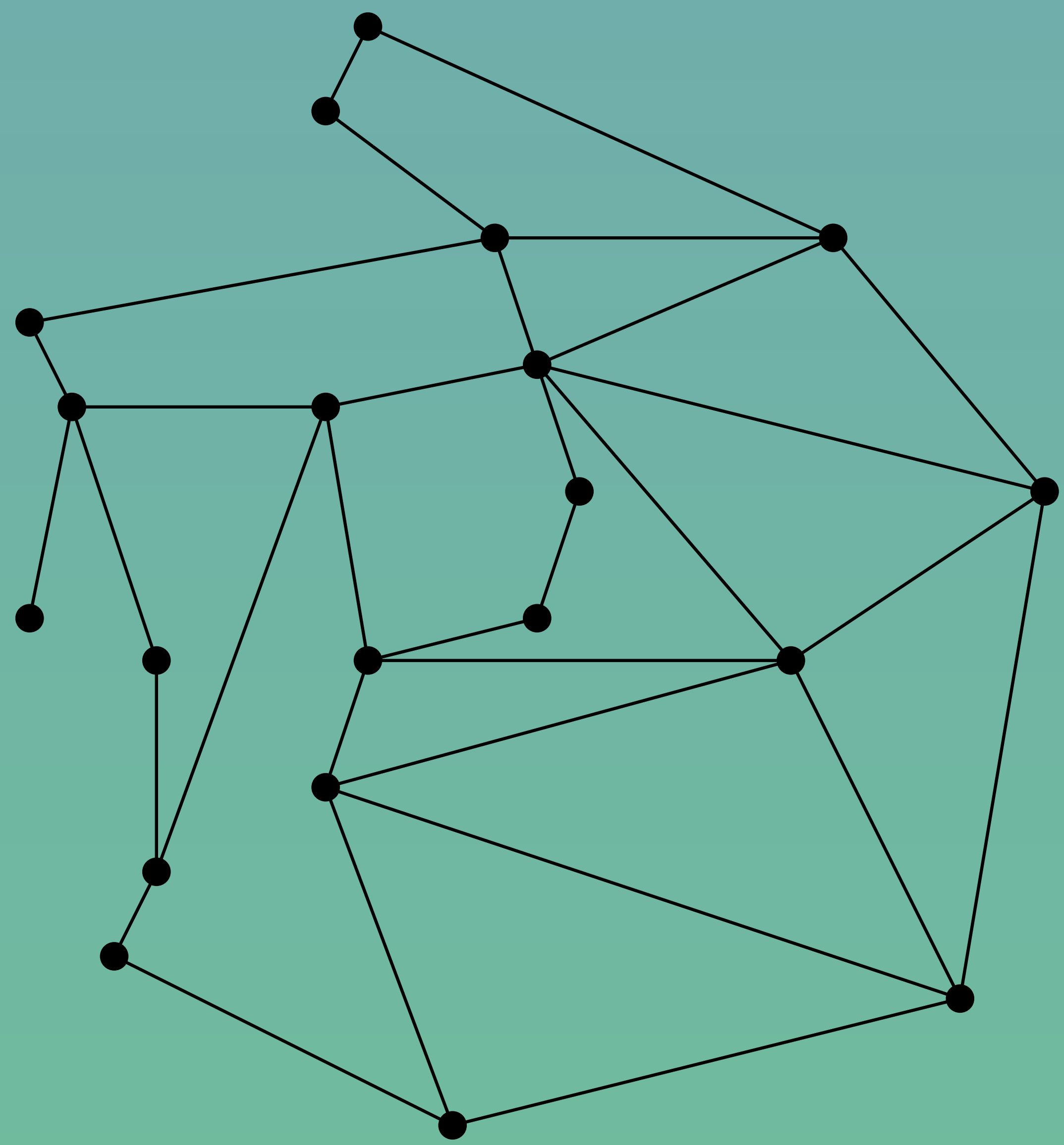
β-Stars
or
On Extending a Drawing of a Connected Subgraph

Tamara Mchedlidze
Karlsruhe Institute of Technology

Jérôme Urhausen
Utrecht University

The Problem



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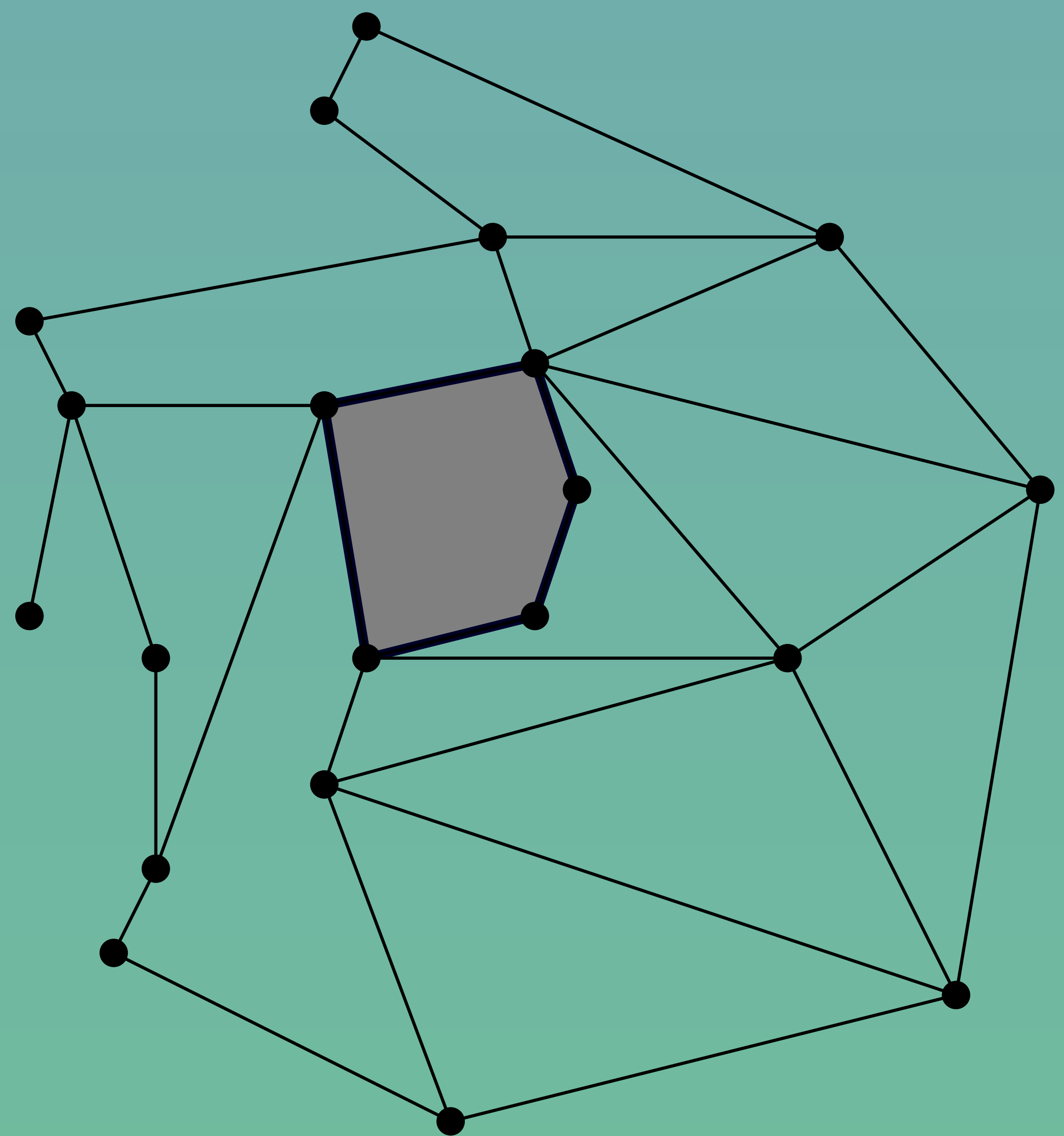

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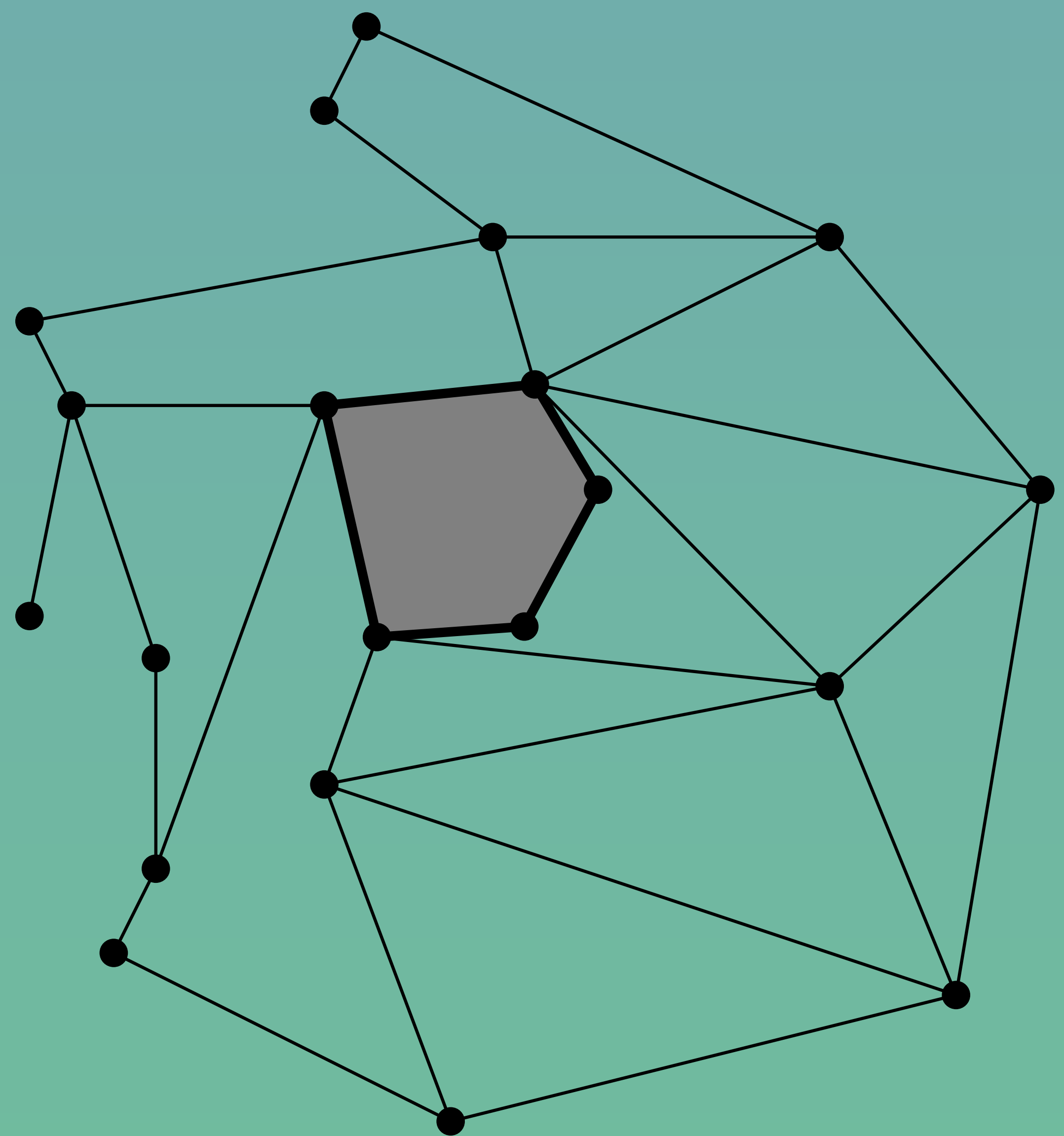

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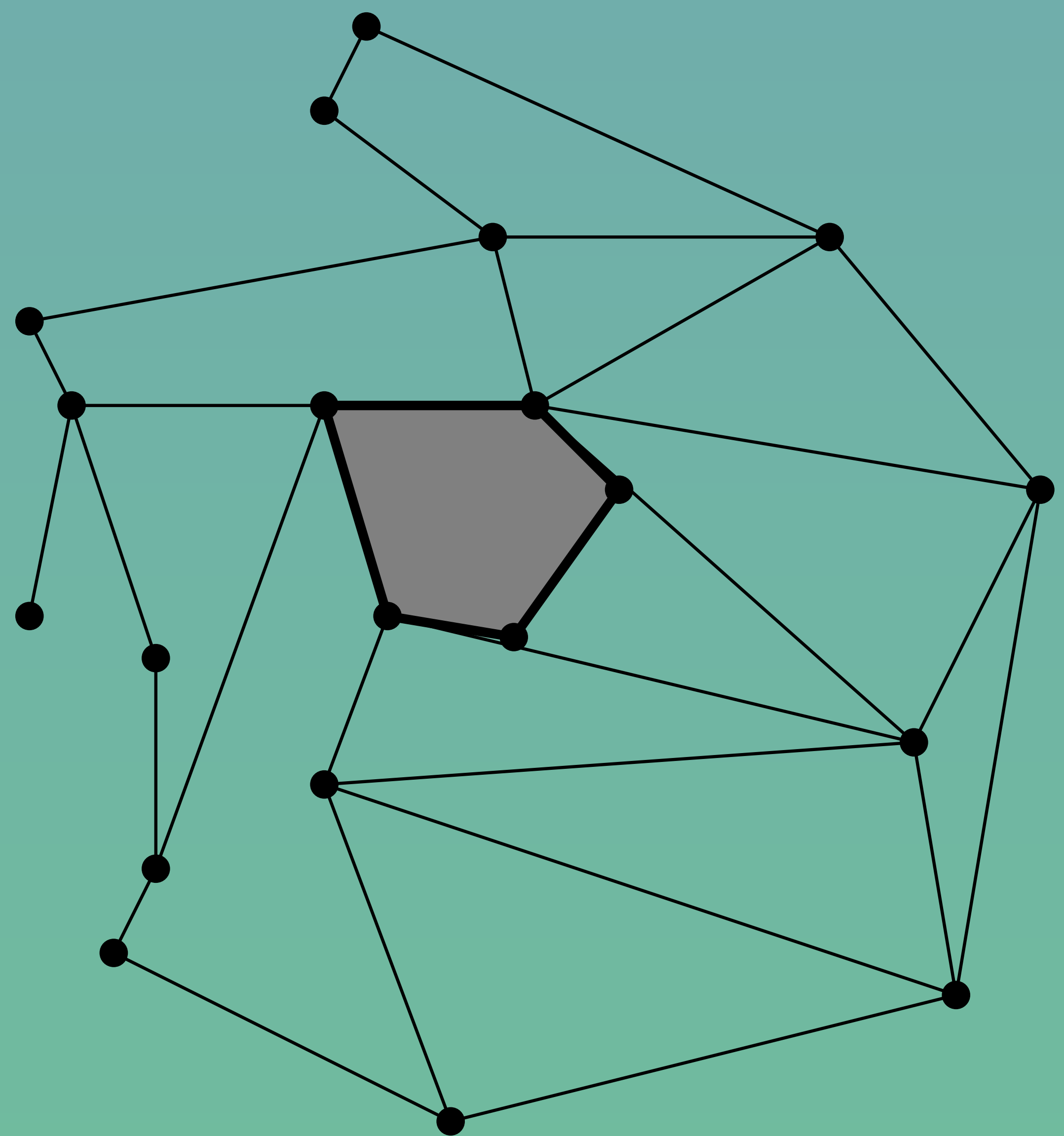

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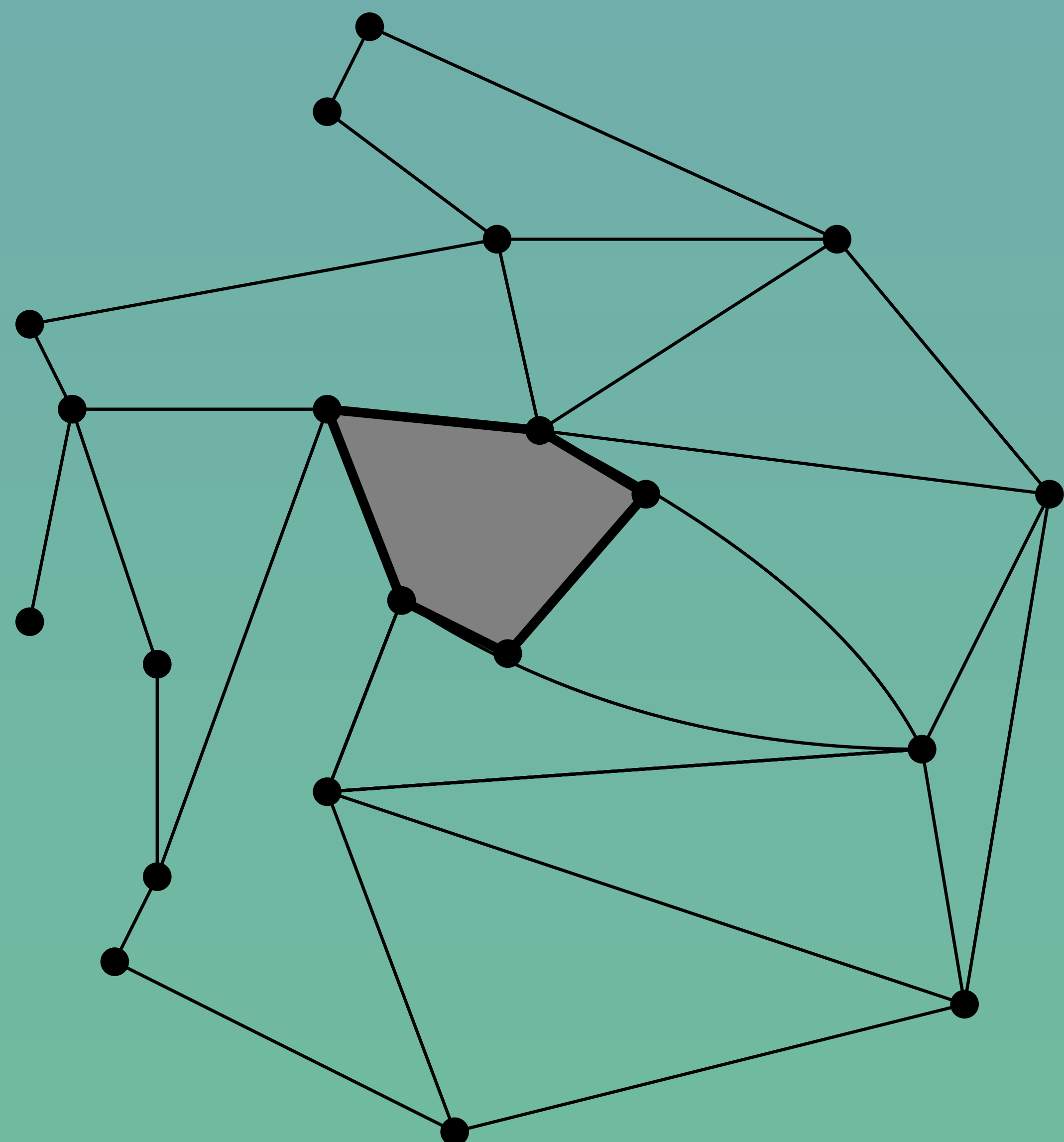

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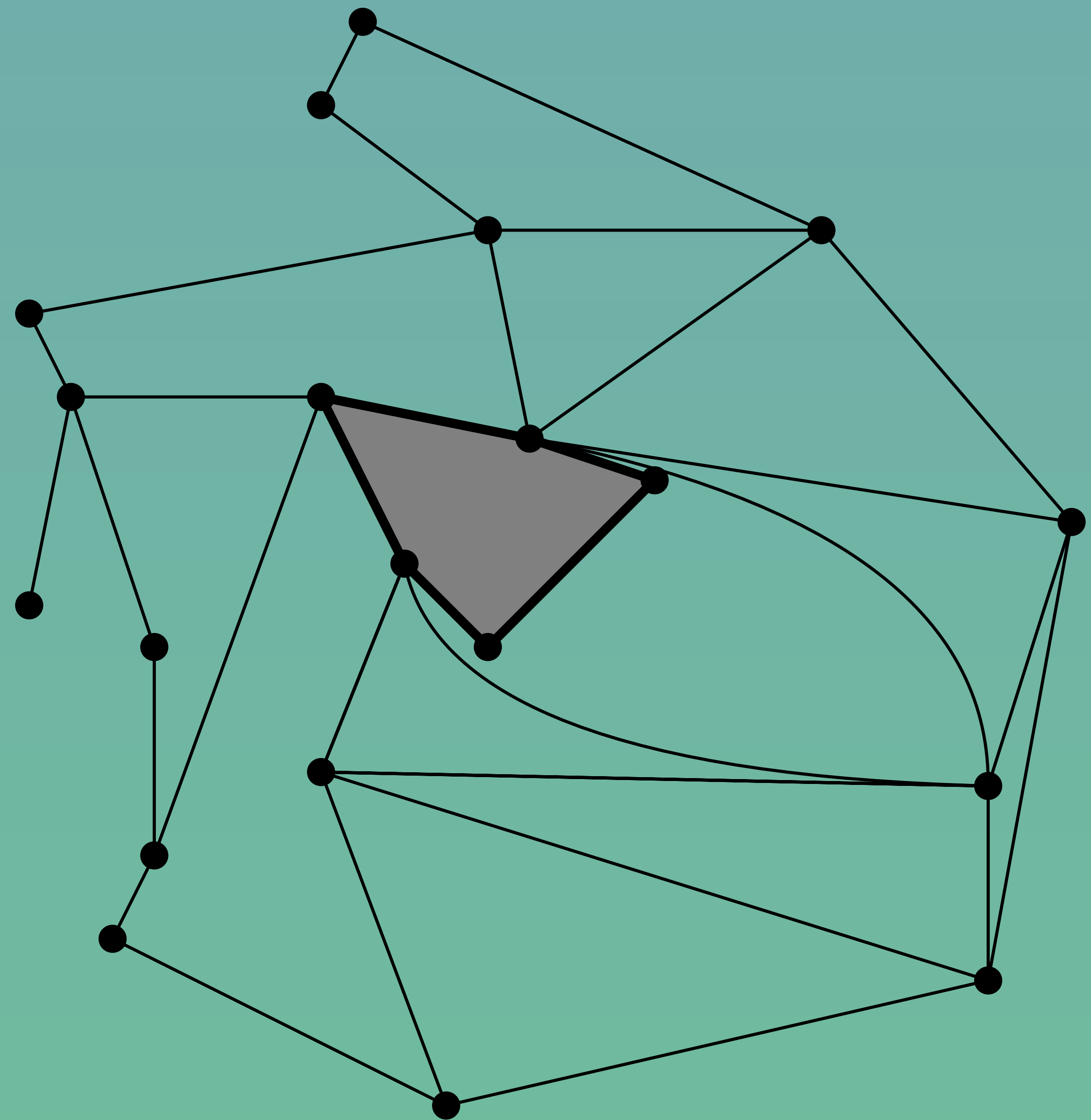

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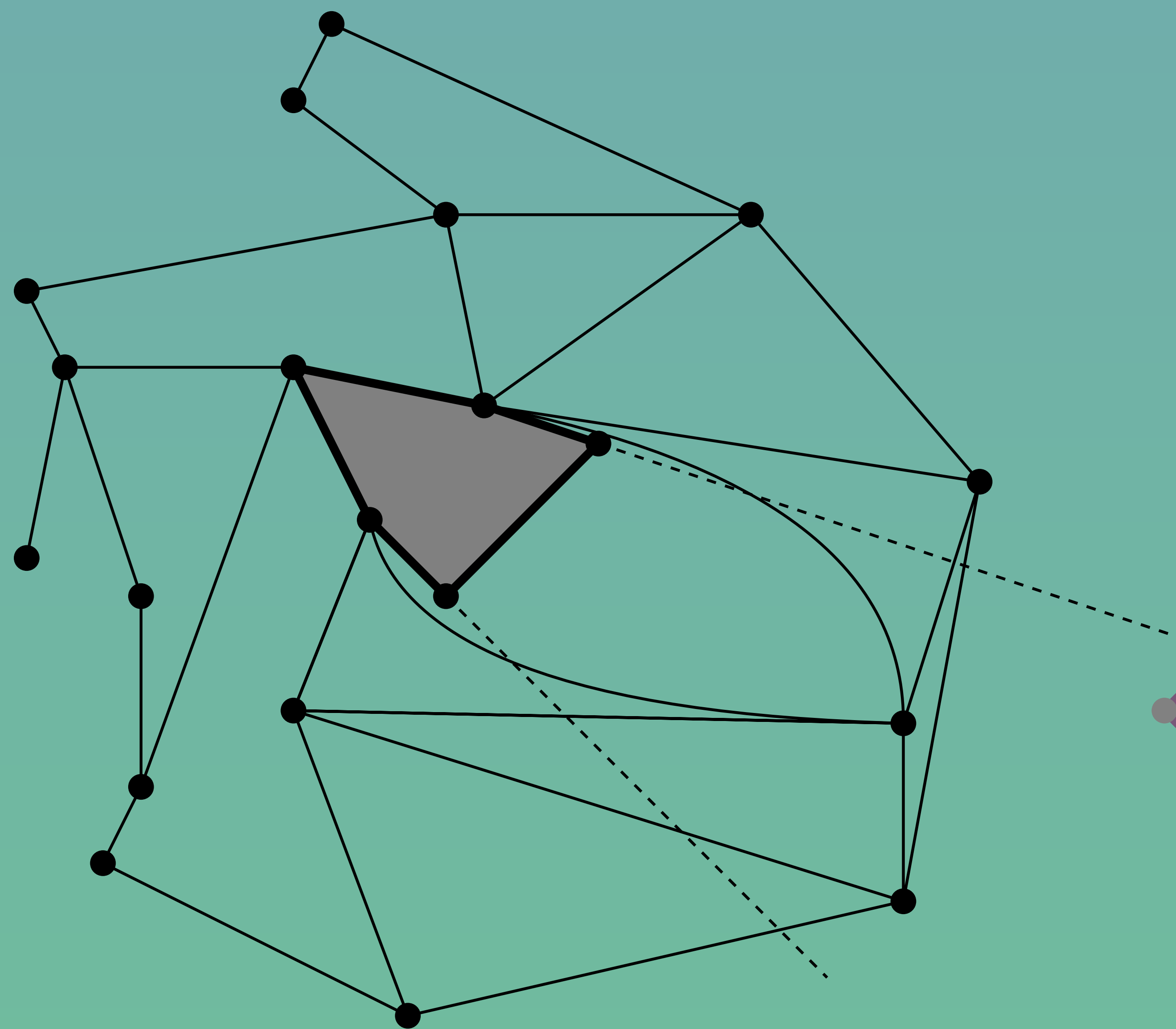

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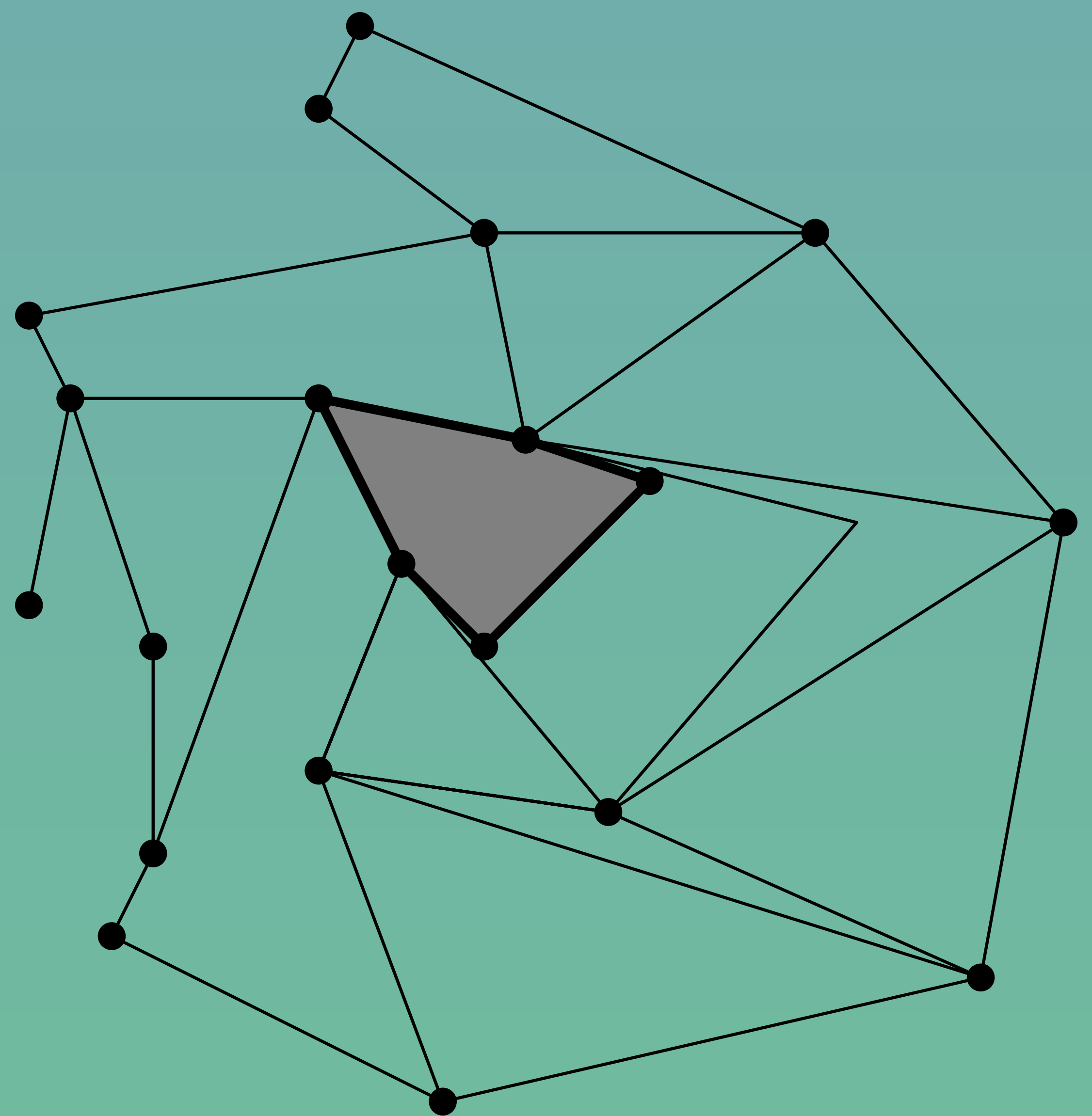

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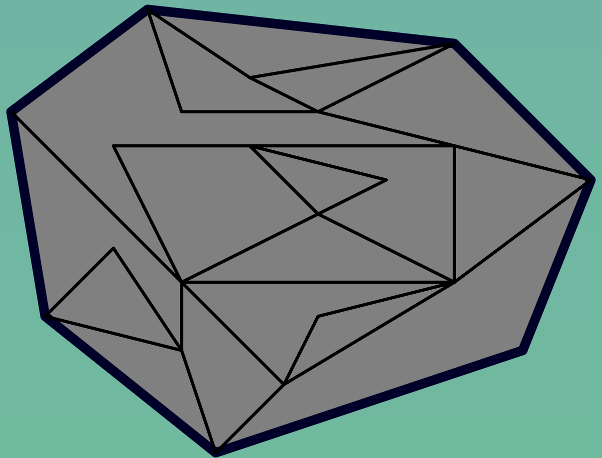

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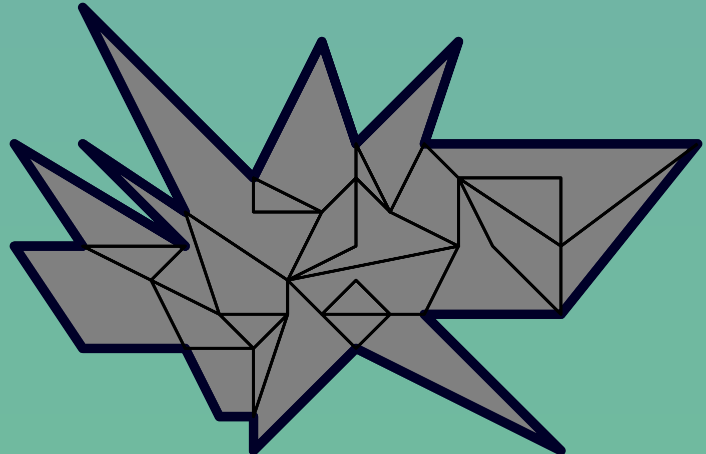

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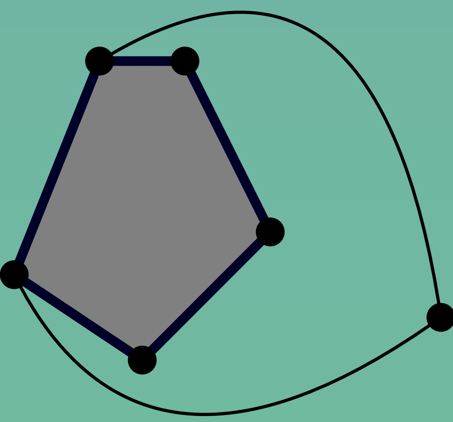

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	# of bends per edge
$H = (V, \emptyset)$	$120\|V\|$ [PW01]

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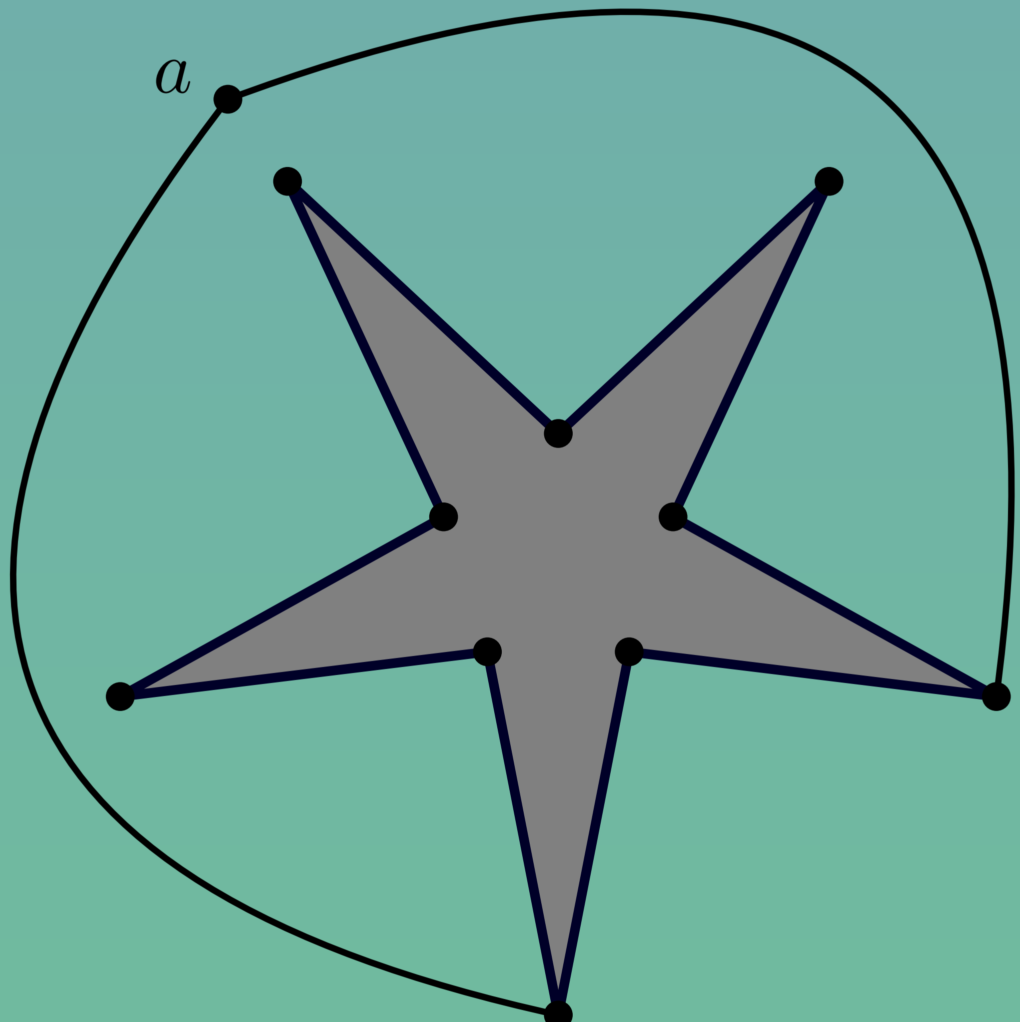
Extending Stars



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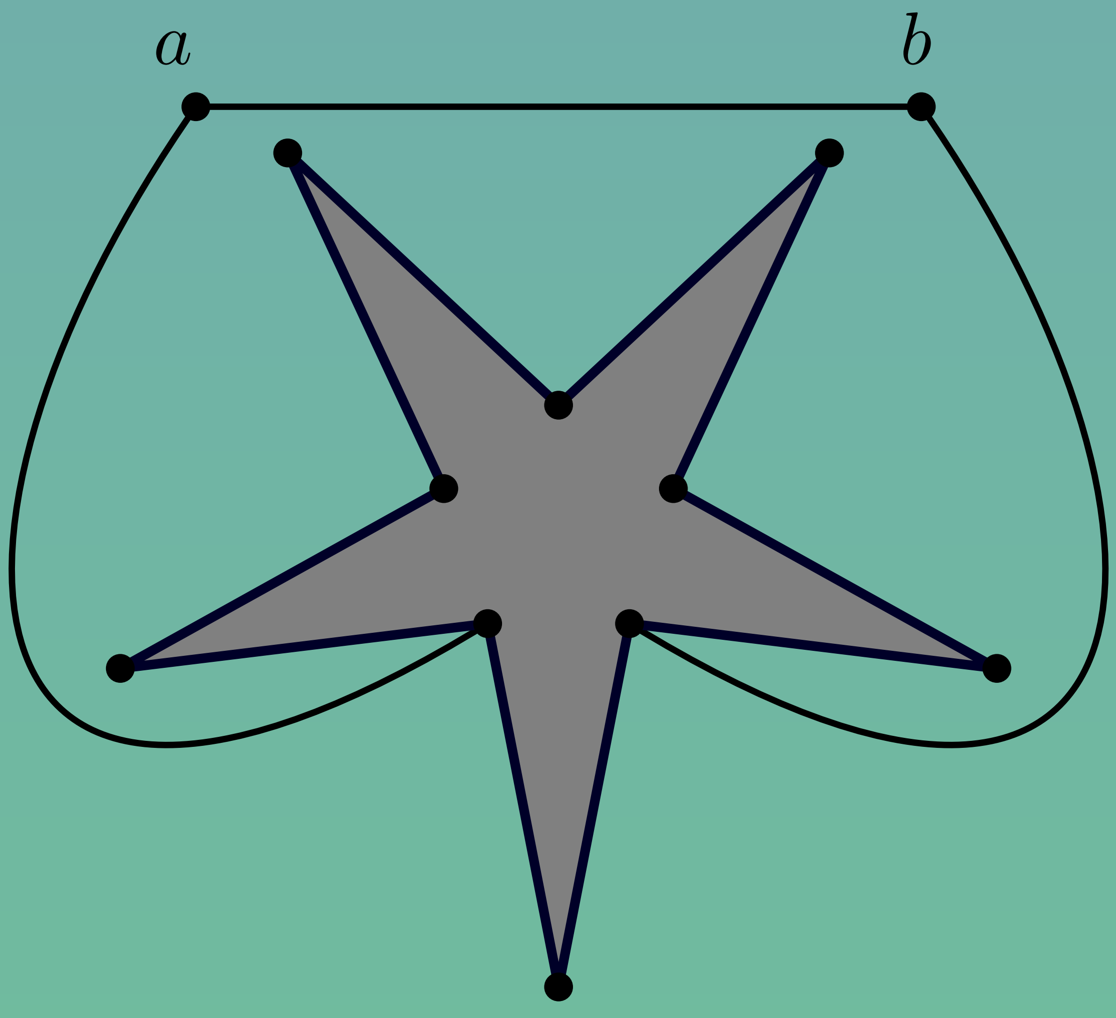

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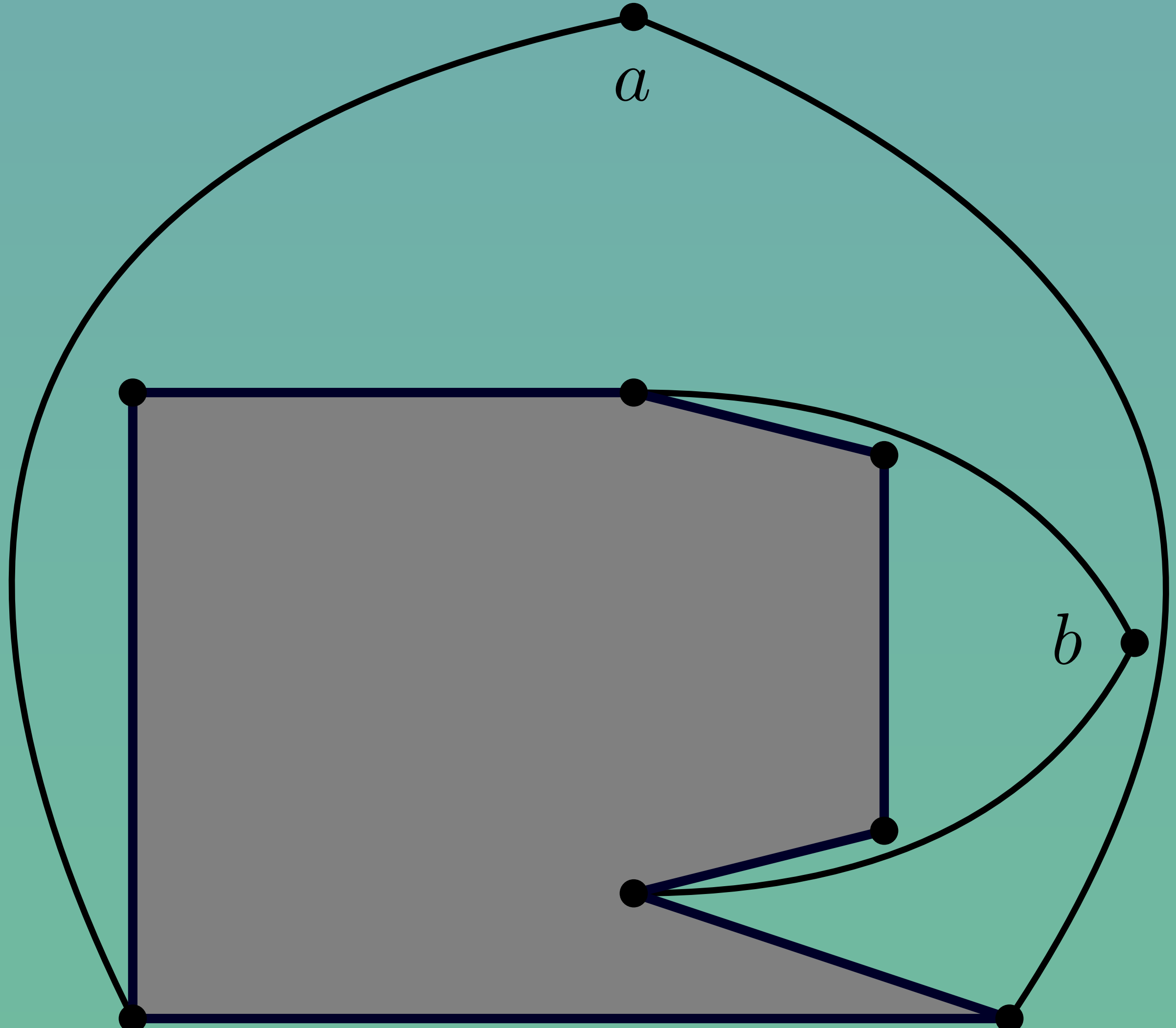

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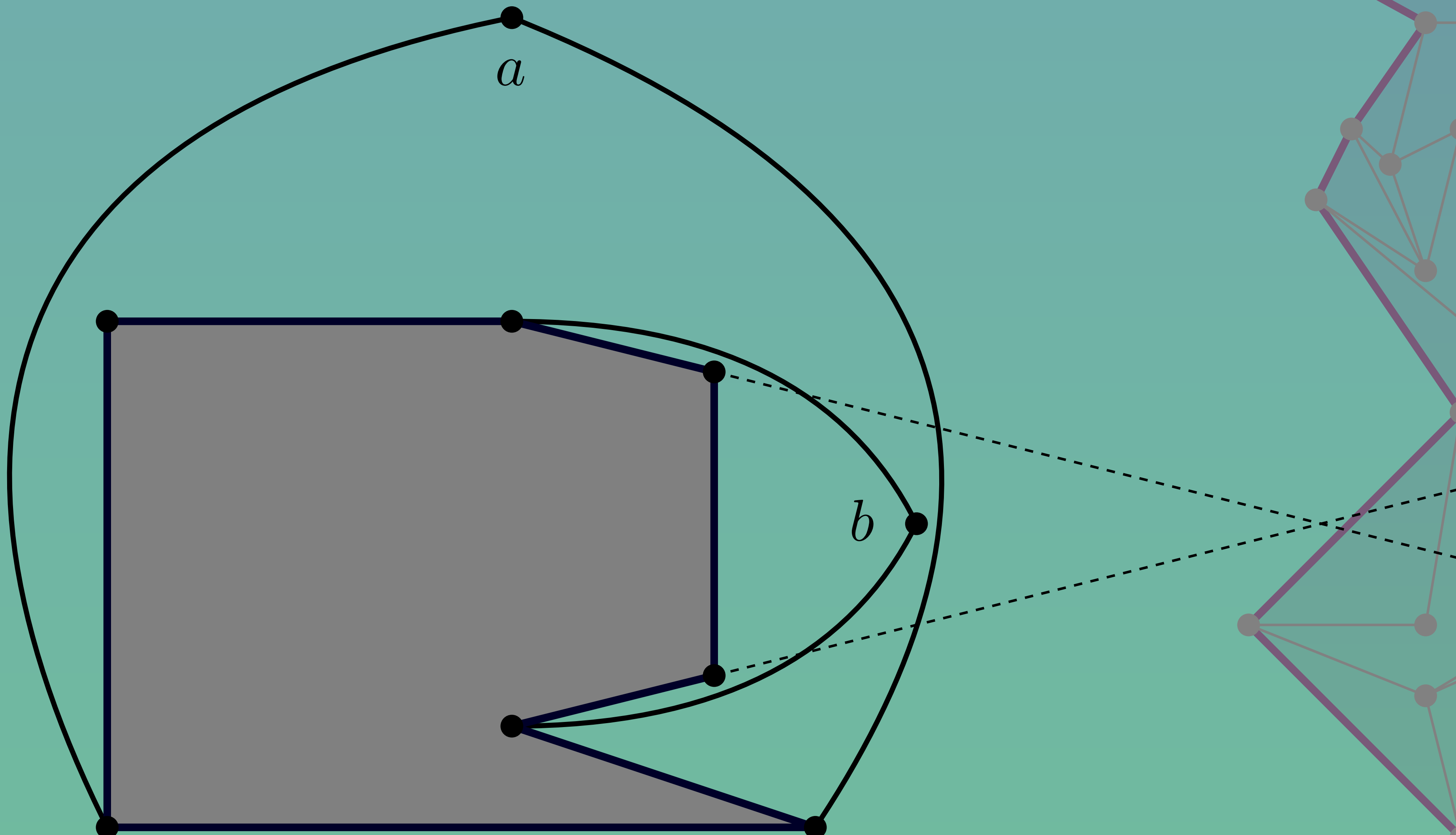

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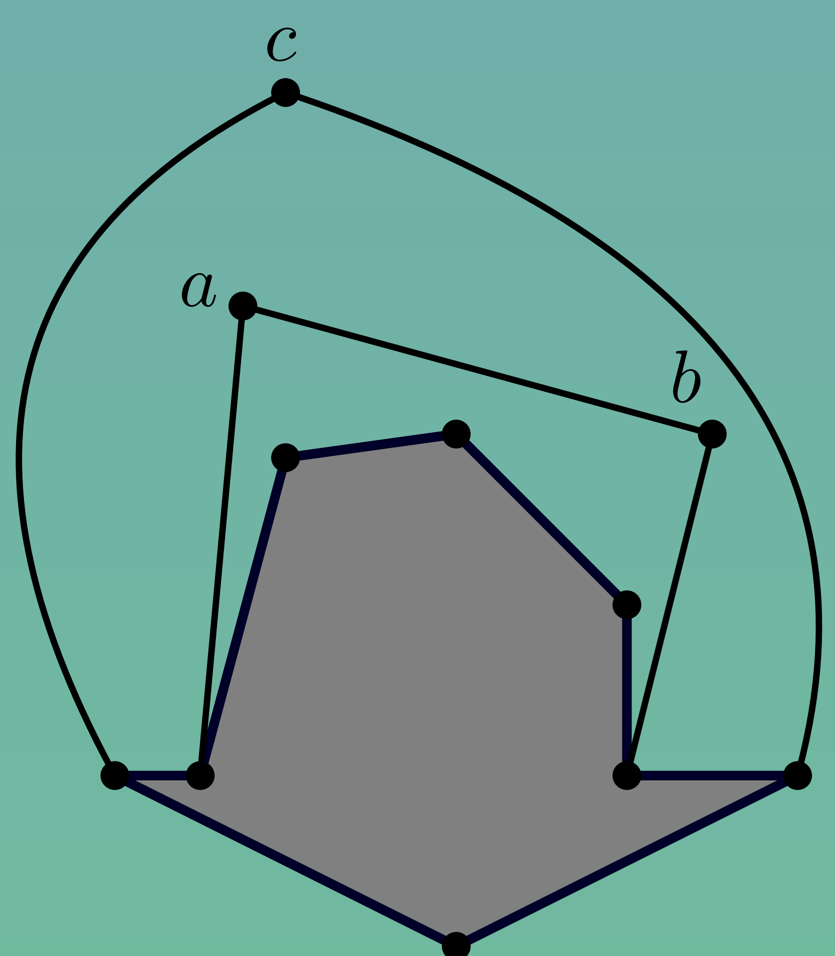

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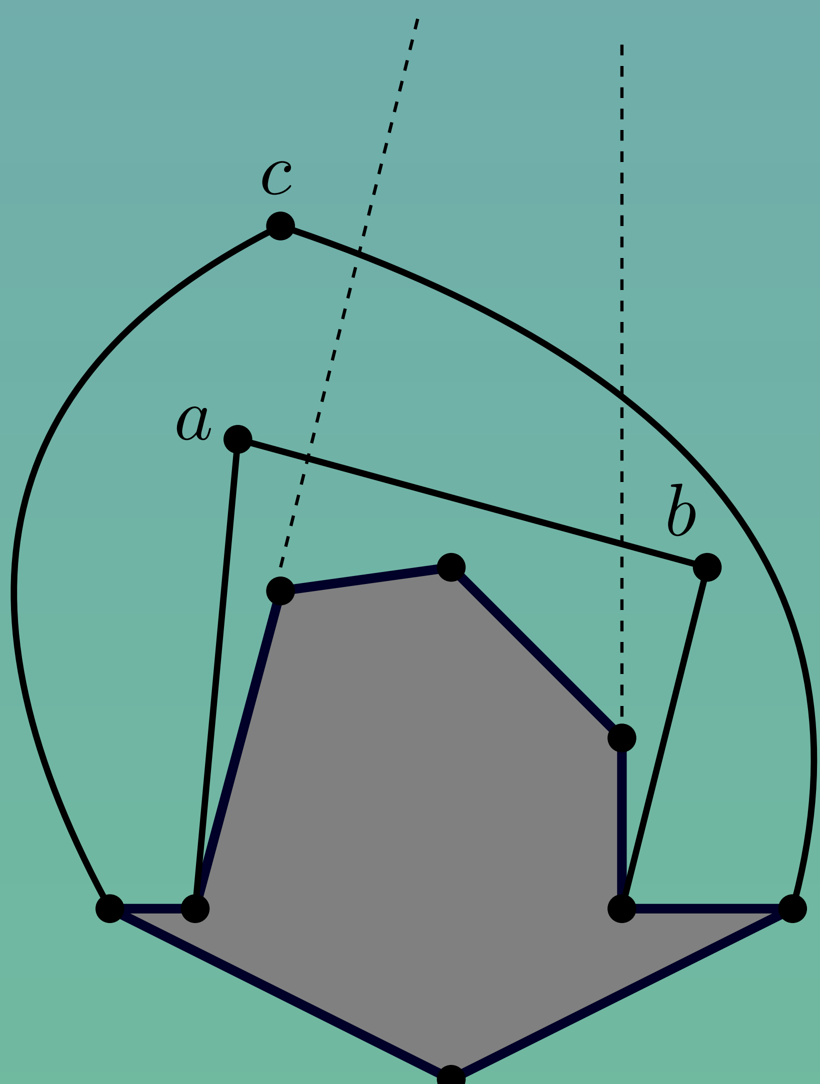

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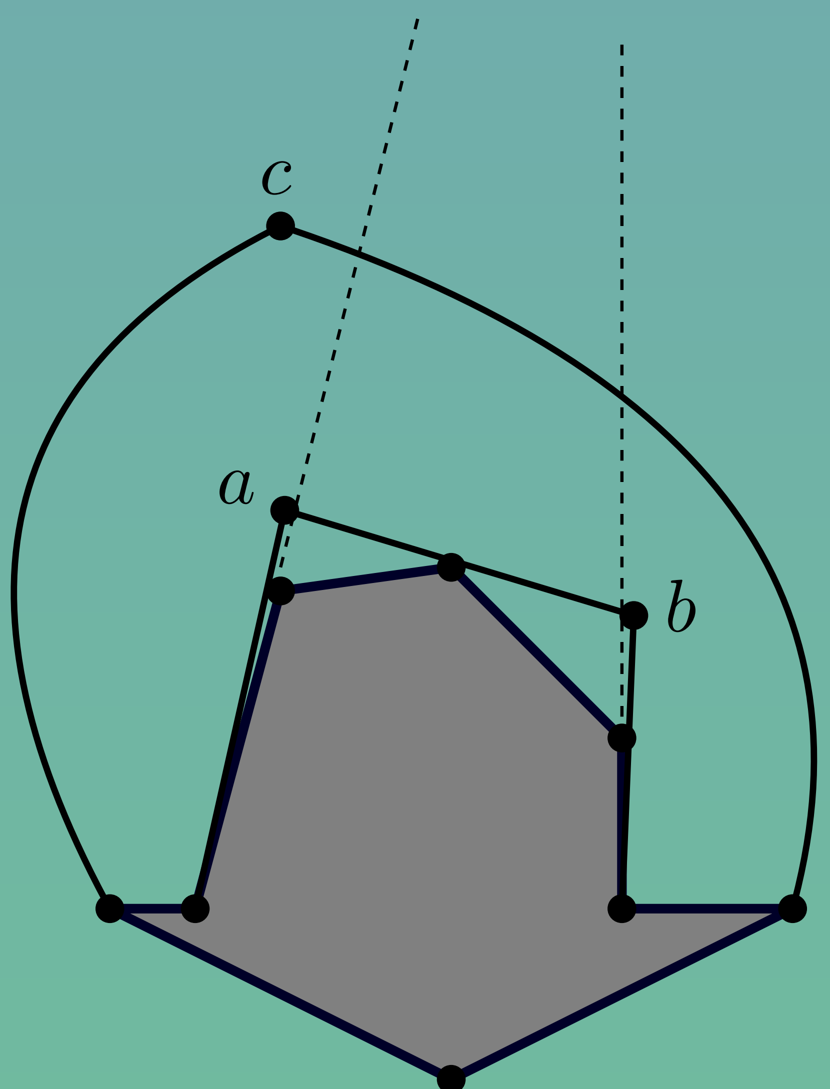

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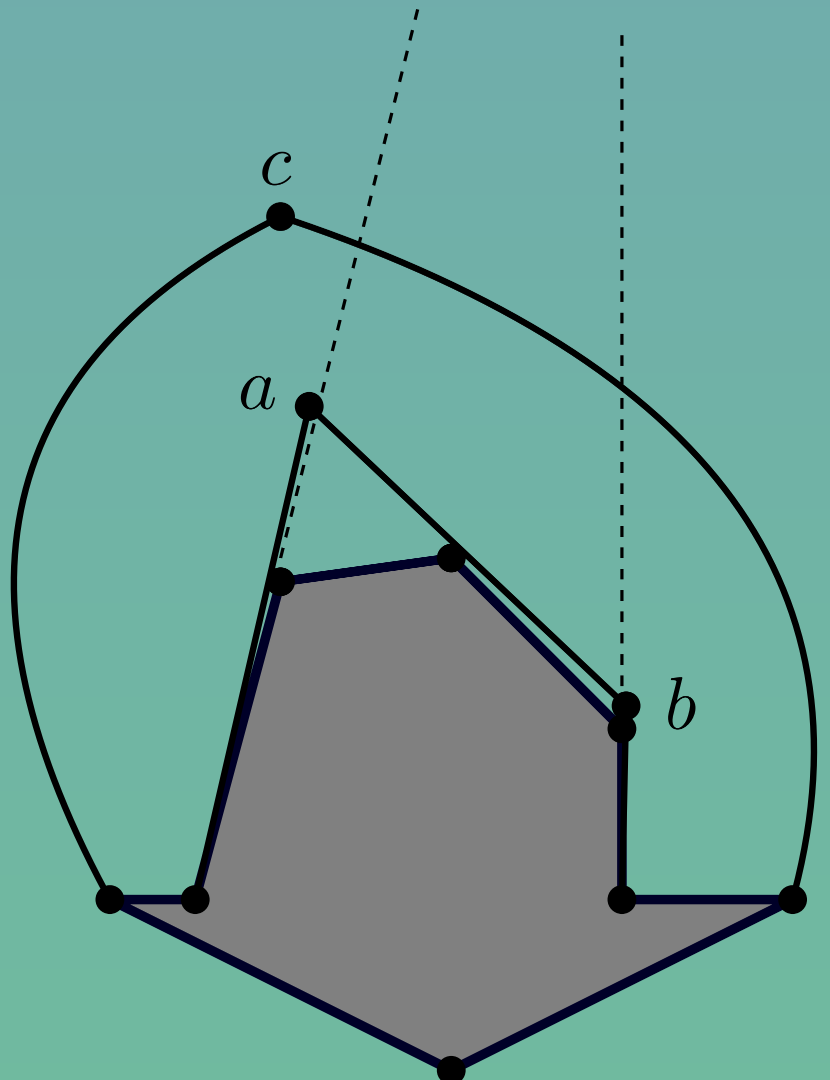

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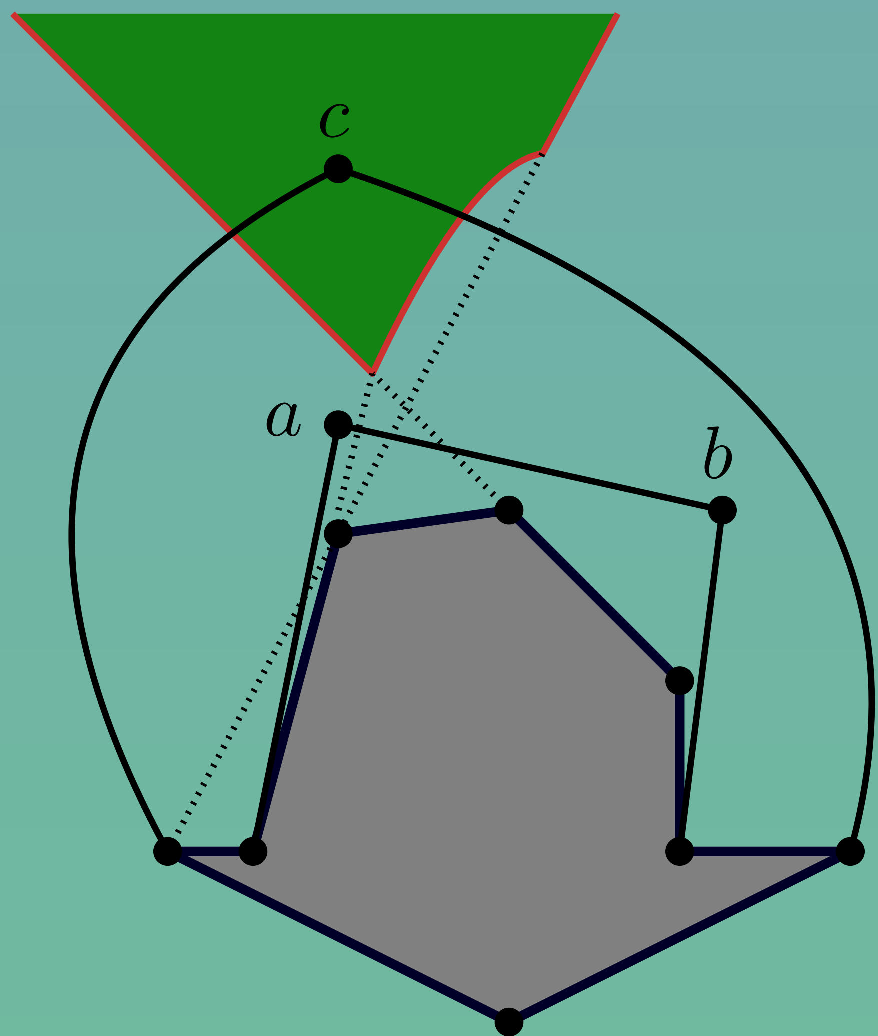

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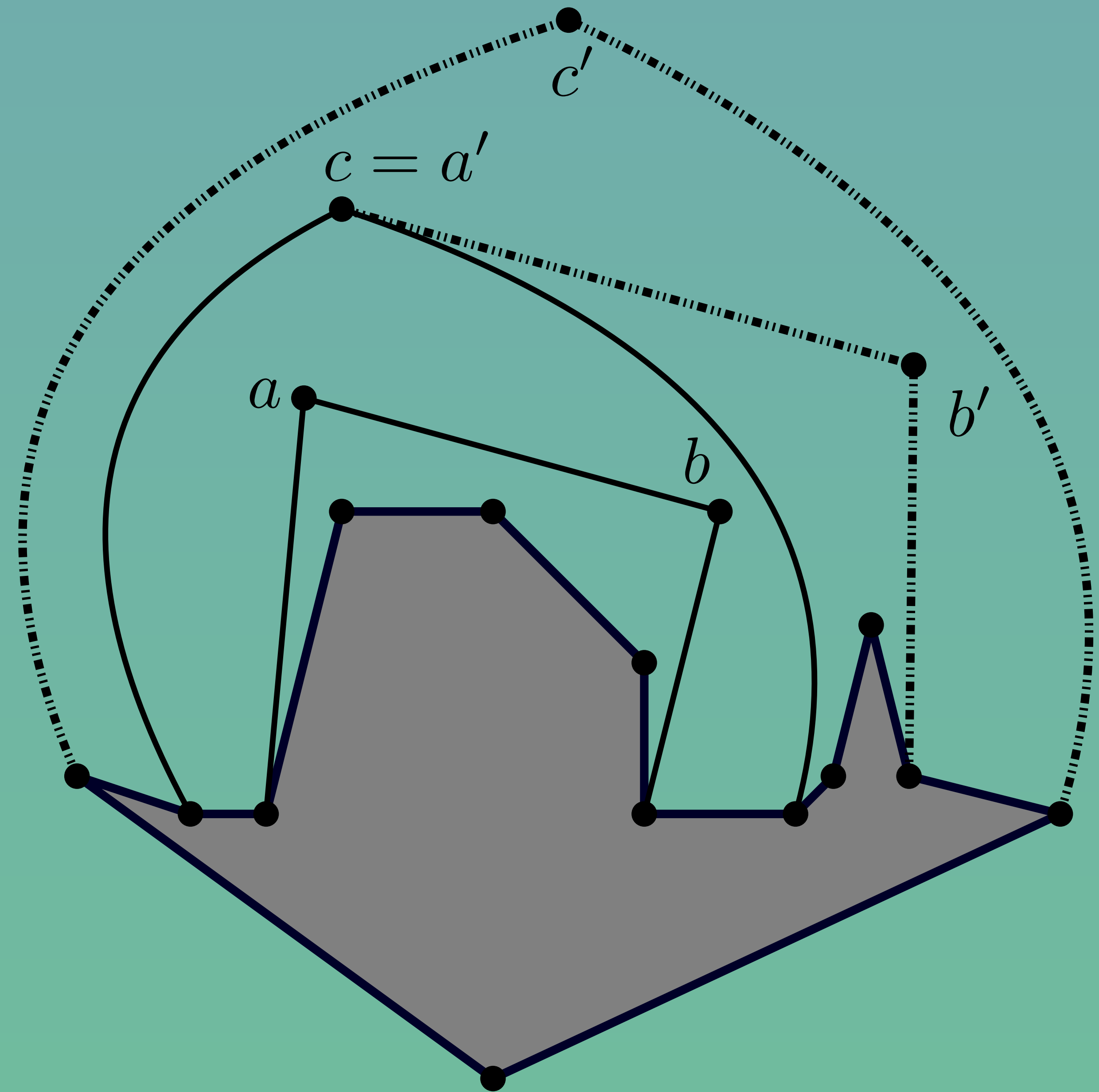

Extending Stars

Theorem

There is an instance (G, Γ_H) where Γ_H is a star-shaped inner face, such that the feasibility area of some vertex $v \in G$ is partially bounded by a curve whose implicit representation is a polynomial of degree $2^{\Omega(|V|)}$.

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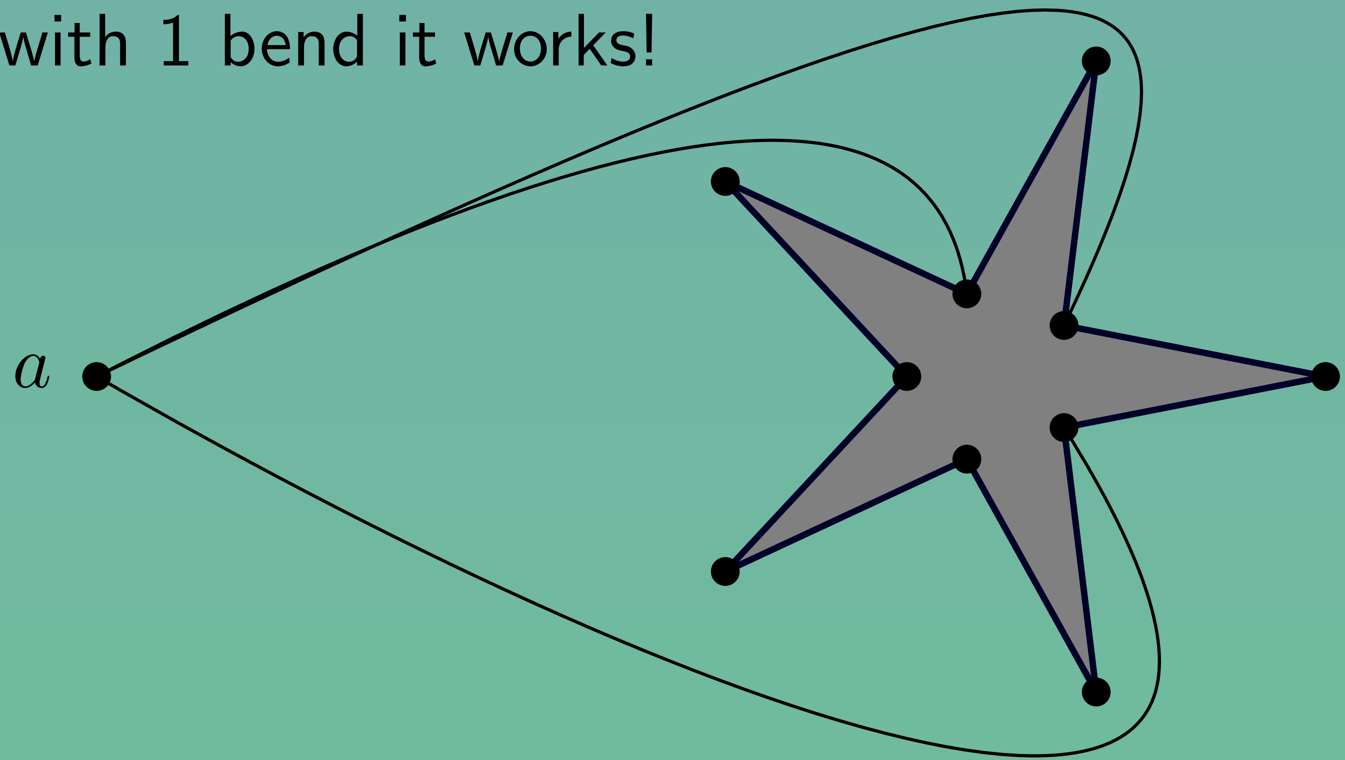

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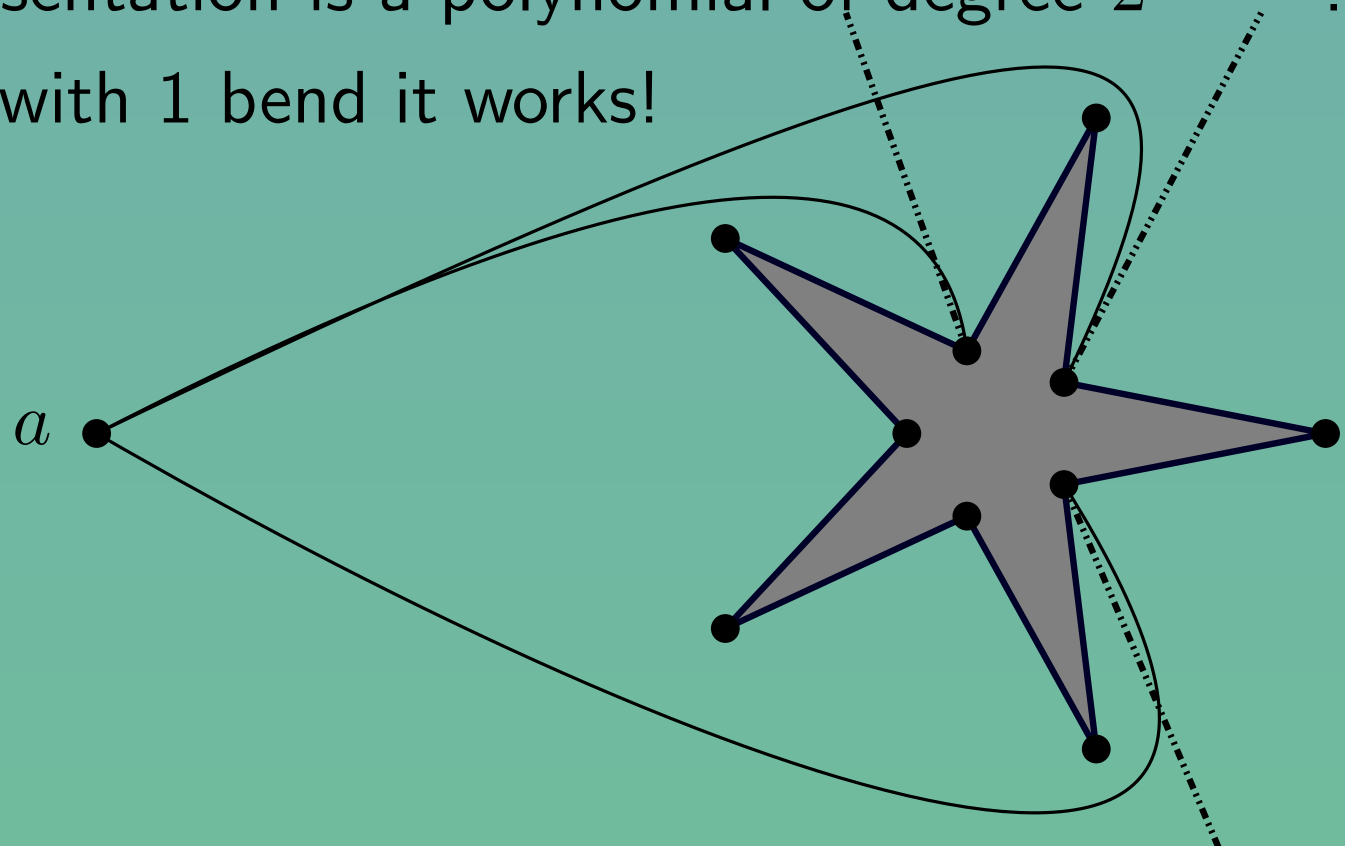

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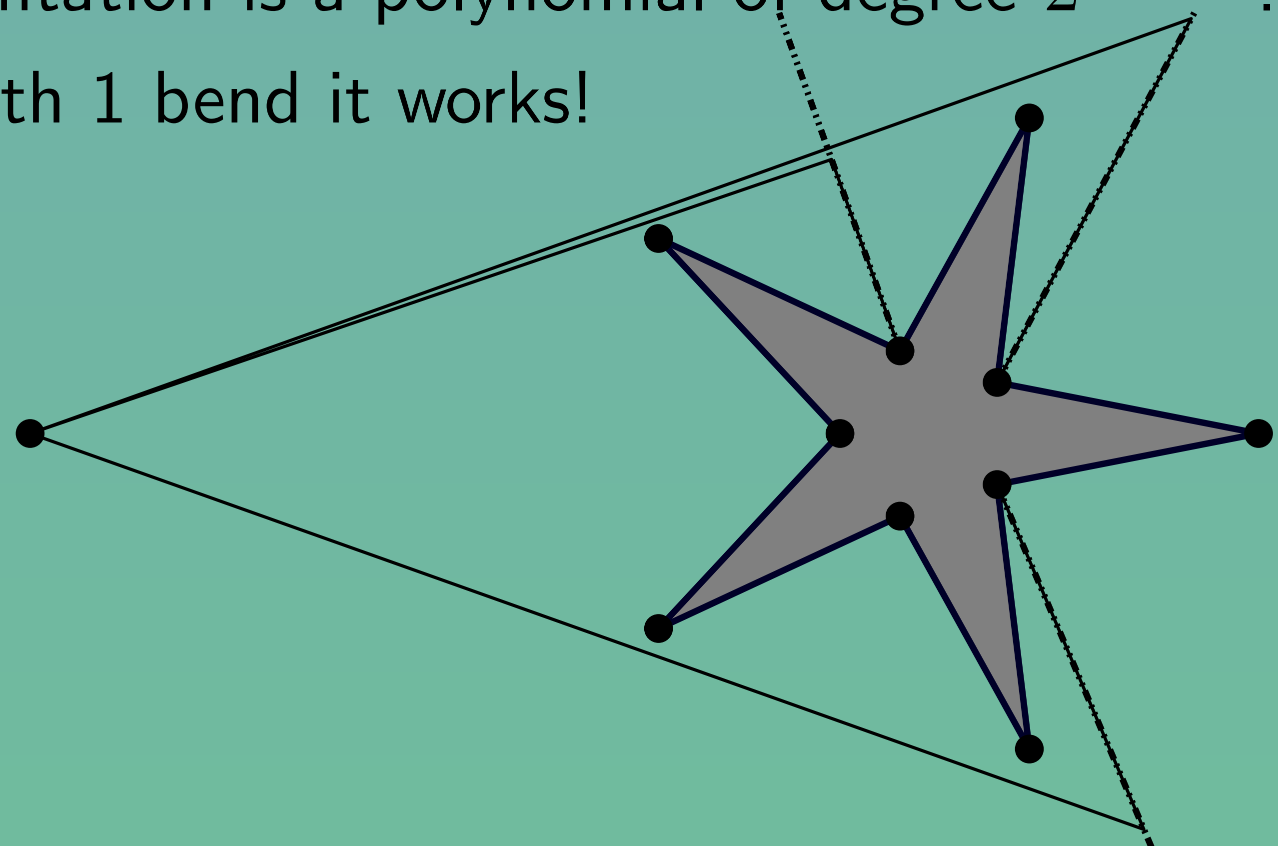

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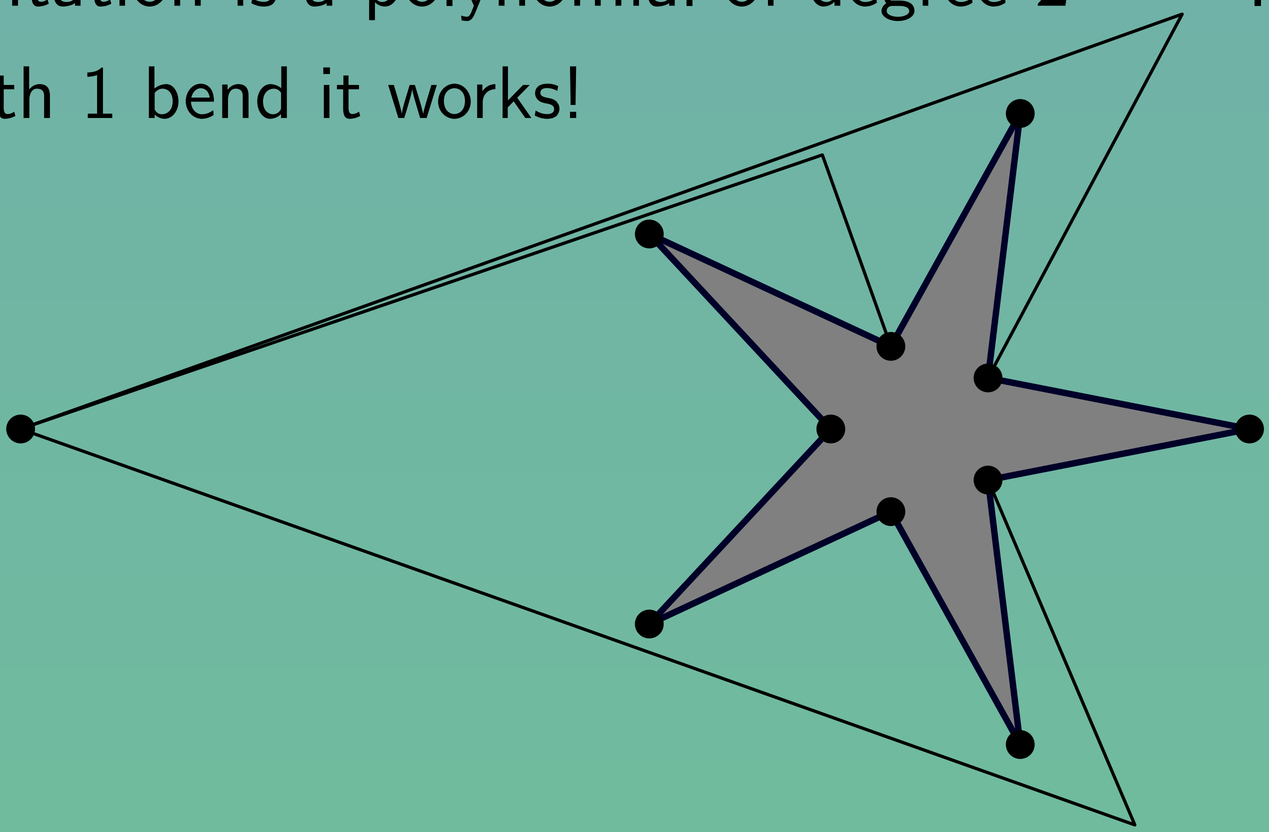
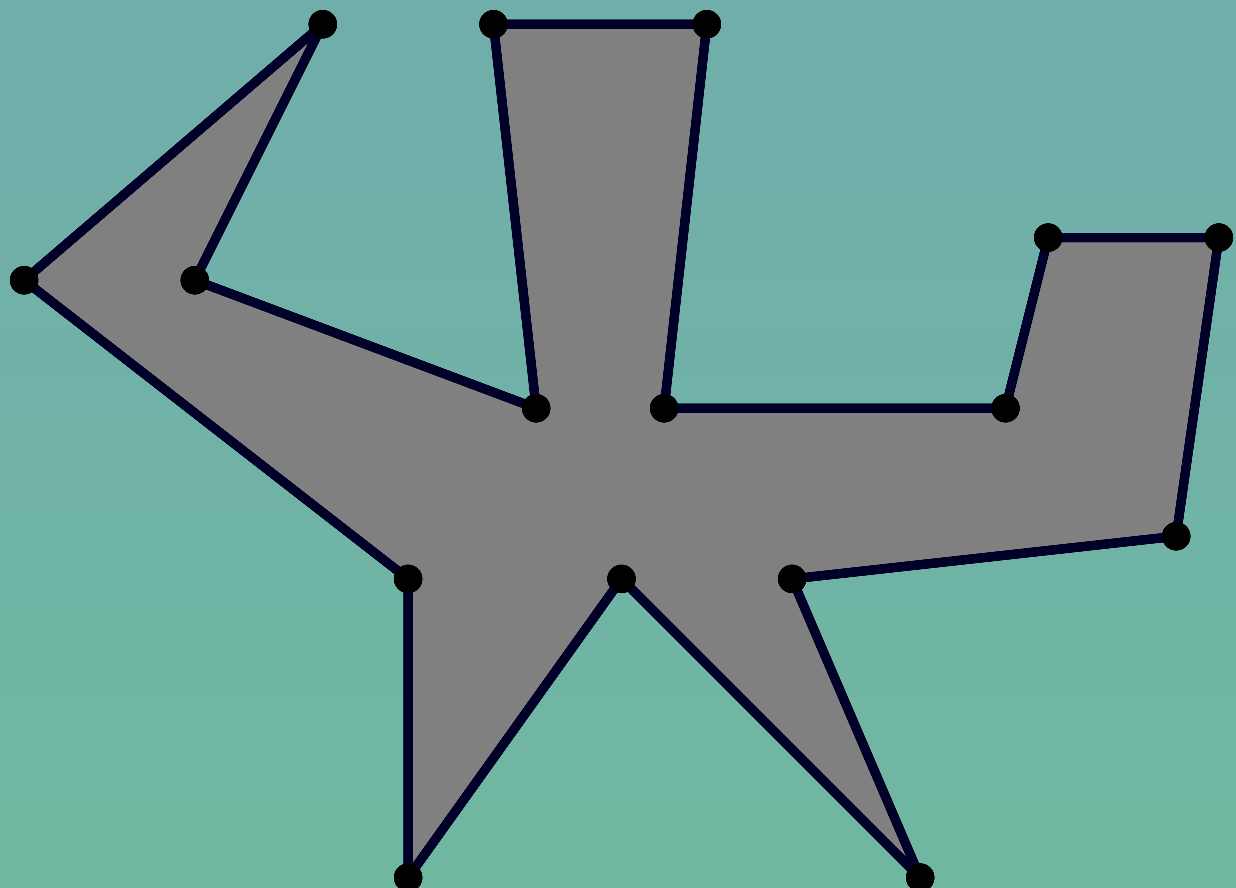


β-stars



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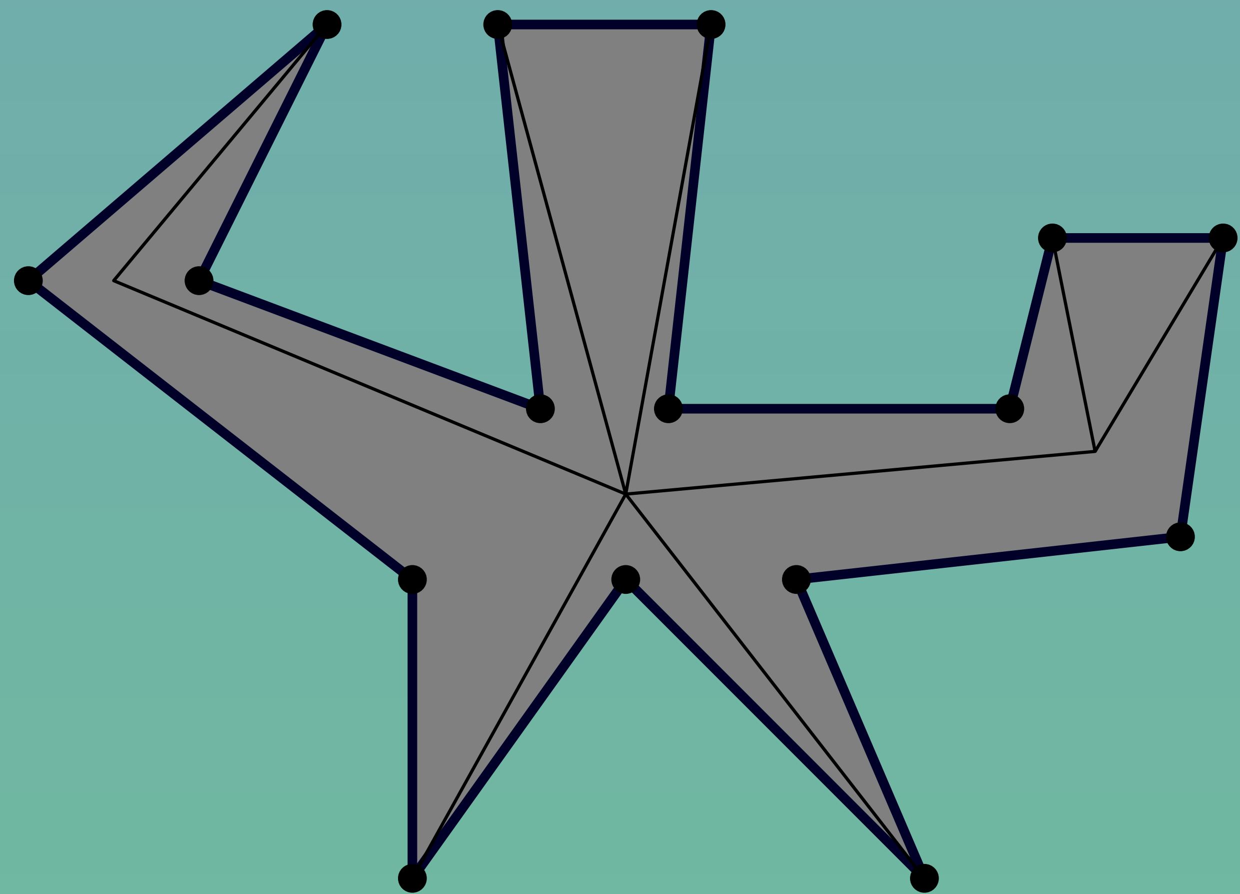

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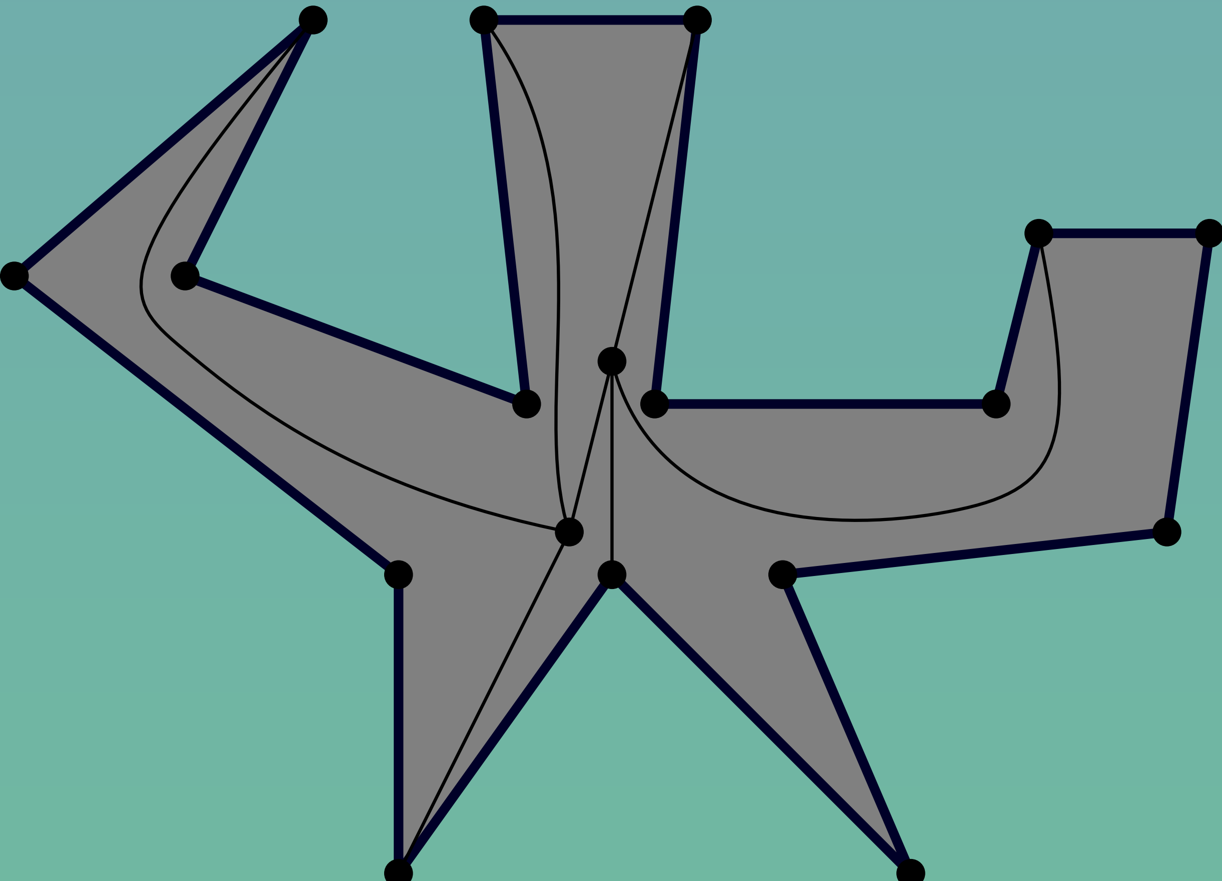

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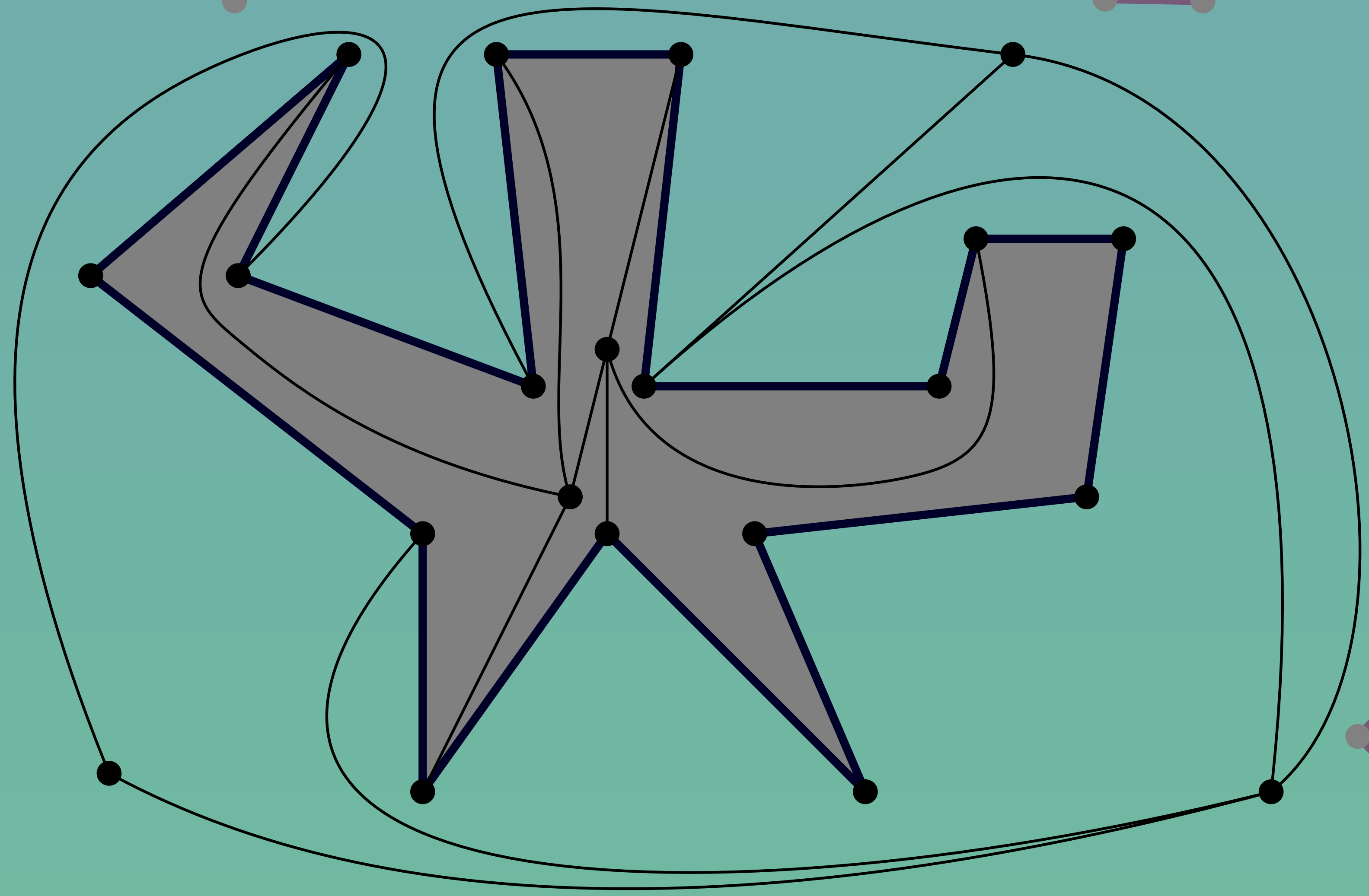

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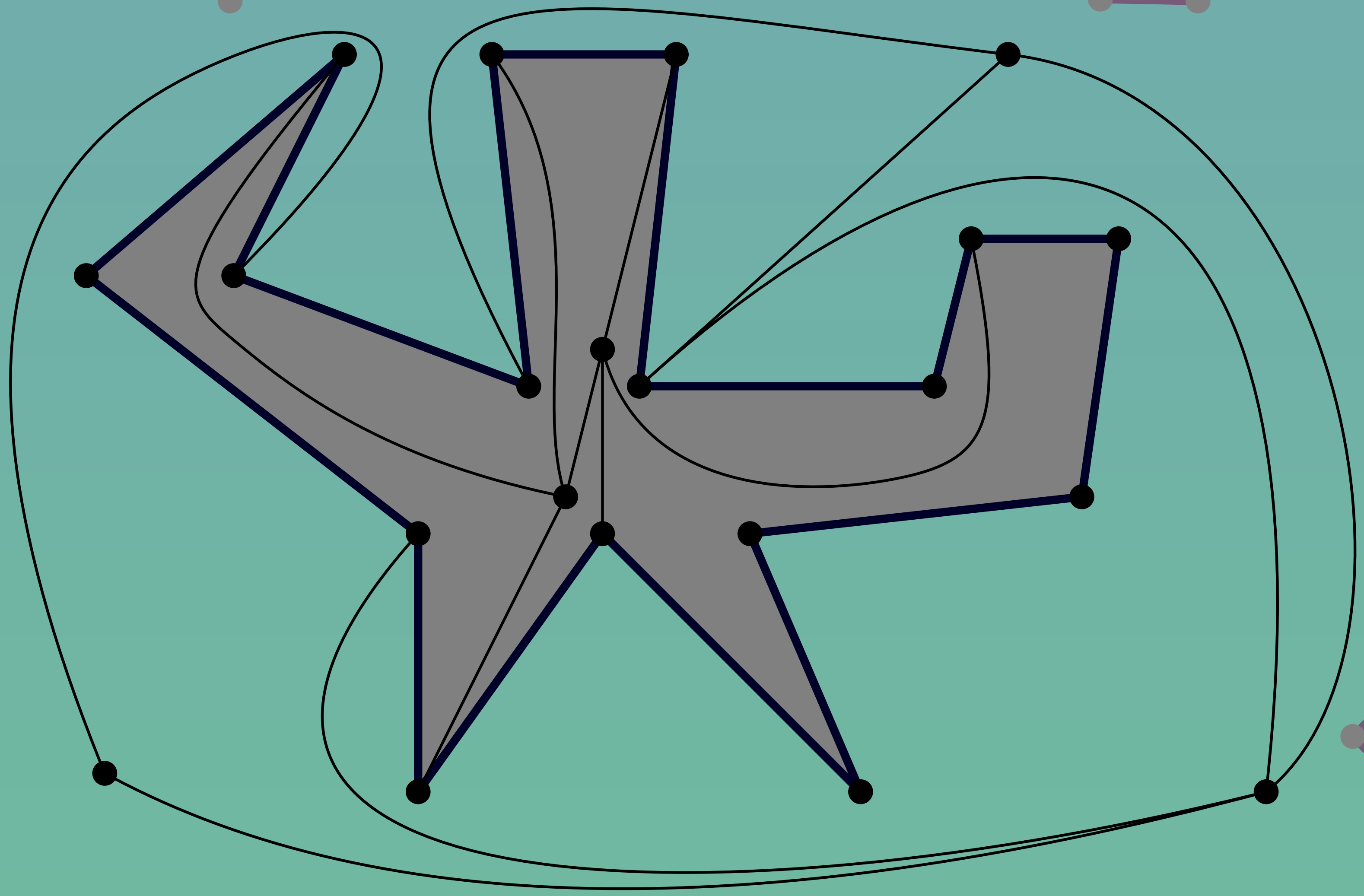
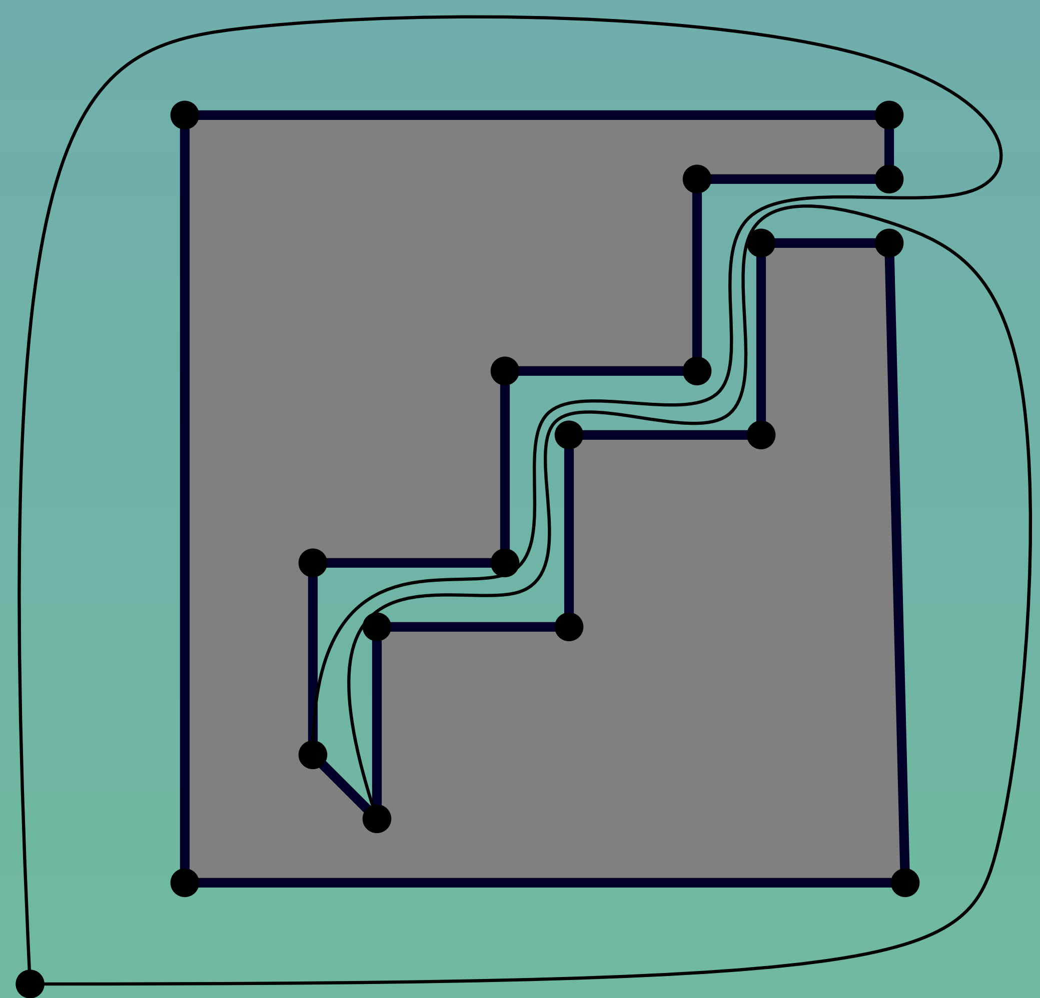


Can each drawing of an outer (inner) face as a β-star be extended with at most $\beta + 1$ bends per edge?

β-stars



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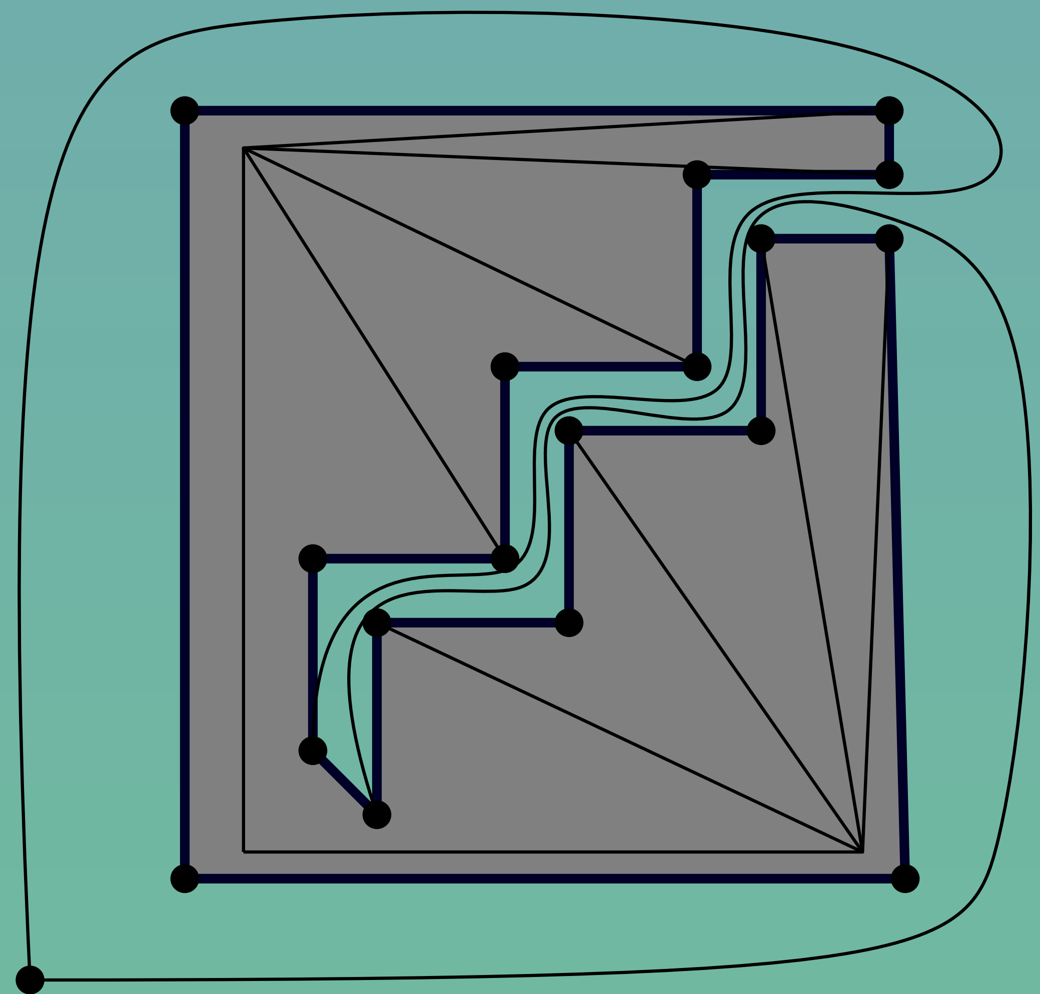

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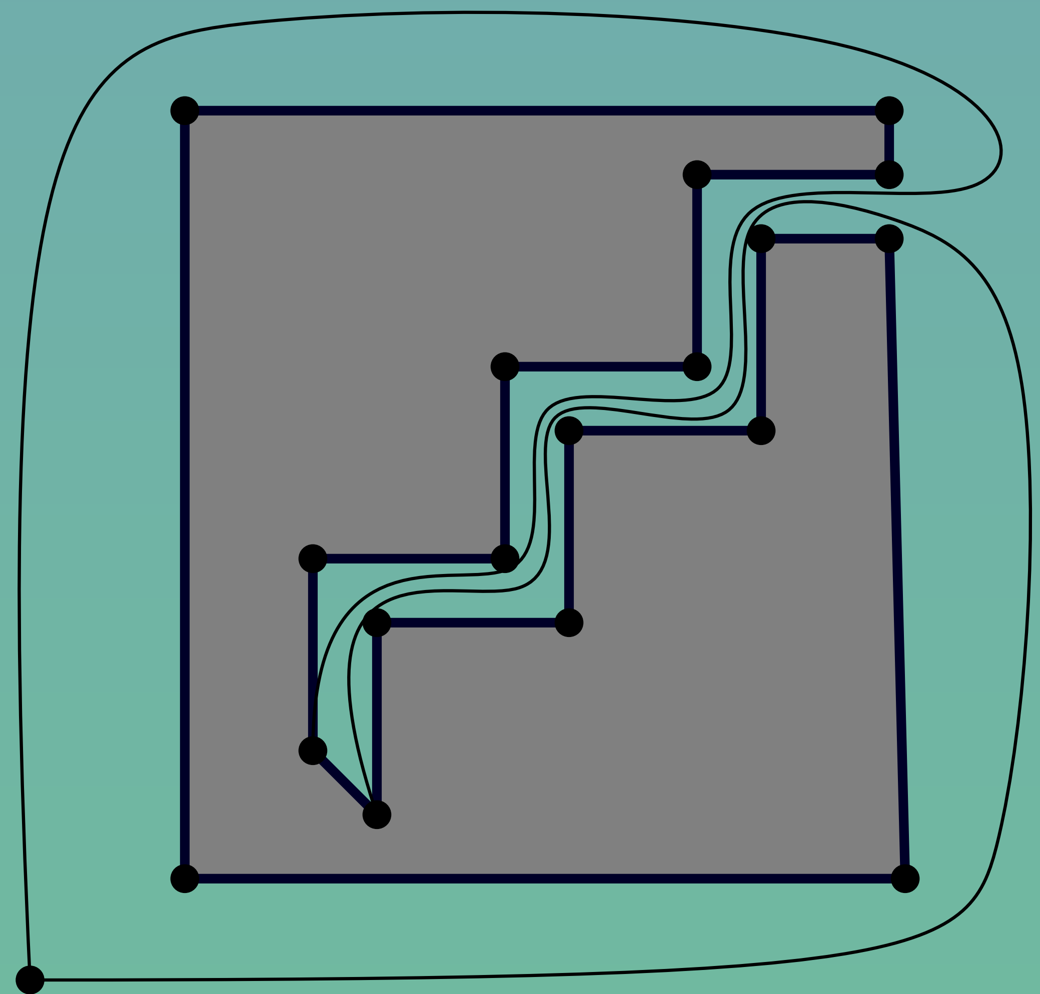
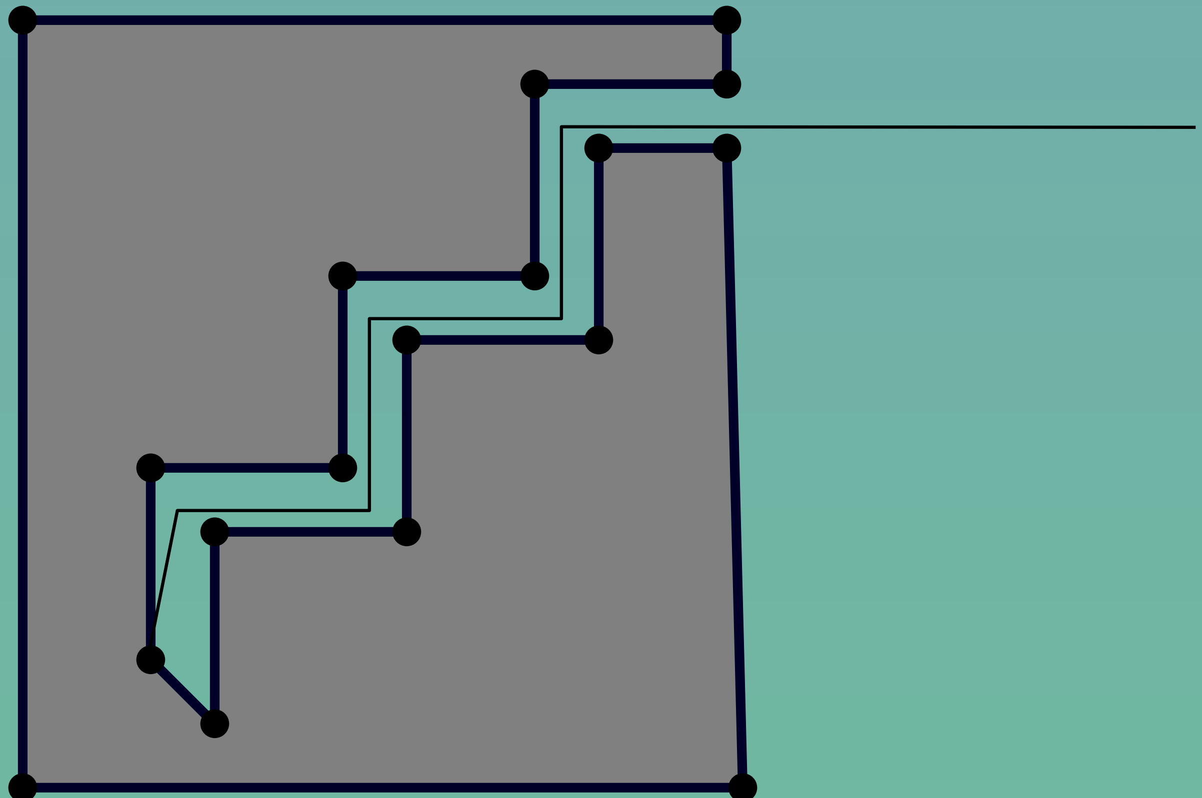


β-stars

5-outer-star



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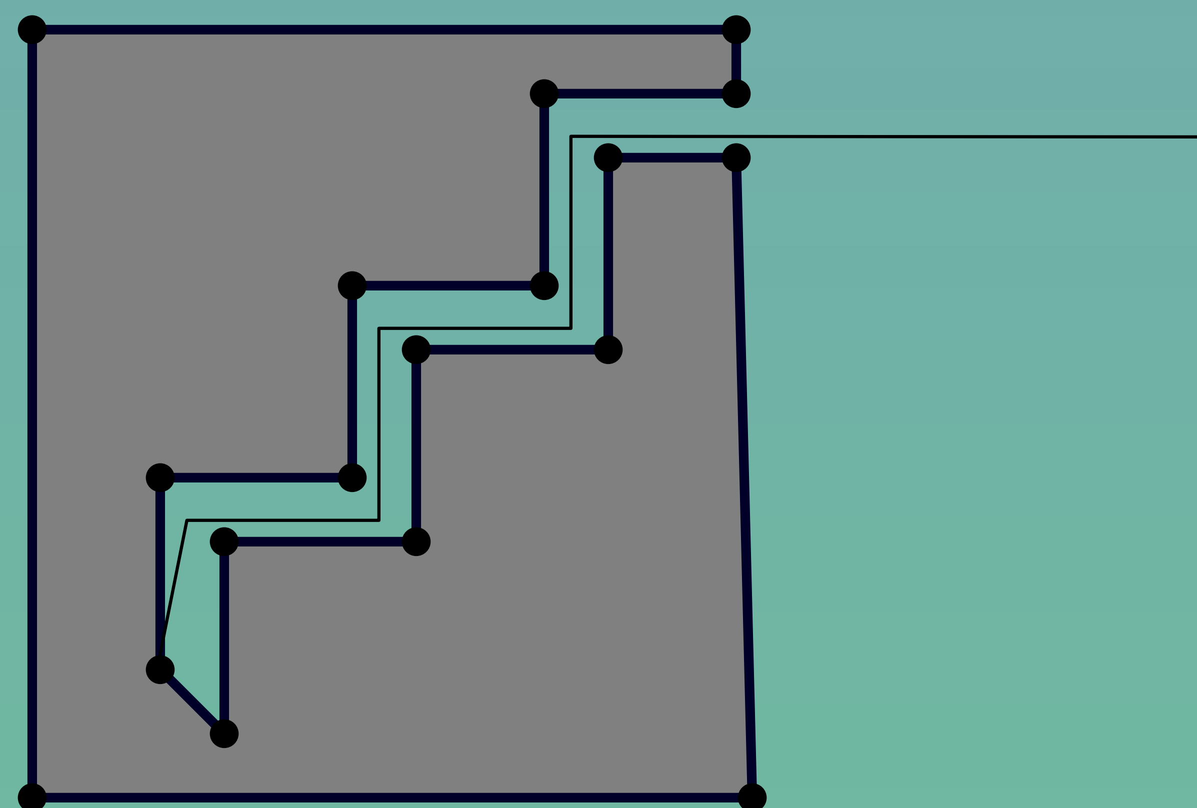

Can each drawing of an outer (inner) face as a β-(outer)-star be extended with at most $\beta + 1$ bends per edge?

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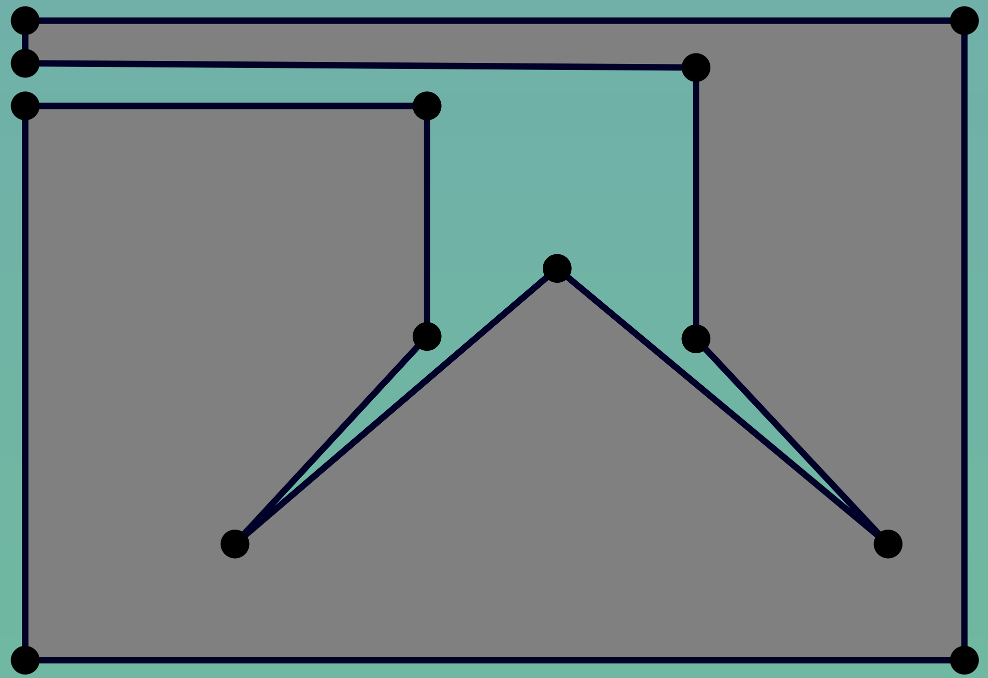
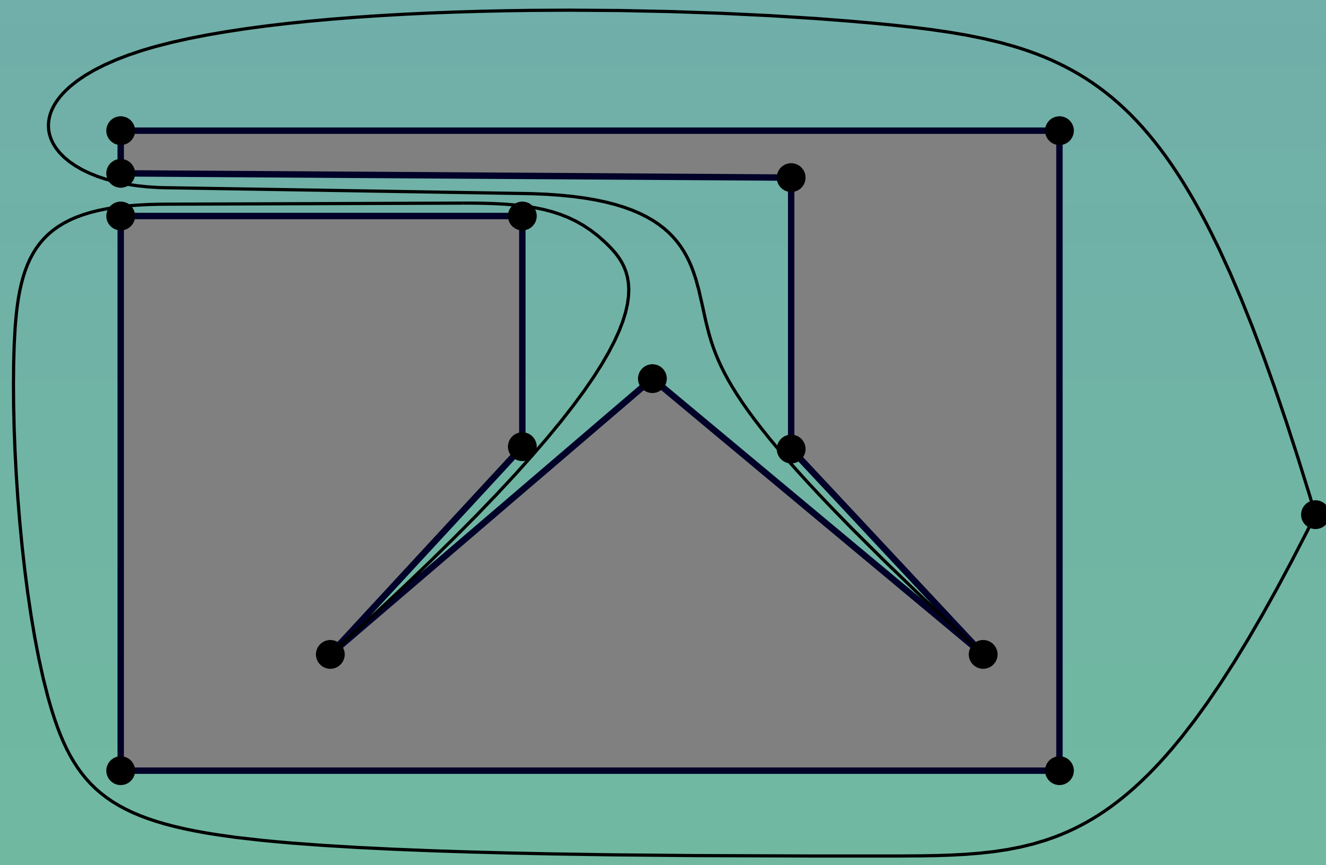
1-outer-star



β-stars

1-outer-star, but 3 bends are needed.



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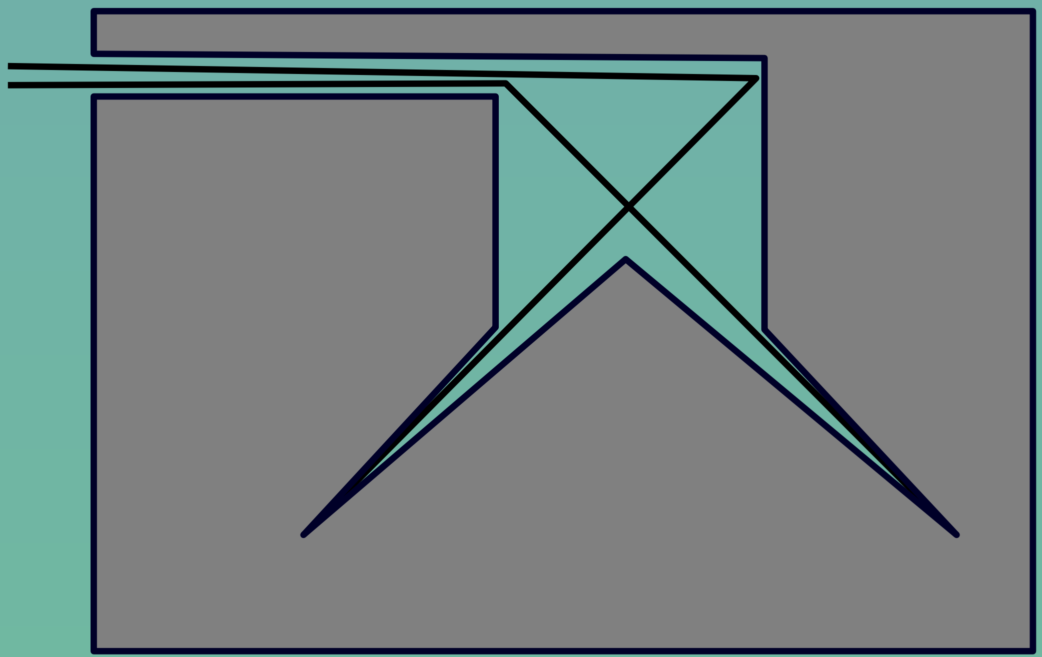
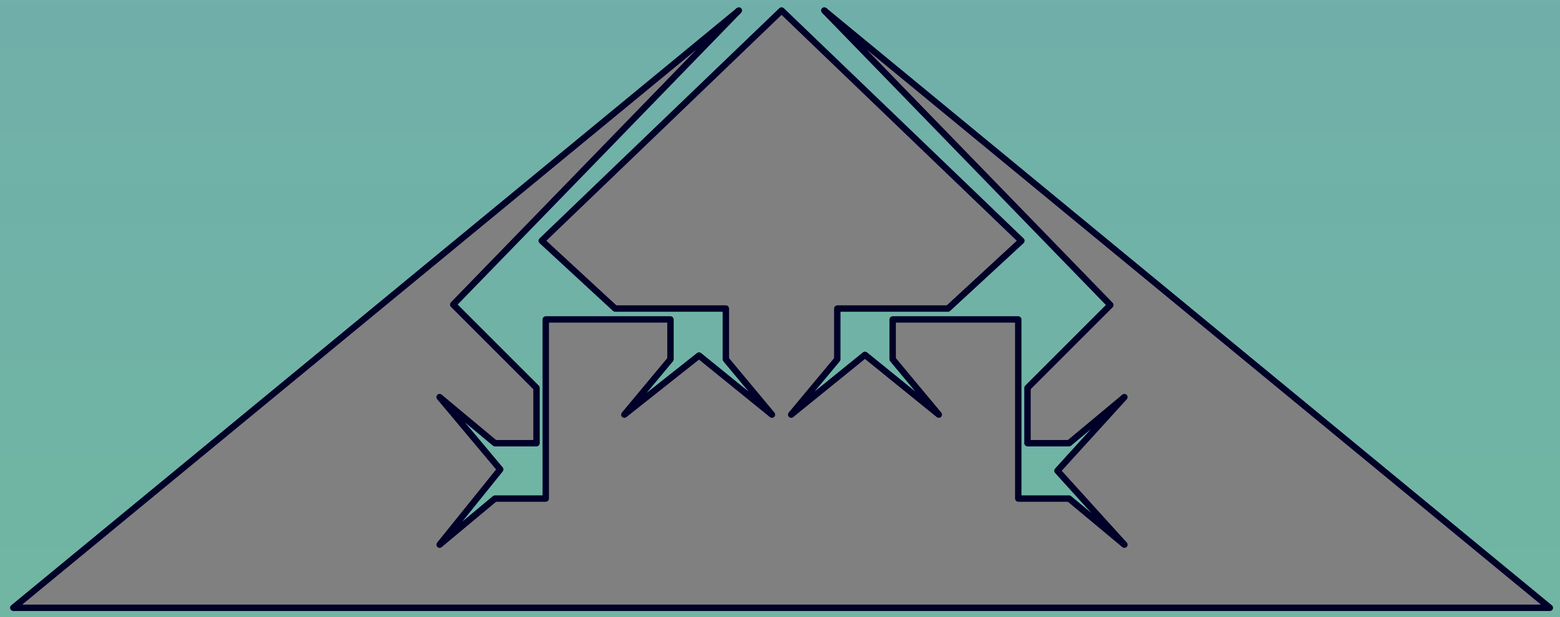


β-stars

2-outer-star, but 6 bends are needed.



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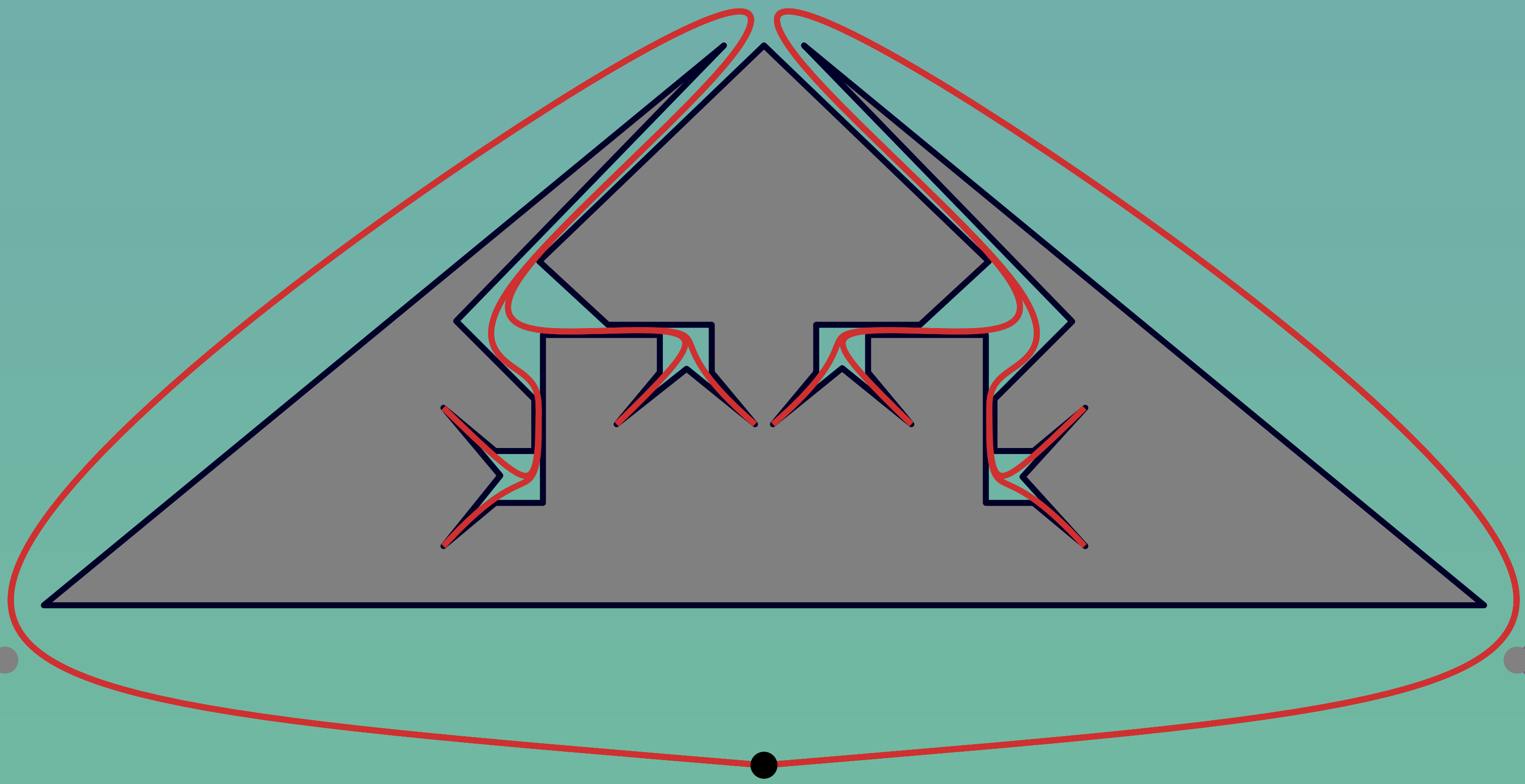

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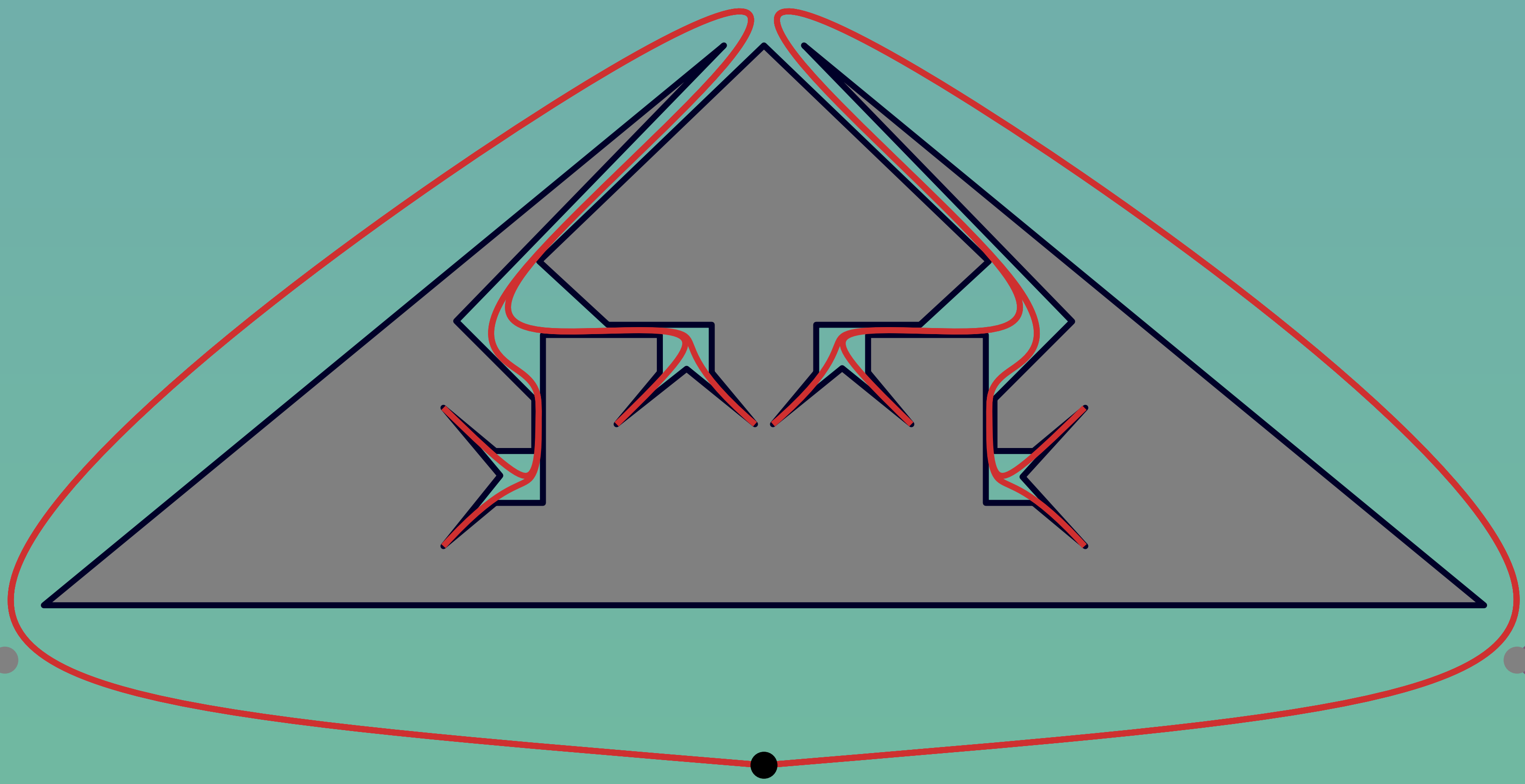
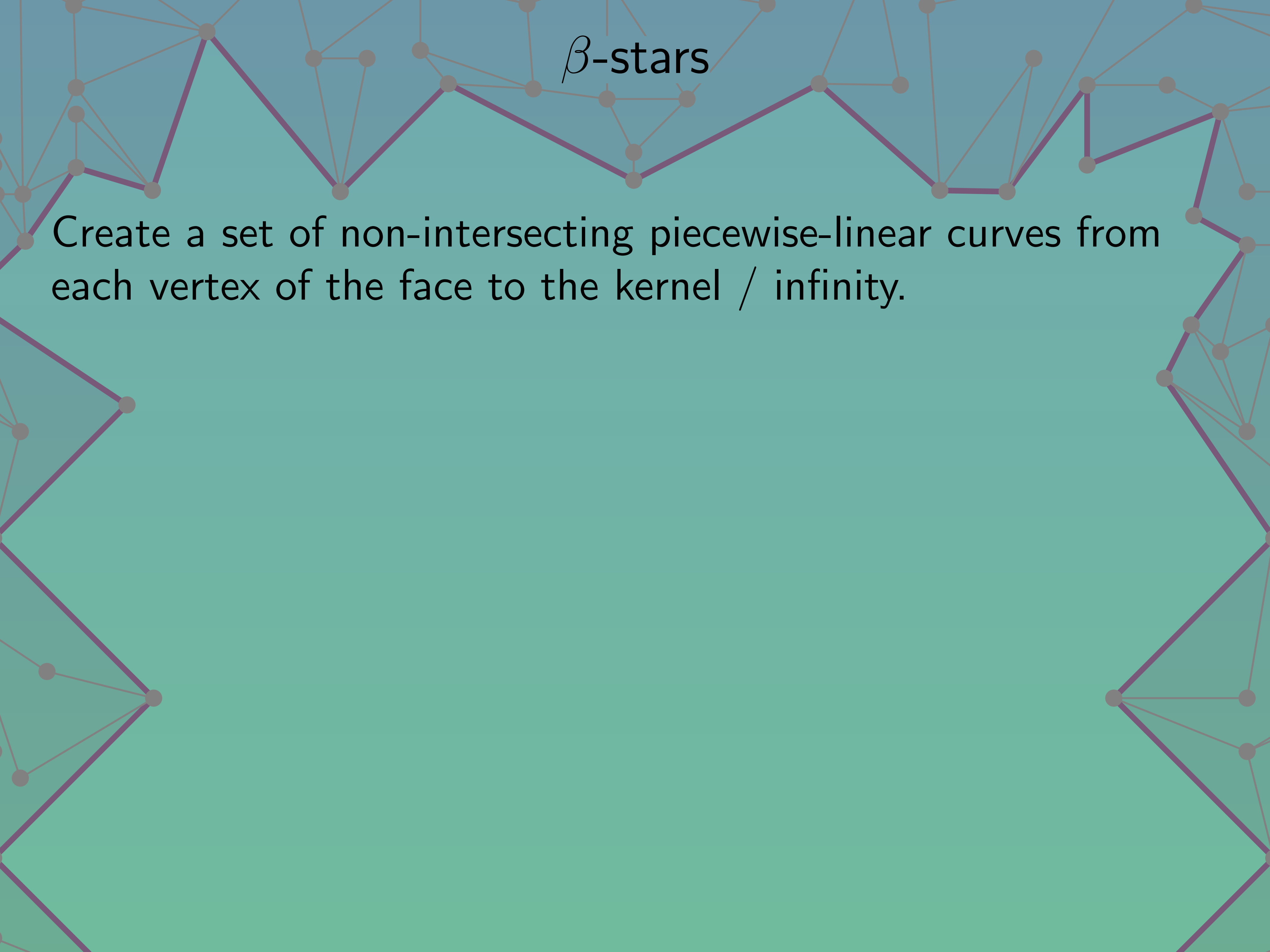


How to get rid of the intersections while adding a minimal number of bends?

β-stars

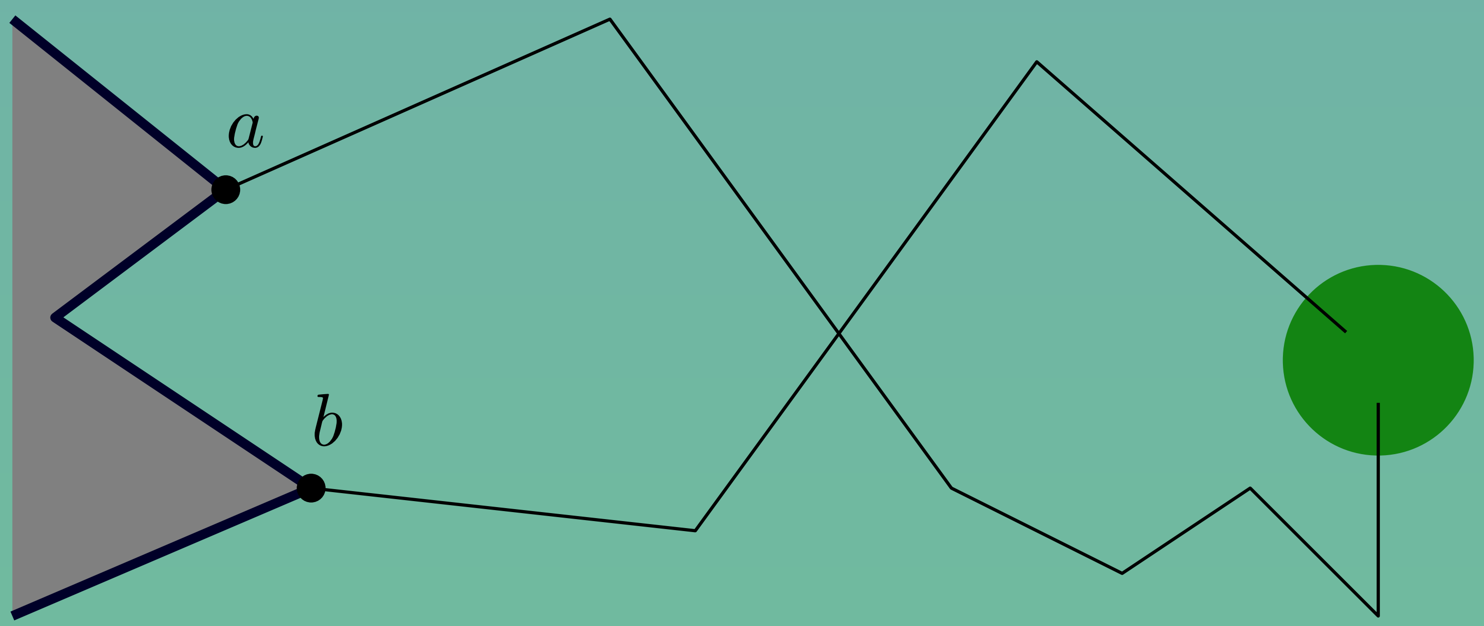
Create a set of non-intersecting piecewise-linear curves from each vertex of the face to the kernel / infinity.



β-stars

Create a set of non-intersecting piecewise-linear curves from each vertex of the face to the kernel / infinity.

1. remove avoidable intersections



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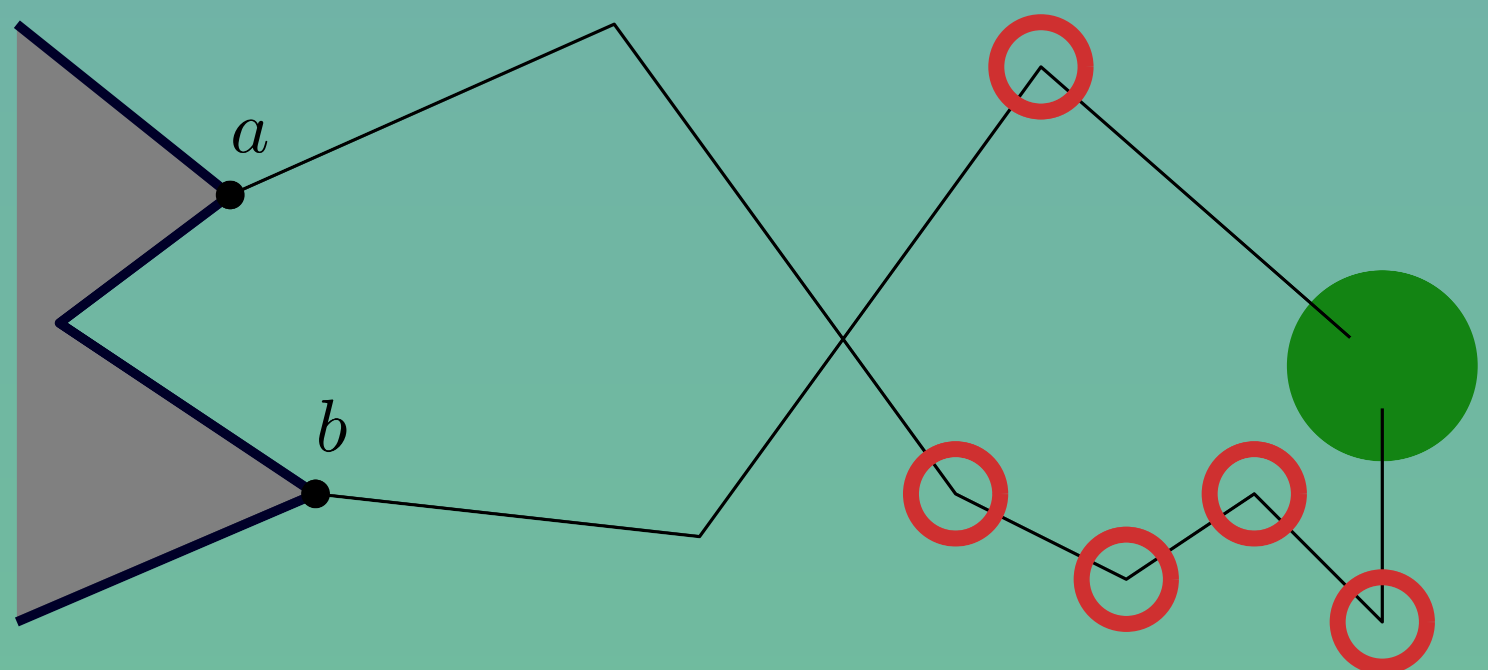

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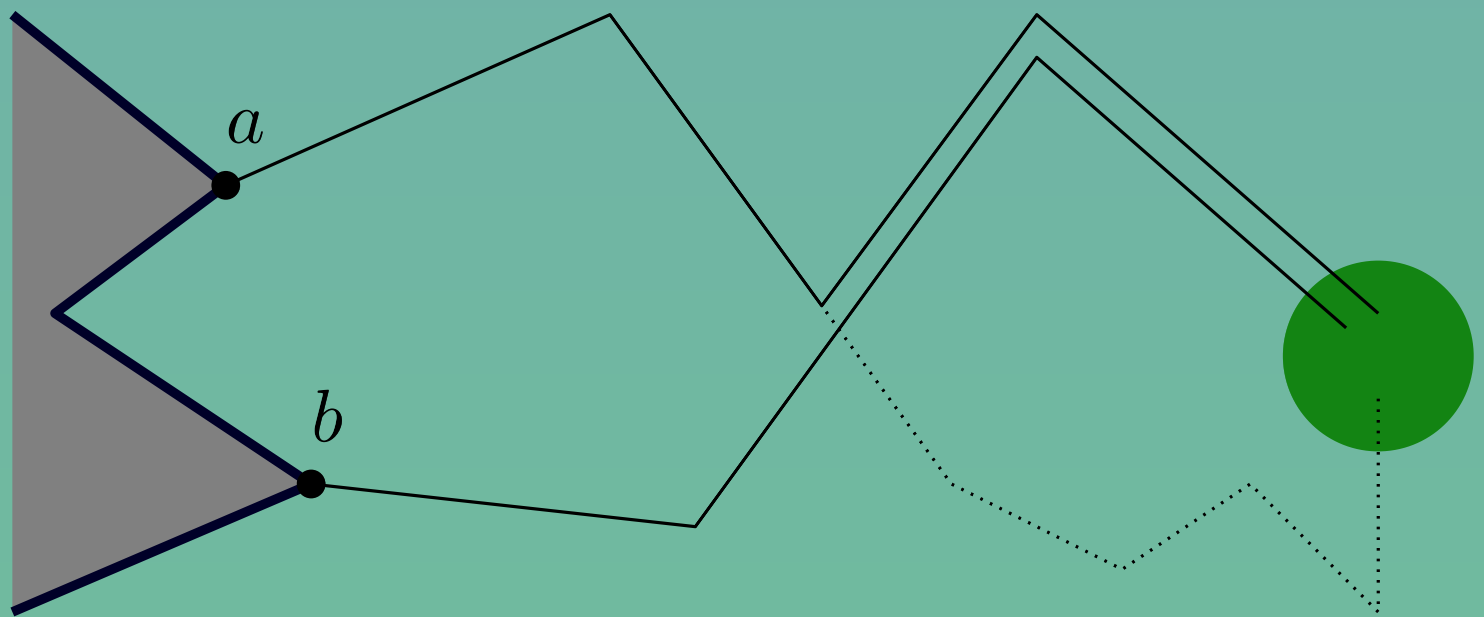
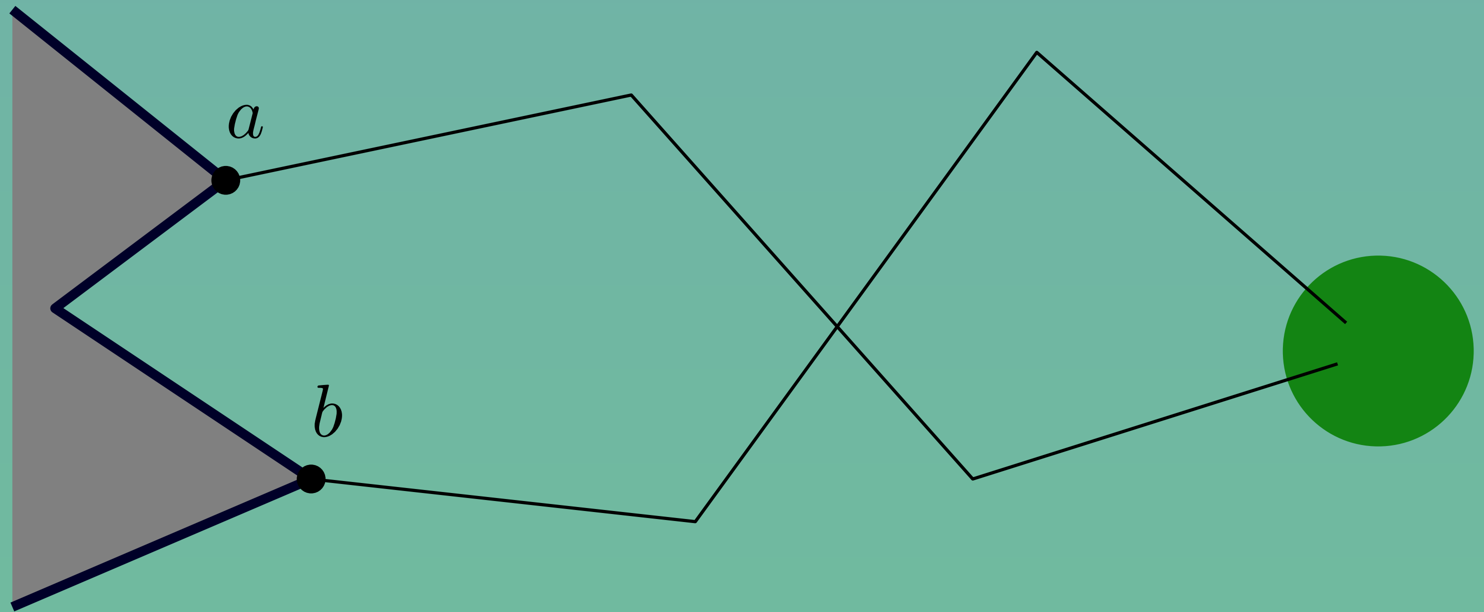


β-stars

Create a set of non-intersecting piecewise-linear curves from each vertex of the face to the kernel / infinity.

1. remove avoidable intersections
2. find two curves that intersect each other first



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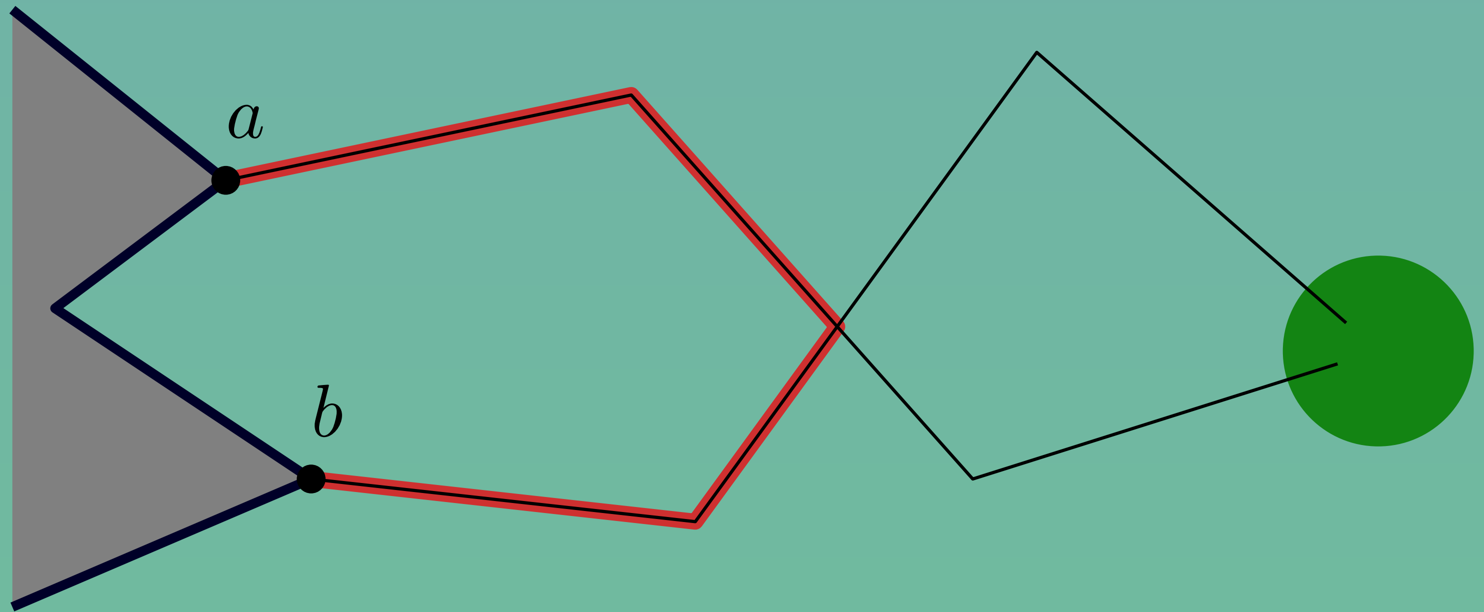

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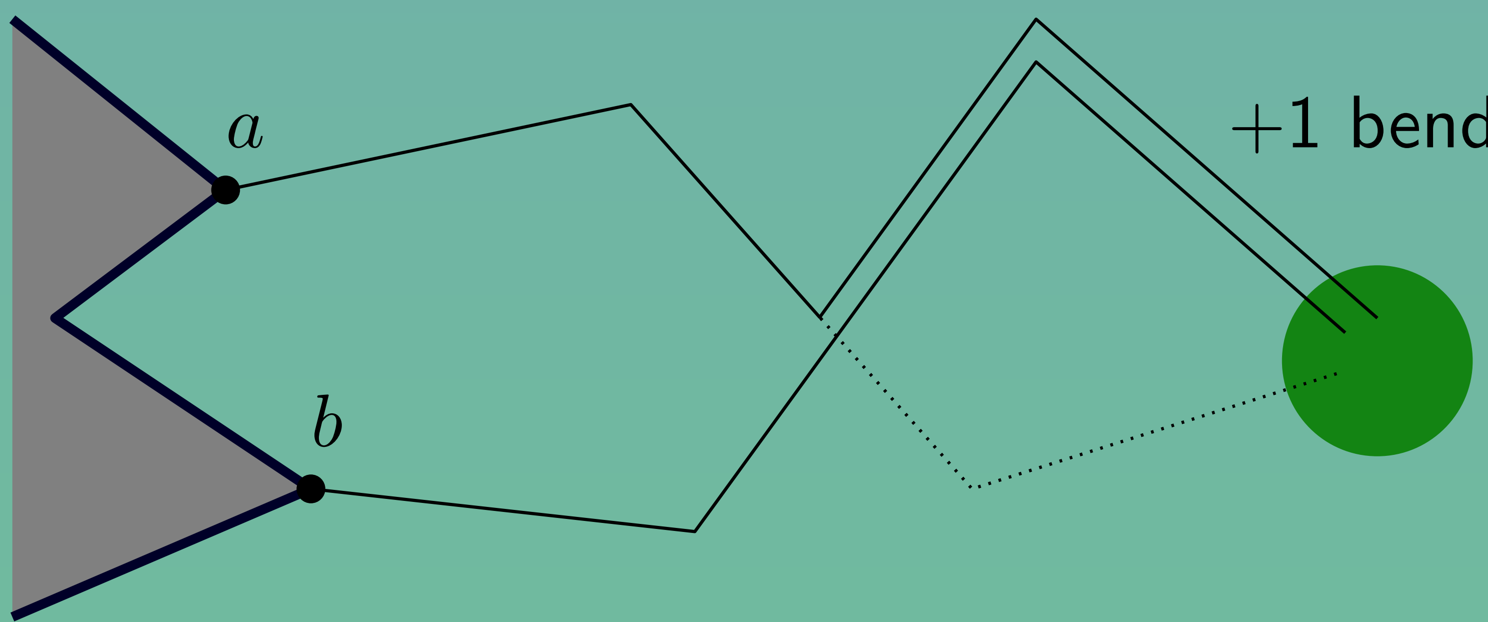
Create a set of non-intersecting piecewise-linear curves from each vertex of the face to the kernel / infinity.

1. remove avoidable intersections
2. find two curves that intersect each other first
3. reroute the curve with less bends



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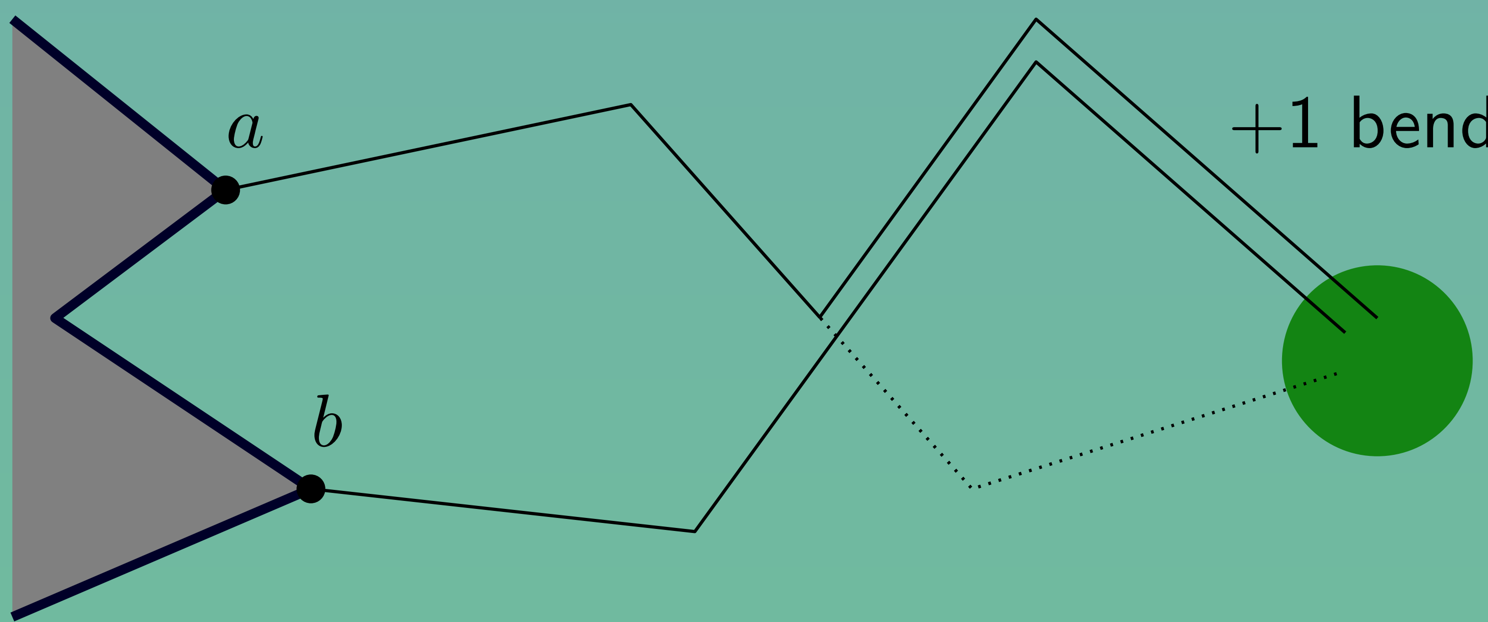
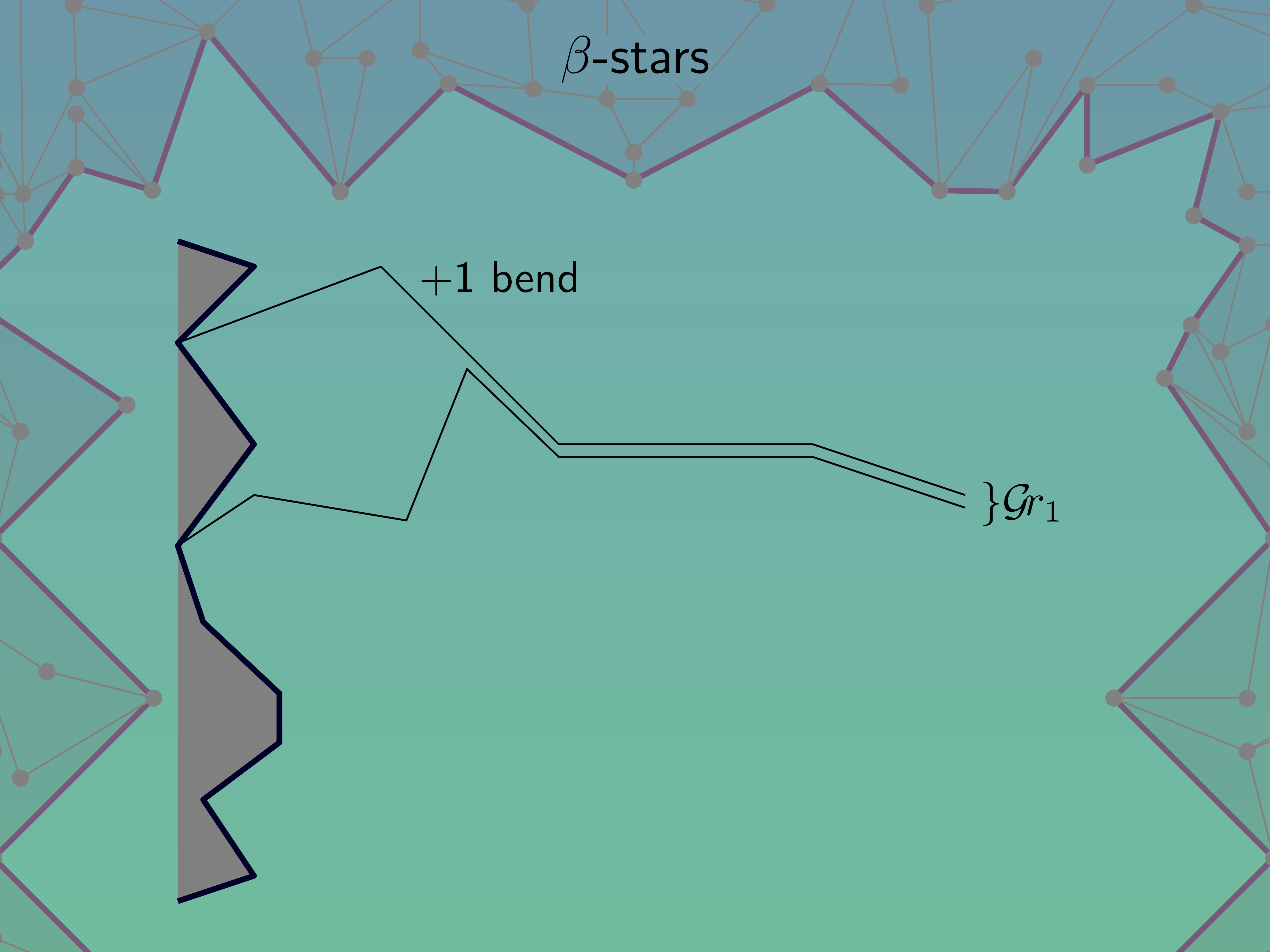
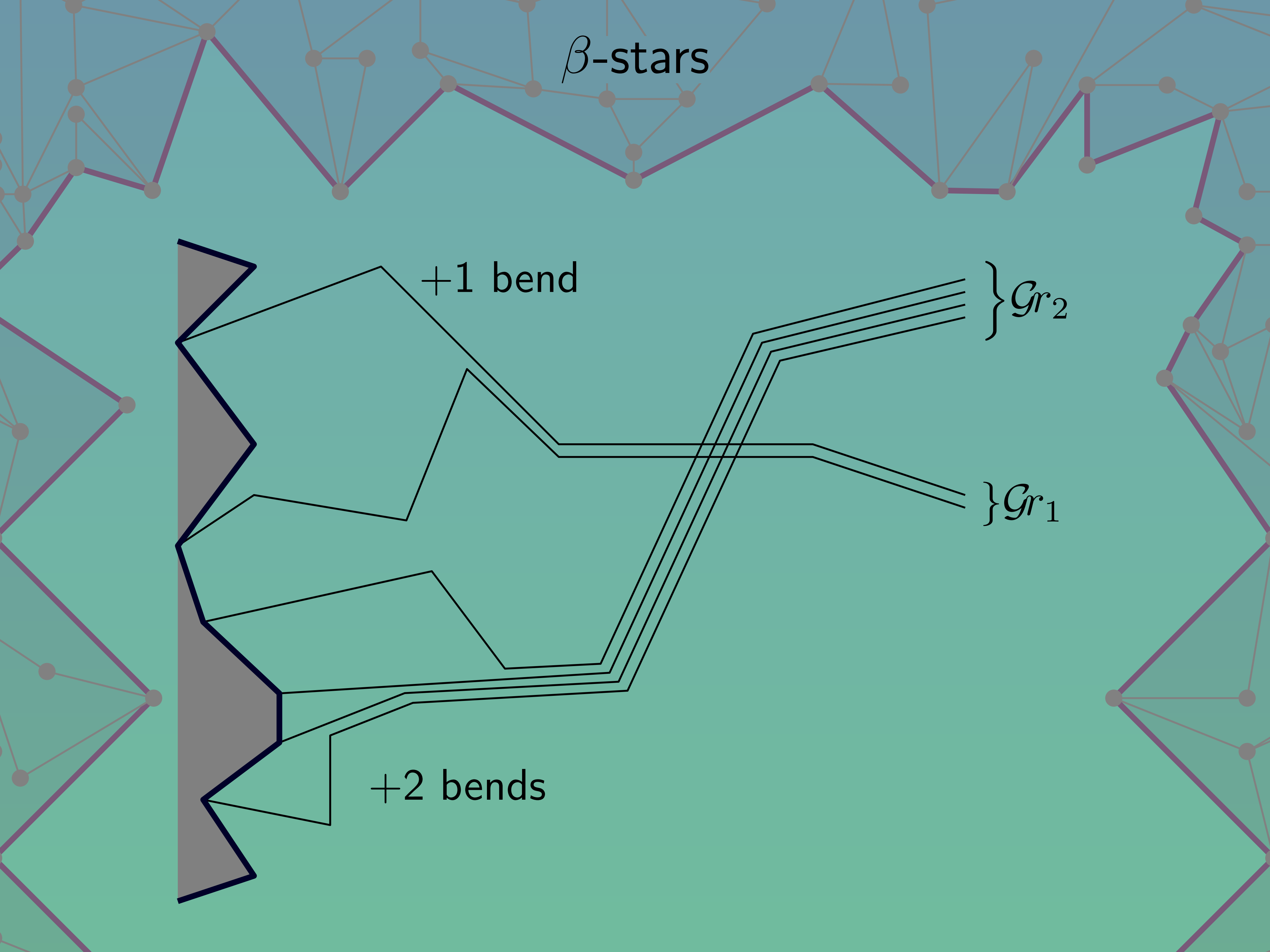
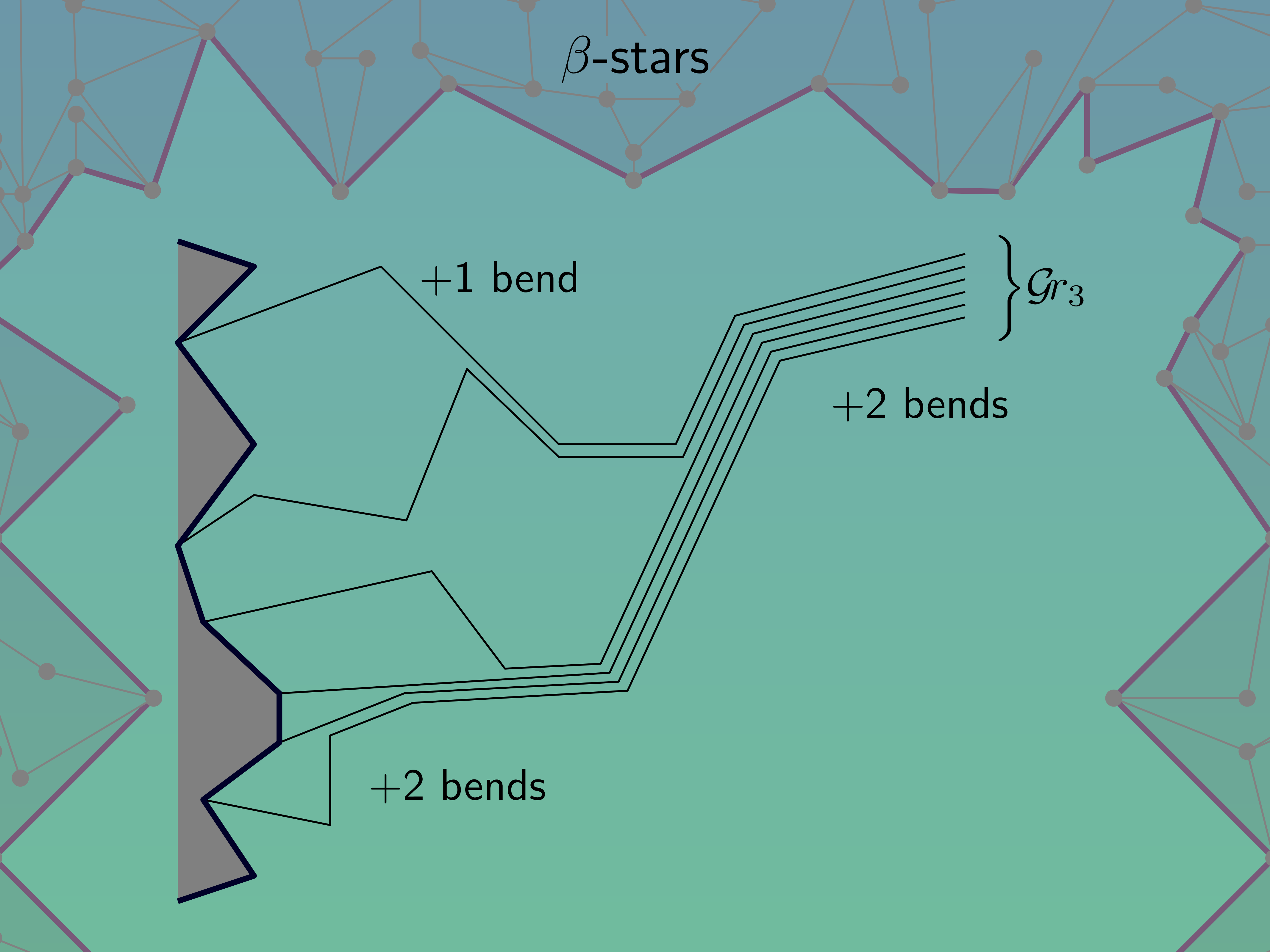


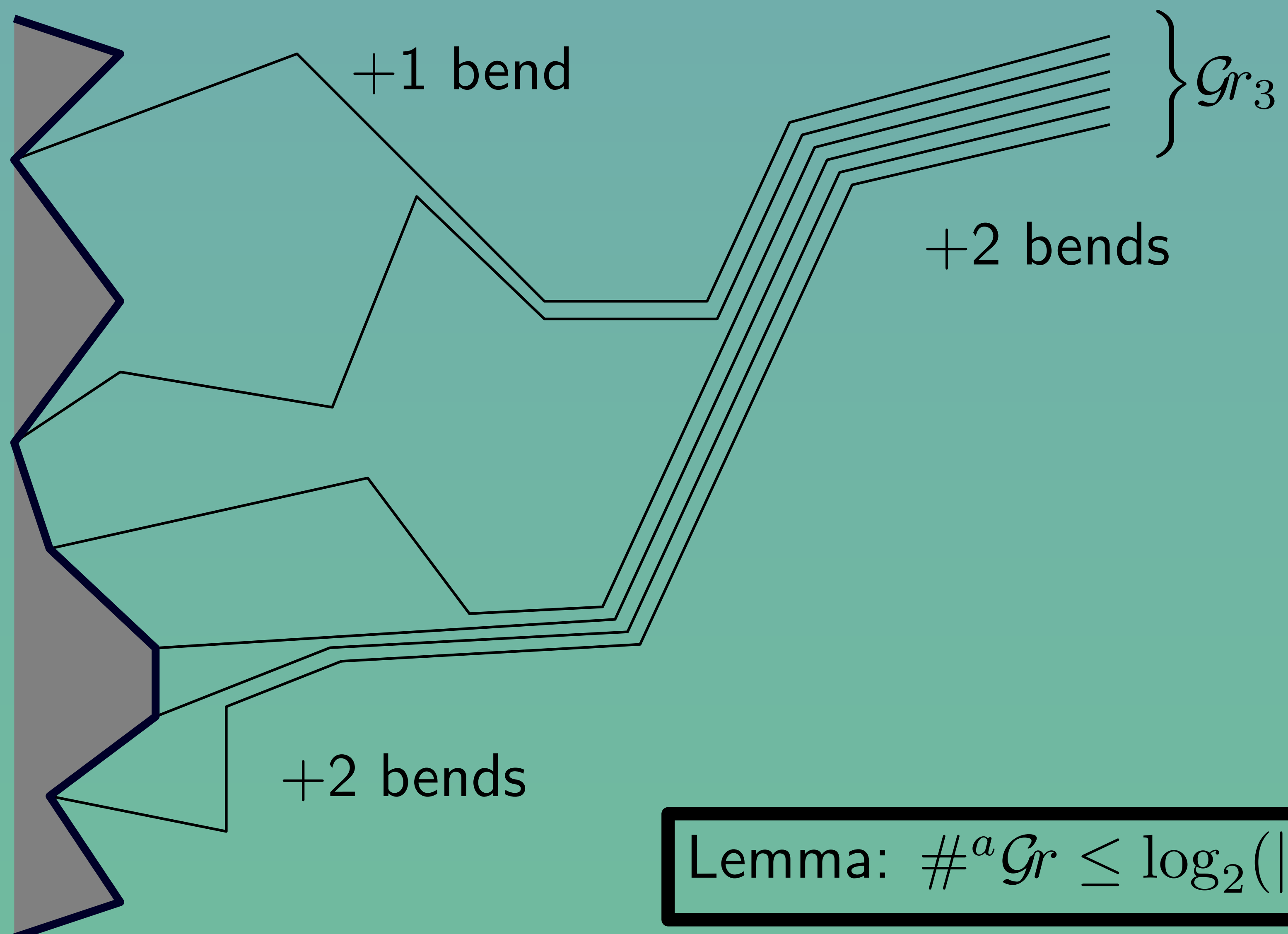

β-stars
+1 bend
$\}\mathcal{G}r_1$


β-stars
+1 bend
$\}\mathcal{G}r_2$
$\}\mathcal{G}r_1$
+2 bends



β-stars
+1 bend
$\mathcal{G}r_3$
+2 bends
+2 bends

β-stars

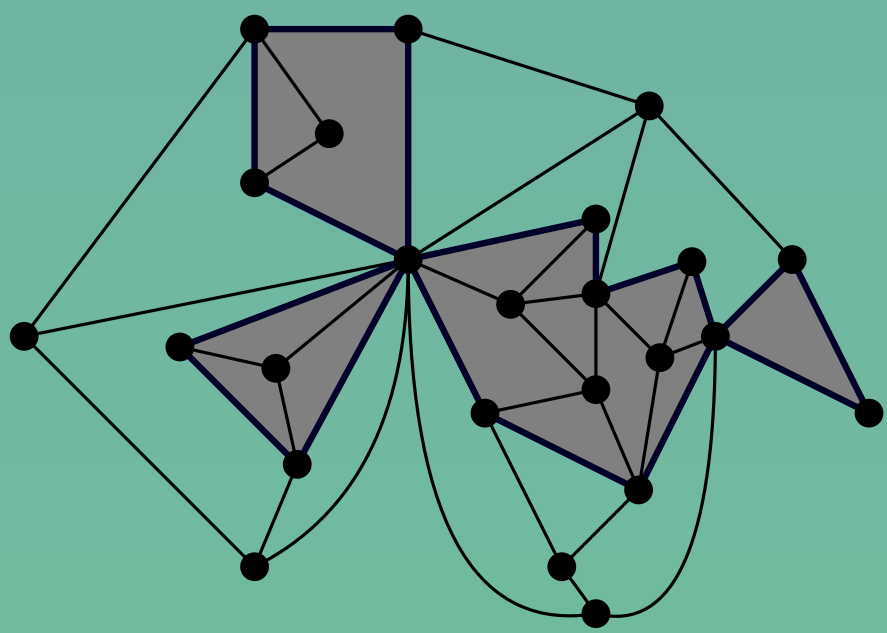


Lemma: $\#^a \mathcal{G}r \leq \log_2(|\mathcal{G}r|)$

β-stars

Theorem

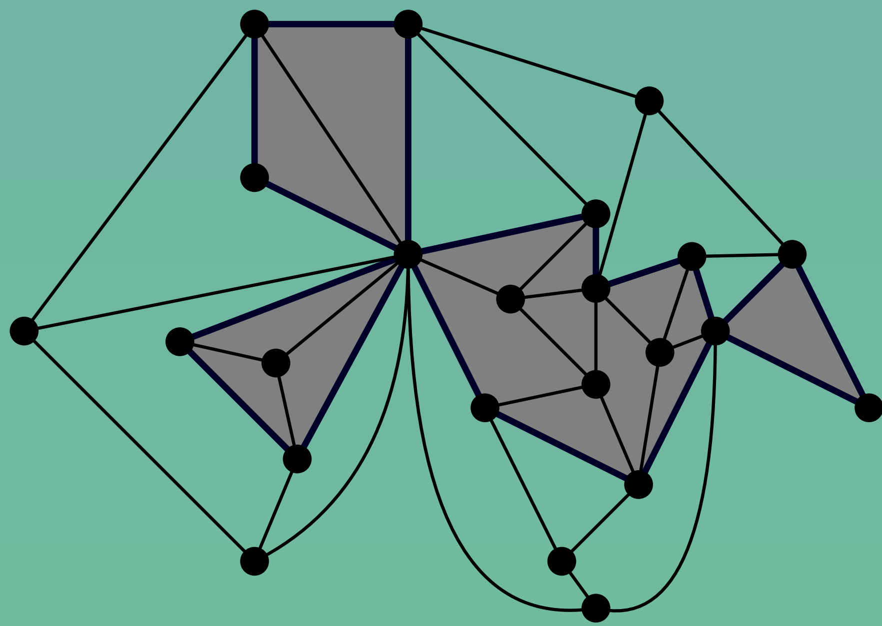
Each instance (G, Γ_H) where H is an induced connected subgraph of G allows a $\min\{|H|/2, \beta + \log_2(|H|) + 1\}$-bend-extension, where β is the maximum (outer) star complexity of a face in Γ_H. This bound is tight up to an additive constant.



β-stars

Theorem

Each instance (G, Γ_H) where H is a connected subgraph of G, allows a $\min\{|H| + 1, 2\beta + 2\log_2(|H|) + 3\}$-bend-extension, where β is the maximum star complexity of a face in Γ_H. This bound is tight up to an additive constant.



Conclusion

Given a plane graph $G = (V, E)$, a drawing of a subgraph H, extend the drawing using as few bends per edge as possible.

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	# of bends per edge
$H = (V, \emptyset)$	$3\|V\| + 2$ $^{[PW01]}$
-	$72\|H\|$ $^{[CFGLMS15]}$
H induced and connected	$\min\{\|H\|/2, \beta + \log_2(\|H\|) + 1\}$