JPEG: A CLEVER CHANGE OF BASIS

from

Matemáticas para las tecnologías de la información y las comunicaciones

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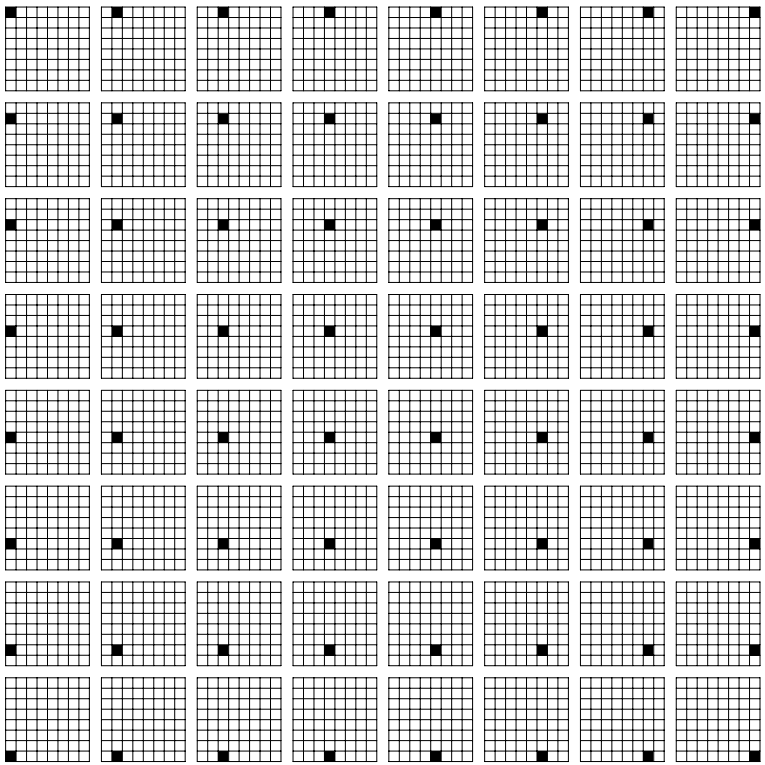
Using transforms in sound and image compressing

1. **Transform:** The temporal (sound) or spacial (image) domain gets transformed into the frequency domain.
2. **Quantization:** The frequency coefficients are stored with variable precision, based on models of human perception. This step produces *loss of information*.
3. **Dequantization:** Approximated values of the frequencies are recomputed.
4. **Inverse transform:** Back to temporal or spacial domain.

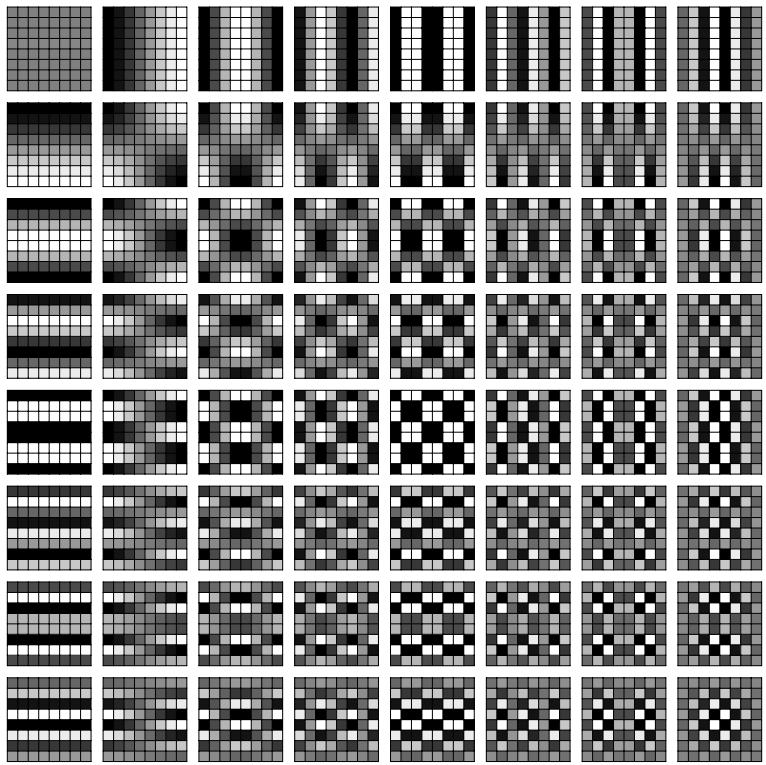
The most frequently used are the cosine transform and the wavelet transform.

Note: the space needed to store the quantized frequencies is additionally reduced using some probabilistic compression.

Bases in the discrete cosine transform



Space domain



Frequency domain

Using the discrete cosine transform in image compressing

$$\begin{vmatrix} 142 & 144 & 130 & 92 & 84 & 77 & 72 & 85 \\ 138 & 150 & 138 & 115 & 105 & 119 & 125 & 130 \\ 136 & 142 & 139 & 135 & 135 & 130 & 133 & 137 \\ 139 & 142 & 144 & 141 & 140 & 143 & 141 & 138 \\ 139 & 138 & 141 & 138 & 142 & 146 & 146 & 147 \\ 142 & 141 & 142 & 150 & 147 & 149 & 149 & 151 \\ 142 & 143 & 149 & 151 & 148 & 152 & 151 & 153 \\ 145 & 148 & 151 & 152 & 151 & 154 & 148 & 157 \end{vmatrix} \rightarrow \begin{vmatrix} 1095 & 24 & 16 & -12 & -9 & -6 & 5 & -2 \\ -99 & 57 & 26 & -9 & -12 & -8 & -1 & 4 \\ -41 & 37 & 15 & -9 & -3 & -7 & 4 & -1 \\ -35 & 20 & 8 & 1 & 0 & 0 & 2 & 1 \\ -21 & 18 & 0 & 2 & 0 & -4 & 4 & 3 \\ -12 & 13 & -8 & 1 & 4 & 0 & 0 & 2 \\ -8 & 7 & -6 & 1 & 9 & -3 & -2 & 1 \\ -7 & 1 & -1 & 0 & 3 & -2 & 1 & 4 \end{vmatrix}$$

$$\downarrow$$

$$\begin{vmatrix} -9 & 3 & 7 & -10 & -1 & -1 & -10 & -2 \\ -7 & 10 & 9 & -2 & -3 & 12 & 12 & 10 \\ -4 & 2 & 2 & 2 & 6 & -1 & -5 & -7 \\ 1 & 1 & 2 & 1 & 3 & 6 & 0 & -6 \\ 3 & -2 & -3 & -6 & 1 & 6 & 4 & 3 \\ 8 & 2 & -3 & 3 & 0 & 1 & -2 & -3 \\ 4 & 1 & 2 & 2 & -1 & 2 & -3 & -5 \\ 1 & 1 & 2 & 3 & 5 & 8 & 0 & 5 \end{vmatrix} \qquad \begin{vmatrix} 68 & 2 & 2 & -1 & 0 & 0 & 0 & 0 \\ -8 & 5 & 2 & 0 & 0 & 0 & 0 & 0 \\ -3 & 3 & 1 & 0 & 0 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

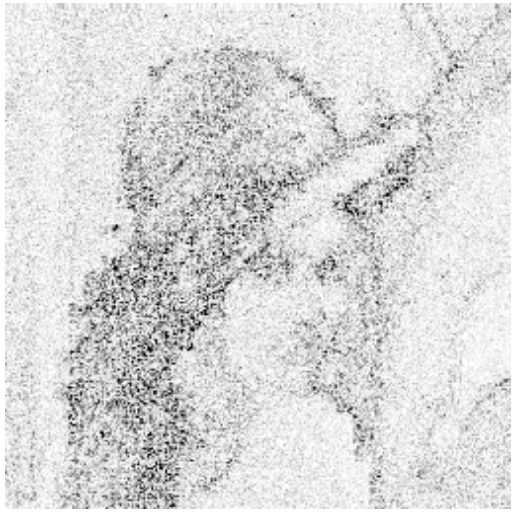
$$\downarrow$$

$$\begin{vmatrix} 151 & 141 & 123 & 102 & 85 & 78 & 82 & 87 \\ 145 & 140 & 129 & 117 & 108 & 107 & 113 & 120 \\ 140 & 140 & 137 & 133 & 129 & 131 & 138 & 144 \\ 138 & 141 & 142 & 140 & 137 & 137 & 141 & 144 \\ 136 & 140 & 144 & 144 & 141 & 140 & 142 & 144 \\ 134 & 139 & 145 & 147 & 147 & 148 & 151 & 154 \\ 138 & 142 & 147 & 149 & 149 & 150 & 154 & 158 \\ 144 & 147 & 149 & 149 & 146 & 146 & 148 & 152 \end{vmatrix} \leftarrow \begin{vmatrix} 1088 & 22 & 20 & -16 & 0 & 0 & 0 & 0 \\ -96 & 60 & 28 & 0 & 0 & 0 & 0 & 0 \\ -42 & 39 & 16 & 0 & 0 & 0 & 0 & 0 \\ -28 & 17 & 0 & 0 & 0 & 0 & 0 & 0 \\ -18 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Example: JPEG compression of an image

Original image

Decompressed image



error$\times 10$