GEOMETRIC TOOLS FOR COMPUTER GRAPHICS (MIRI)

Computing Euler angles: an example

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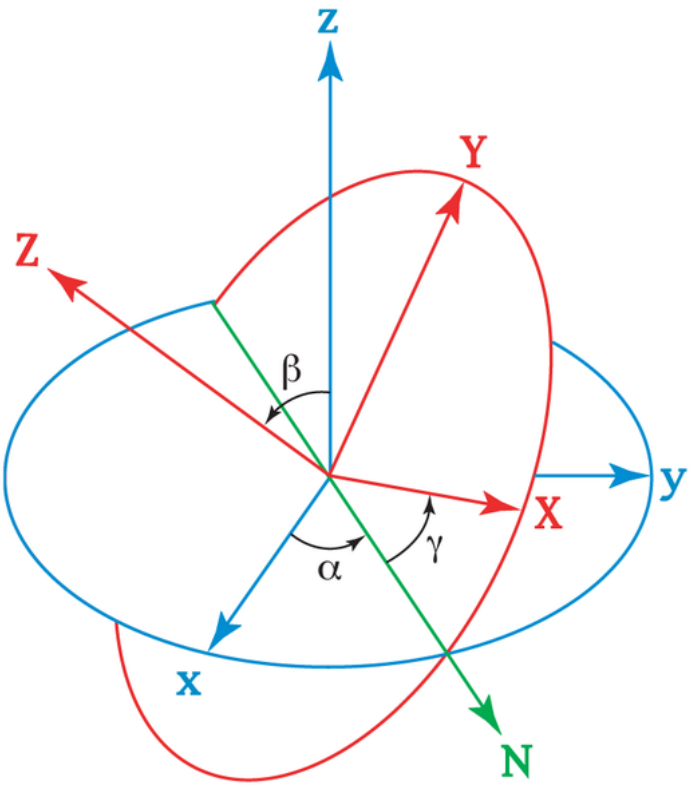

The old and the new frames

```
e1 = {1, 0, 0};
```

```
e2 = {0, 1, 0};
```

```
e3 = {0, 0, 1};
```

```
v3 = {1, 1, 1}
```

```
{1, 1, 1}
```

```
u3 = v3 / Sqrt[v3.v3]
```

$$\left\{\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}$$

```
v2 = Cross[v3, {1, 2, 4}]
```

```
{2, -3, 1}
```

```
u2 = v2 / Sqrt[v2.v2]
```

$$\left\{\sqrt{\frac{2}{7}}, -\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right\}$$

```
u1 = Cross[u2, u3]
```

$$\left\{-2\sqrt{\frac{2}{21}}, -\frac{1}{\sqrt{42}}, \frac{5}{\sqrt{42}}\right\}$$

The line of nodes

```
n = Cross[e3, u3]
```

$$\left\{-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, 0\right\}$$

```
nn = n / Sqrt[n.n]
```

$$\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right\}$$

The angles

```
cosGamma = nn.u1
```

$$\frac{\sqrt{\frac{3}{7}}}{2}$$

```
signGamma = Sign[Det[{nn, u1, {0, 0, 1}}]]
```

```
1
```

```
sinGamma = signGamma Sqrt[1 - cosGamma^2]
```

$$\frac{5}{2\sqrt{7}}$$

```
cosBeta = e3.u3
```

$$\frac{1}{\sqrt{3}}$$

```
signBeta = 1
```

```
1
```

```
sinBeta = signBeta Sqrt[1 - cosBeta^2]
```

$$\sqrt{\frac{2}{3}}$$

```
cosAlpha = e1.nn
```

$$-\frac{1}{\sqrt{2}}$$

```
signAlpha = Sign[Det[{e1, nn, {0, 0, 1}}]]
```

```
1
```

```
sinAlpha = signAlpha Sqrt[1 - cosAlpha^2]
```

$$\frac{1}{\sqrt{2}}$$

The rotations

```
MatrixForm[RotOzGamma = {{cosGamma, -sinGamma, 0}, {sinGamma, cosGamma, 0}, {0, 0, 1}}]
```

$$\begin{pmatrix} \frac{\sqrt{\frac{3}{7}}}{2} & -\frac{5}{2\sqrt{7}} & 0 \\ \frac{5}{2\sqrt{7}} & \frac{\sqrt{\frac{3}{7}}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
MatrixForm[RotOxBeta = {{1, 0, 0}, {0, cosBeta, -sinBeta}, {0, sinBeta, cosBeta}}]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}} \\ 0 & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

```
MatrixForm[RotOzAlpha = {{cosAlpha, -sinAlpha, 0}, {sinAlpha, cosAlpha, 0}, {0, 0, 1}}]
```

$$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The result

```
MatrixForm[Result = Simplify[RotOzAlpha.RotOxBeta.RotOzGamma]]
```

$$\begin{pmatrix} -2\sqrt{\frac{2}{21}} & \sqrt{\frac{2}{7}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{42}} & -\frac{3}{\sqrt{14}} & \frac{1}{\sqrt{3}} \\ \frac{5}{\sqrt{42}} & \frac{1}{\sqrt{14}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

```
MatrixForm[matrixA = Transpose[{u1, u2, u3}]]
```

$$\begin{pmatrix} -2\sqrt{\frac{2}{21}} & \sqrt{\frac{2}{7}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{42}} & -\frac{3}{\sqrt{14}} & \frac{1}{\sqrt{3}} \\ \frac{5}{\sqrt{42}} & \frac{1}{\sqrt{14}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

```
MatrixForm[Simplify[Result - matrixA]]
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$