Perception of Symmetries in Drawings of Graphs

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Graph Drawing, 2018

Outline

Symmetries
in nature

A symmetric layout shows the repetition of a pattern along one or more axes.



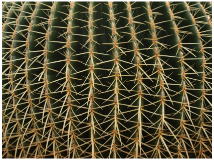



Symmetries

... types of symmetry

A symmetric layout shows the repetition of a pattern along one or more axes.

- **Vertical**: Reflection across a vertical axis (**mirror** symmetry)
- **Horizontal**: Reflection across an horizontal axis
- **Translational**: A pattern is repeated and shifted in the space
- **Rotational**: Repetition across radial axes with a given angle



Symmetries

... types of symmetry

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- **Vertical**: Reflection across a vertical axis (**mirror** symmetry)
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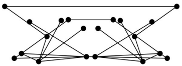
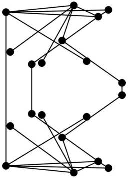
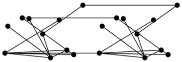
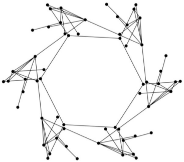








Related work

... psychology

Gestalt Principles

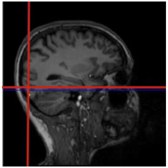
Similarity

Closure

Proximity

Symmetry

Psychology

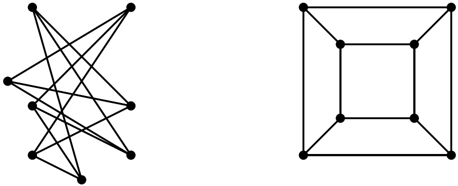


Vertical sym activates a specific brain region (preattentively) [Cattaneo 2017]

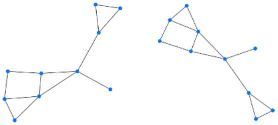
Related work

... graph drawing

Graph Drawing



Readability [Purchase 1997]



Klapaukh [2014] and Purchase [2002] metrics

Perception of symmetry

Experimental study

Rank how symmetries in drawings of graphs are perceived.

- reflective (vertical and horizontal) symmetry
- translational symmetry
- rotational symmetry

Objective:

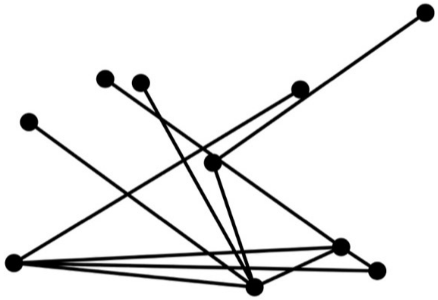
1. Rank reflective and translational symmetries
2. Rank rotational symmetries based on the number of axes

Layout generation: Symmetric graph drawings by duplicating a graph with 10 vertices and 11 edges drawn with a random layout.

Perception of symmetry

Experimental setup

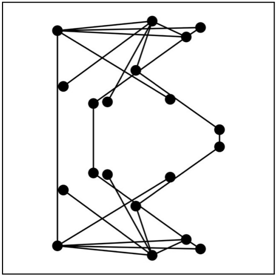
Layout generation: Symmetric graph drawings by duplicating a graph with 10 vertices and 11 edges drawn with a random layout.



What is the relative ranking of reflective and translational symmetries for drawings of graphs?

Reflective and translational symmetric versions:

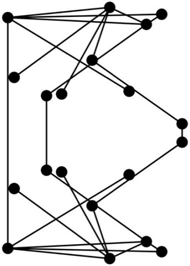
- Horizontal (H)

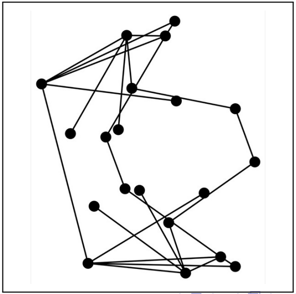


What is the relative ranking of reflective and translational symmetries for drawings of graphs?

Reflective and translational symmetric versions:

- Horizontal (H)
- Horizontal with rotation (Hr)

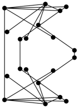


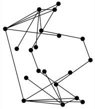


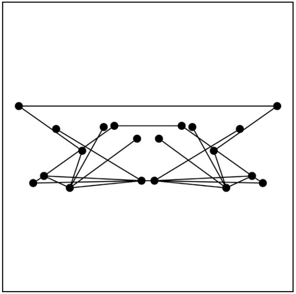
What is the relative ranking of reflective and translational symmetries for drawings of graphs?

Reflective and translational symmetric versions:

- Horizontal (H)
- Horizontal with rotation (Hr)
- Vertical (V)







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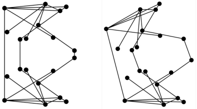
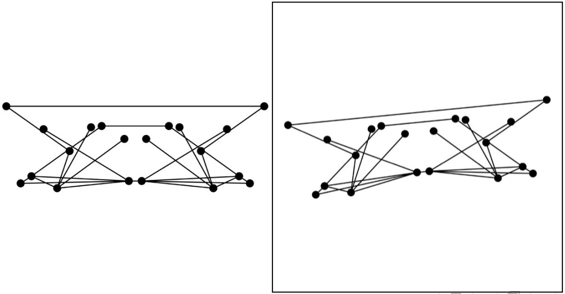


Reflective and translational symmetric versions:

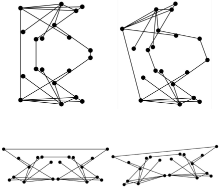
- Horizontal (H)
- Horizontal with rotation (Hr)
- Vertical (V)
- Vertical with rotation (Vr)

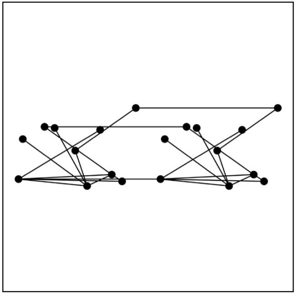


What is the relative ranking of reflective and translational symmetries for drawings of graphs?

Reflective and translational symmetric versions:

- Horizontal (H)
- Horizontal with rotation (Hr)
- Vertical (V)
- Vertical with rotation (Vr)
- Translational (T)





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Reflective and translational symmetric versions:

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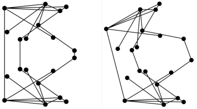
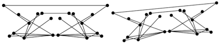
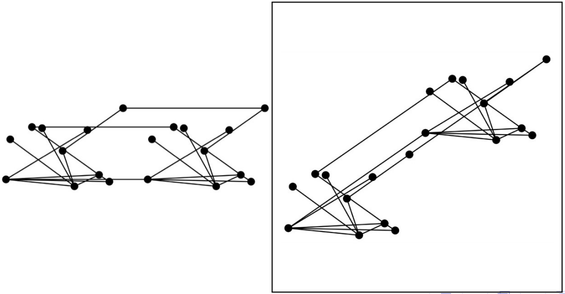
- Translational (T)
- Translational with rotation (Tr)



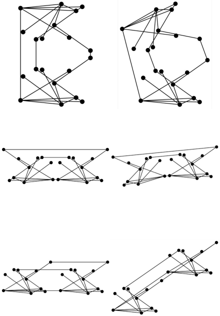


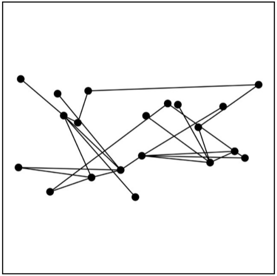


What is the relative ranking of reflective and translational symmetries for drawings of graphs?

Reflective and translational symmetric versions:

- Horizontal (H)
- Horizontal with rotation (Hr)
- Vertical (V)
- Vertical with rotation (Vr)
- Translational (T)
- Translational with rotation (Tr)
- Non symmetric version (NS)

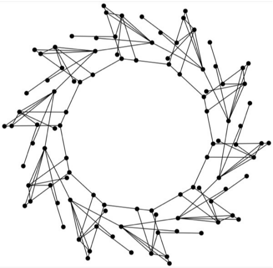


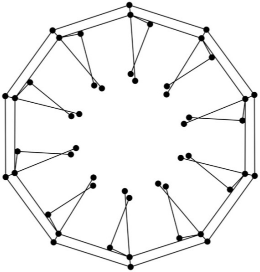


What is the impact of the number of axes (order) for rotational symmetry?

Two types of rotational layouts to take into account the number of vertices.

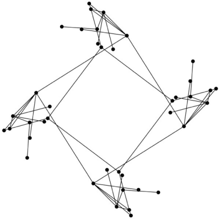
- Rotational with fixed component
- Rotational with fixed vertices (maximum 50 vertices)



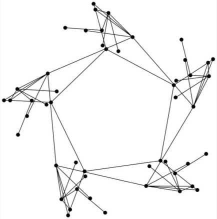


Rotational with fixed component

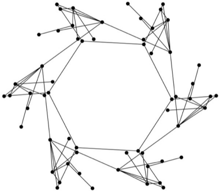
Rotational Stimuli



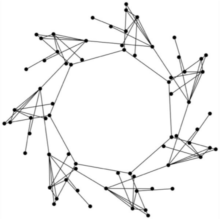
4 axes



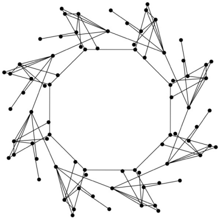
5 axes



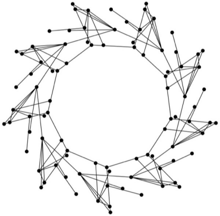
6 axes



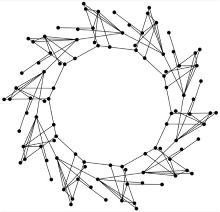
7 axes



8 axes



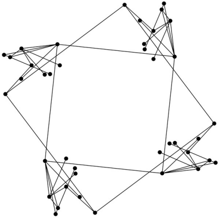
9 axes



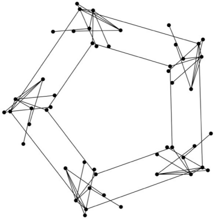
10 axes

Rotational with fixed vertices

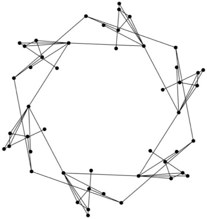
Rotational Stimuli



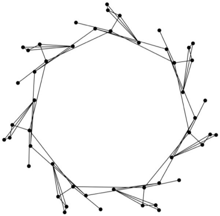
4 axes



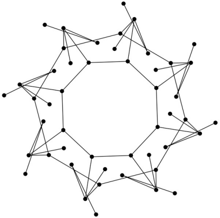
5 axes



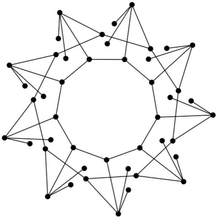
6 axes



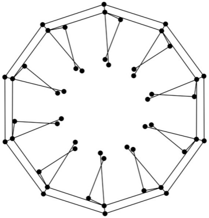
7 axes



8 axes



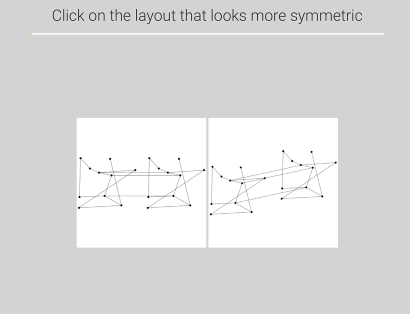
9 axes



10 axes

Experimental methodology
Tool

- **Participants gathering**: Reddit; Personal Communication
- **Methodology**: 'two-alternative forced choice'
- **Task**: Select the layout that looks more symmetric.
- **Presented layouts per task (per participant)**: 210 in random order



Experimental methodology
Tasks

We conducted three separate experiments:

1 Which type of symmetry among *H, V, T, Hr, Vr, Tr* is most recognizable as symmetry?

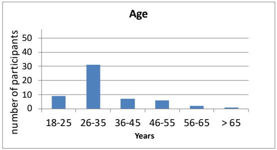
2a How many rotations is most recognizable as rotational symmetry, using the *fixed-component* generation method?

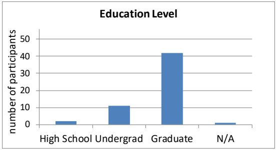
2b How many rotations is most recognizable as rotational symmetry, using the *fixed-vertices* generation method?

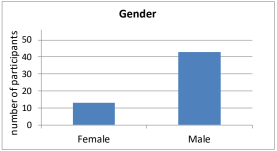
Participants

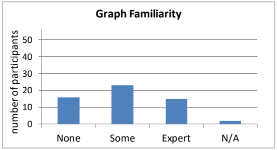
Gathered participants:

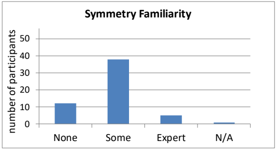
- Total participants: 97
- Incomplete tasks: 39
- Removed participants: 2
  - high number of non symmetric choices
  - same votes to all conditions in RFV











Results

Conditions per task:

- 7 symmetric versions

Votes per participants per task:

- 210

Analysis:

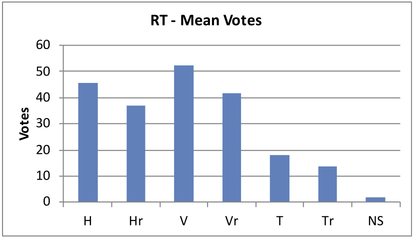
- ANOVA
- Adjusted post-hoc pair-wise

Significance level 0.05

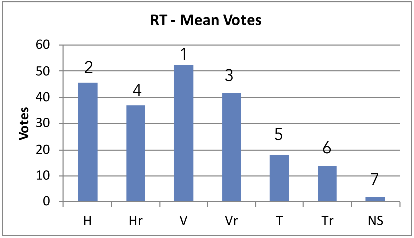
Which conditions are:

- favored over the others

Reflective and Translational
Results



Avg votes: Significant difference ($F = 240.5$, $df = 6$, $\underline{p < 0.001}$).



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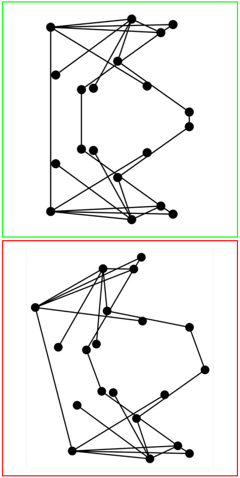
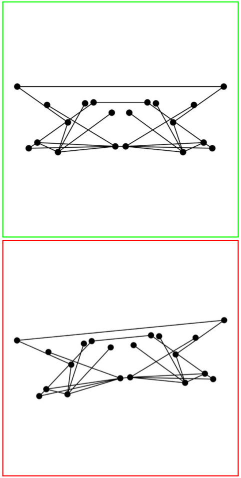
Reflective and Translational

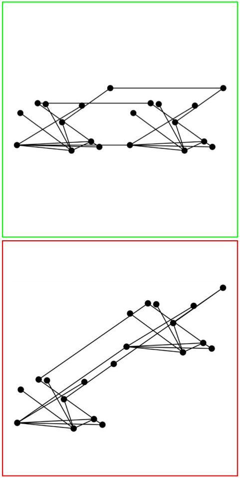
Results



$H \succ Hr$



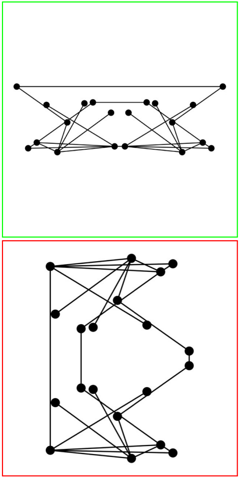
$V \succ Vr$



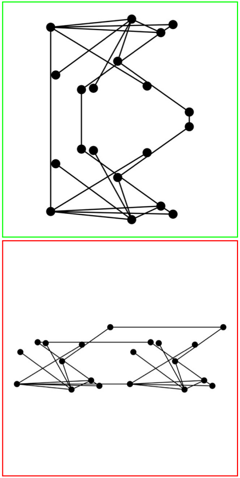
$T \succ Tr$

Reflective and Translational

Results



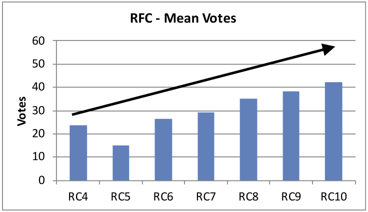
$V \succ H$



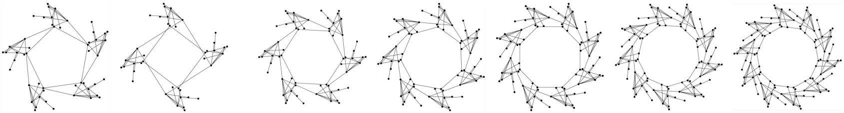
$H \succ T$

Rotational with fixed components

Results

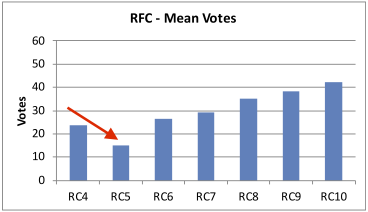


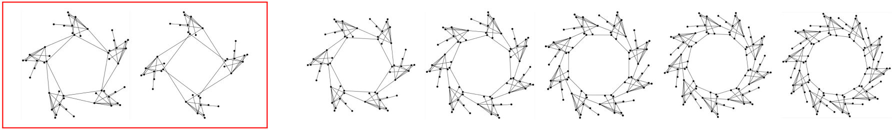
Avg votes: Significant difference ($F = 12.2$, $df = 6$, $p < 0.001$).



Rotational with fixed components

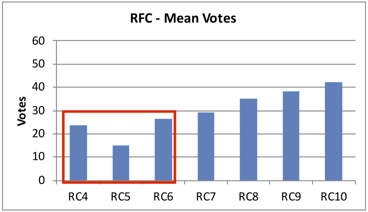
Results

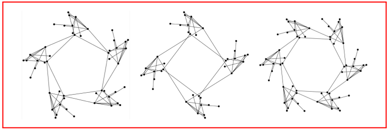




Rotational with fixed components

Results

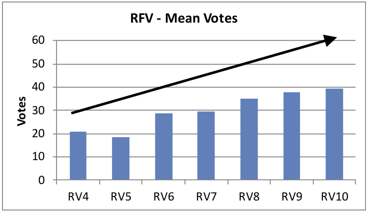




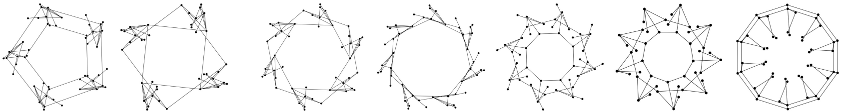
Two pairwise comparisons (at adjusted $p = 0.025$):

$\text{RC6} \succ \text{RC5}$

$\text{RC4} \not\succ \text{RC5}$ ($p = 0.050$)

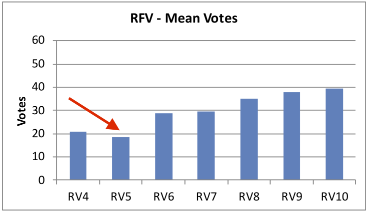


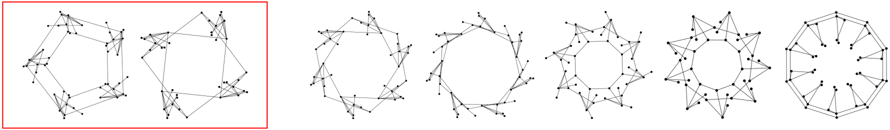
Avg votes: Significant difference ($F = 10.9$, $df = 6$, $p < 0.001$).



Rotational with fixed vertices

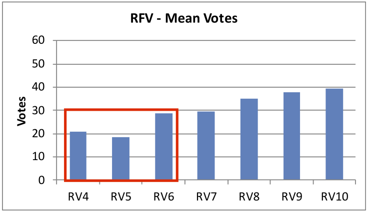
Results

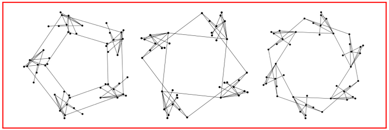




Rotational with fixed vertices

Results





Two pairwise comparisons (at adjusted $p = 0.025$):

$\text{RV6} \succ \text{RV5}$

$\text{RV4} \not\succ \text{RV5}$
$(p = 0.63)$

Findings

Reflective and Translational

1: *Which type of symmetry among H, V, T, Hr, Vr, Tr is most recognizable as symmetry?*

A: Statistically significant effects confirm that **mirror symmetry** is more recognizable as symmetry followed by horizontal and translational.

Findings

Rotational

2a: *How many rotations is most recognizable as rotational symmetry, using the fixed-component generation method?*

2b: *How many rotations is most recognizable as rotational symmetry, using the fixed-vertices generation method?*

A: Evidence of a greater symmetry recognition for **high number of rotation axes** with the exception *RC4* that is considered more symmetric than *RC5* which goes against the general trend.

Conclusion and future work

Can these findings can help guide algorithms that identify features to be displayed using these types of symmetries?

- show vertical symetry to call attention to isomorphic pairs of subgraphs
- layout cycles as *n*-gons with rotational symmetry to highlight them

Future Research:

- Is rotational 4 axes more recognizable than 5 axes because it is perceived as a combination of H and V?
- Ranking among rotational and the reflective symmetries

Conclusion and future work

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- Is rotational 4 axes more recognizable than 5 axes because it is perceived as a combination of H and V?
- Ranking among rotational and the reflective symmetries

Thank You!
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