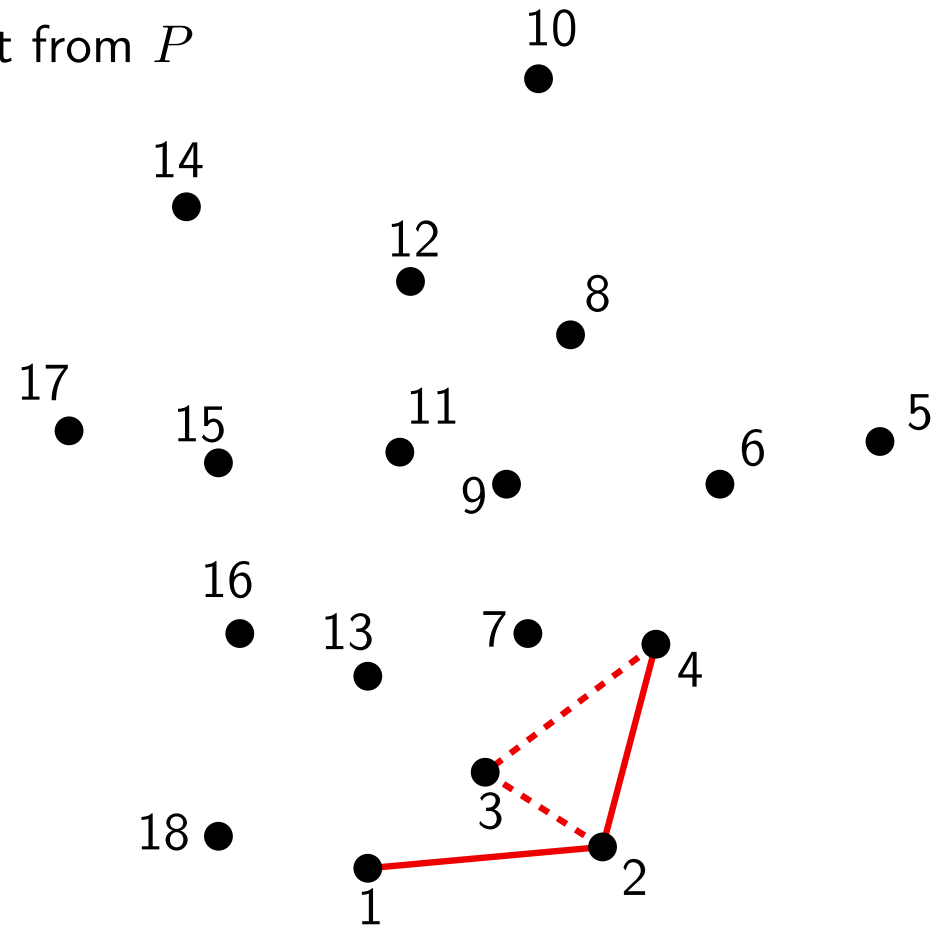
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

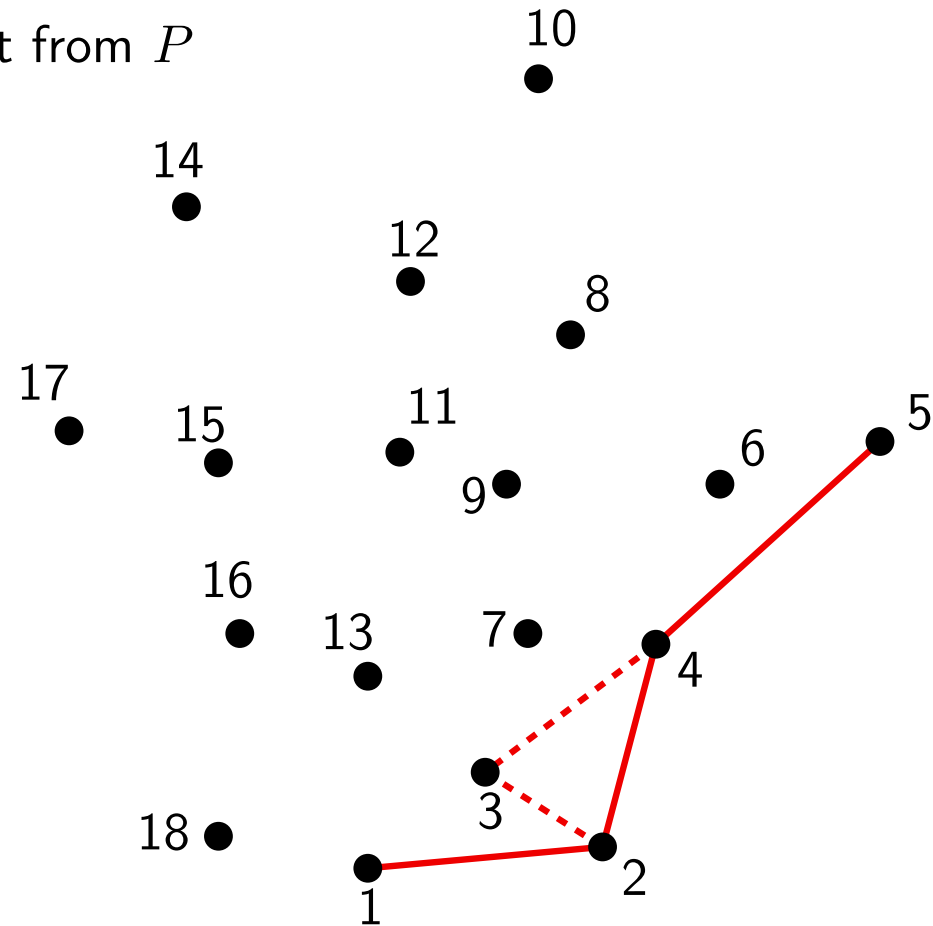
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

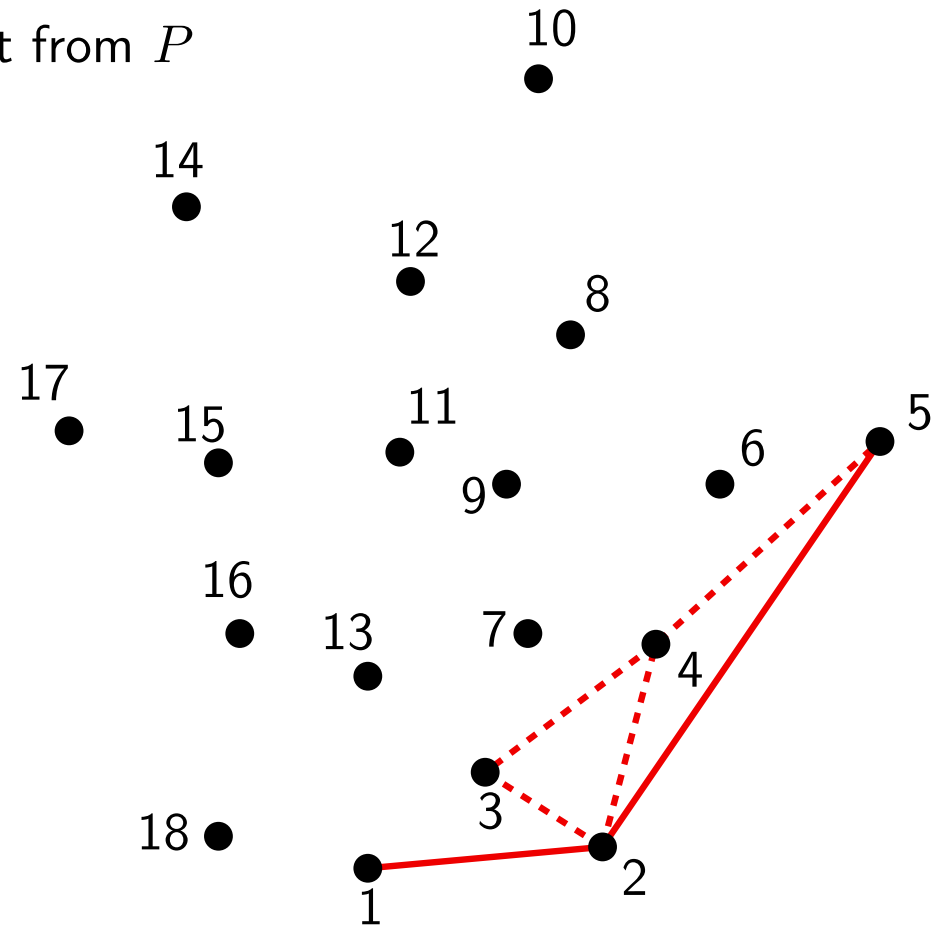
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

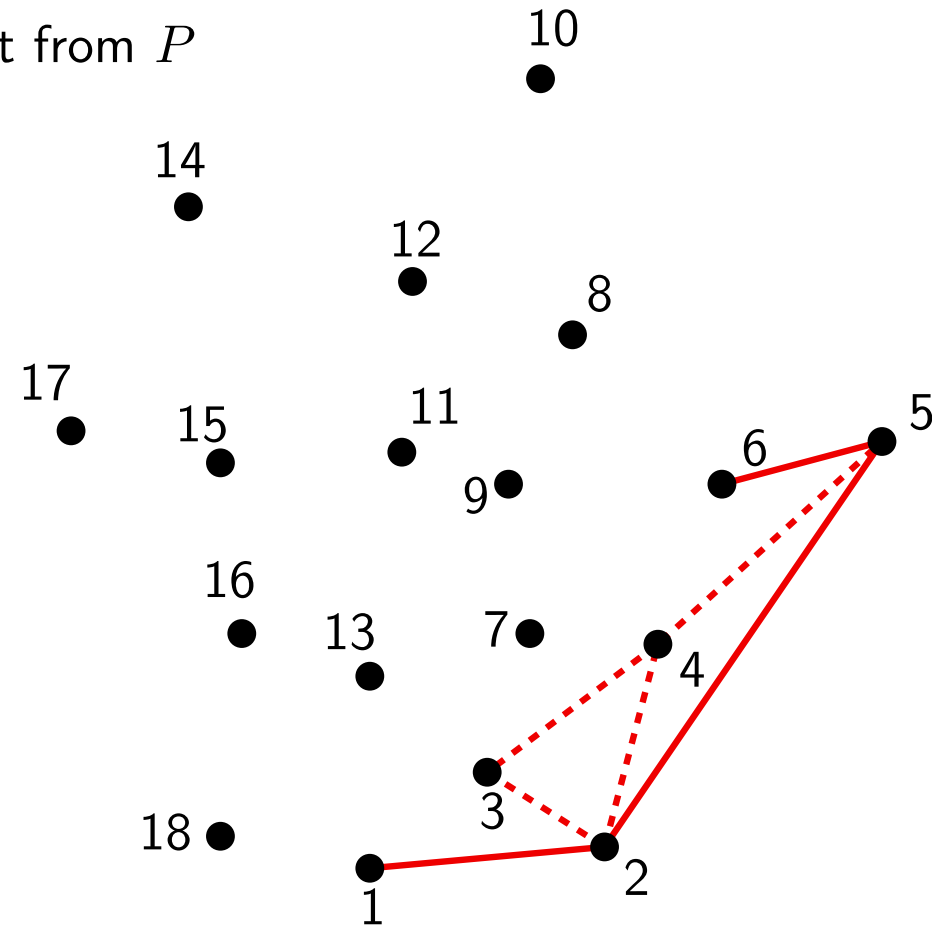
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

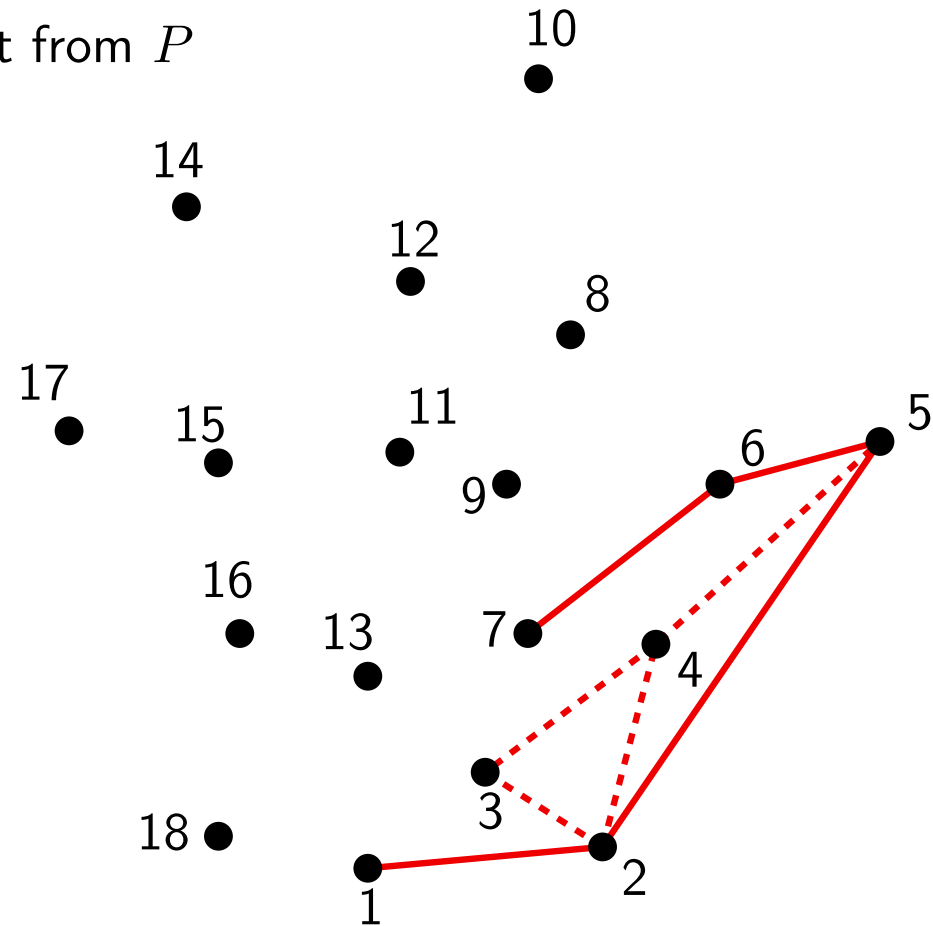
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

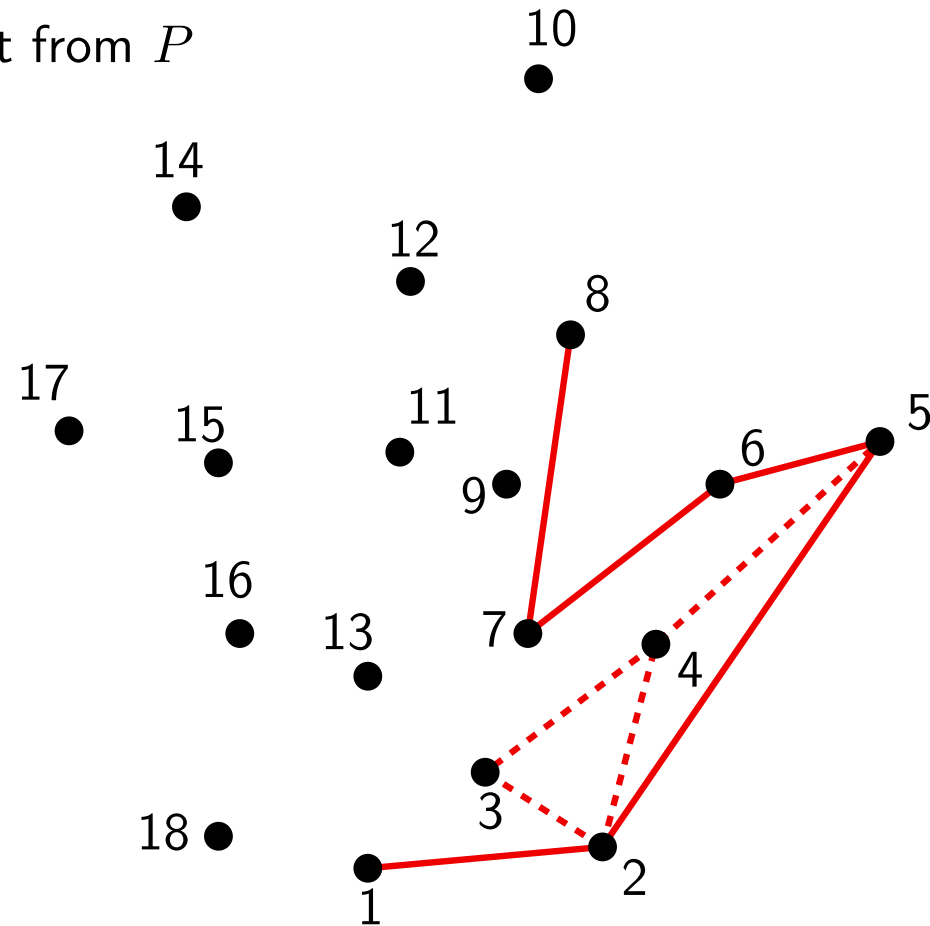
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

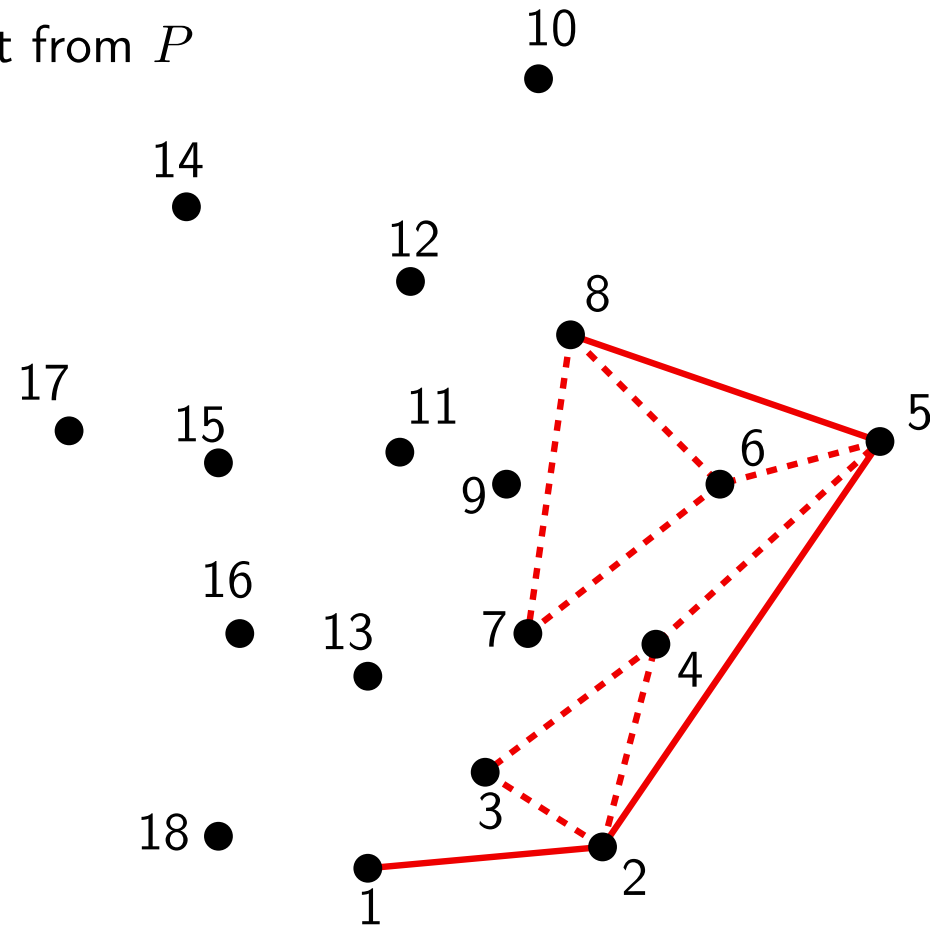
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

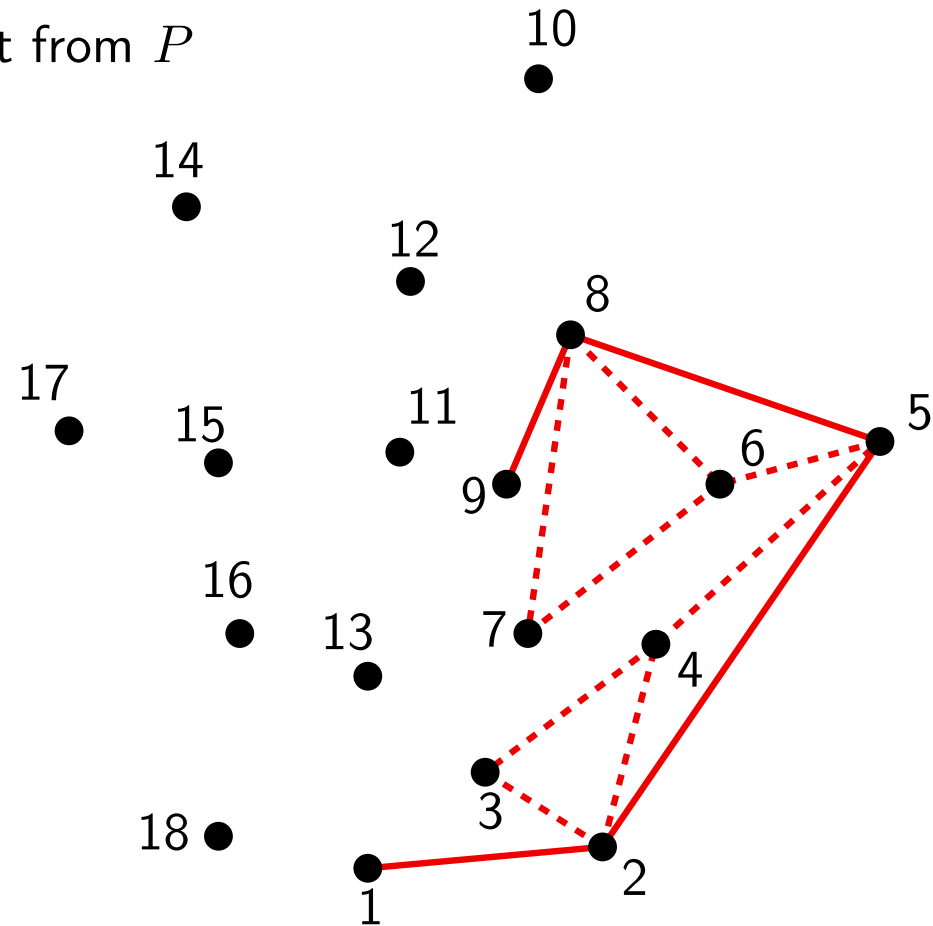
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

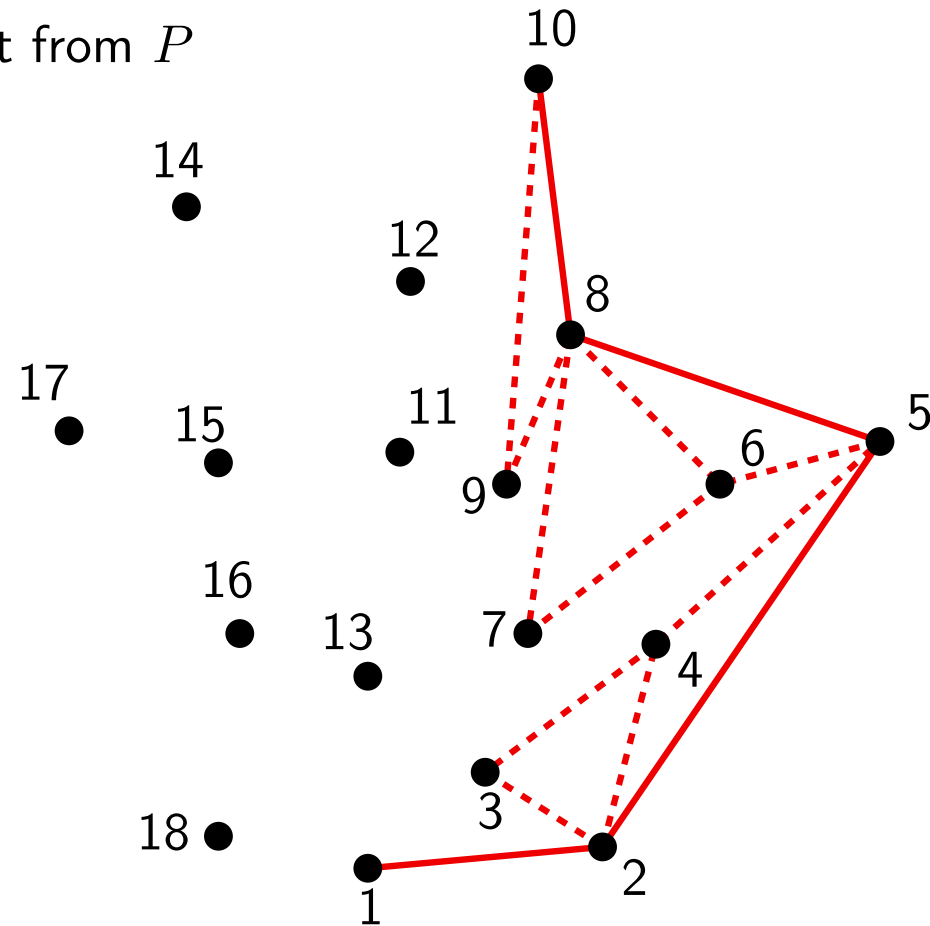
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

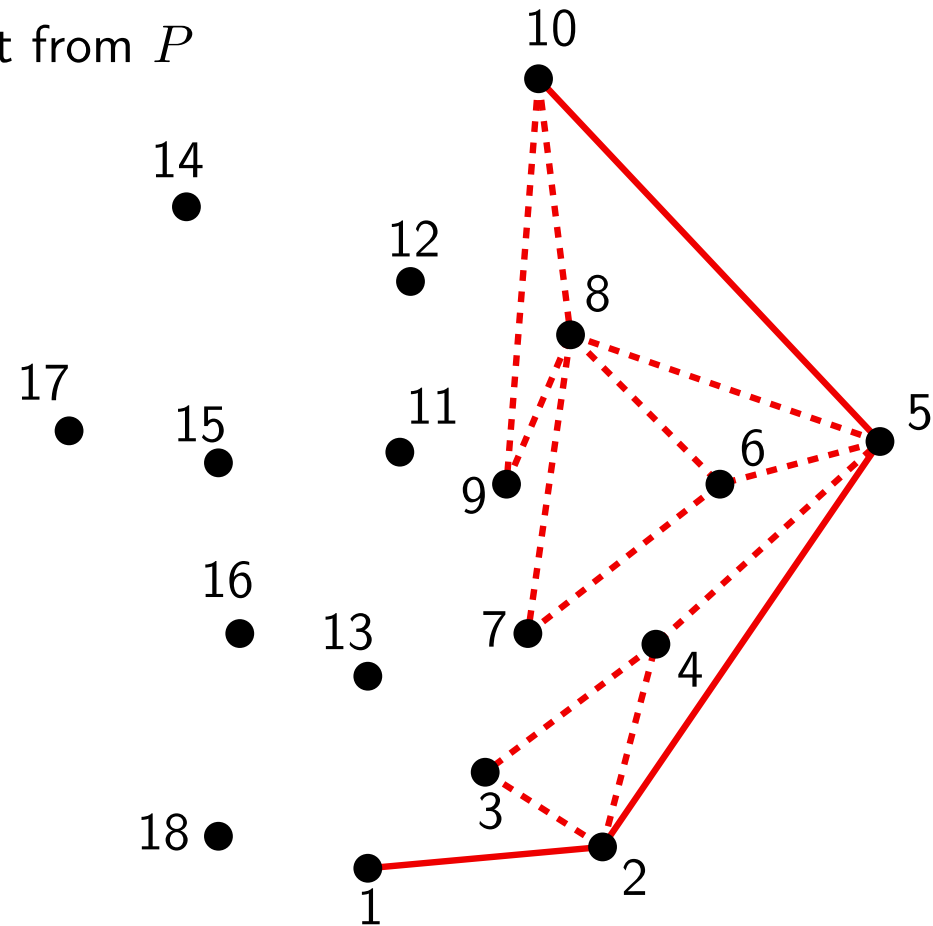
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

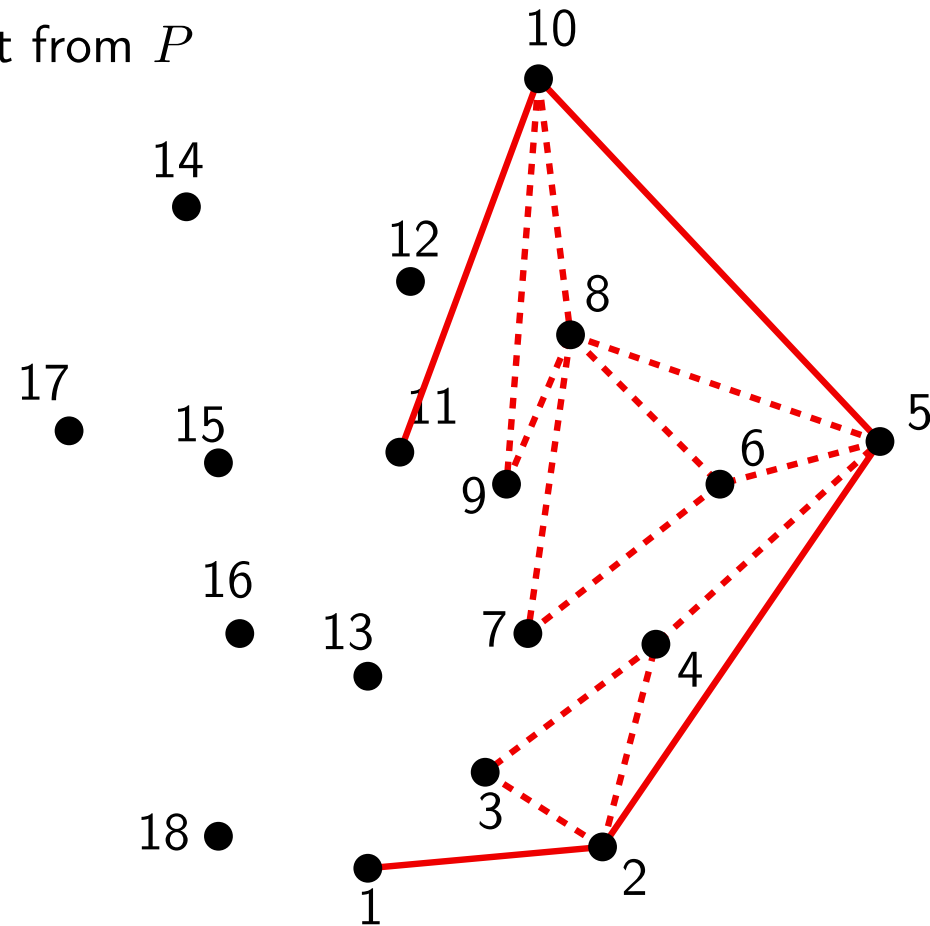
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

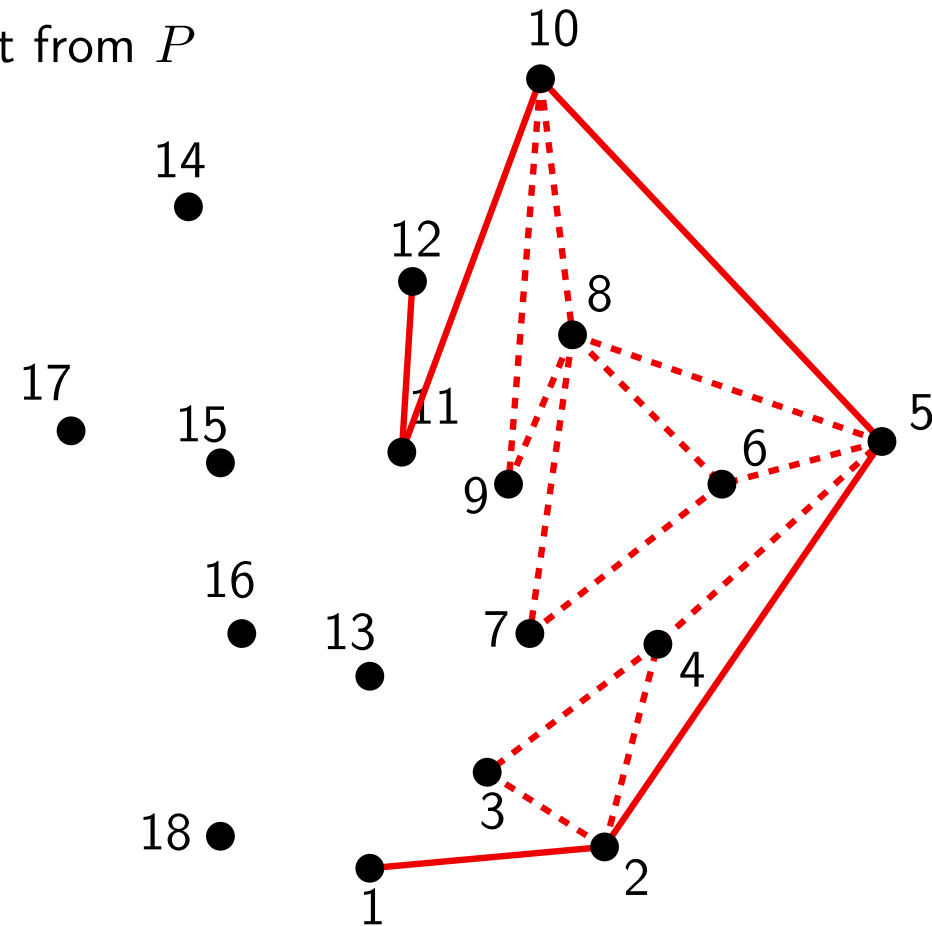
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

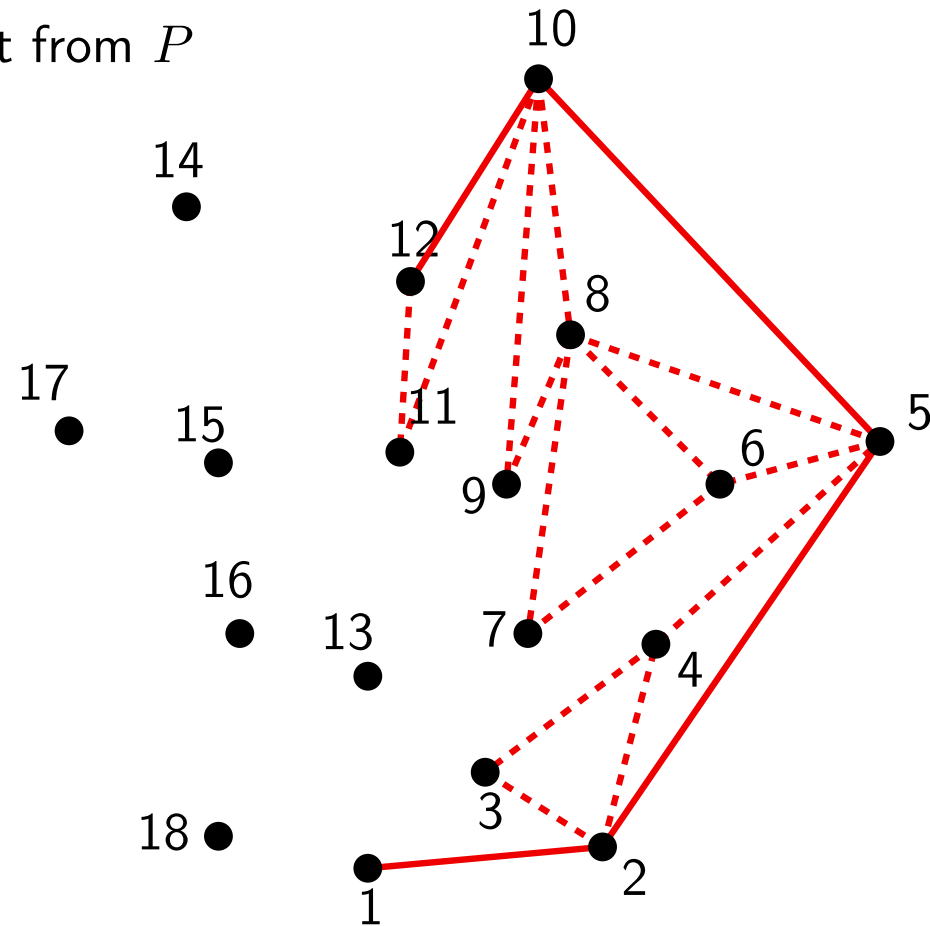
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

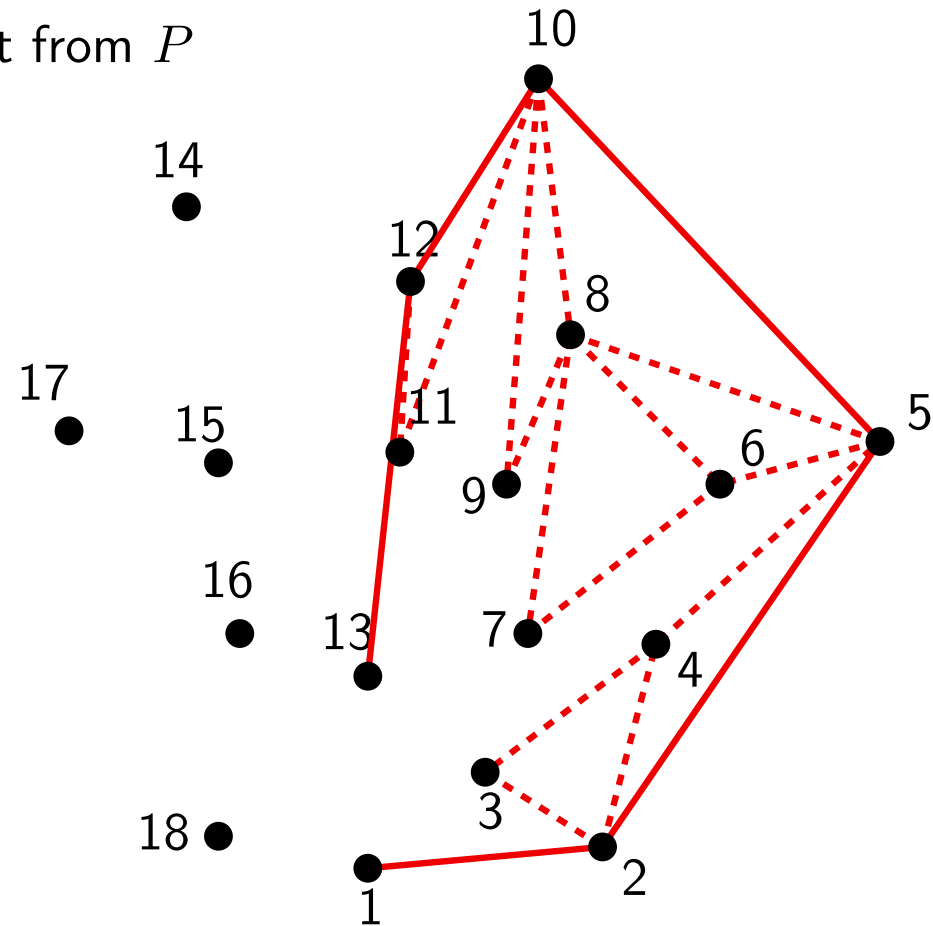
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

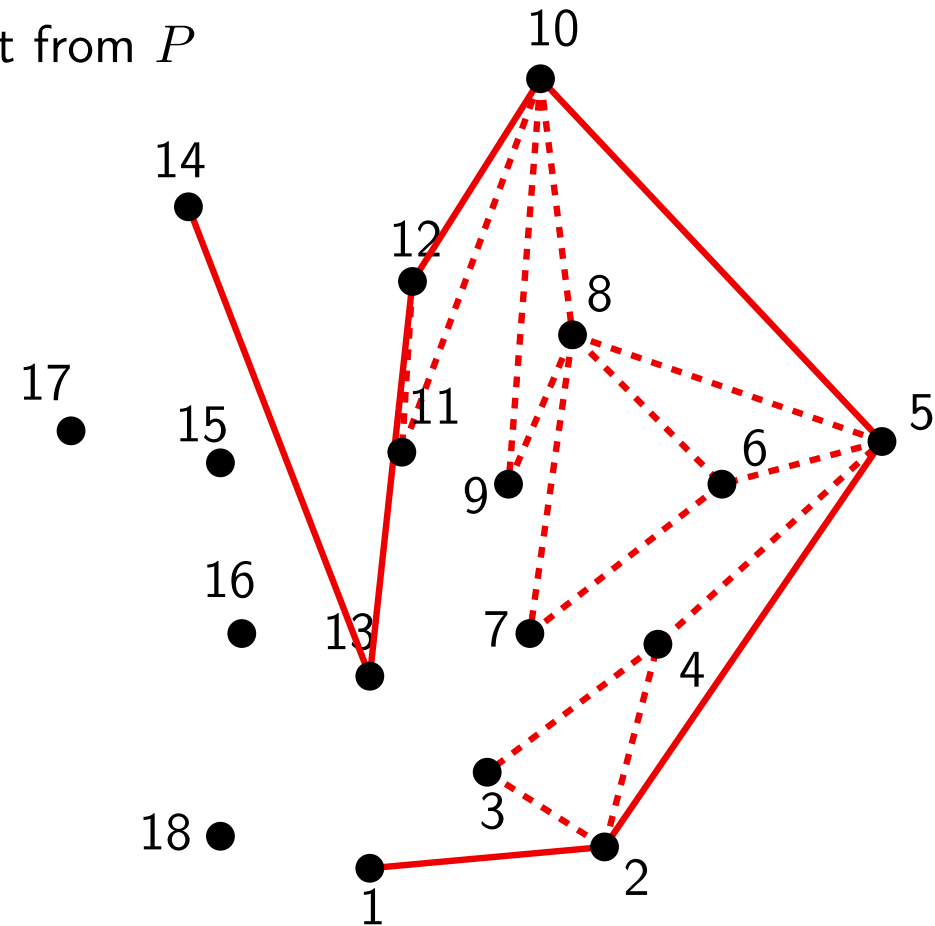
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

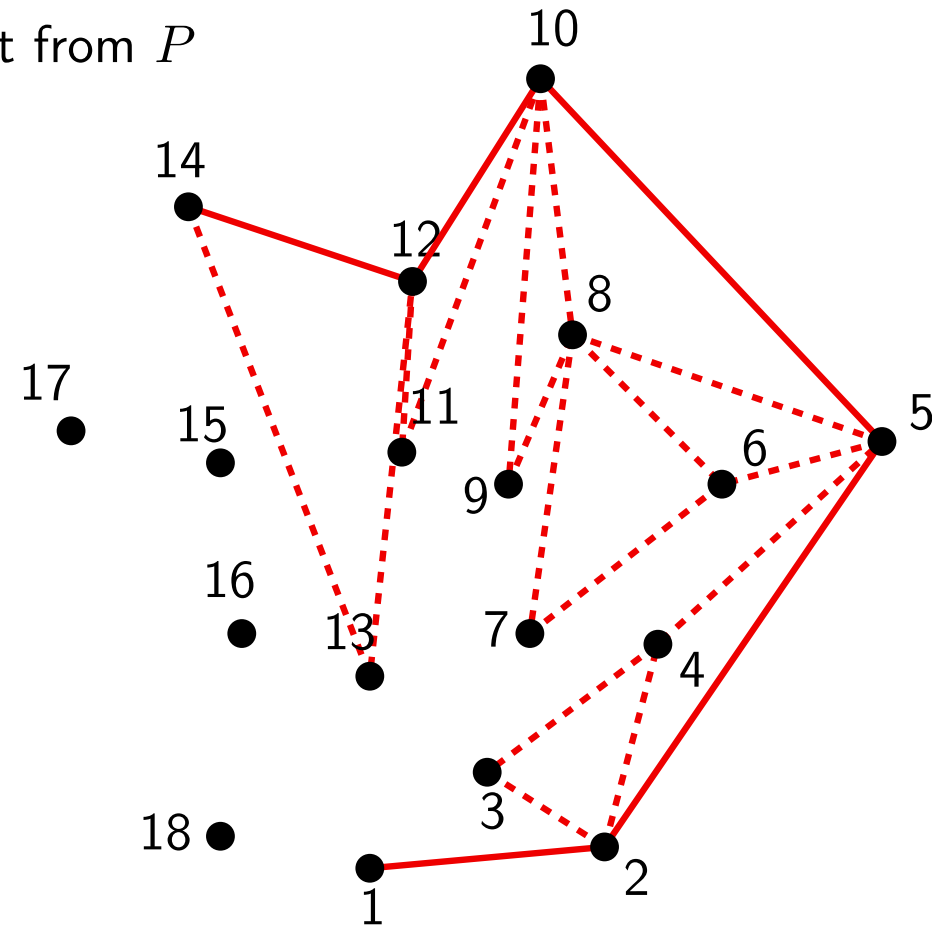
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

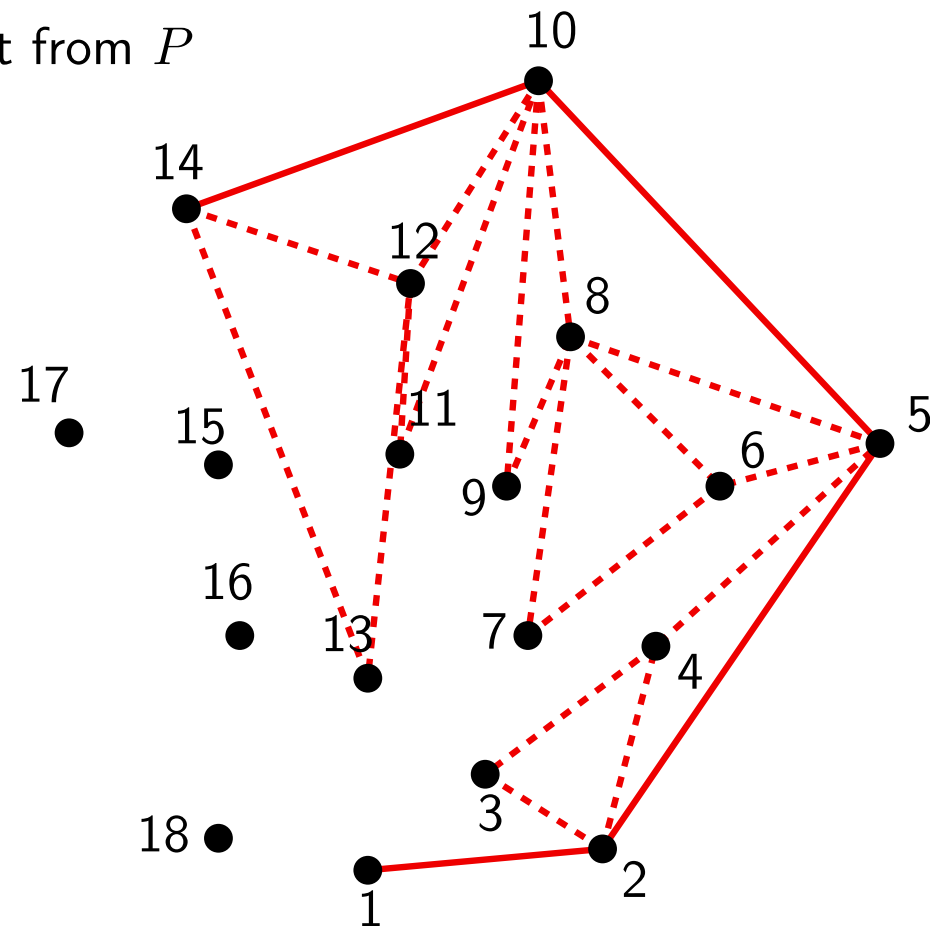
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

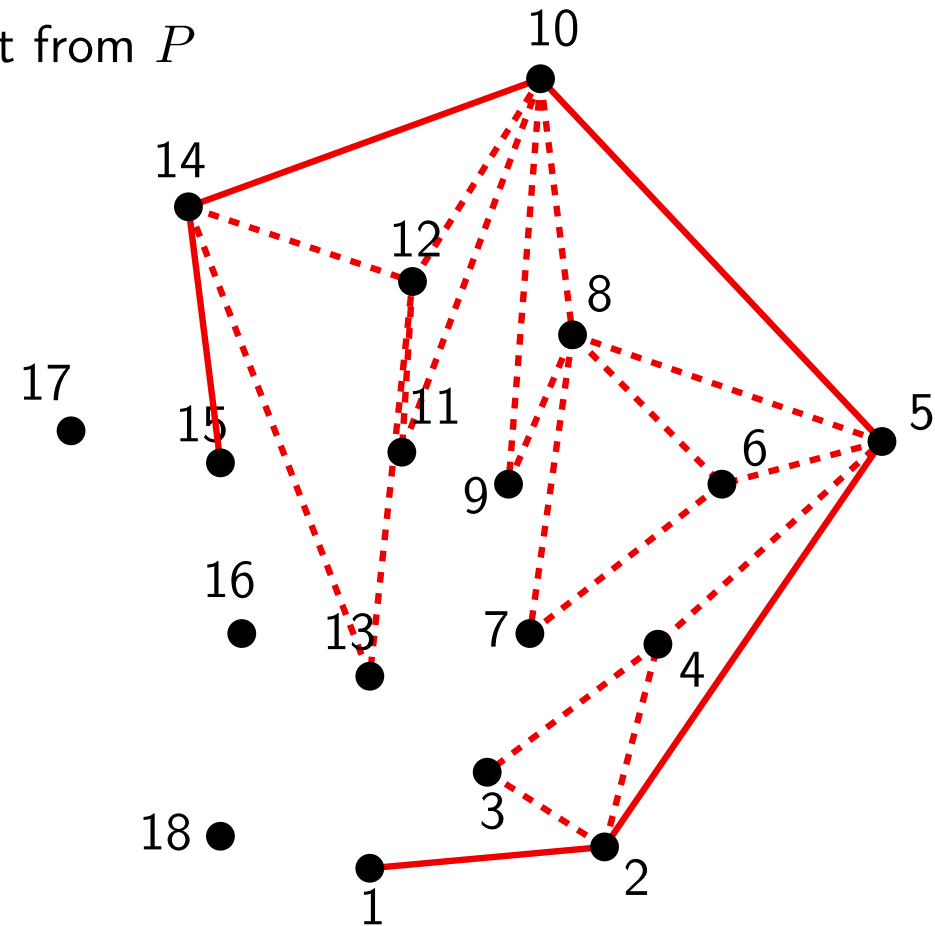
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

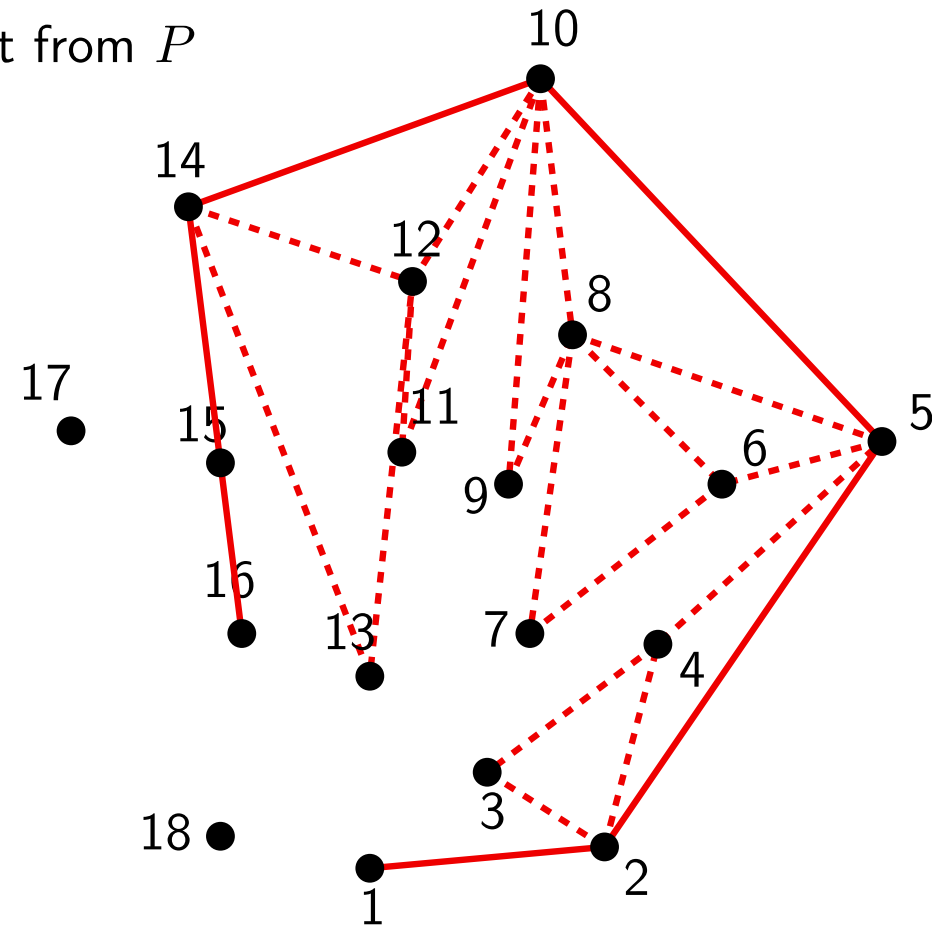
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

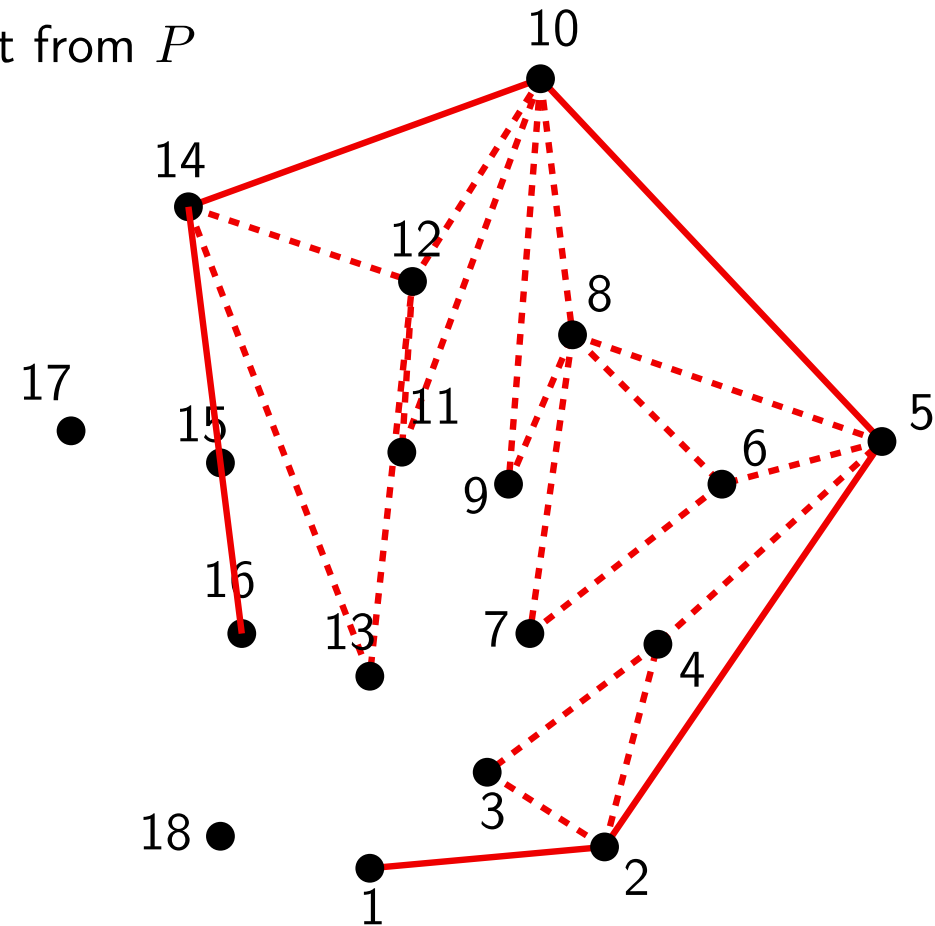
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

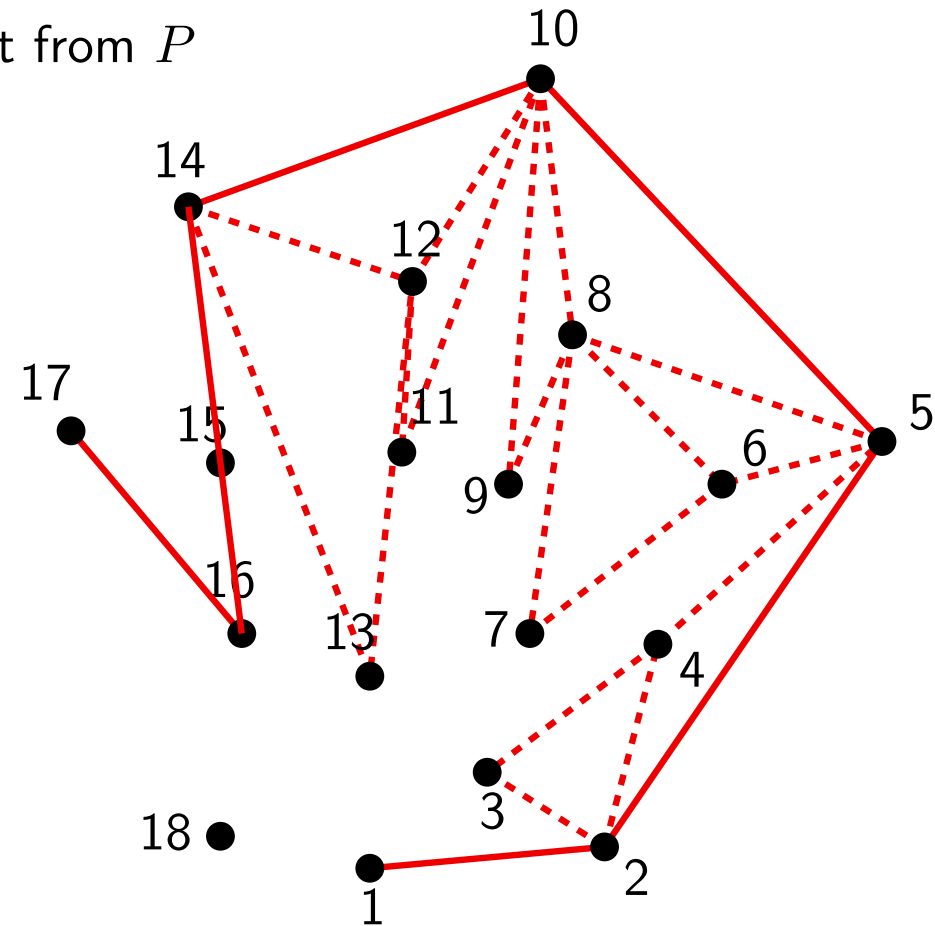
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

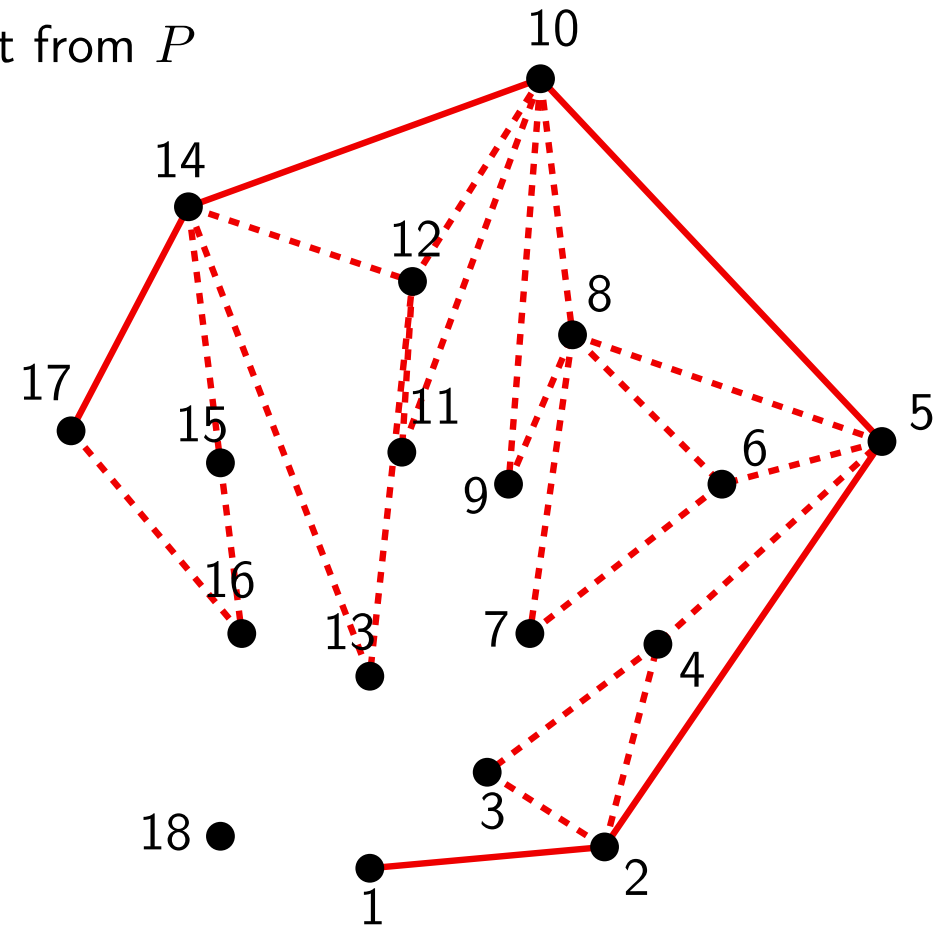
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

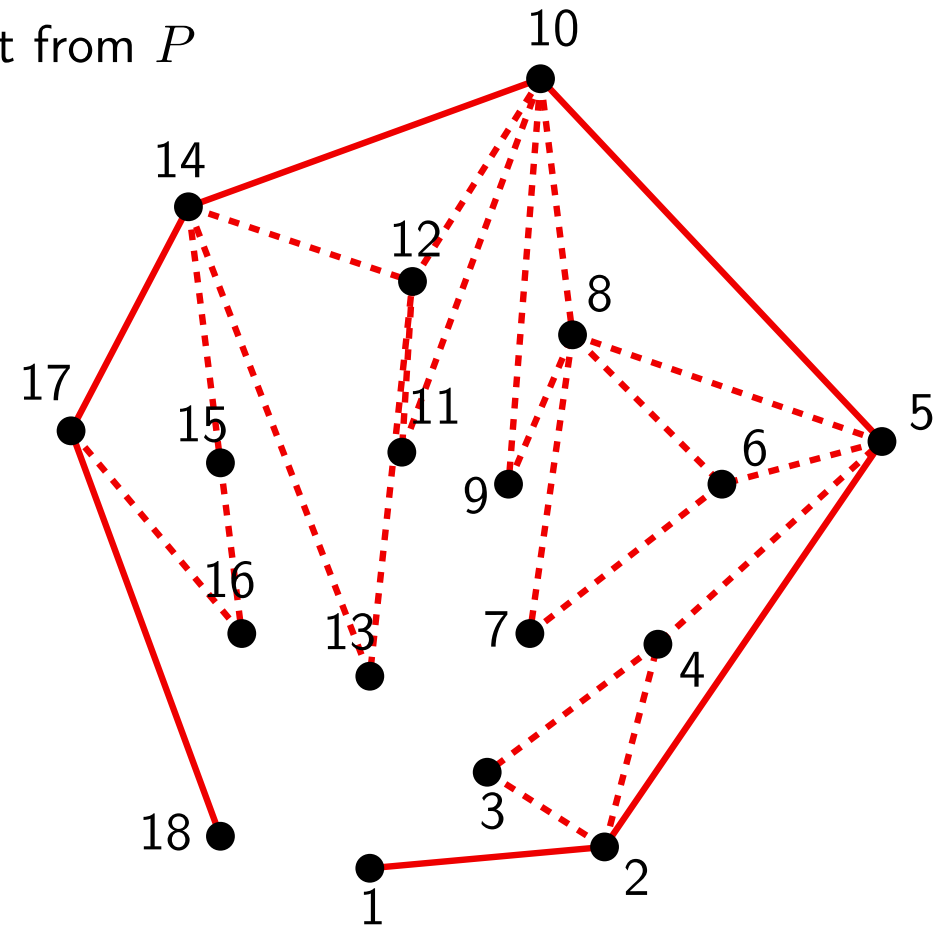
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$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

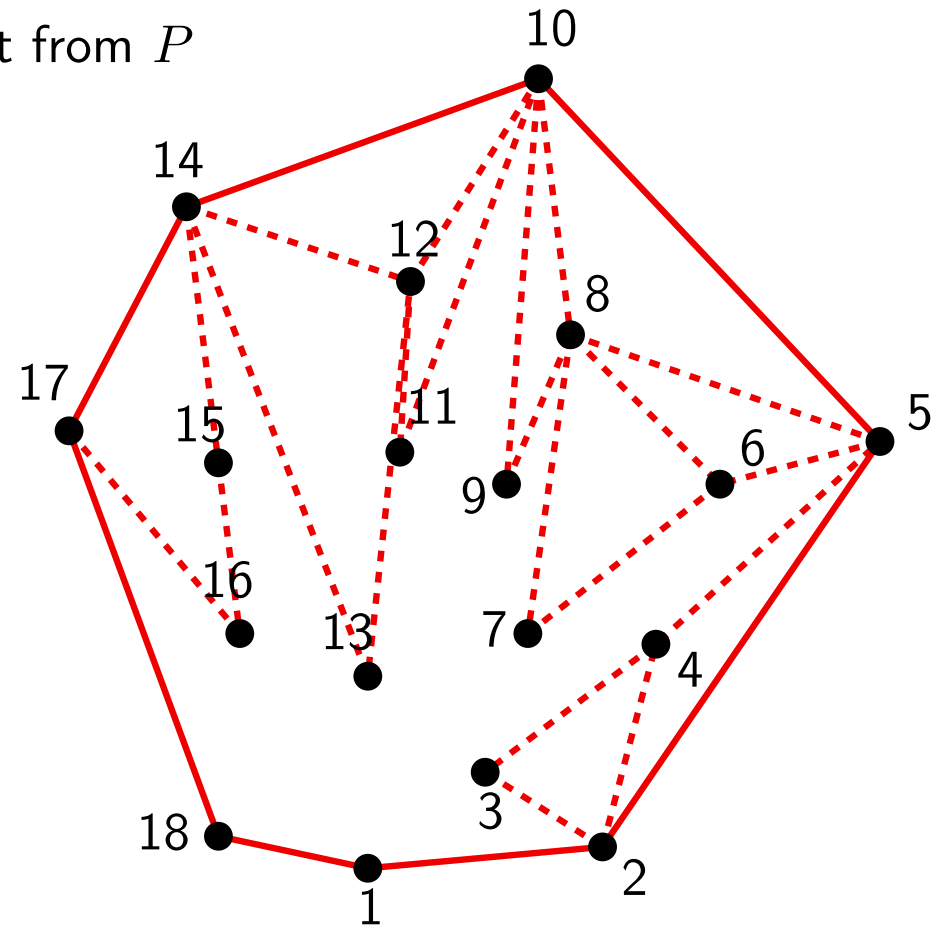
While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
 - Push p_i in l
 - Advance i
- Else:
 - Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:

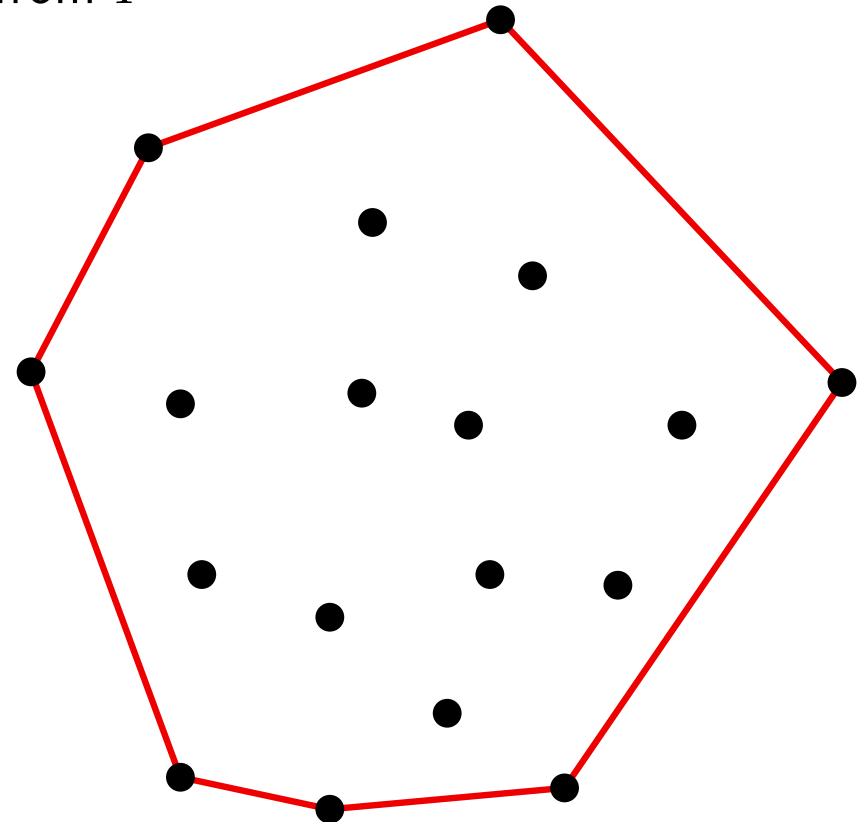
- Push p_i in l

- Advance i

- Else:

- Pop p from l

Return l



CONVEX HULL IN 2D

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete it from P

Advance

While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:

- Push p_i in l

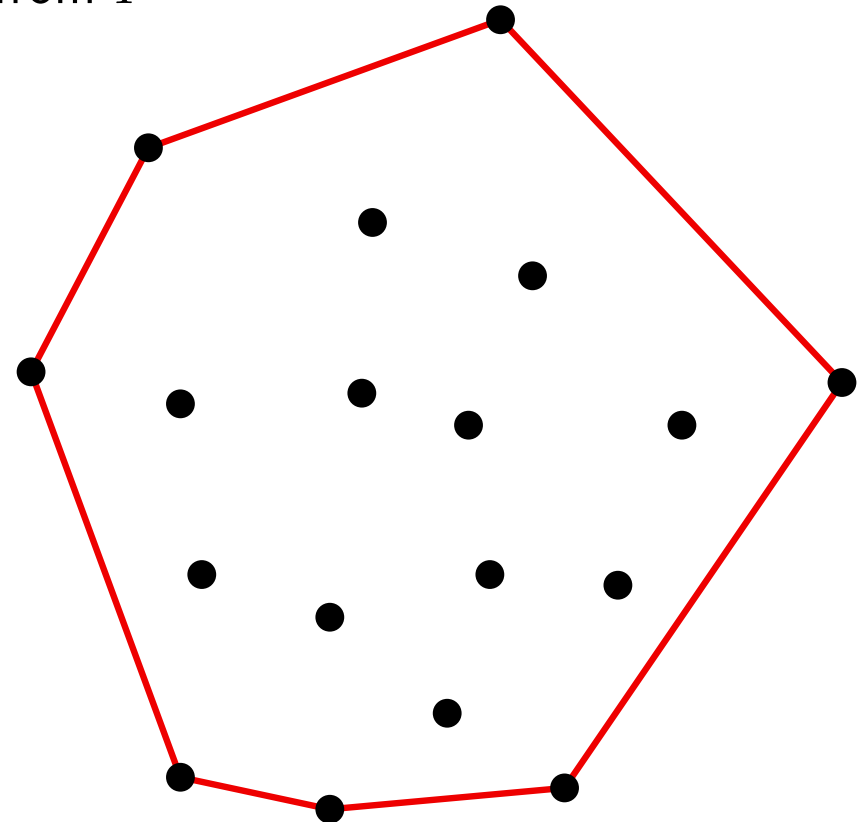
- Advance i

- Else:

- Pop p from l

Return l

Running time: $O(n \log n)$



CONVEX HULL IN 2D

Incremental algorithm

CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l

CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

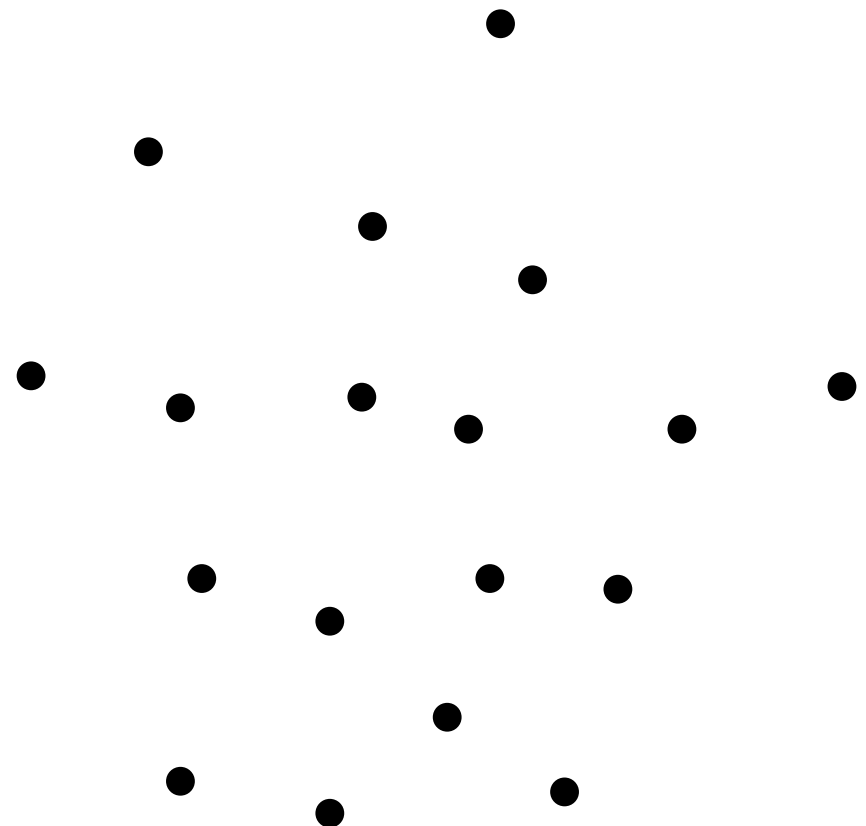
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

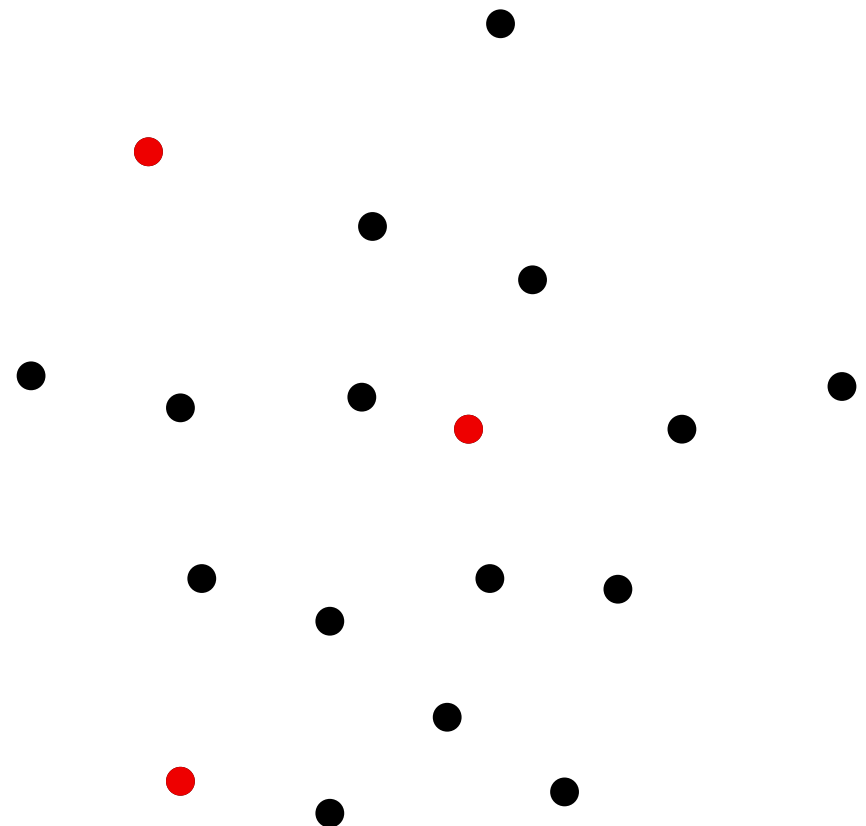
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From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

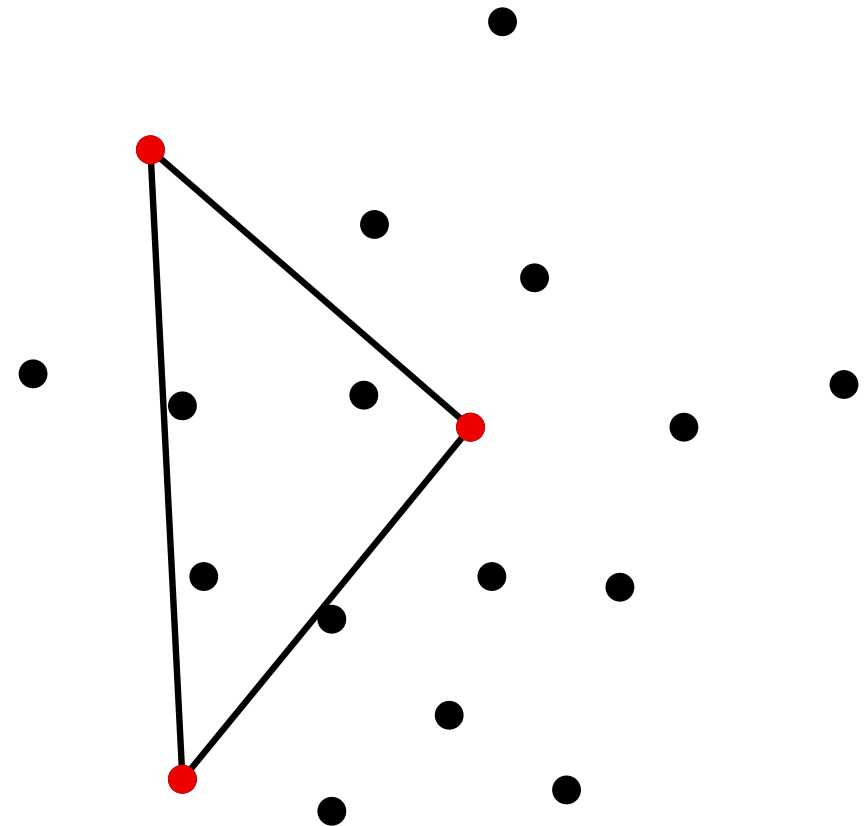
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

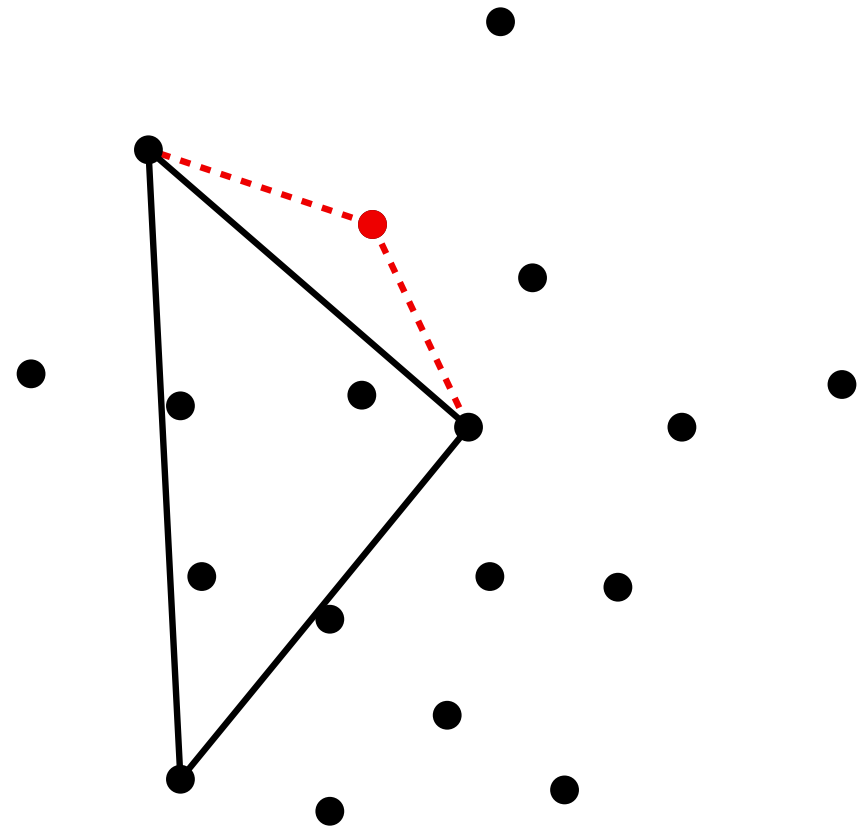
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From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

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- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

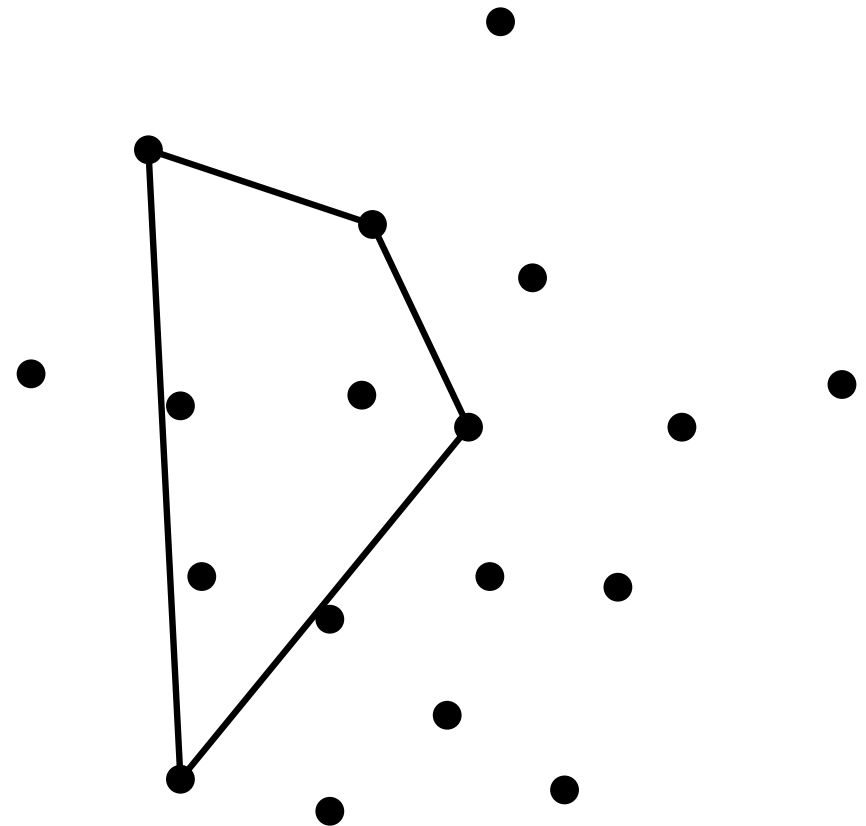
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If p_i lies in the exterior of the polygon defined by l :

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Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

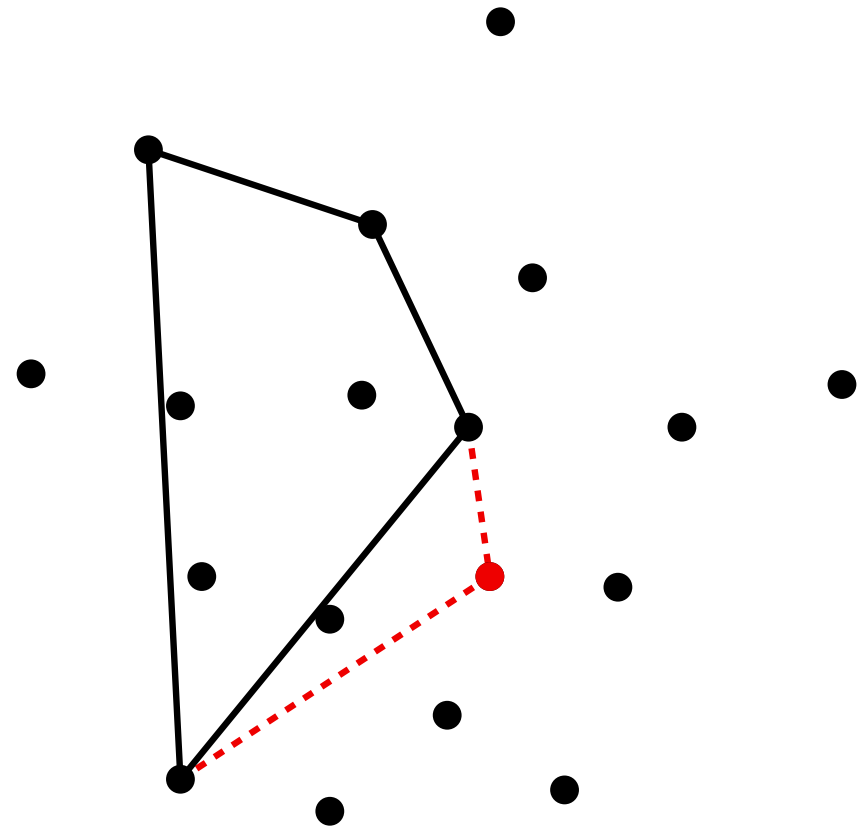
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If p_i lies in the exterior of the polygon defined by l :

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Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

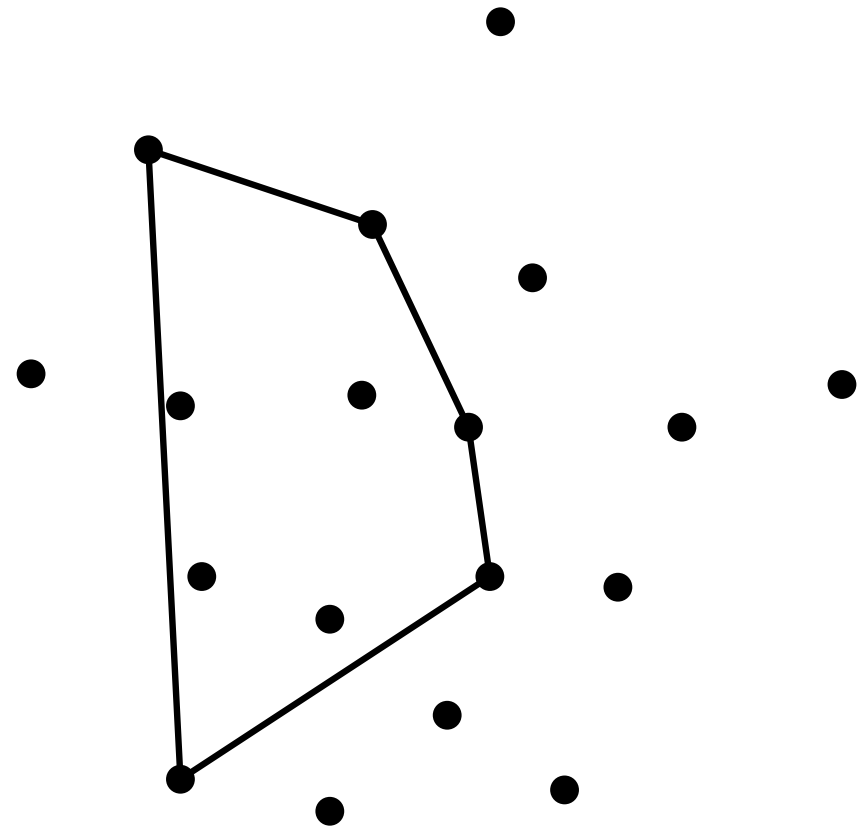
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From $i = 4$ to n , do:

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Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

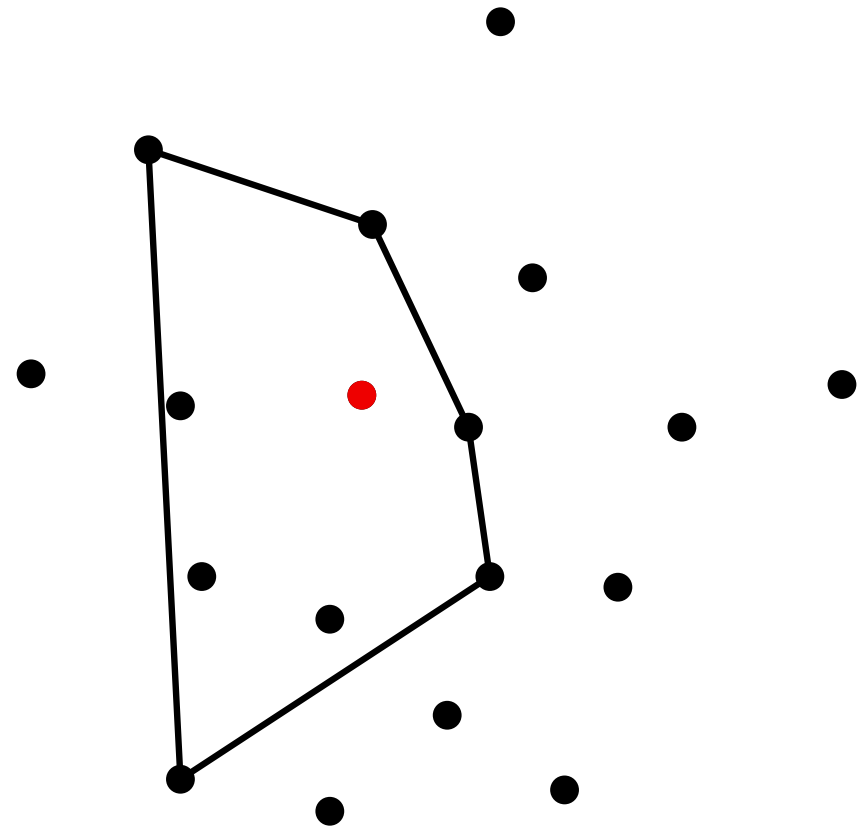
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

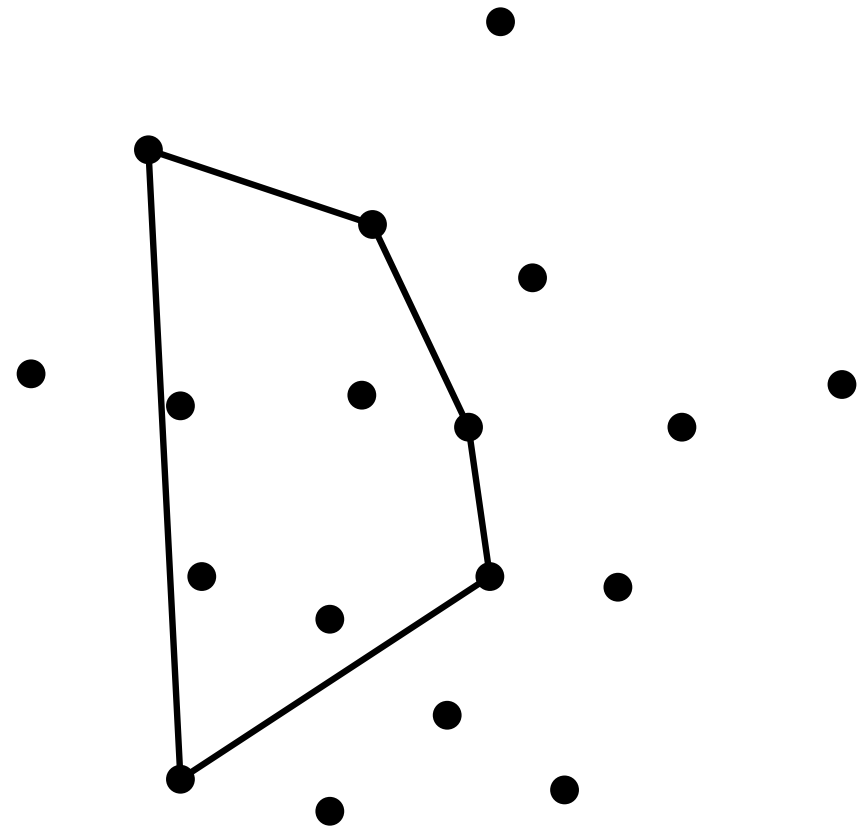
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Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

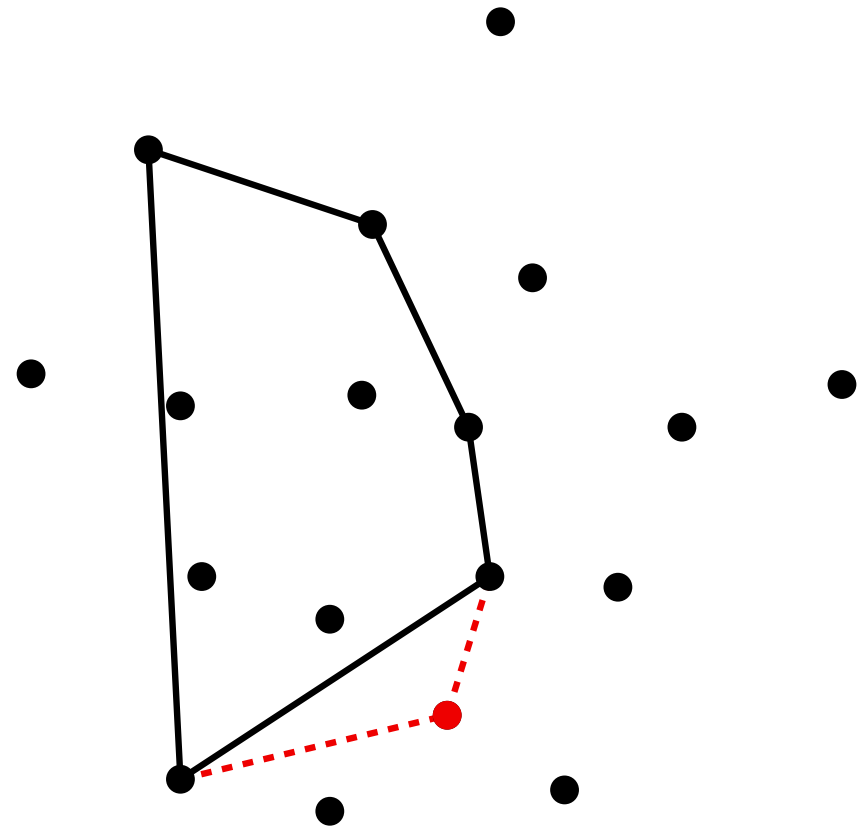
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

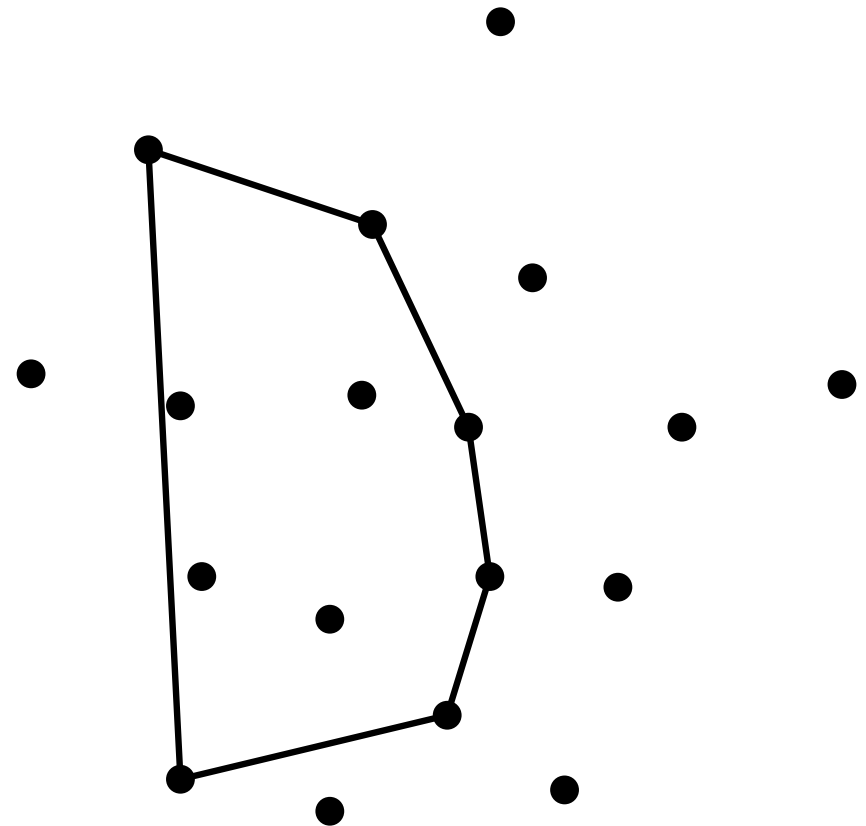
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

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Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

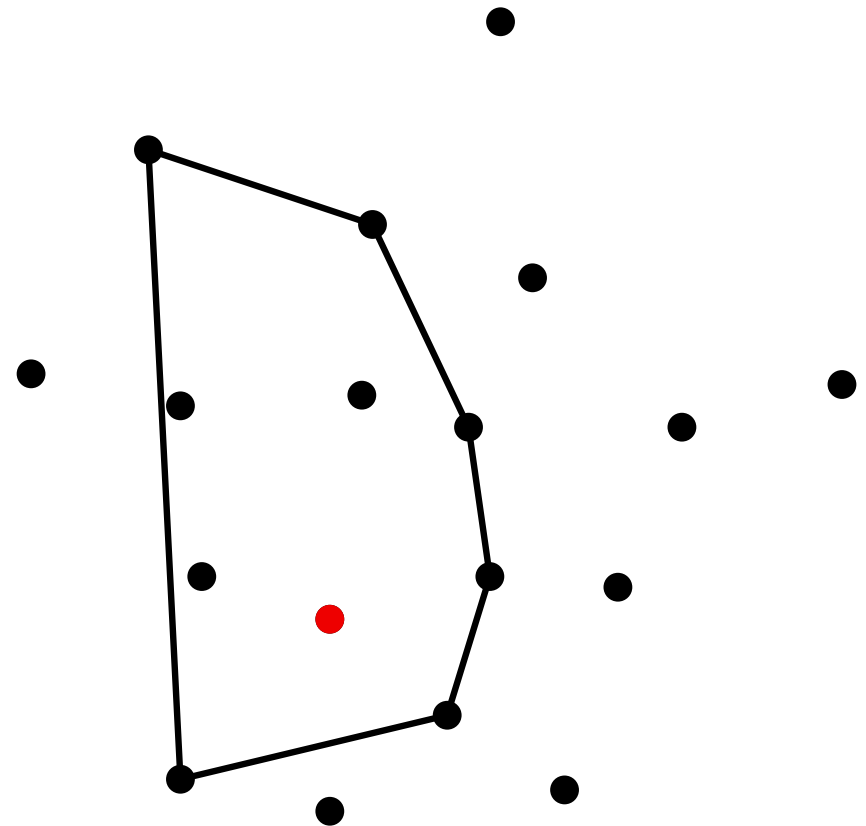
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

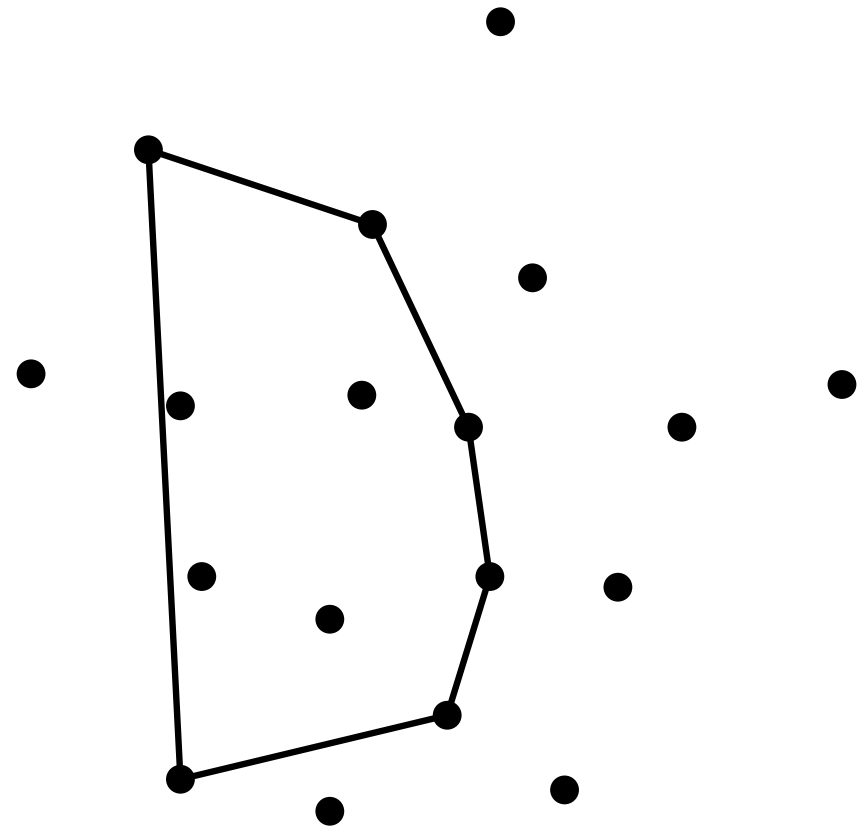
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

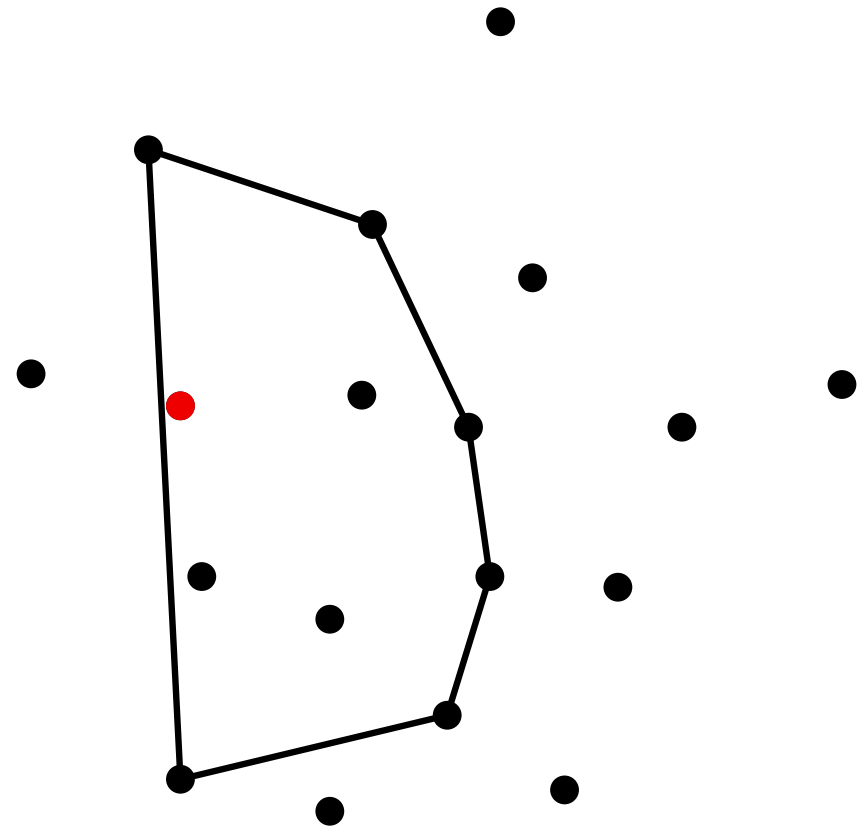
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

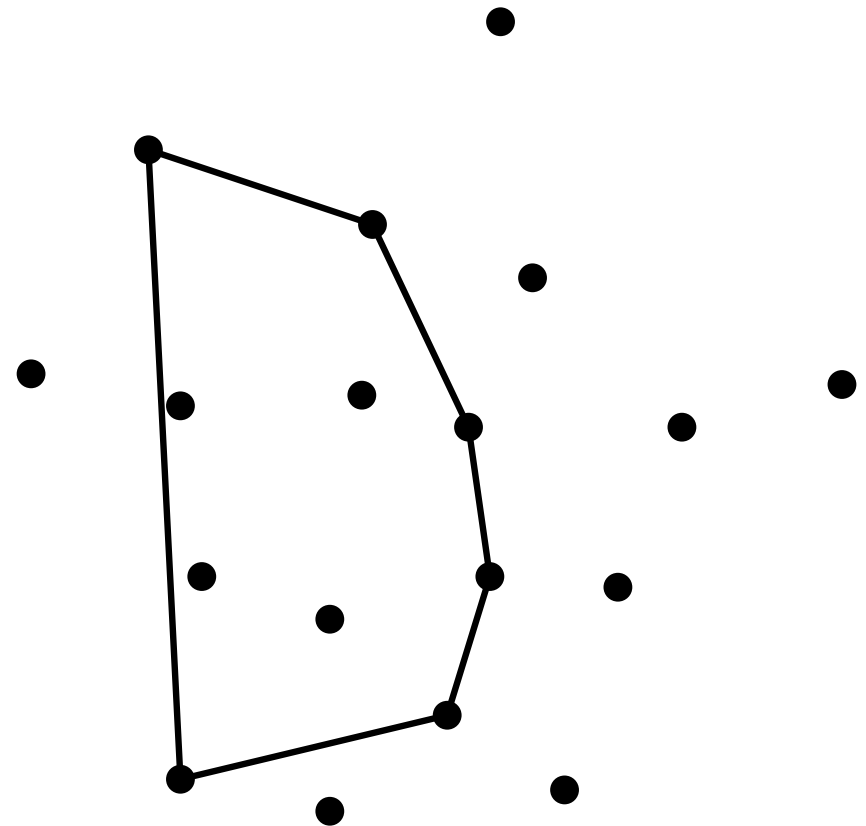
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

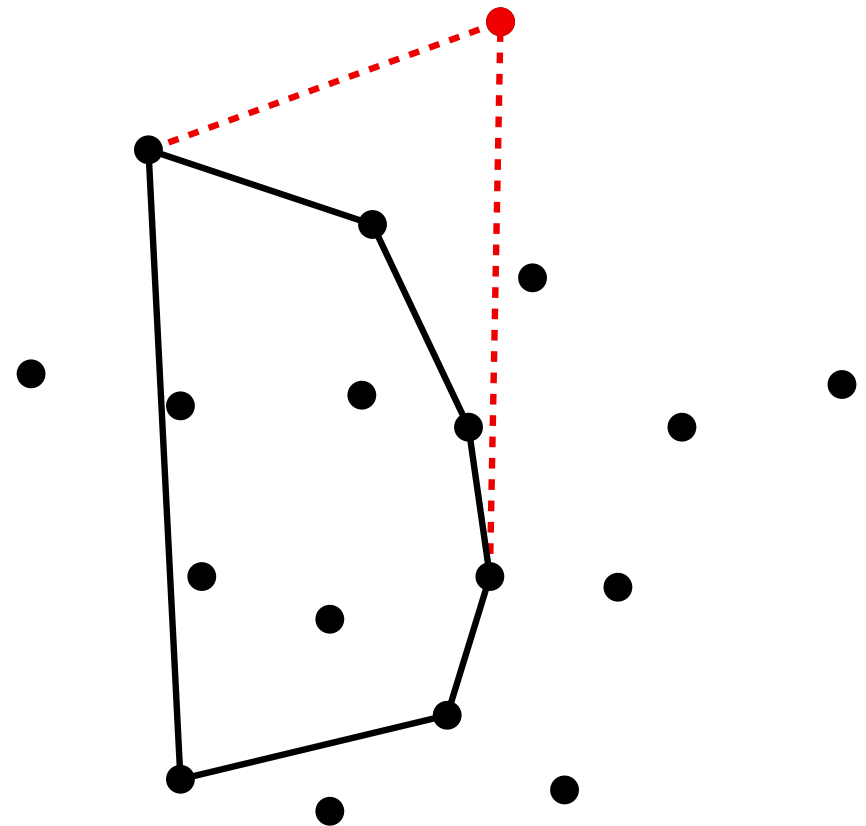
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

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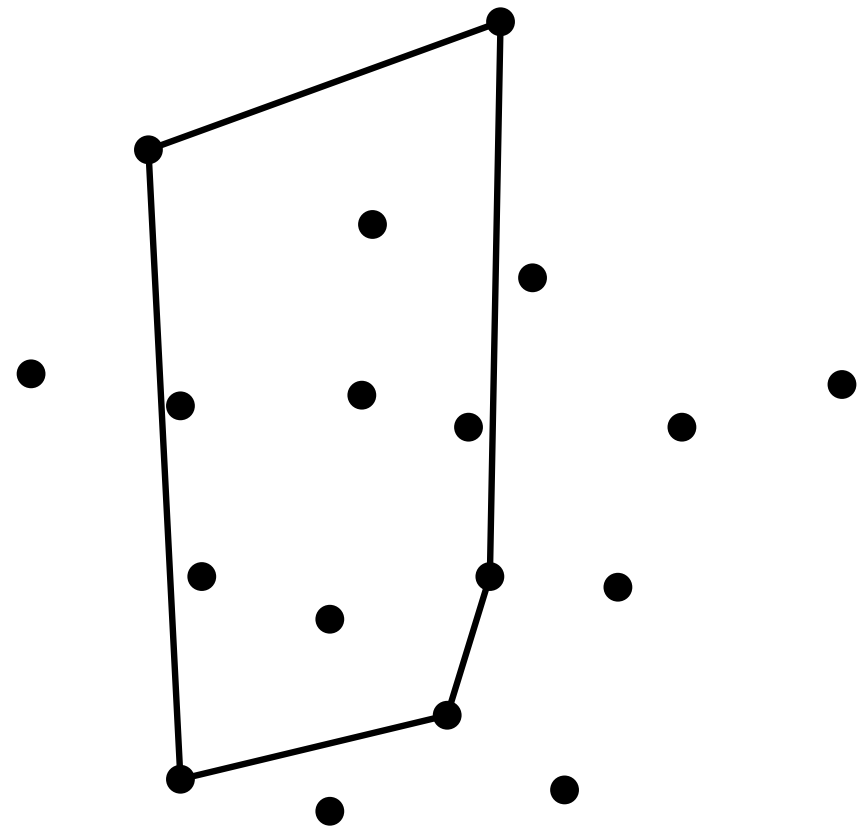
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Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

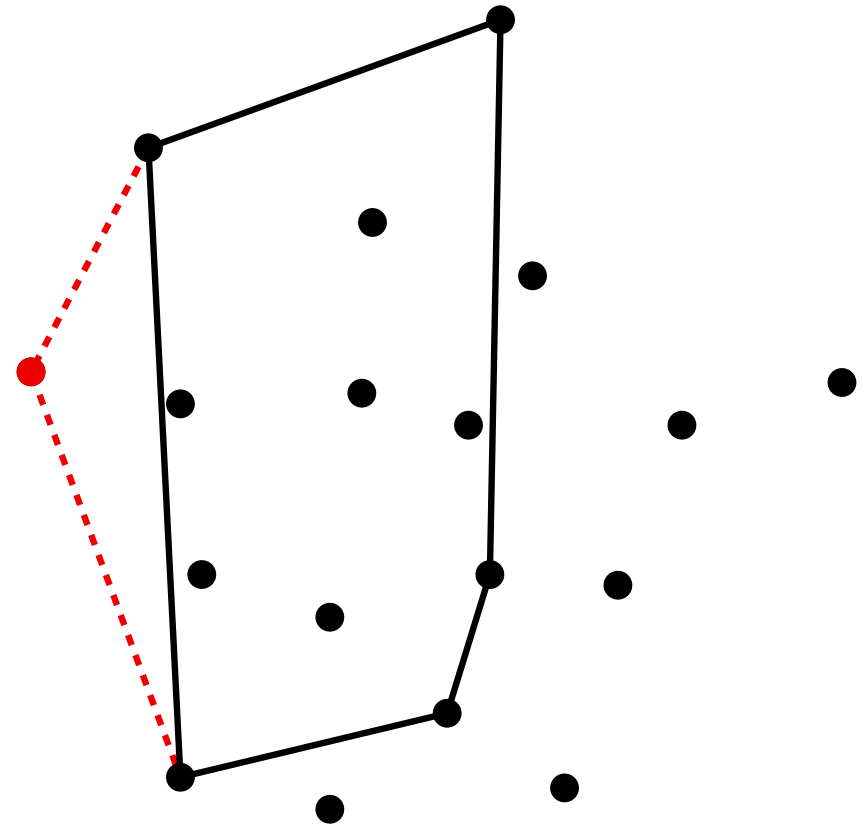
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

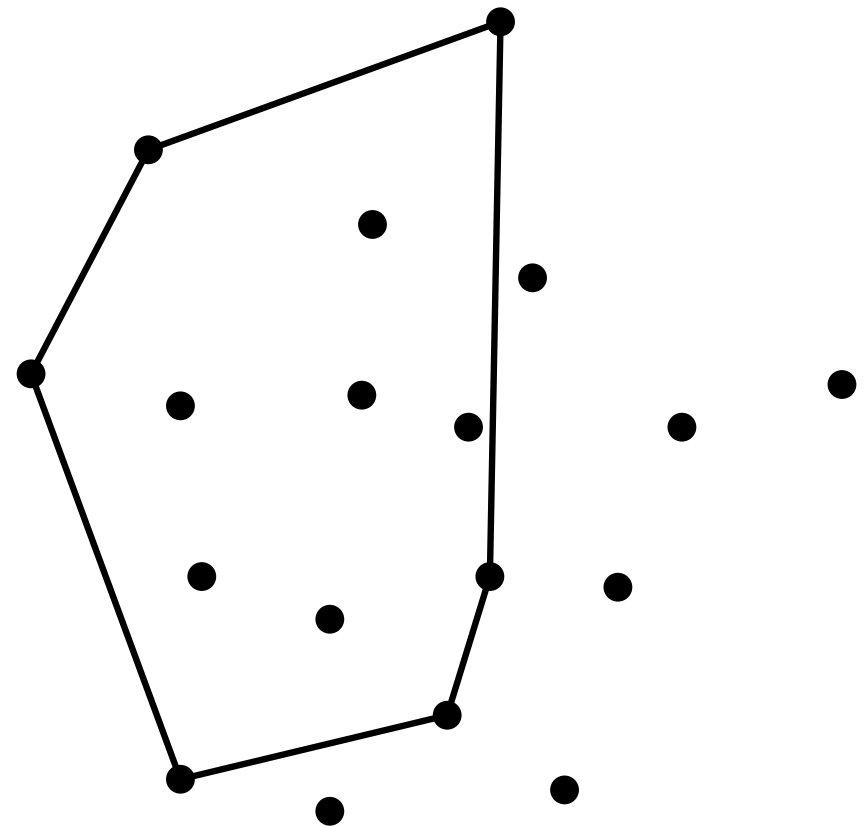
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From $i = 4$ to n , do:

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CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

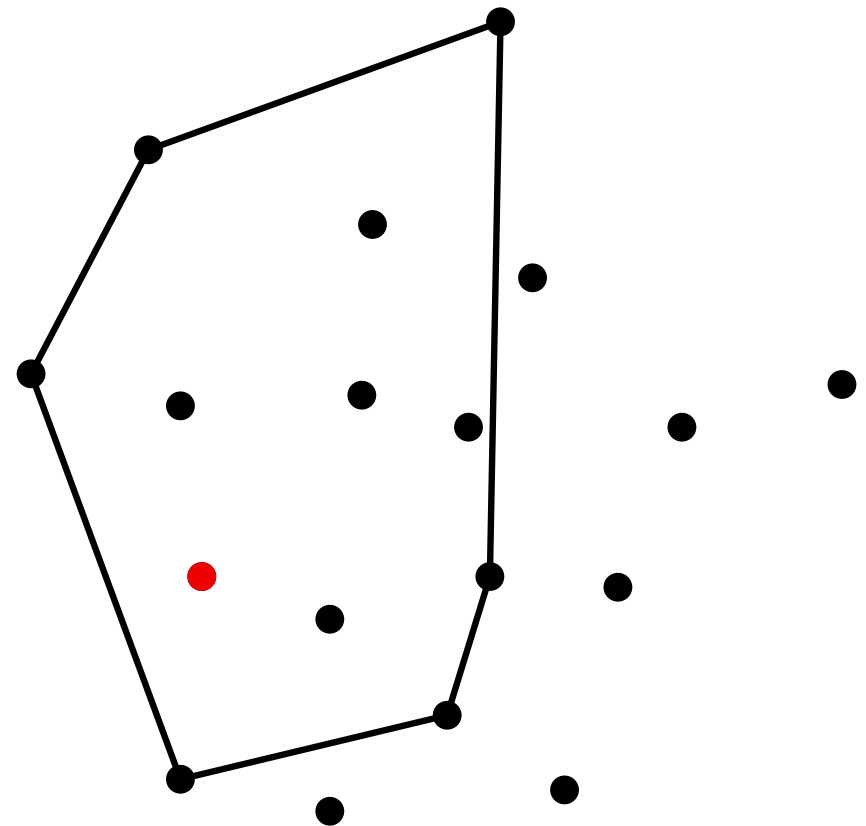
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From $i = 4$ to n , do:

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CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

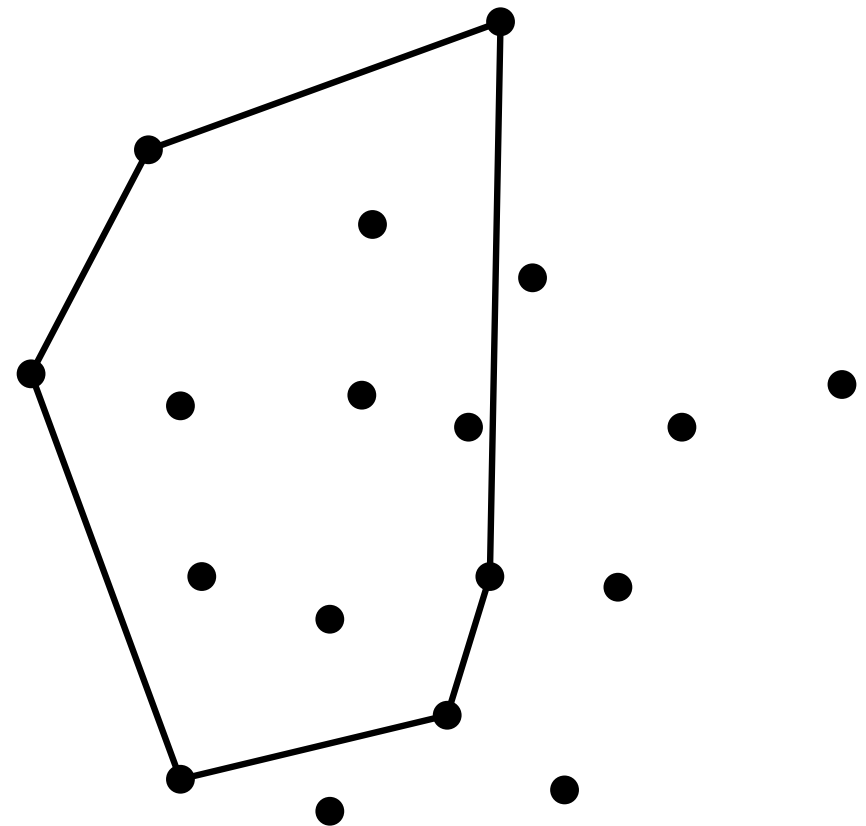
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Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

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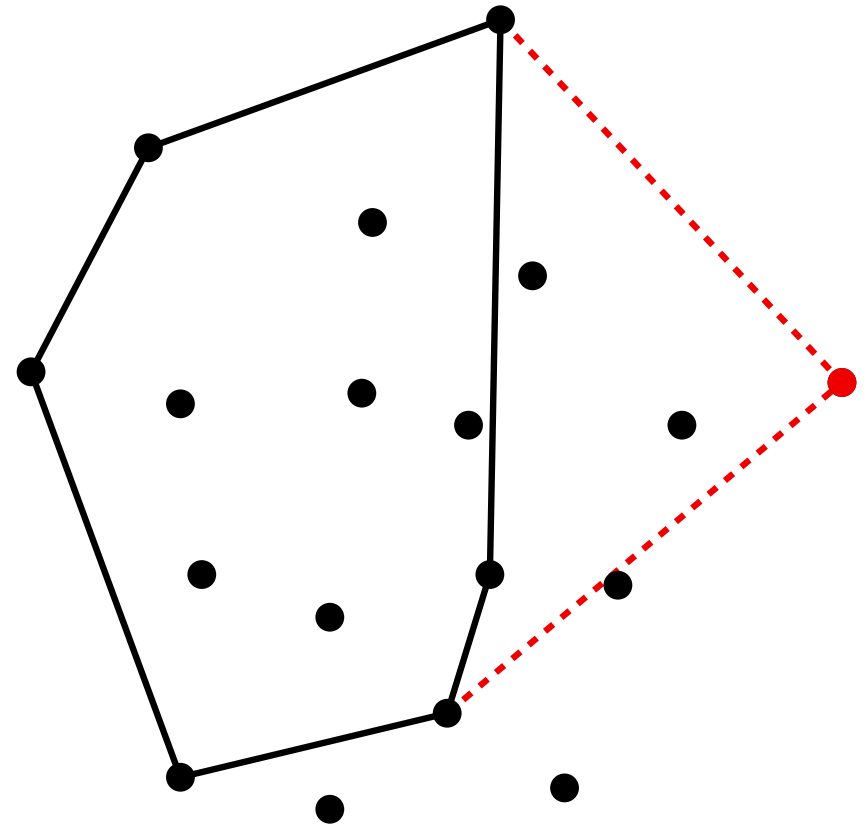
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Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

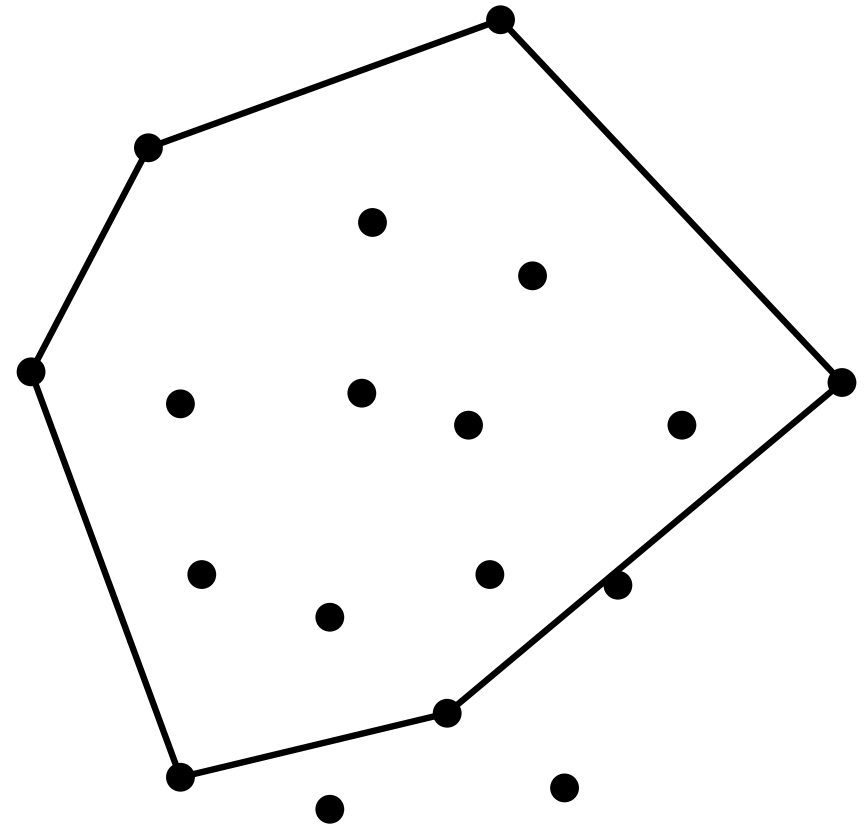
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

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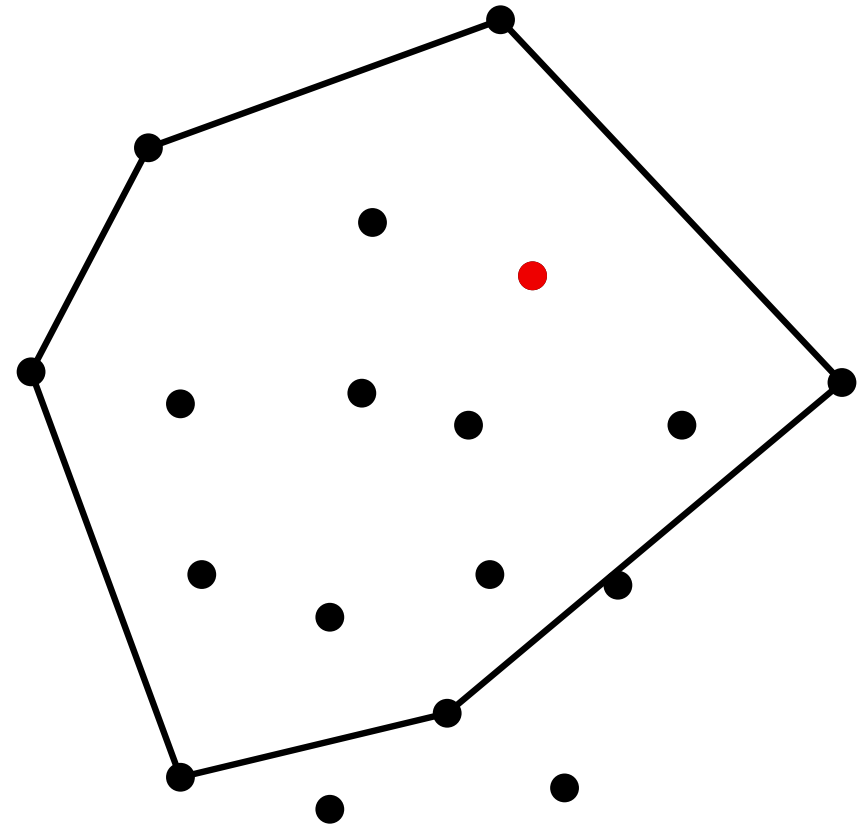
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CONVEX HULL IN 2D

Incremental algorithm

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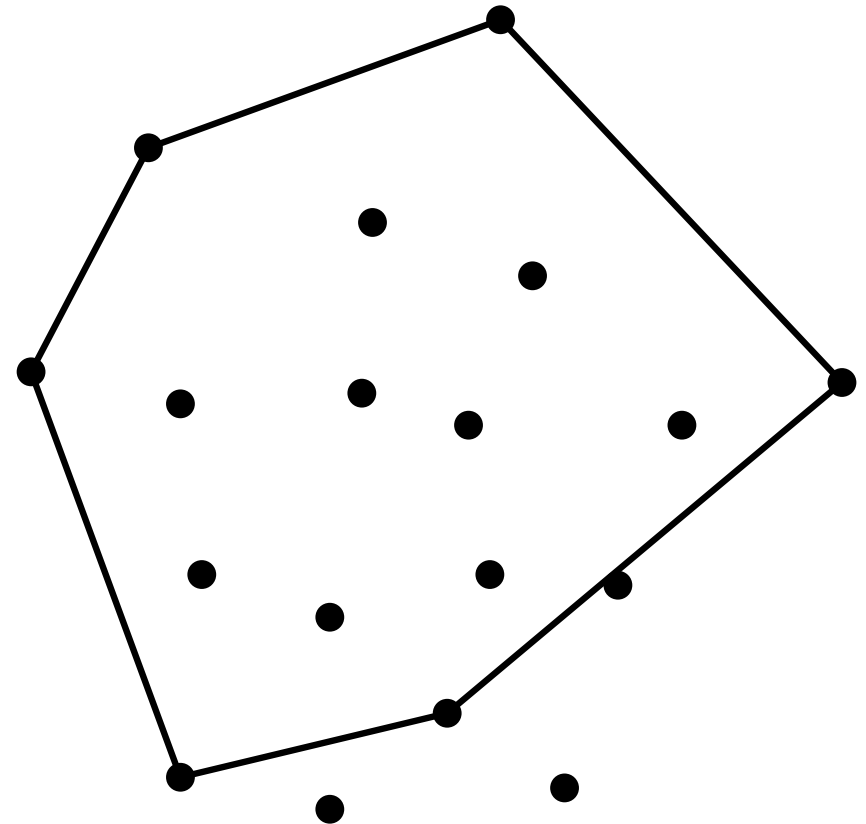
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CONVEX HULL IN 2D

Incremental algorithm

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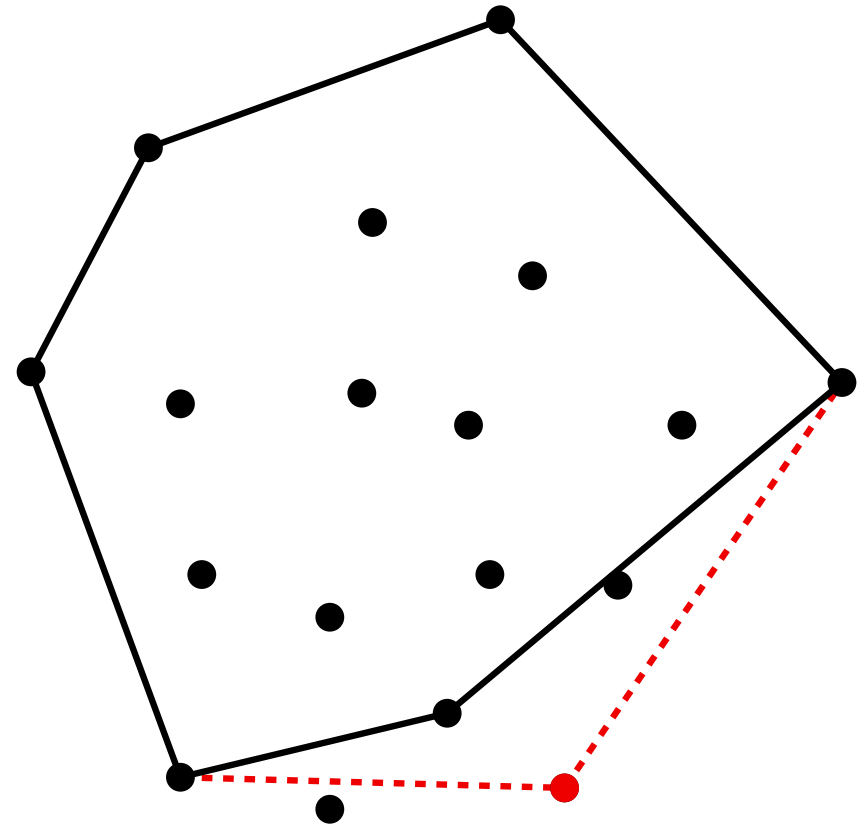
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CONVEX HULL IN 2D

Incremental algorithm

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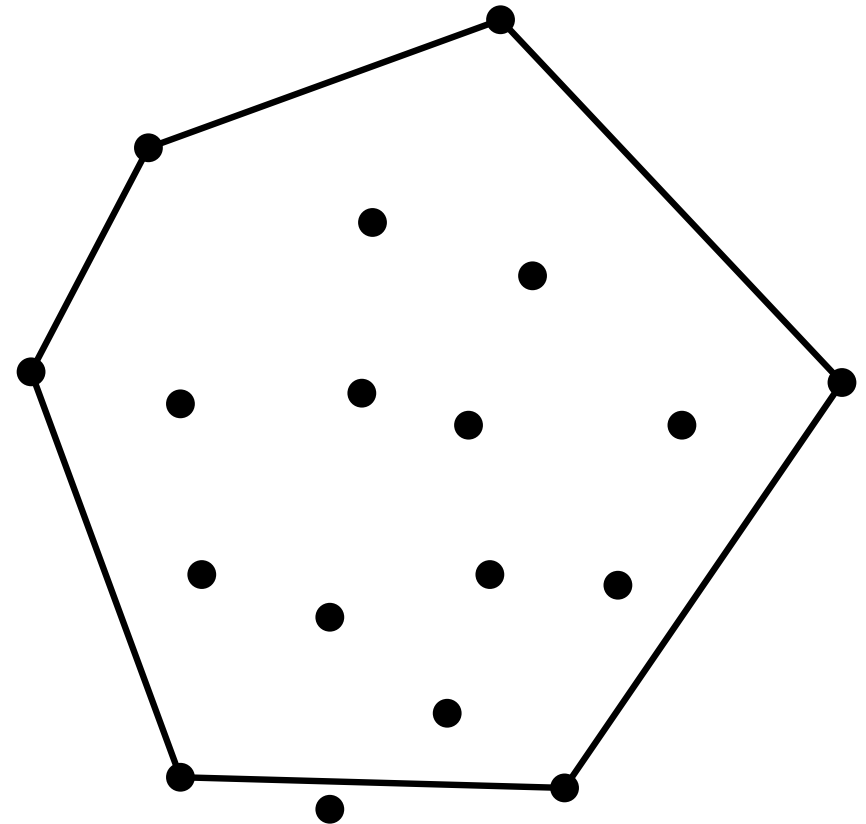
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CONVEX HULL IN 2D

Incremental algorithm

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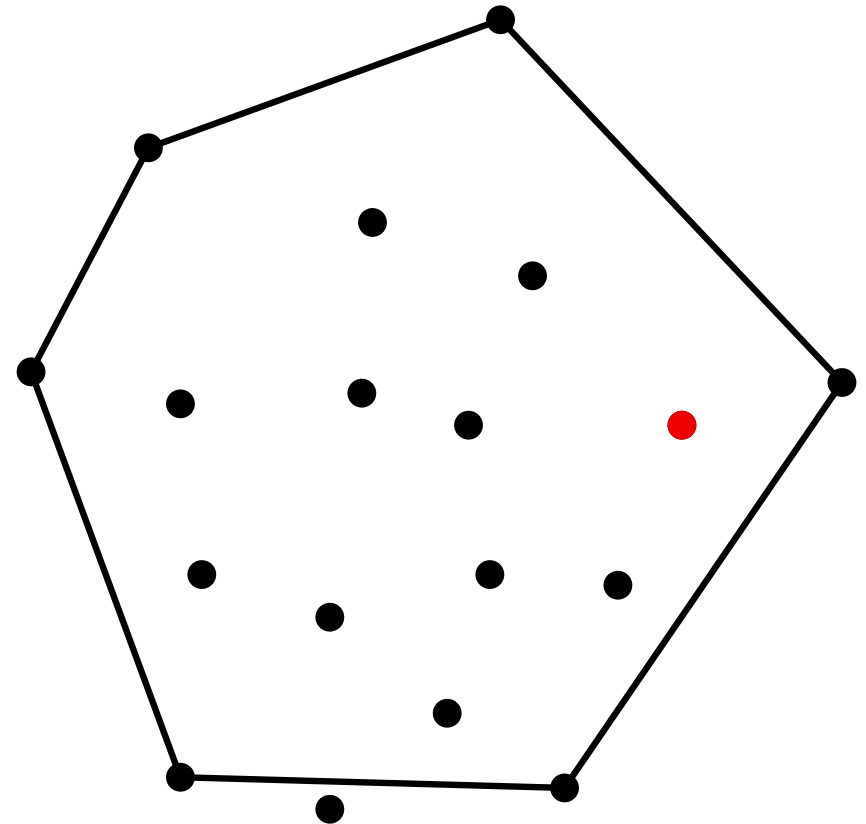
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CONVEX HULL IN 2D

Incremental algorithm

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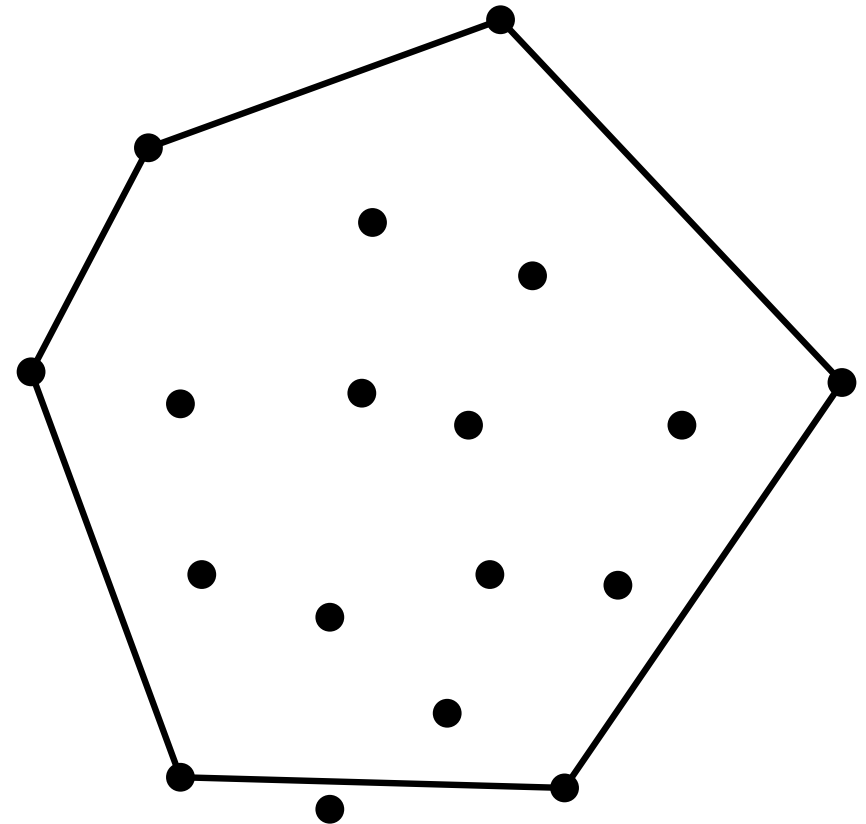
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Initialization

$$l = p_1, p_2, p_3$$

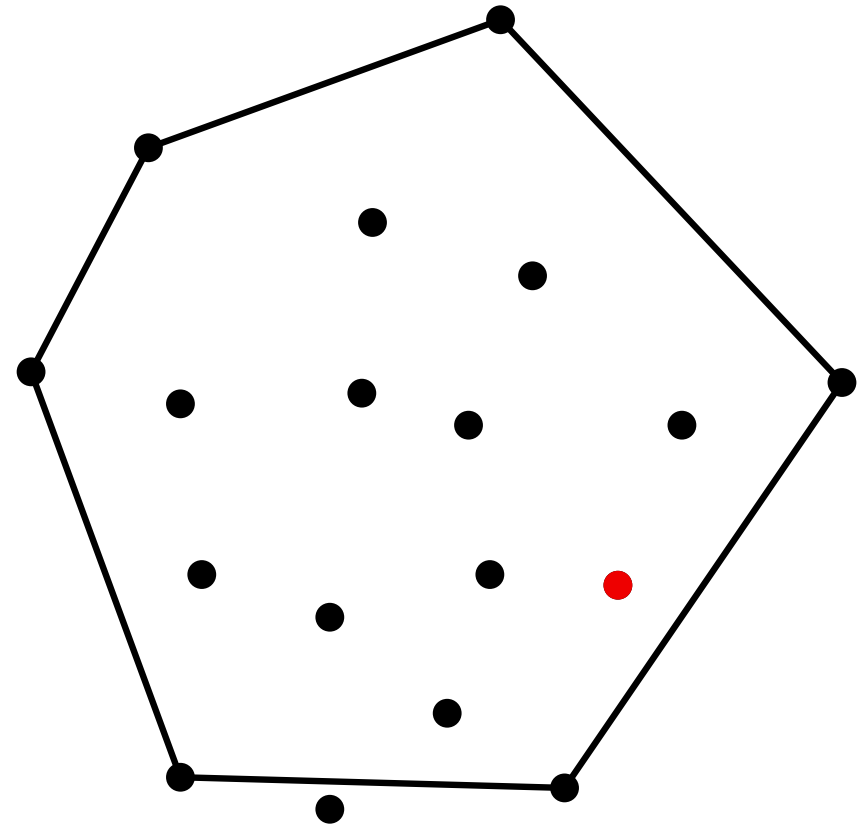
Advance

From $i = 4$ to n , do:

If p_i lies in the exterior of the polygon defined by l :

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

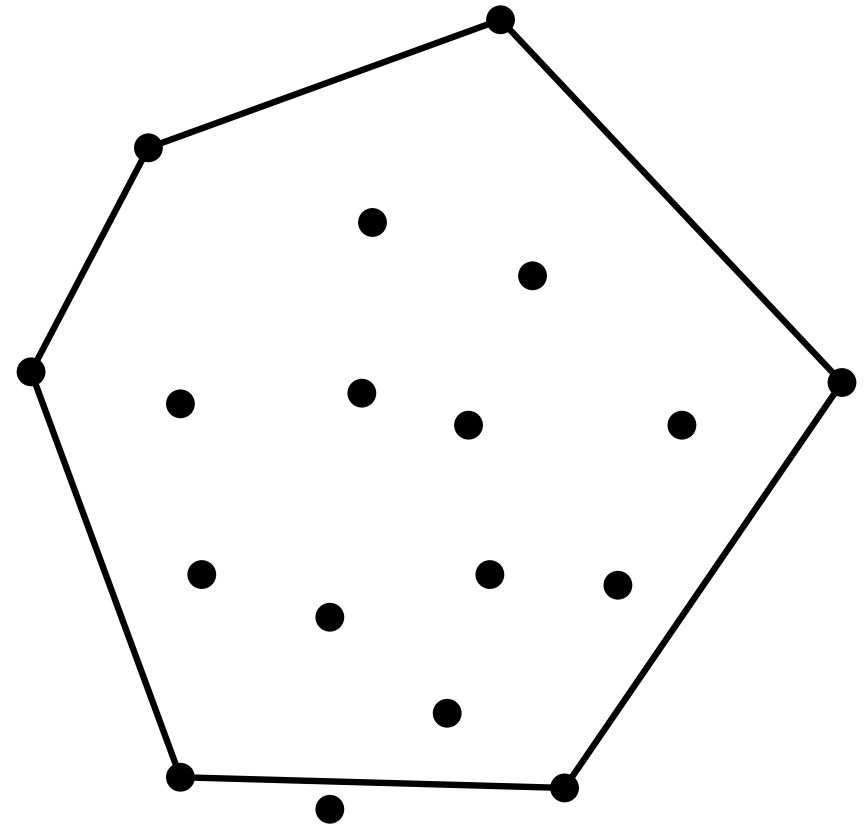
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

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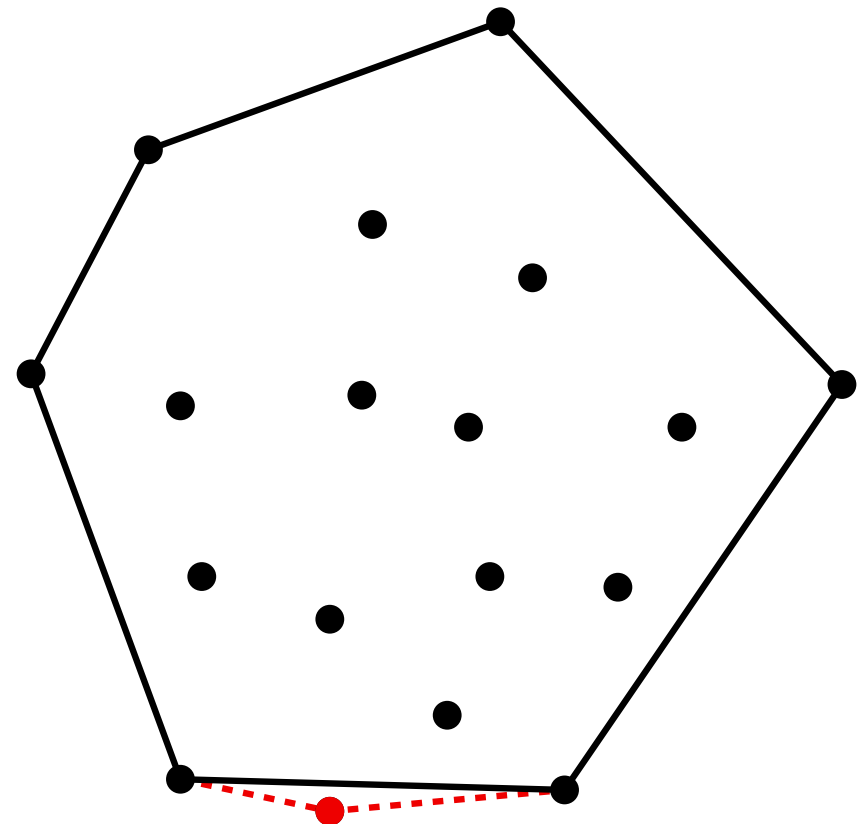
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CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

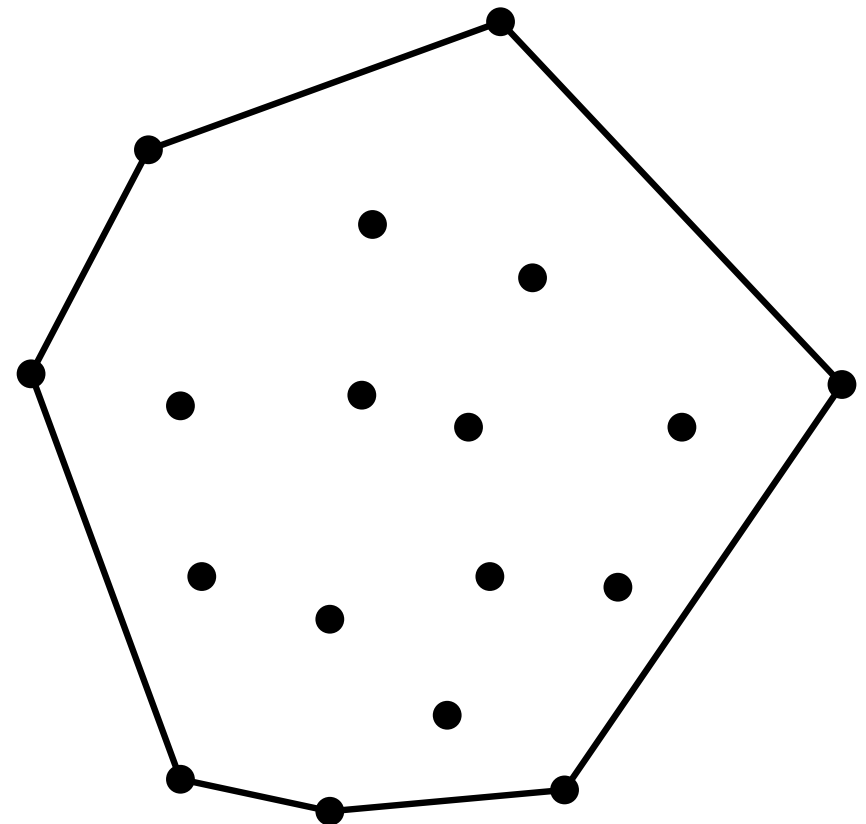
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CONVEX HULL IN 2D

Incremental algorithm

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Advance

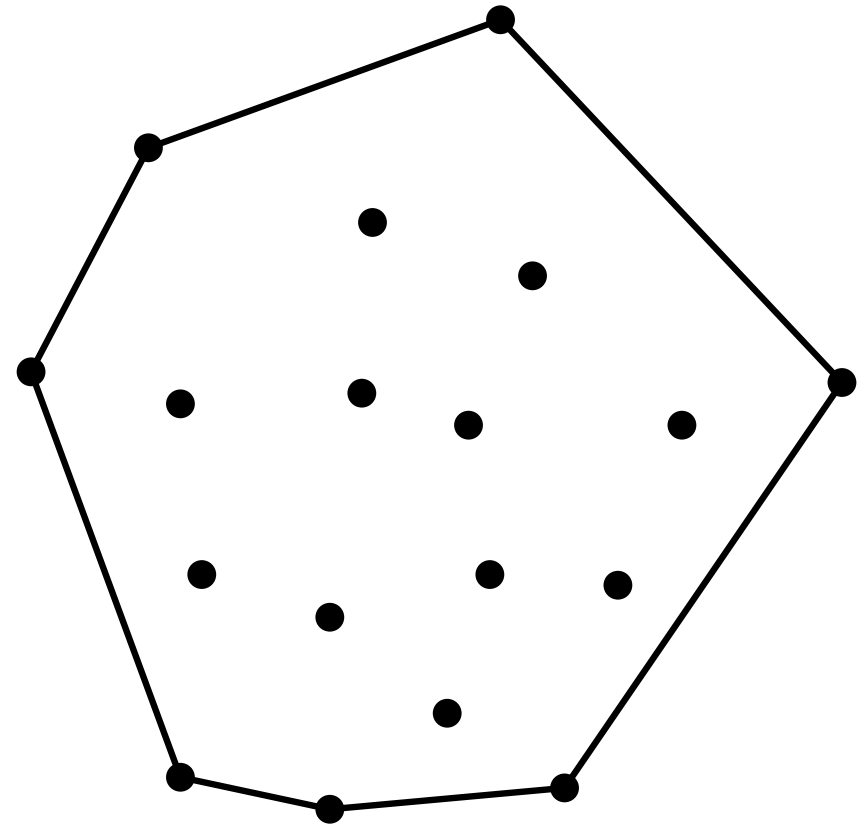
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Return l

Running time: $O(n \log n)$



CONVEX HULL IN 2D

Incremental algorithm

Initialization

$$l = p_1, p_2, p_3$$

Advance

From $i = 4$ to n , do:

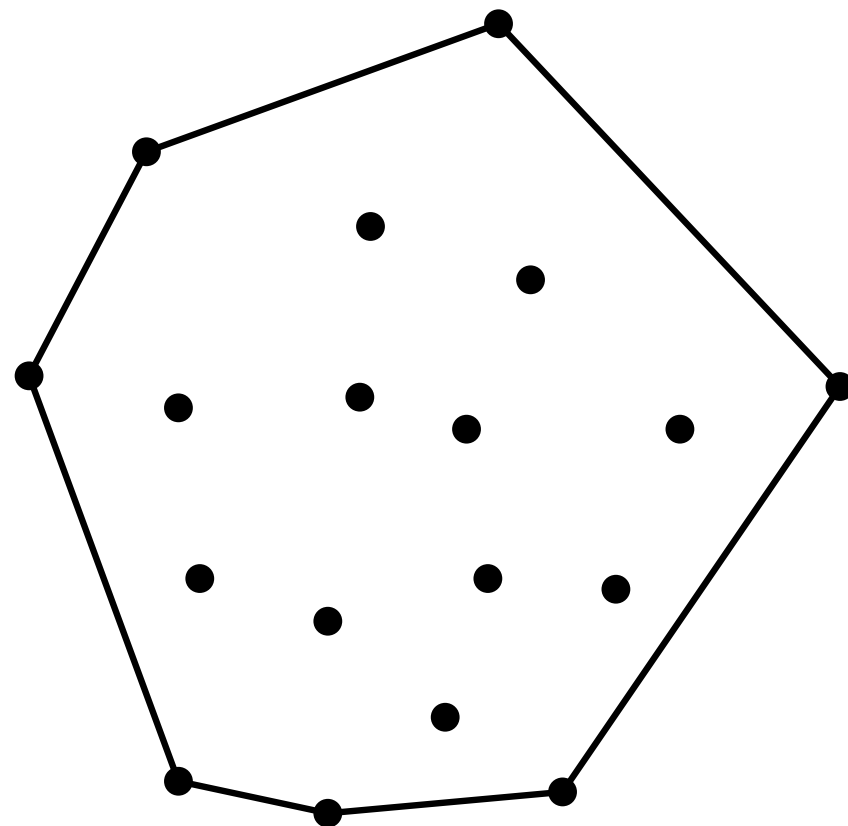
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- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain p_l, \dots, p_r in l with the chain p_l, p_i, p_r

Return l

Running time: $O(n \log n)$

By storing l in a structure allowing binary search and updatings (insertions and deletions) in $O(\log n)$ time.



CONVEX HULL IN 2D

Divide-and-conquer algorithm

CONVEX HULL IN 2D

Divide-and-conquer algorithm

Initialization

1. Sort the points by abscissae

CONVEX HULL IN 2D

Divide-and-conquer algorithm

Initialization

1. Sort the points by abscissae

Division

1. Divide the points (x_i, y_i) into two subsets, wrt the median value of the abscissae

CONVEX HULL IN 2D

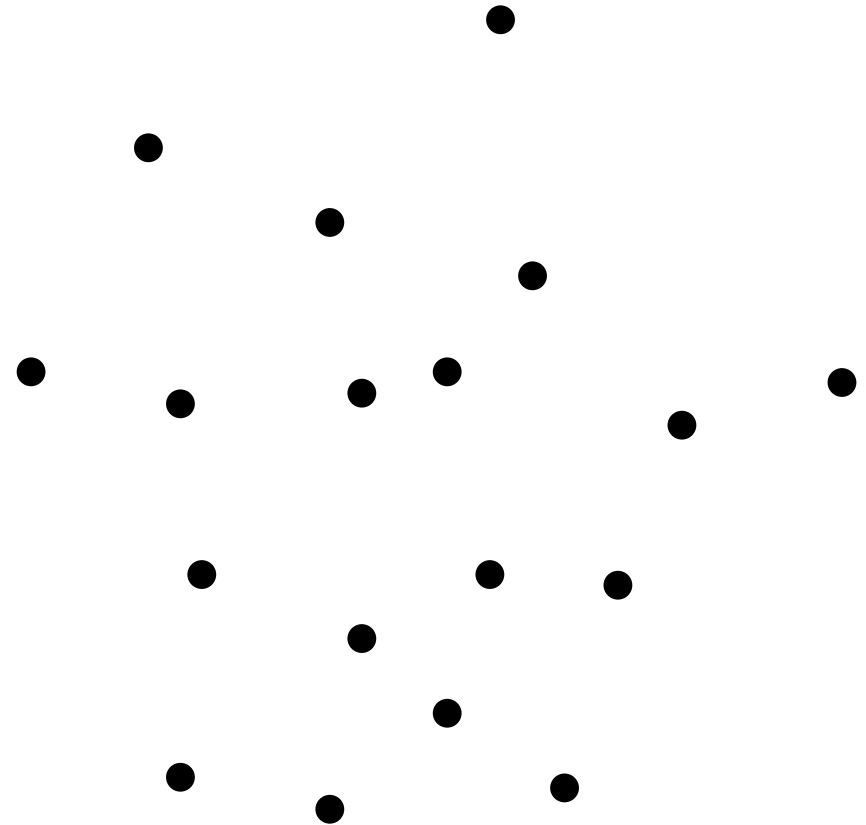
Divide-and-conquer algorithm

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CONVEX HULL IN 2D

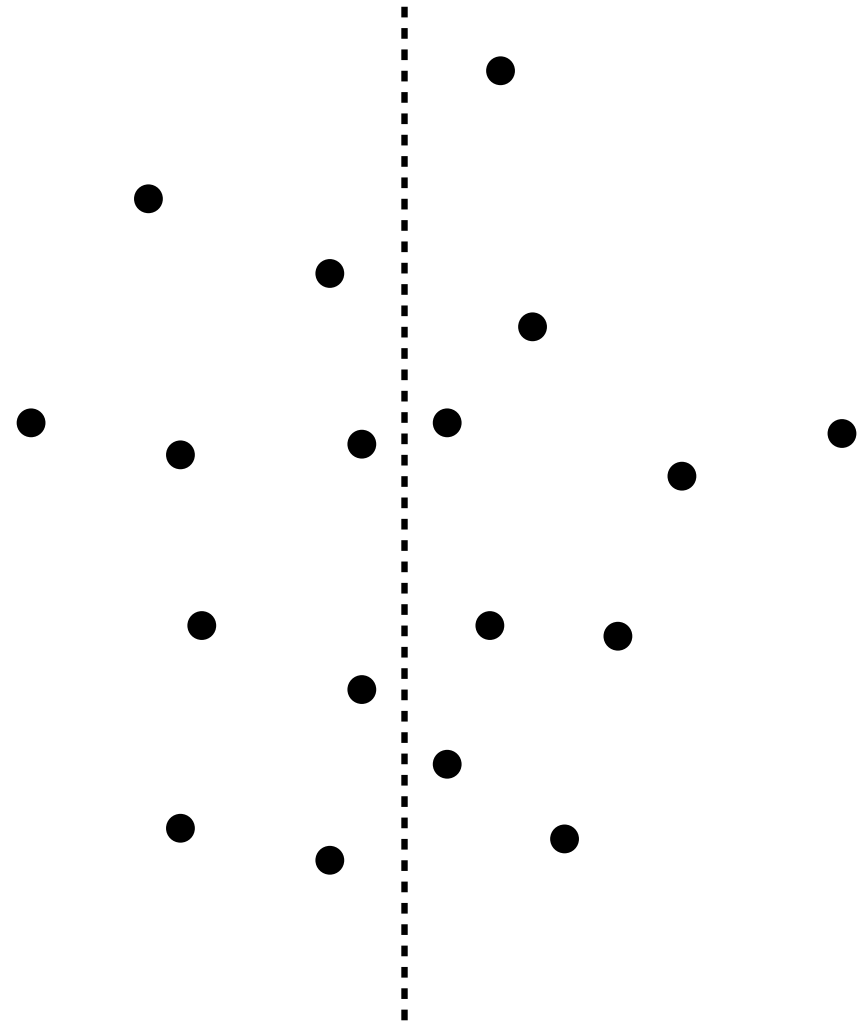
Divide-and-conquer algorithm

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CONVEX HULL IN 2D

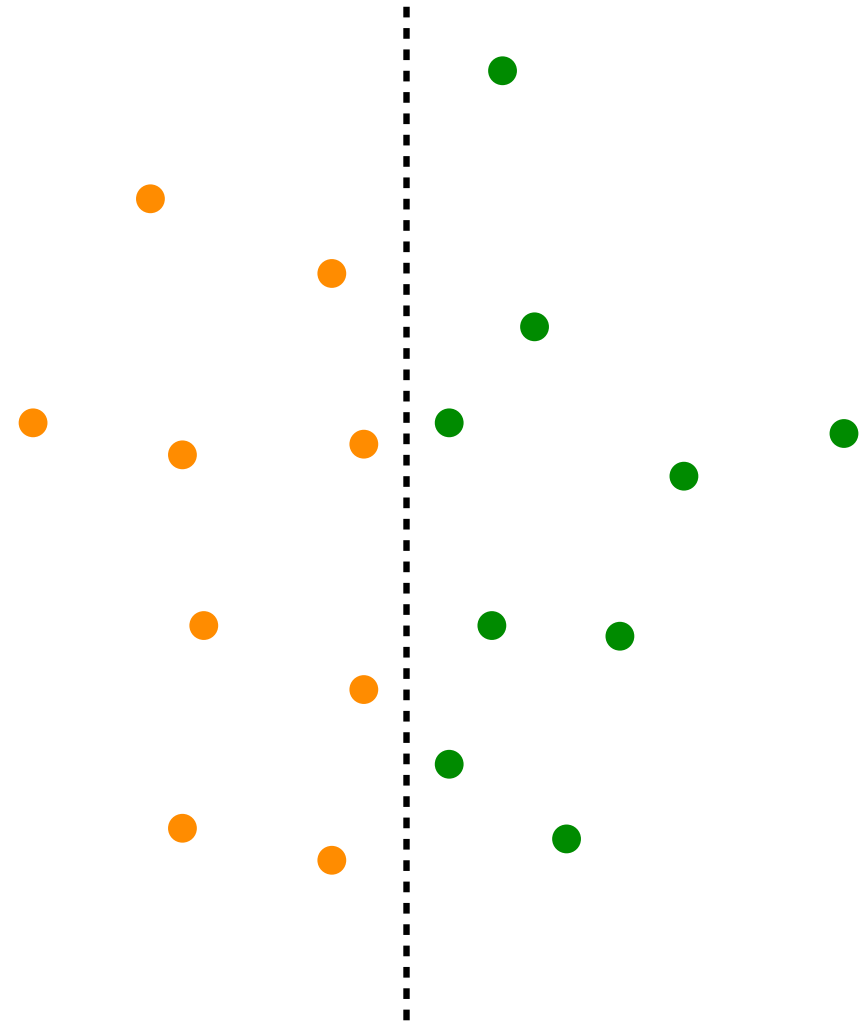
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CONVEX HULL IN 2D

Divide-and-conquer algorithm

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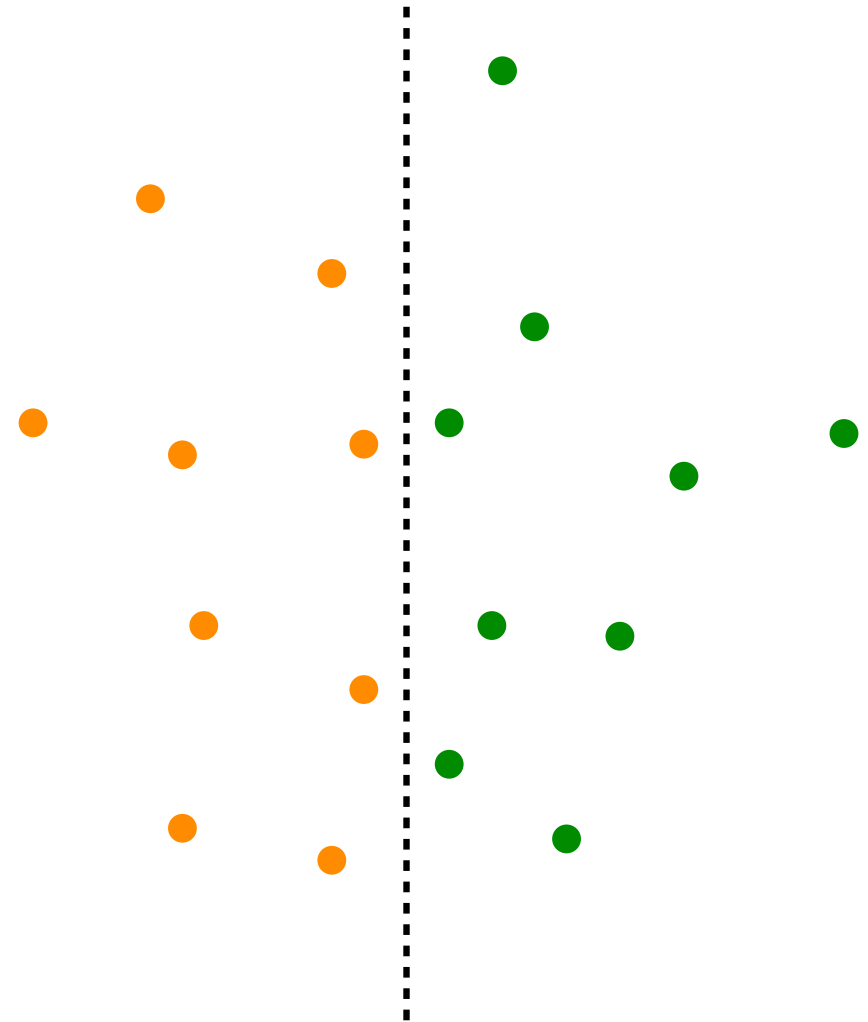
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Division

1. Divide the points (x_i, y_i) into two subsets, wrt the median value of the abscissae

Recursion

1. Recursively compute the convex hull of the two subsets



CONVEX HULL IN 2D

Divide-and-conquer algorithm

Initialization

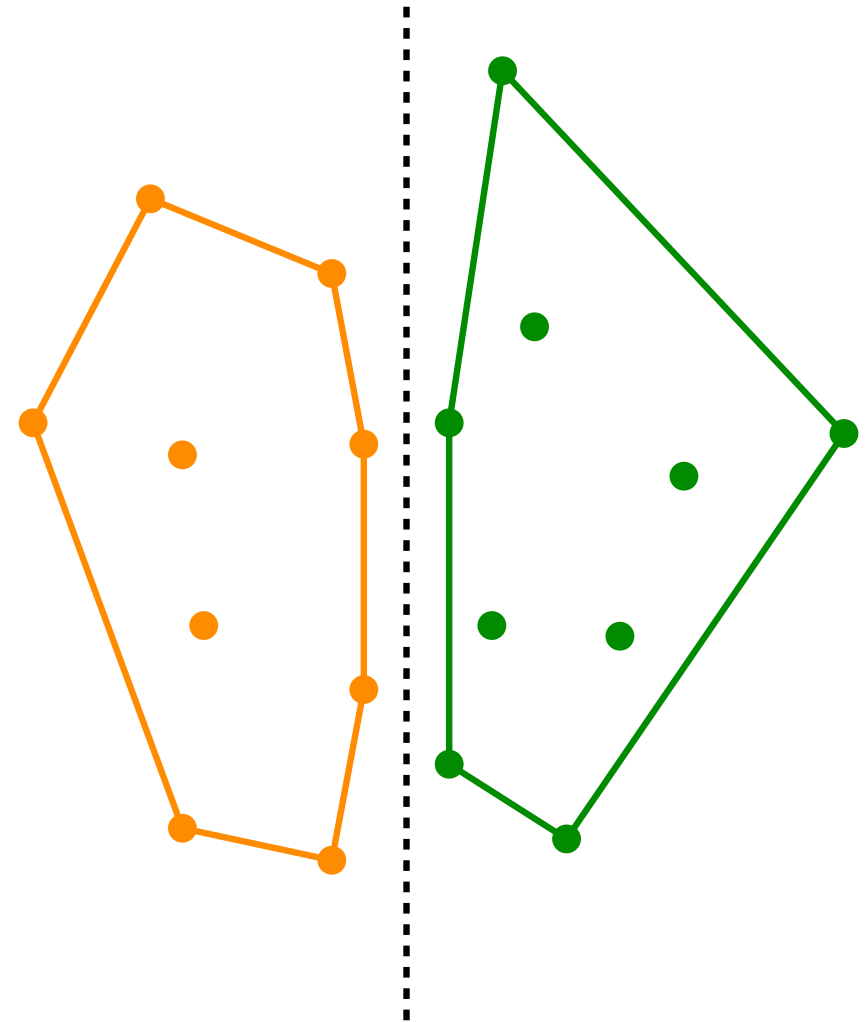
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CONVEX HULL IN 2D

Divide-and-conquer algorithm

Initialization

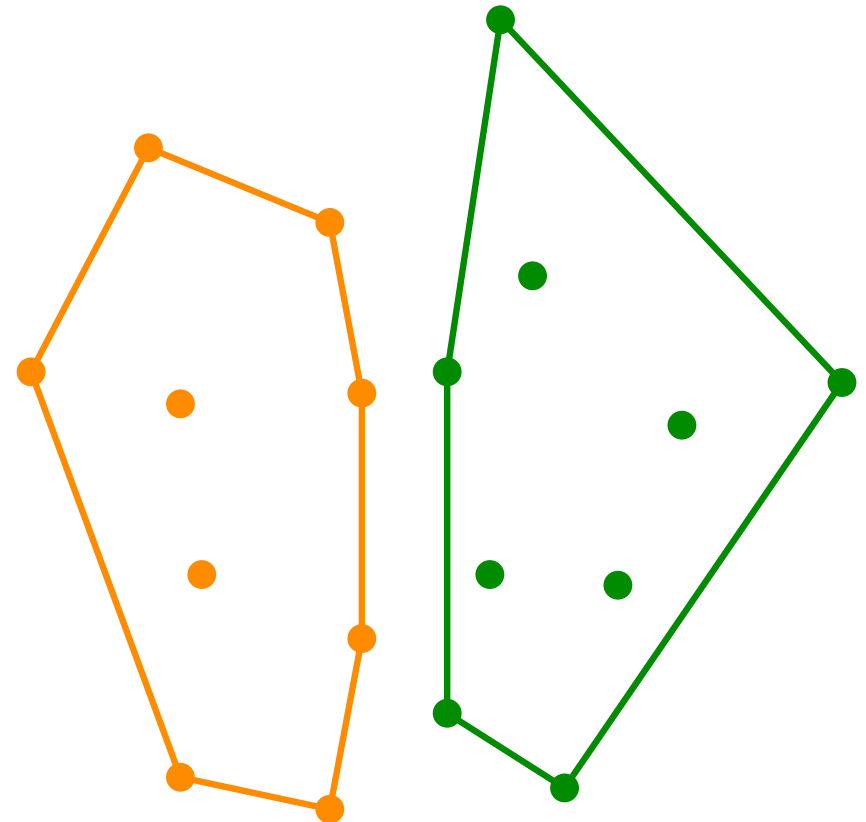
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CONVEX HULL IN 2D

Divide-and-conquer algorithm

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Division

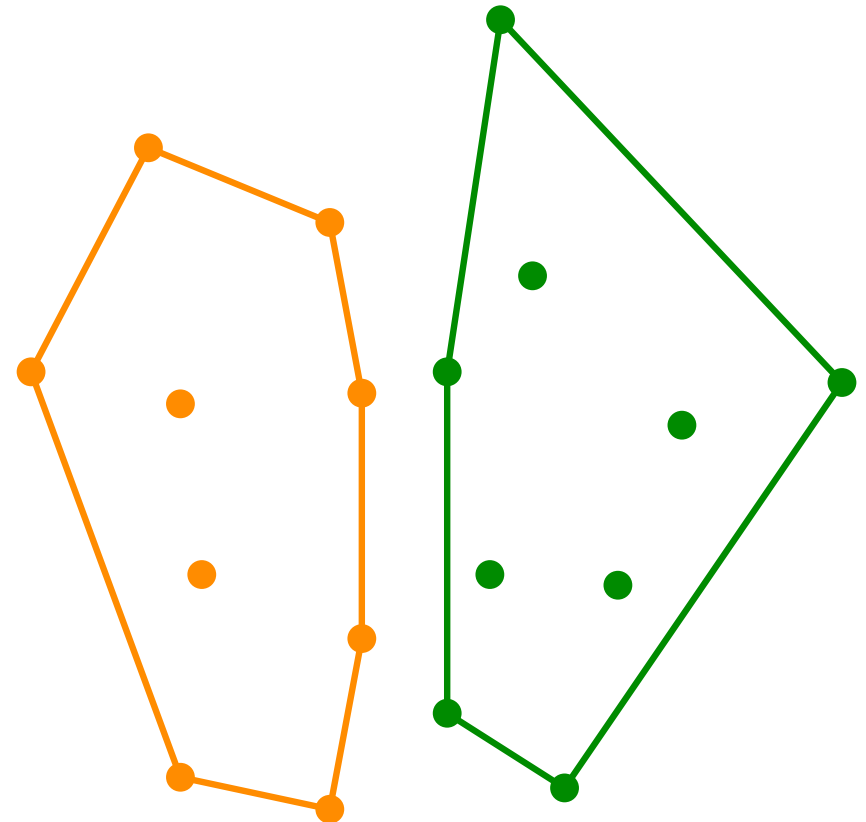
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Recursion

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Merge

1. Compute the external common tangents of the two convex polygons
2. Delete the interior chains of the two polygons and join the external chains through the supporting segments



CONVEX HULL IN 2D

Divide-and-conquer algorithm

Initialization

1. Sort the points by abscissae

Division

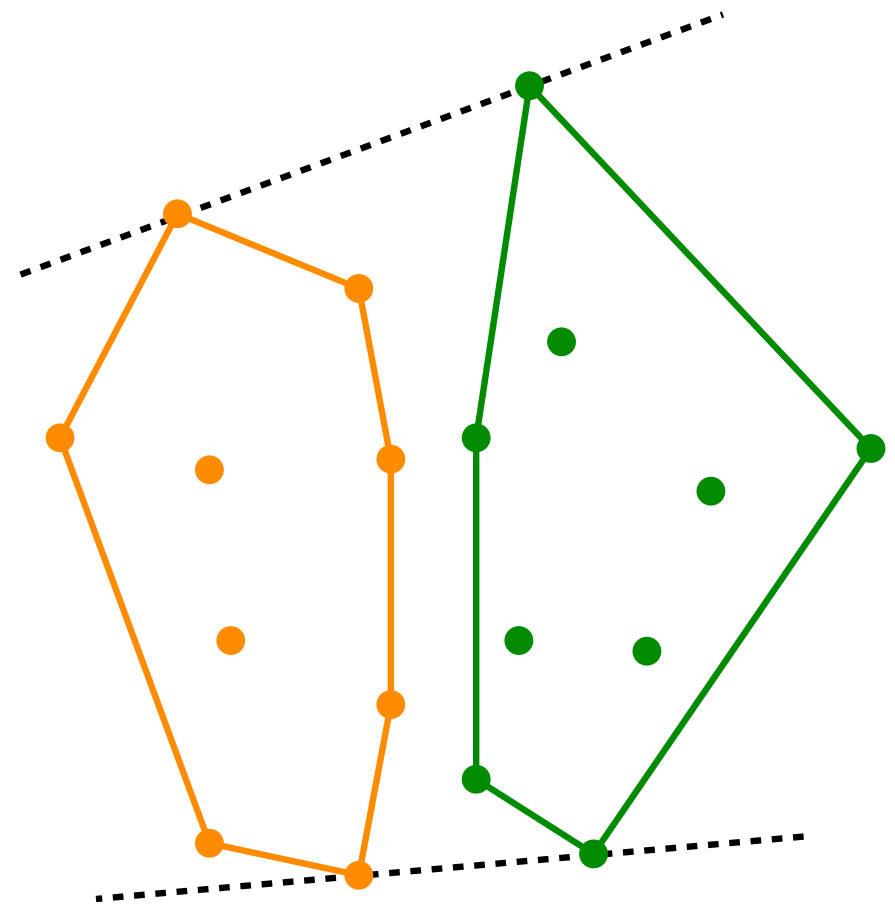
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CONVEX HULL IN 2D

Divide-and-conquer algorithm

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Division

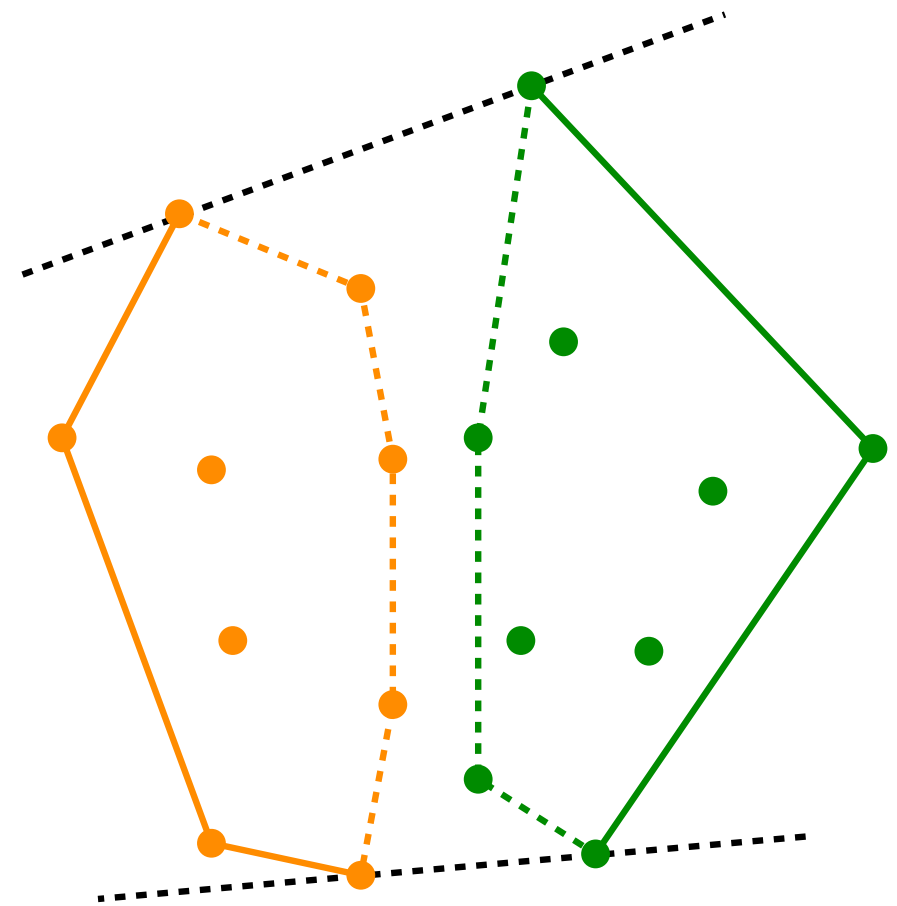
1. Divide the points (x_i, y_i) into two subsets, wrt the median value of the abscissae

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CONVEX HULL IN 2D

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Division

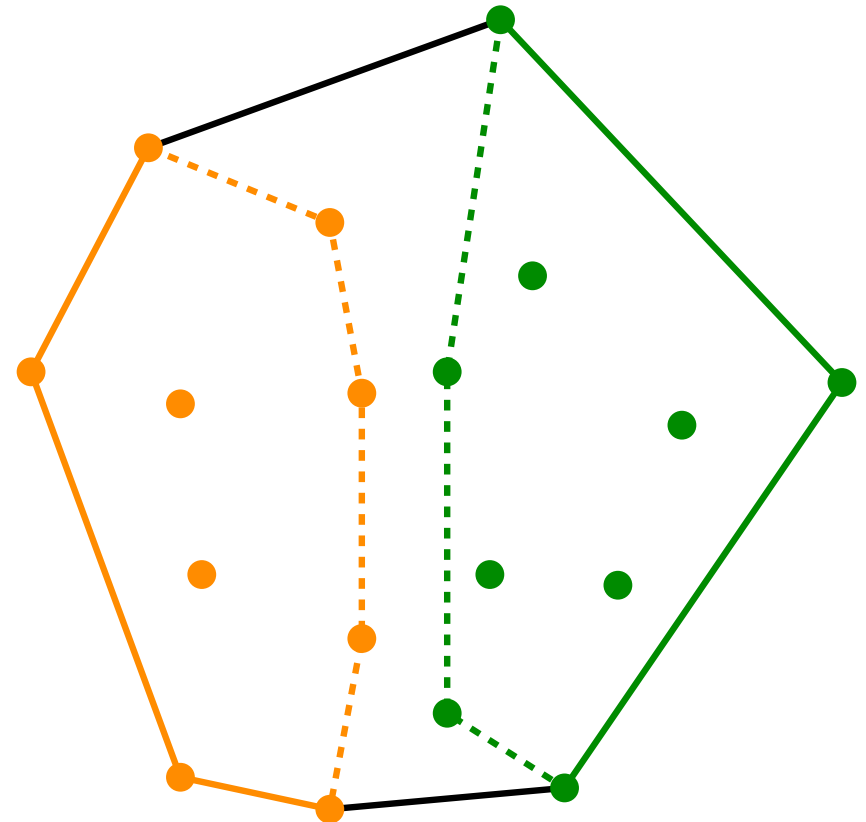
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CONVEX HULL IN 2D

Divide-and-conquer algorithm

Initialization

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Division

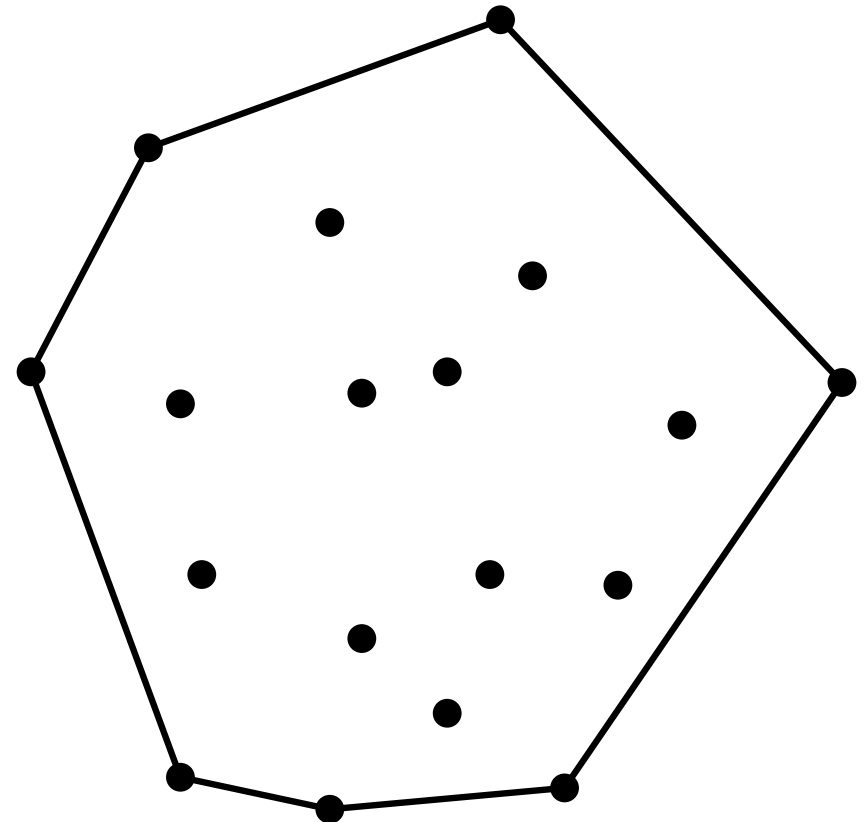
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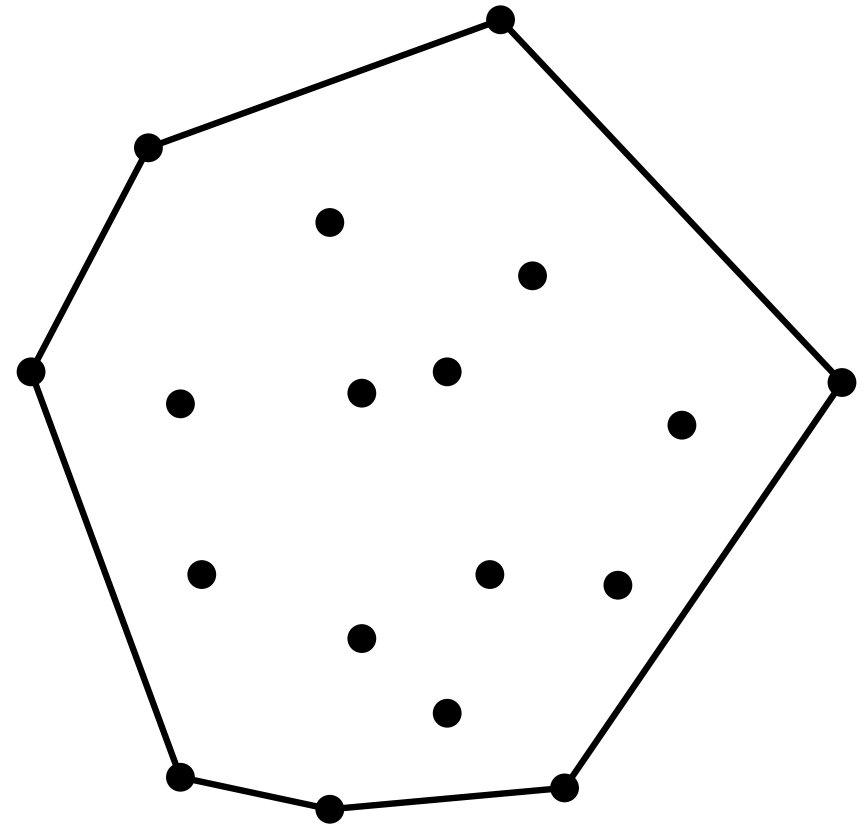


CONVEX HULL IN 2D

Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)



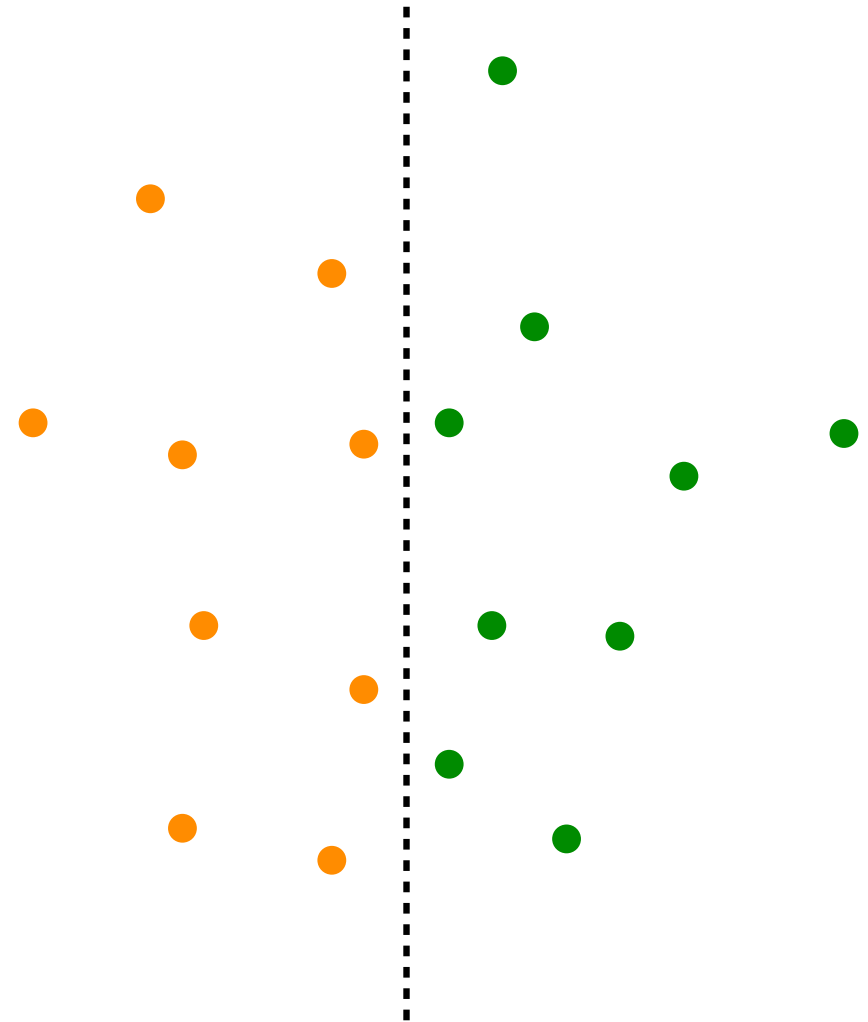
CONVEX HULL IN 2D

Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

Division: $O(n)$



CONVEX HULL IN 2D

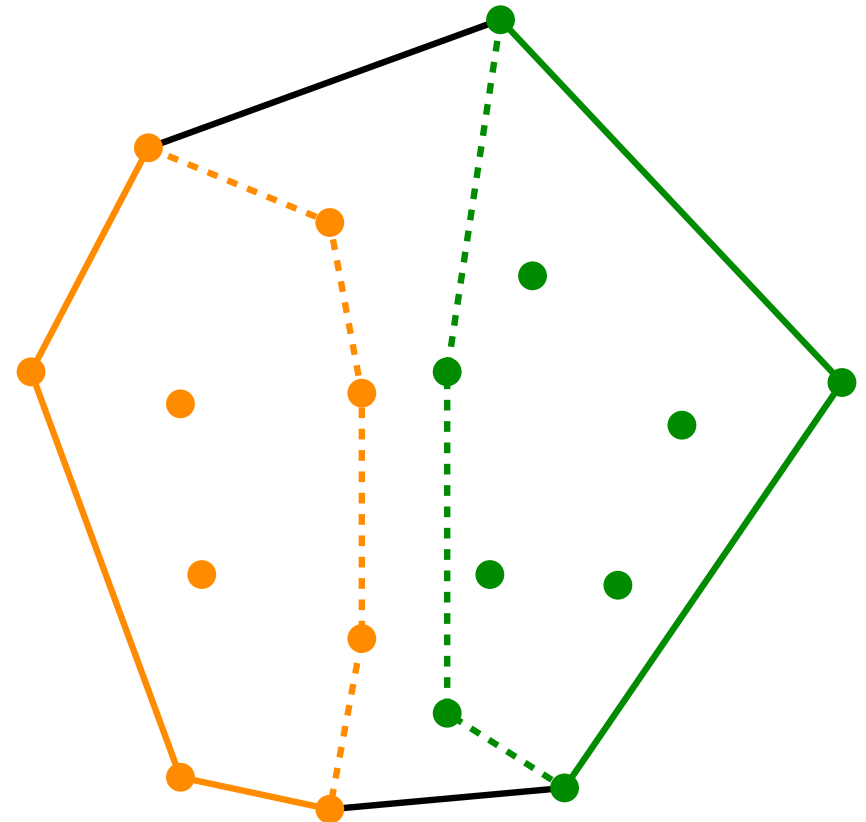
Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

Division: $O(n)$

Merge: $O(n)$



CONVEX HULL IN 2D

Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

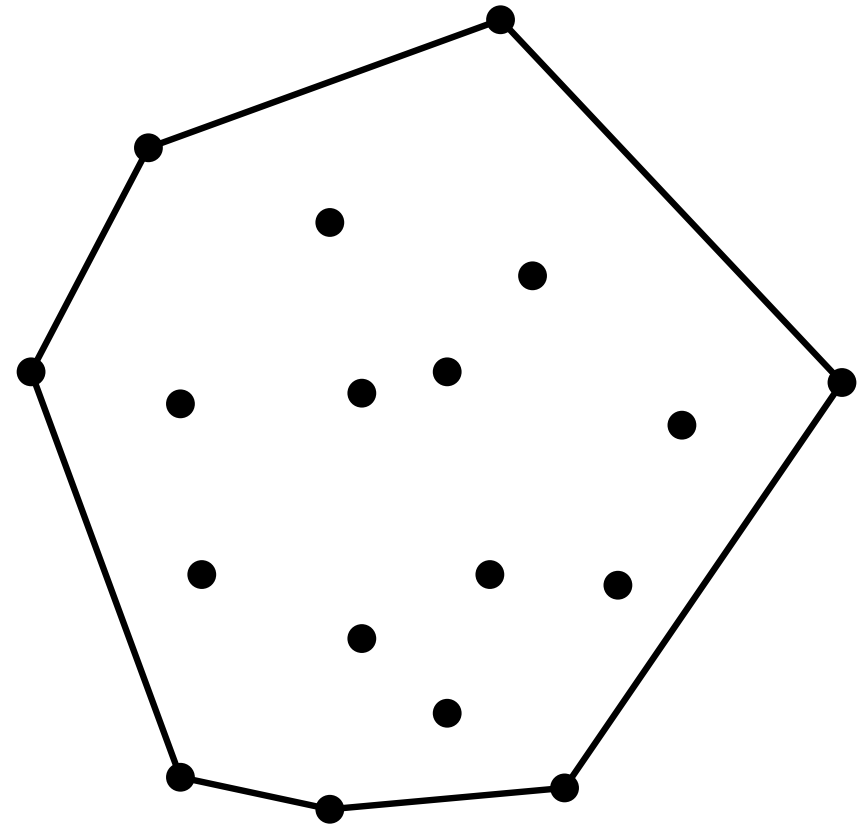
Division: $O(n)$

Merge: $O(n)$

Advance:

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n) = O(n \log n)$$

Overall: $O(n \log n)$



CONVEX HULL IN 2D

Lower bound

CONVEX HULL IN 2D

Lower bound

Input: n real numbers

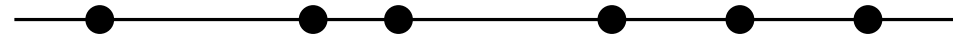
x_1, \dots, x_n real numbers

CONVEX HULL IN 2D

Lower bound

Input: n real numbers

x_1, \dots, x_n real numbers



CONVEX HULL IN 2D

Lower bound

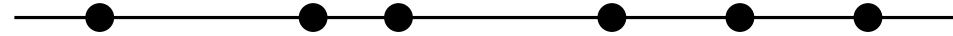
Input: n real numbers

x_1, \dots, x_n real numbers



Input: n points

p_1, \dots, p_n , with $p_i = (x_i, x_i^2)$



CONVEX HULL IN 2D

Lower bound

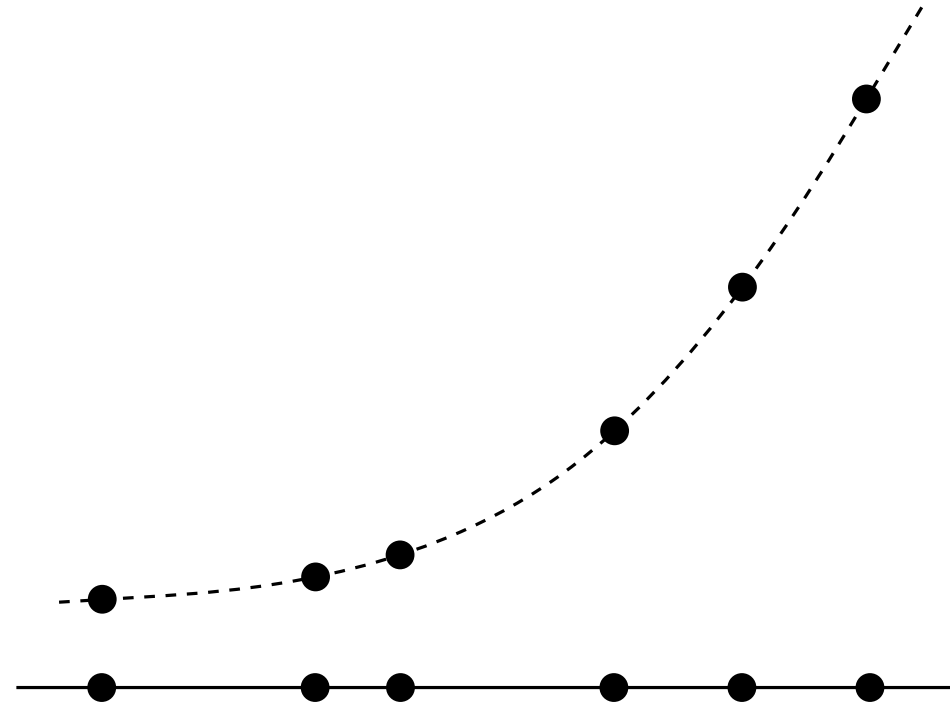
Input: n real numbers

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Input: n points

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CONVEX HULL IN 2D

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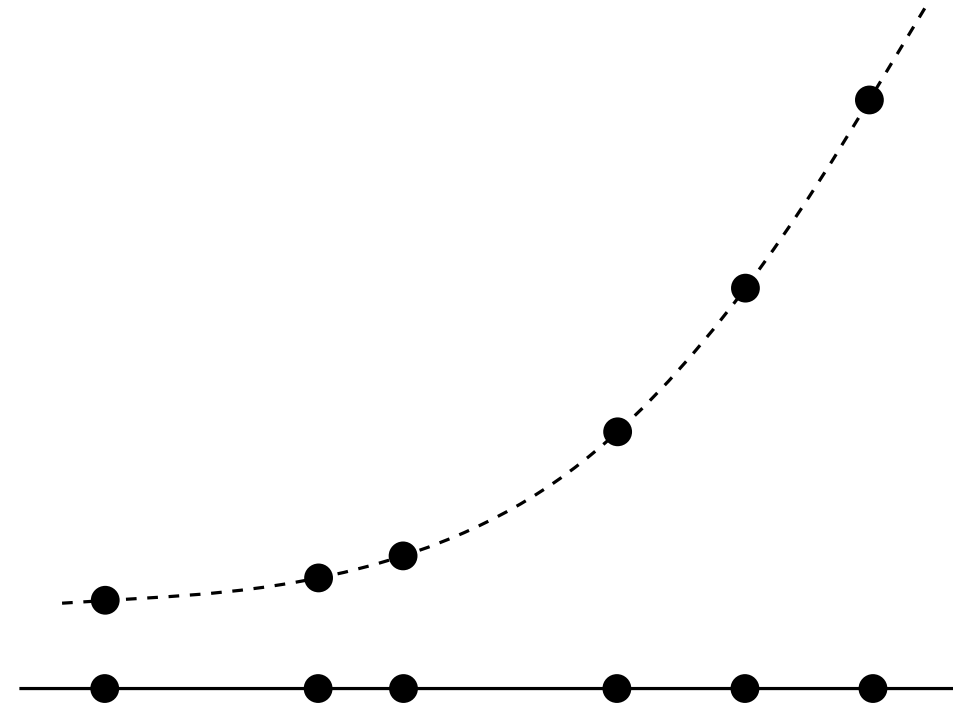
Input: n points

p_1, \dots, p_n , with $p_i = (x_i, x_i^2)$



Output: convex hull of the points

Sorted list of the vertices of the convex hull



CONVEX HULL IN 2D

Lower bound

Input: n real numbers

x_1, \dots, x_n real numbers



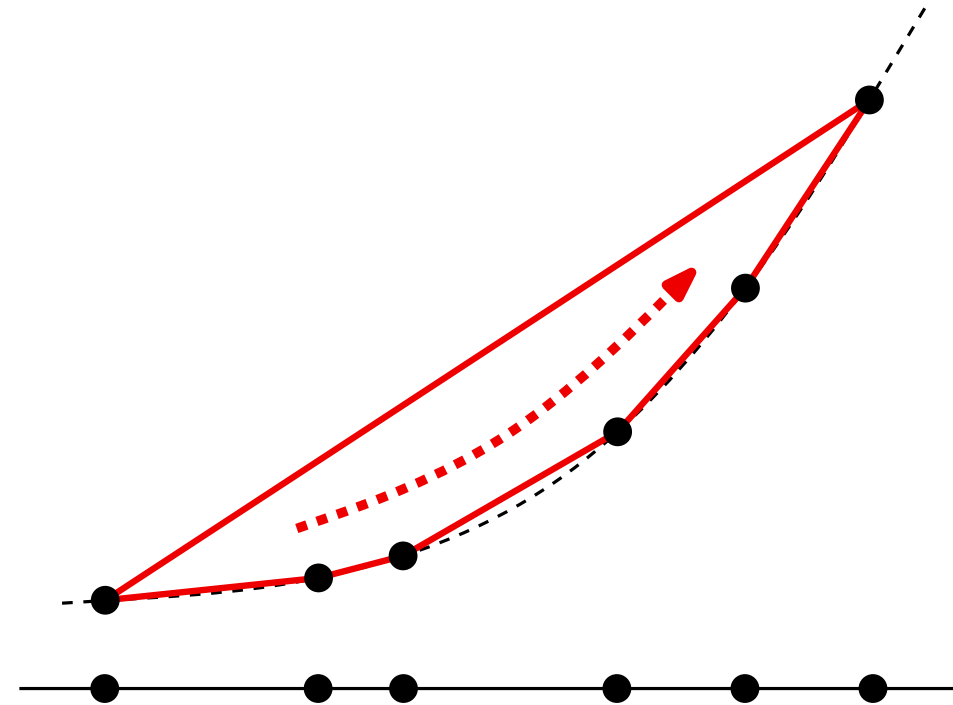
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CONVEX HULL IN 2D

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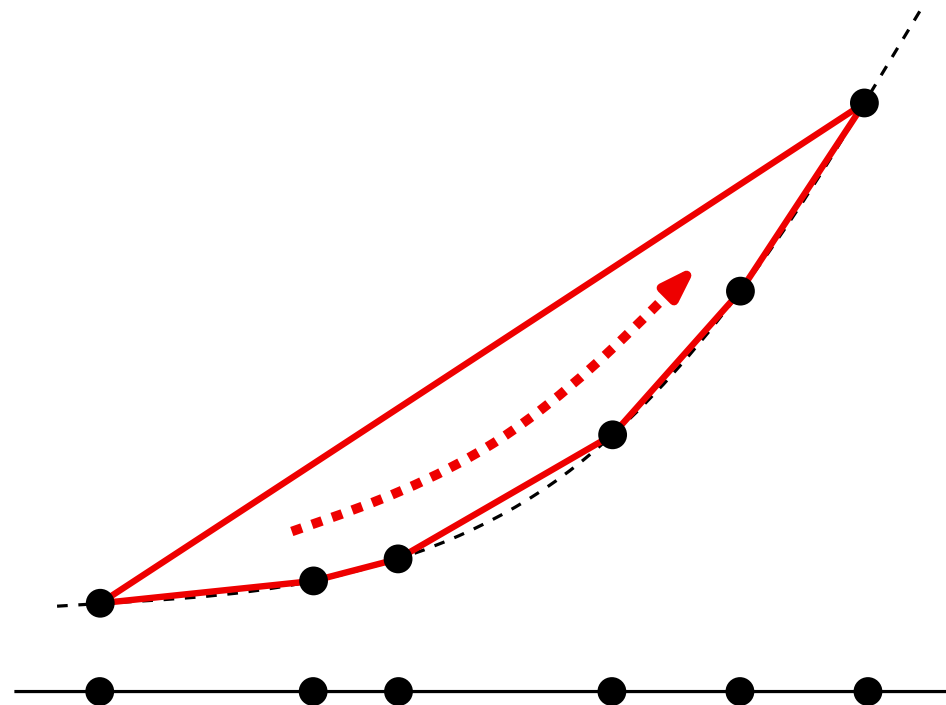
Output: convex hull of the points

Sorted list of the vertices of the convex hull



Output: sorting the numbers

Sorted list of the numbers x_1, \dots, x_n



CONVEX HULL IN 2D

Lower bound

Input: n real numbers

x_1, \dots, x_n real numbers



Input: n points

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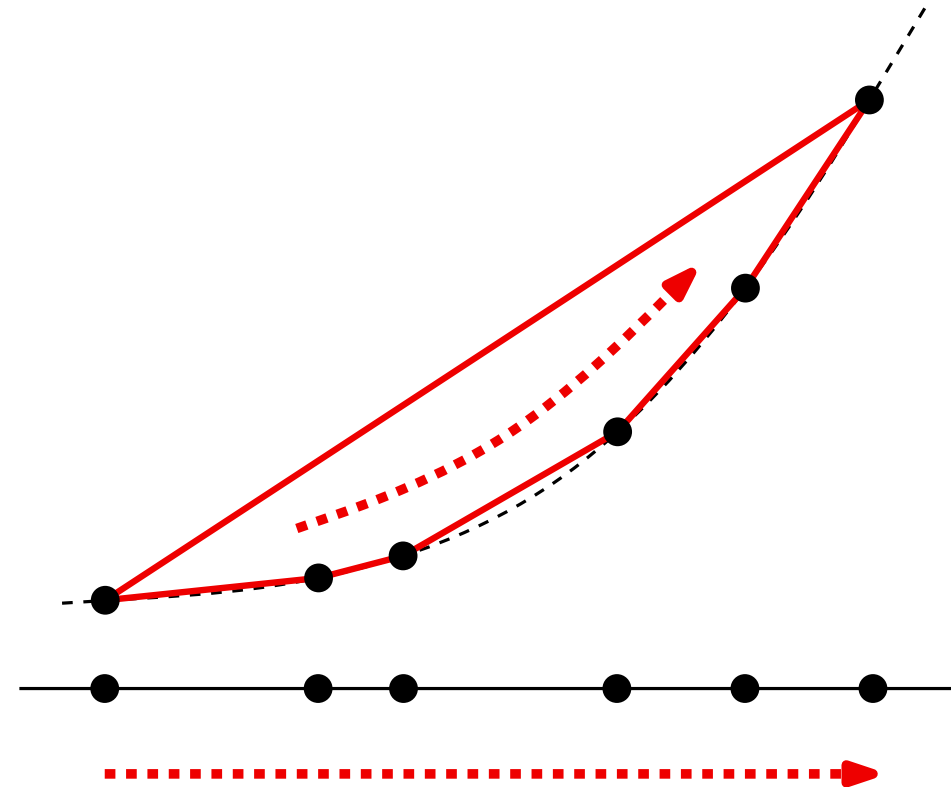
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Output: sorting the numbers

Sorted list of the numbers x_1, \dots, x_n

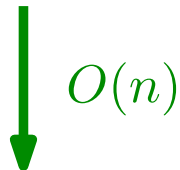


CONVEX HULL IN 2D

Lower bound

Input: n real numbers

x_1, \dots, x_n real numbers



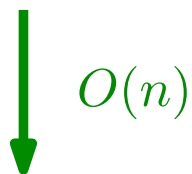
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p_1, \dots, p_n , with $p_i = (x_i, x_i^2)$



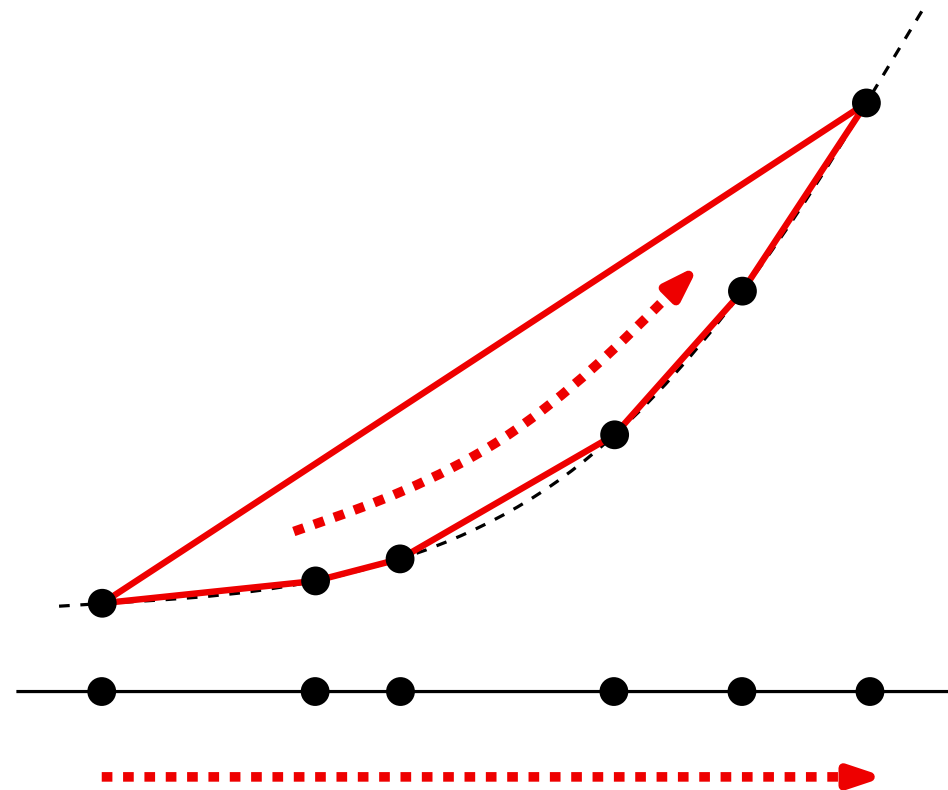
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Output: sorting the numbers

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CONVEX HULL IN 2D

Lower bound

Input: n real numbers

x_1, \dots, x_n real numbers

$O(n)$

Input: n points

p_1, \dots, p_n , with $p_i = (x_i, x_i^2)$

Output: convex hull of the points

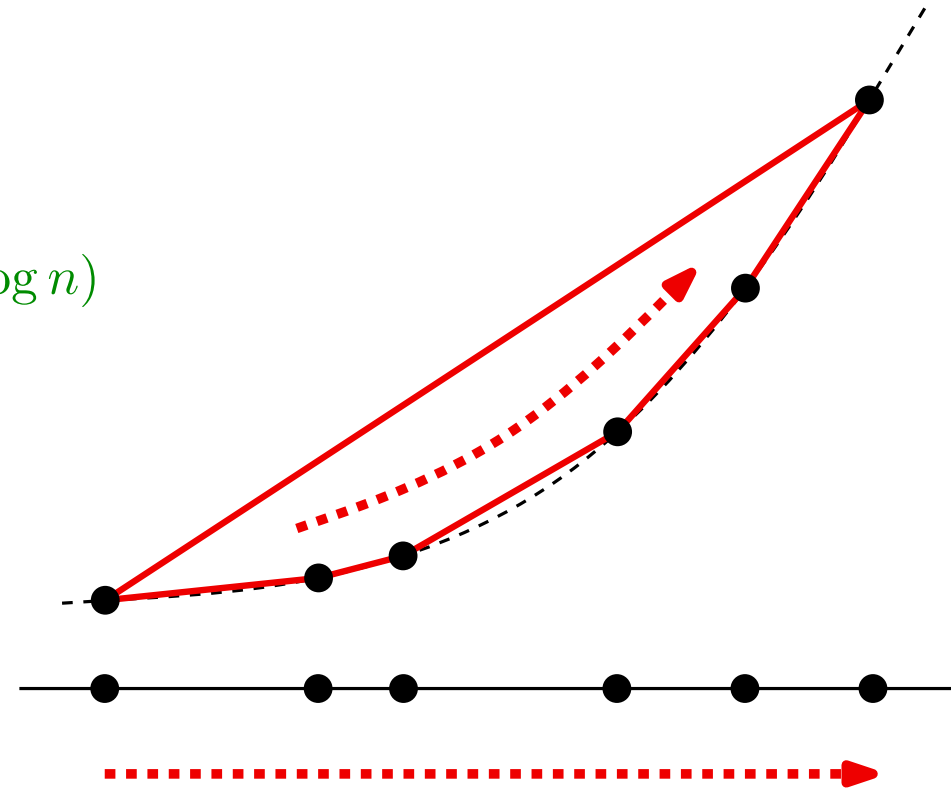
Sorted list of the vertices of the convex hull

$O(n)$

Output: sorting the numbers

Sorted list of the numbers x_1, \dots, x_n

$\Omega(n \log n)$



CONVEX HULL IN 2D

Lower bound

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x_1, \dots, x_n real numbers

$O(n)$

Input: n points

p_1, \dots, p_n , with $p_i = (x_i, x_i^2)$

$\Omega(n \log n)$

Output: convex hull of the points

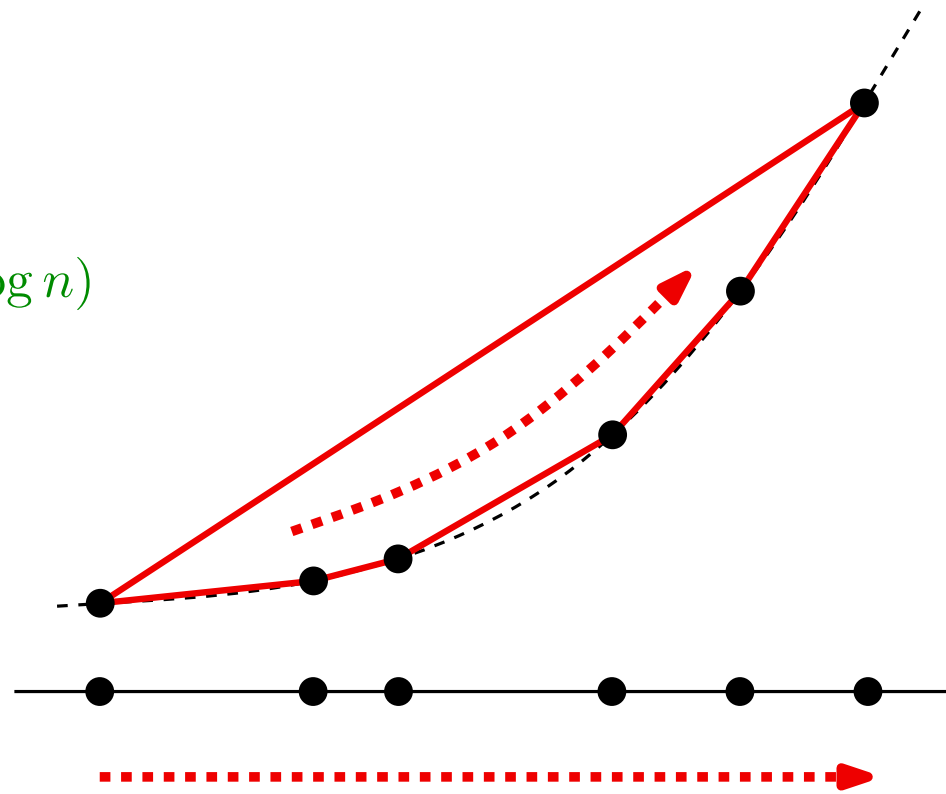
Sorted list of the vertices of the convex hull

$O(n)$

Output: sorting the numbers

Sorted list of the numbers x_1, \dots, x_n

$\Omega(n \log n)$



CONVEX HULL IN 2D

Extension: convex hull of a simple polygon

- Is it possible to design an $o(n \log n)$ time algorithm by exploiting the order of the vertices of the polygon?
- Is it possible, for example, to apply Graham's algorithm using the order of the vertices of the polygon?

CONVEX HULL IN 2D

Extension: convex hull of a simple polygon

- Is it possible to design an $o(n \log n)$ time algorithm by exploiting the order of the vertices of the polygon?
- Is it possible, for example, to apply Graham's algorithm using the order of the vertices of the polygon?

Extension: do the previous strategies extend to the 3D case?

- Is it possible to design a 3-dimensional gift wrap convex hull algorithm?
- Is it possible to design a 3-dimensional incremental convex hull algorithm?
- Is it possible to design a 3-dimensional divide-and-conquer convex hull algorithm?

CONVEX HULL IN 2D

FURTHER READING

- J. O'Rourke, **Computational Geometry in C (2nd ed.)**, Cambridge University Press, 1998.
- F. Preparata, M. Shamos, **Computational Geometry: An introduction (revised ed.)**, Springer, 1993.

...AND PLAYING

In 2D:

http://www.dma.fi.upm.es/recursos/aplicaciones/geometria_computacional_y_grafos/

In 3D:

<http://www.cse.unsw.edu.au/~lambert/java/3d>