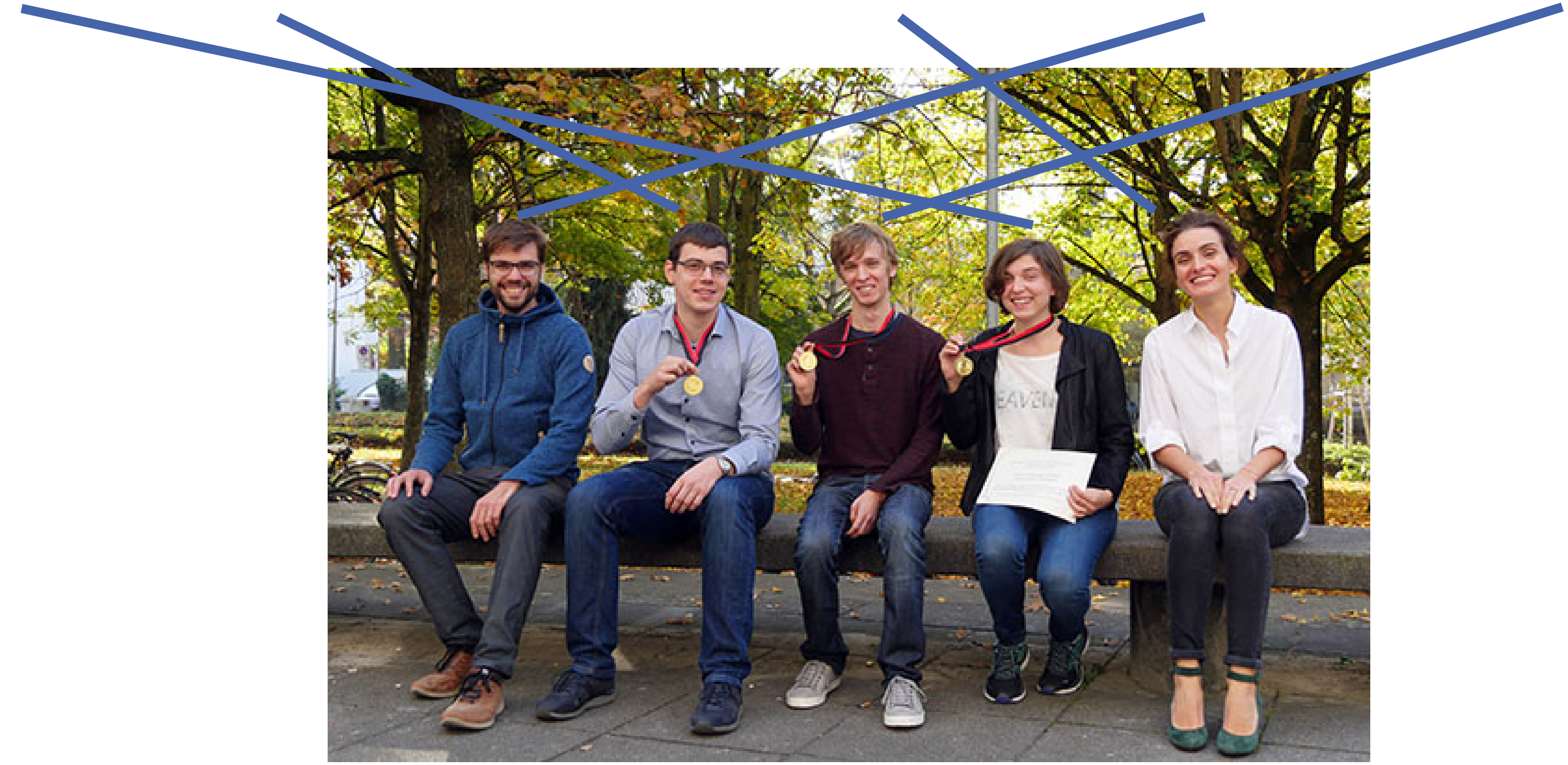
A Greedy Heuristic for Crossing-Angle Maximization

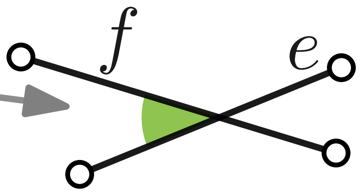
Graph Drawing 2018

Almut Demel
Dominik Dürrschnabel
Tamara Mchedlidze
Marcel Radermacher
Lasse Wulff



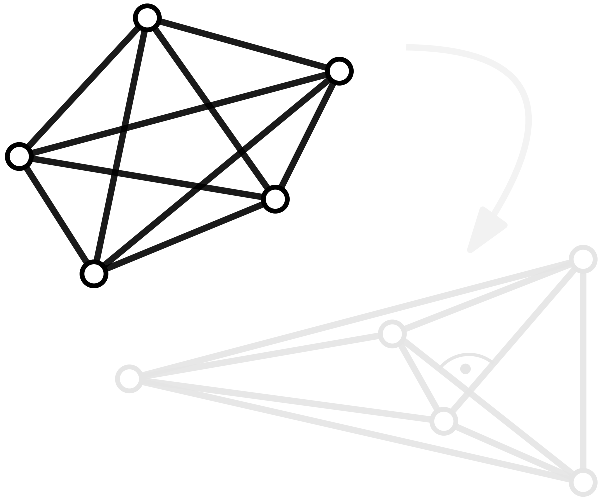
Crossing-Angle Maximization

crossing angle $\mathrm{cr}\text{-}\angle(e, f)$



crossing angle $\mathrm{cr}\text{-}\angle(\Gamma)$ of a drawing:

smallest crossing angle of two crossing edges



Crossing-Angle Maximization

Compute a straight-line drawing Γ of G that maximizes $\mathrm{cr}\text{-}\angle(\Gamma)$

A Lot of Theory

NP-Hardness

1, 2, 3 - bends per edge

Counting edges of RAC graphs

…

[Argyriou et al., Arikushi et al, Didimo et al, Djumović et al., …]

Practice before GD'17 contest

2 force-directed algorithms

[Argyriou et al. '13, Huang et al.'10]

Crossing-Angle Maximization

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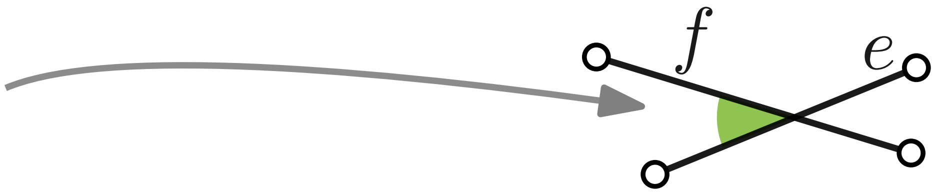

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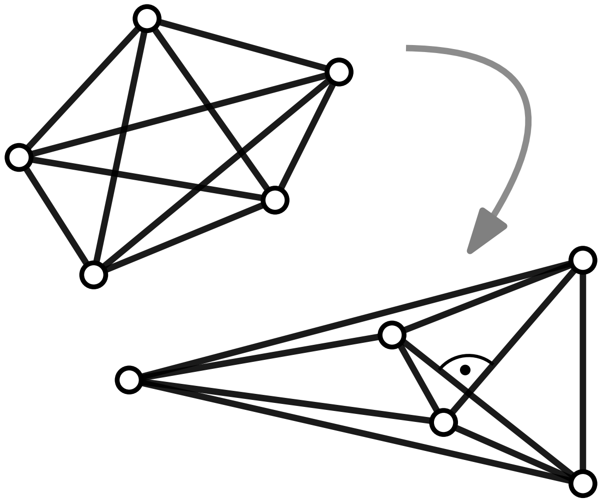

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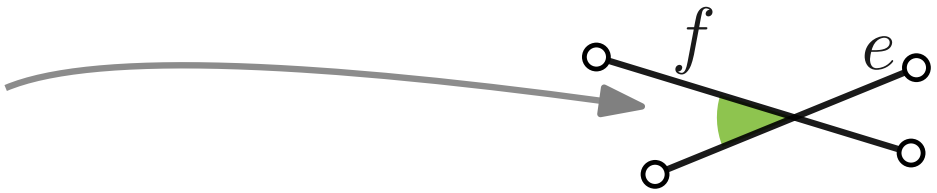

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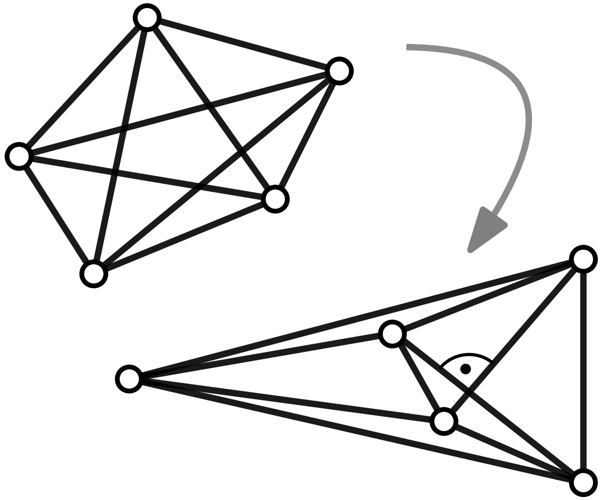

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Our Heuristic

Motivation: Win Graph Drawing Contest

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Challenge: No Restriction on Input

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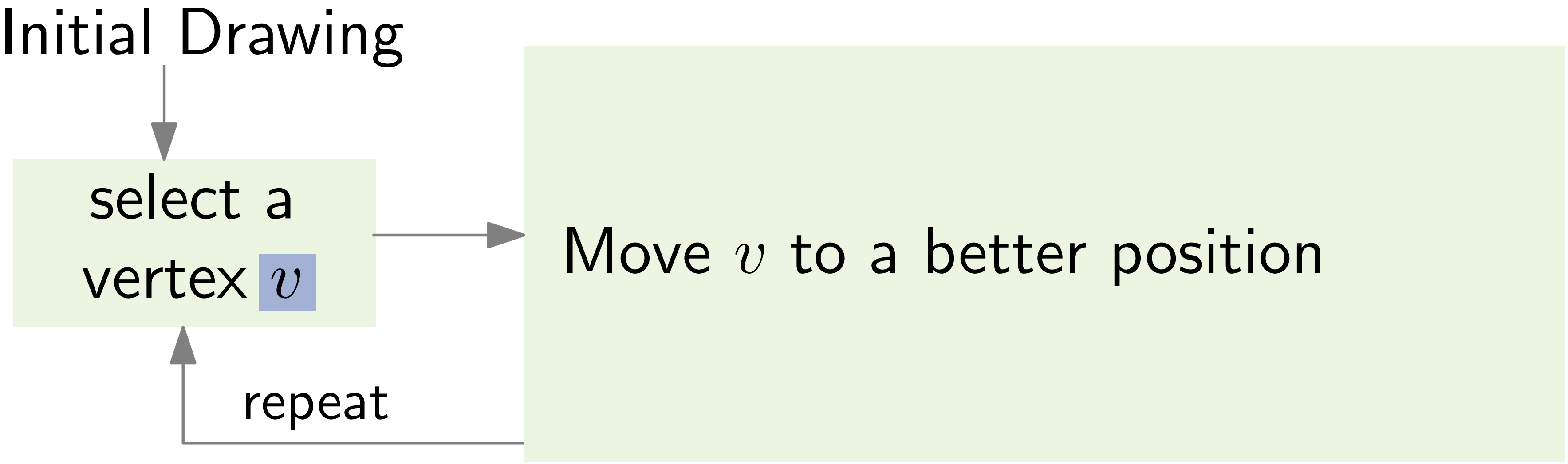
Our Heuristic

Design Goals

- Fast
- Generic
- Easy to implement

Our Heuristic

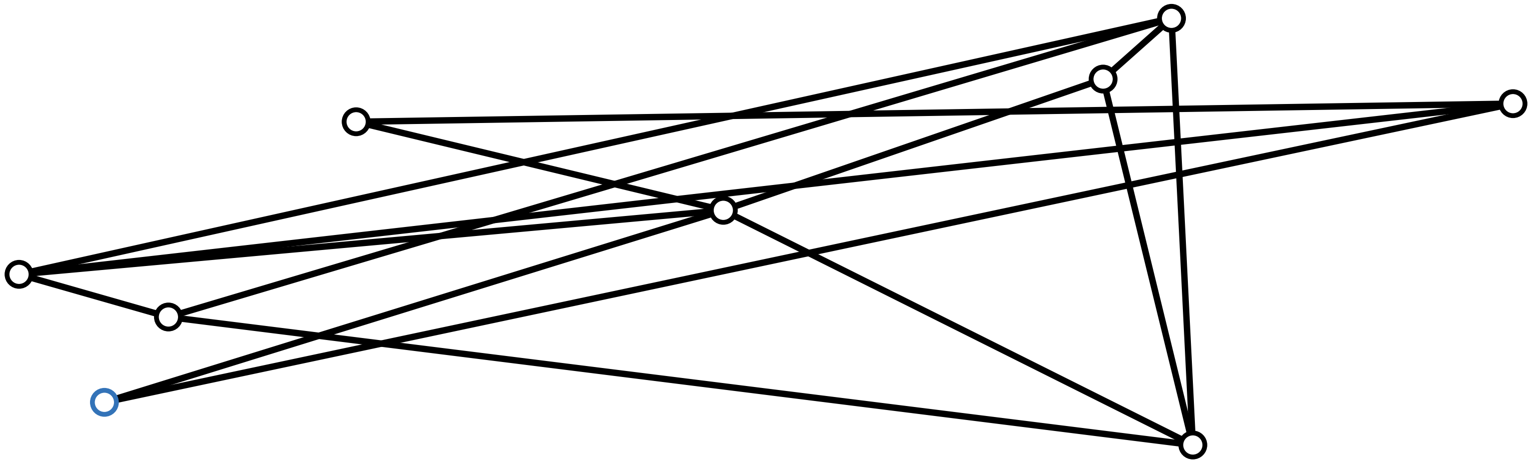


Initial Drawing

select a vertex v

Move v to a better position

repeat



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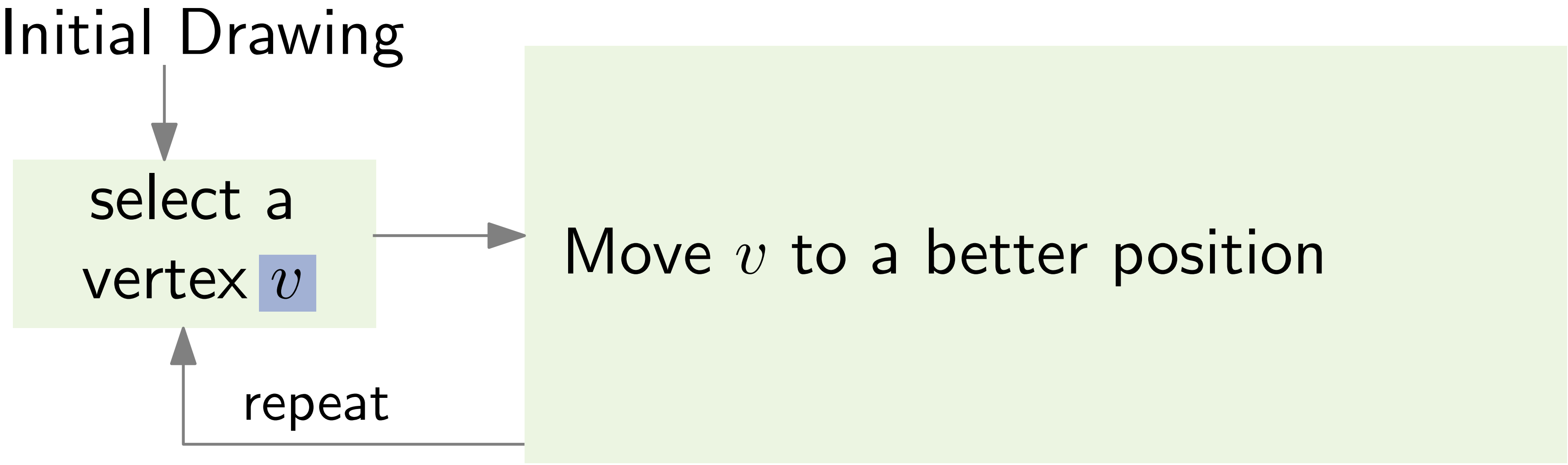

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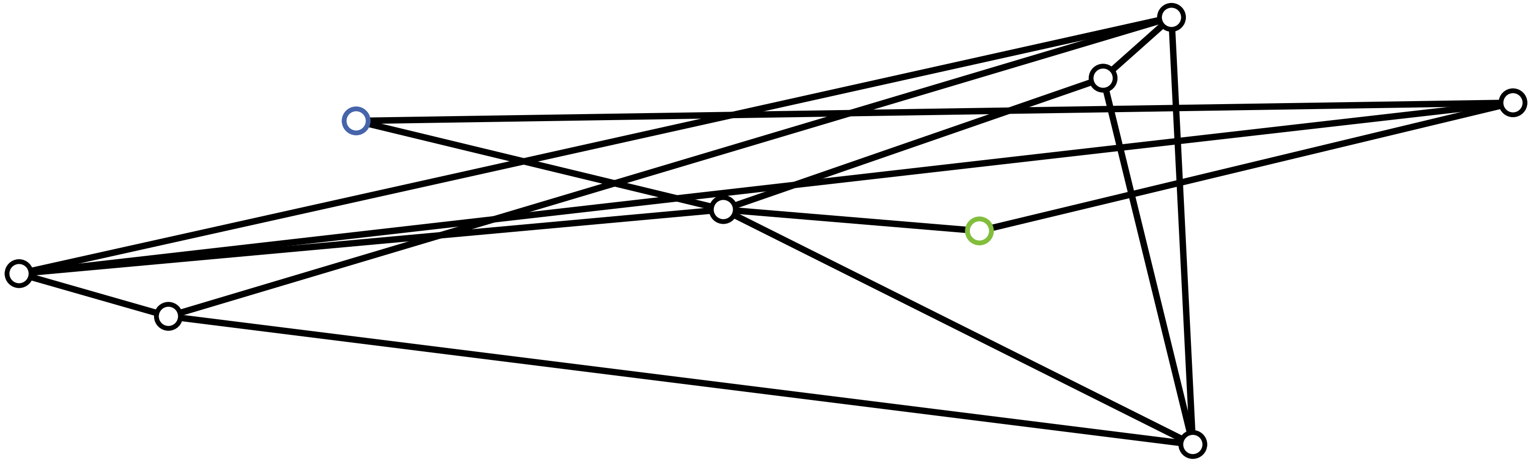

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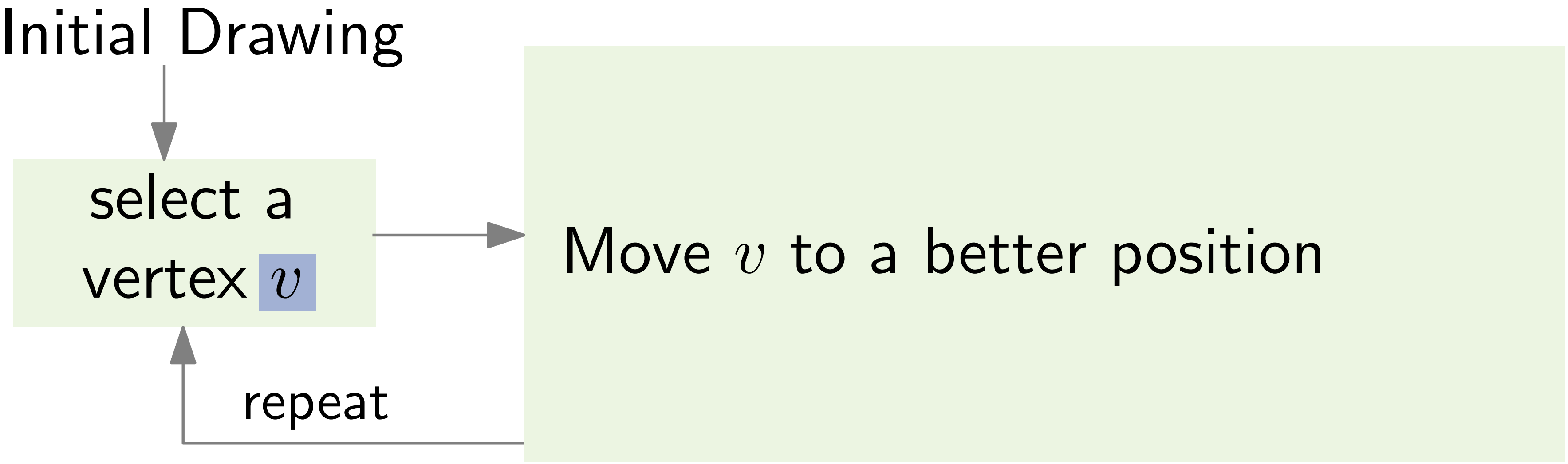

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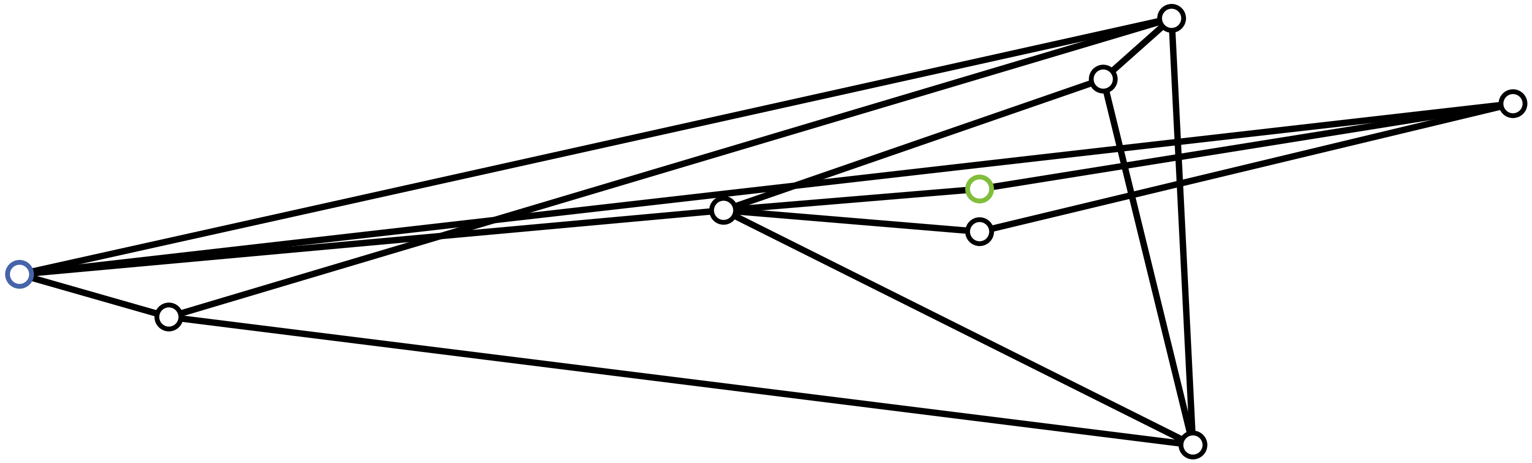

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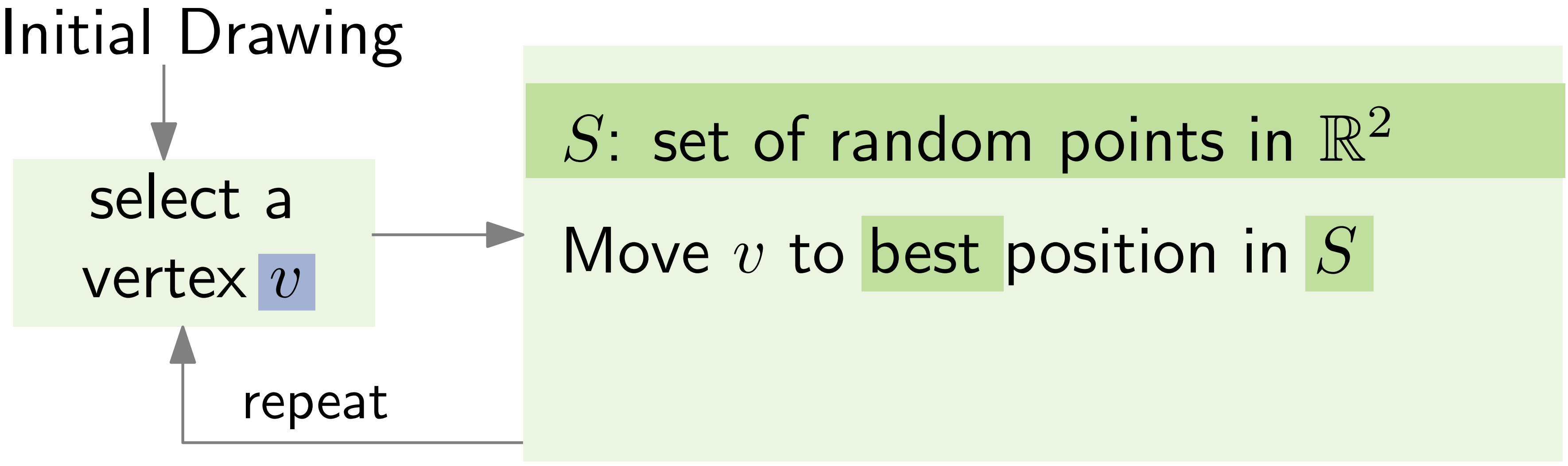

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S: set of random points in $\mathbb{R}^2$

Move v to best position in S

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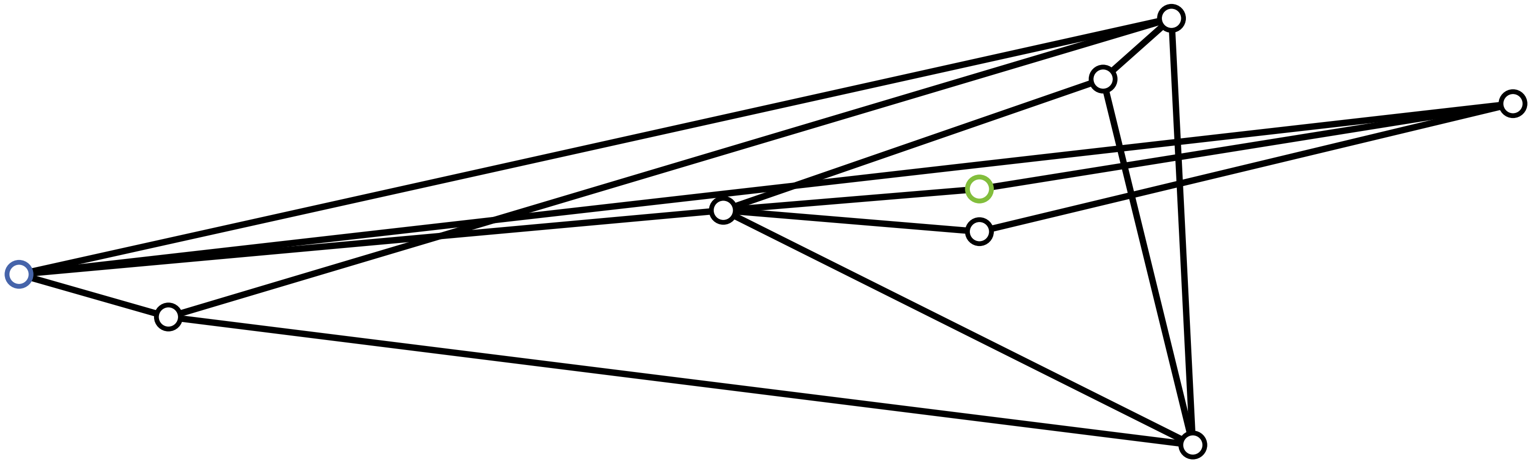

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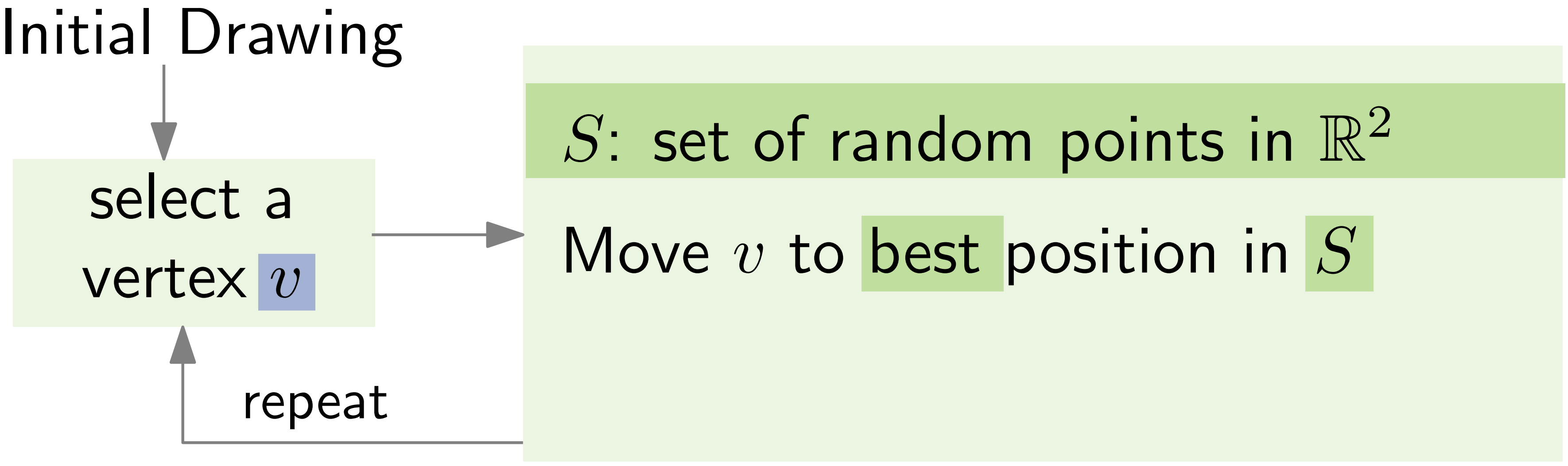

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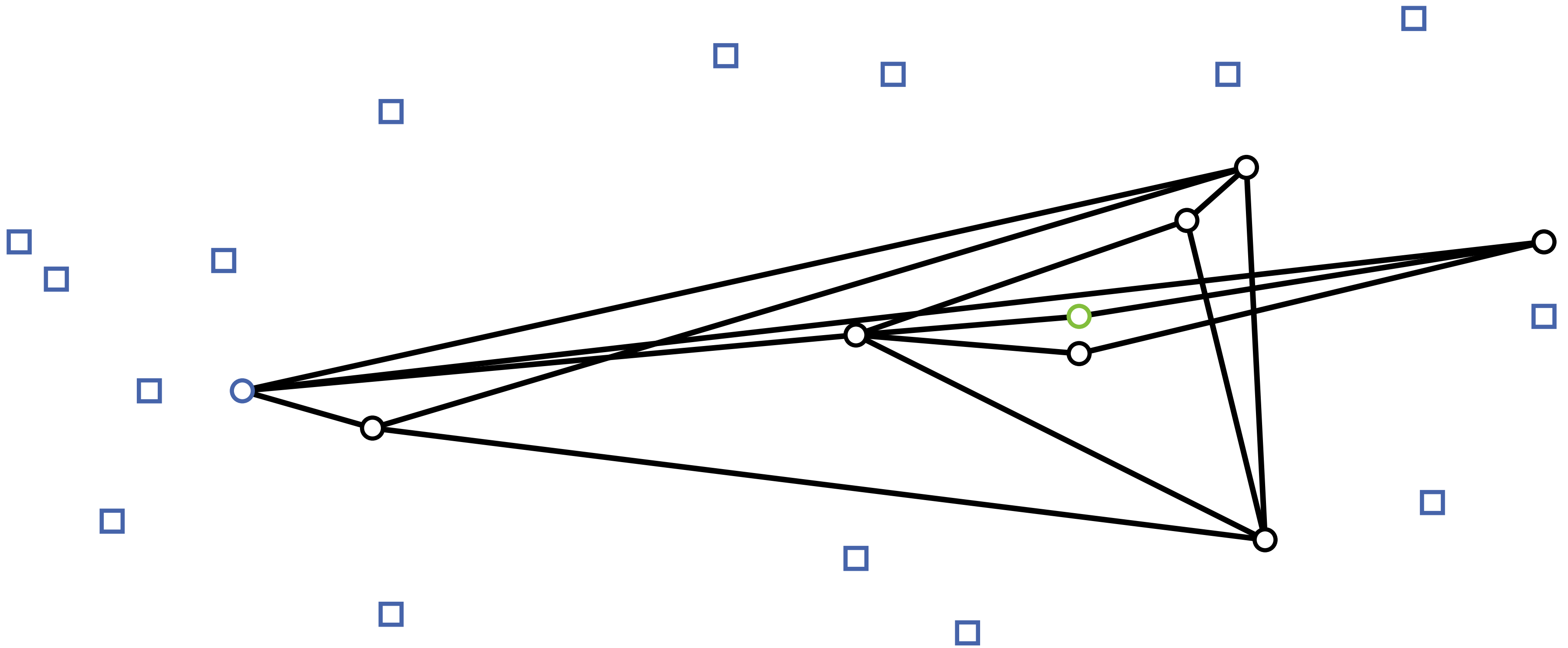

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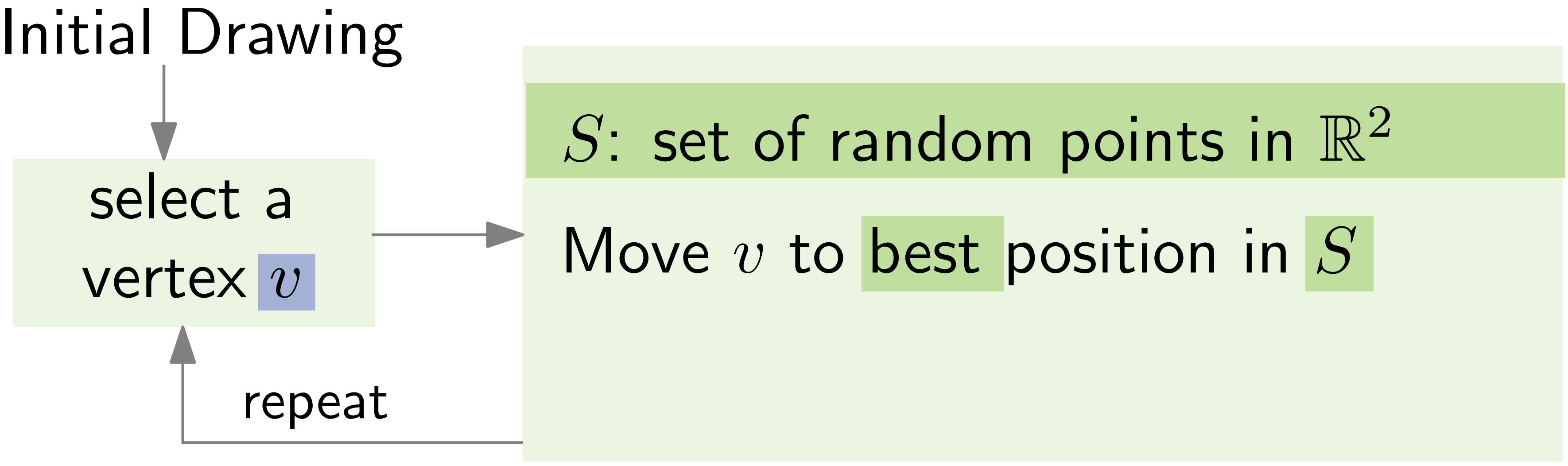

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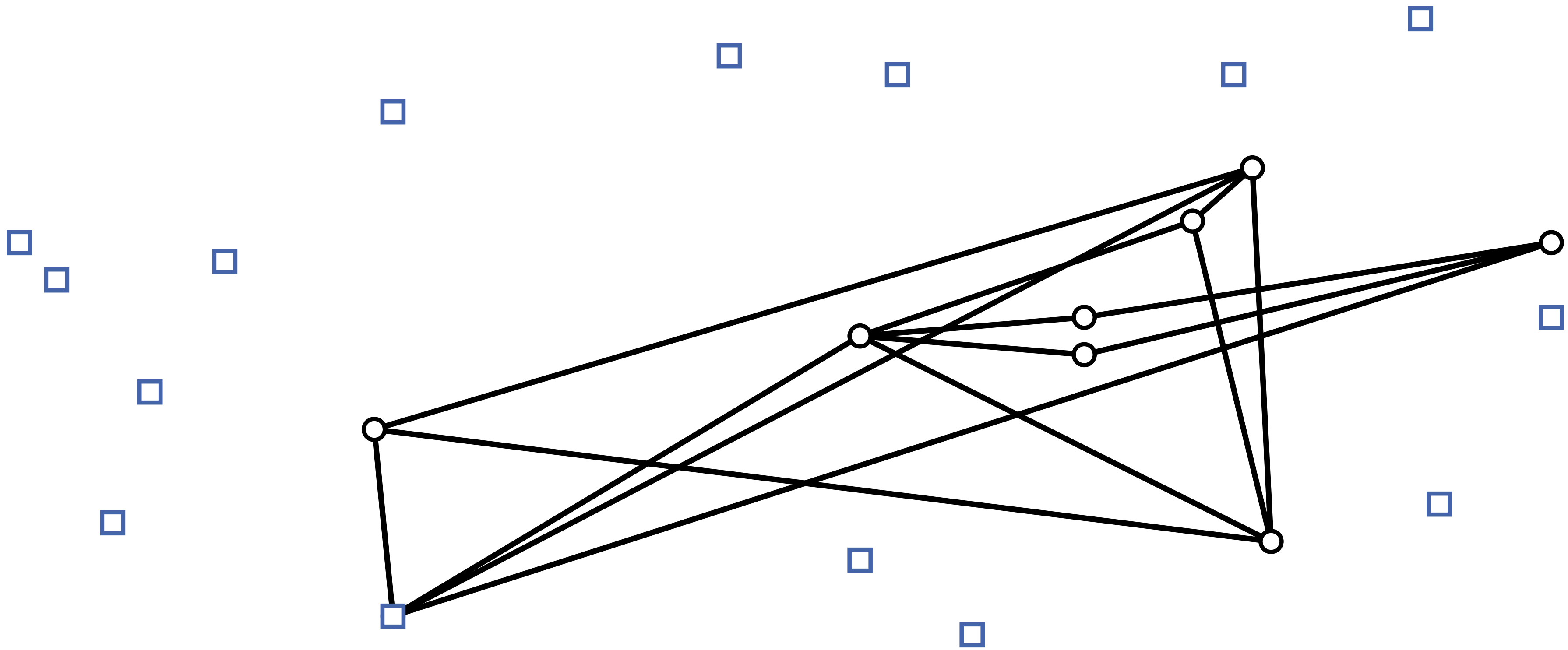

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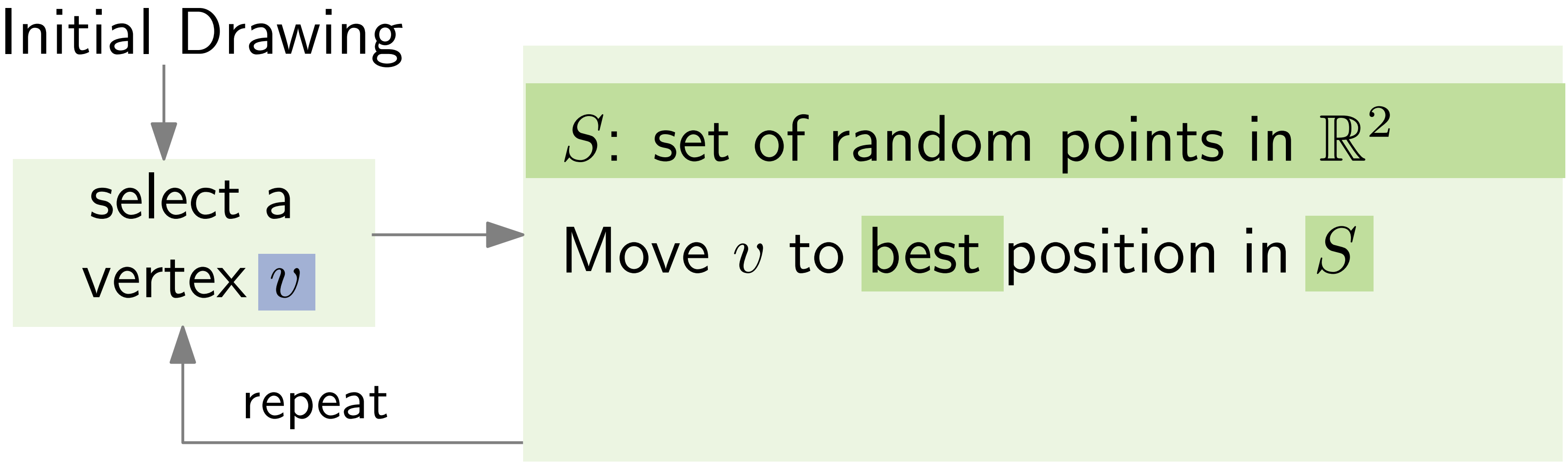

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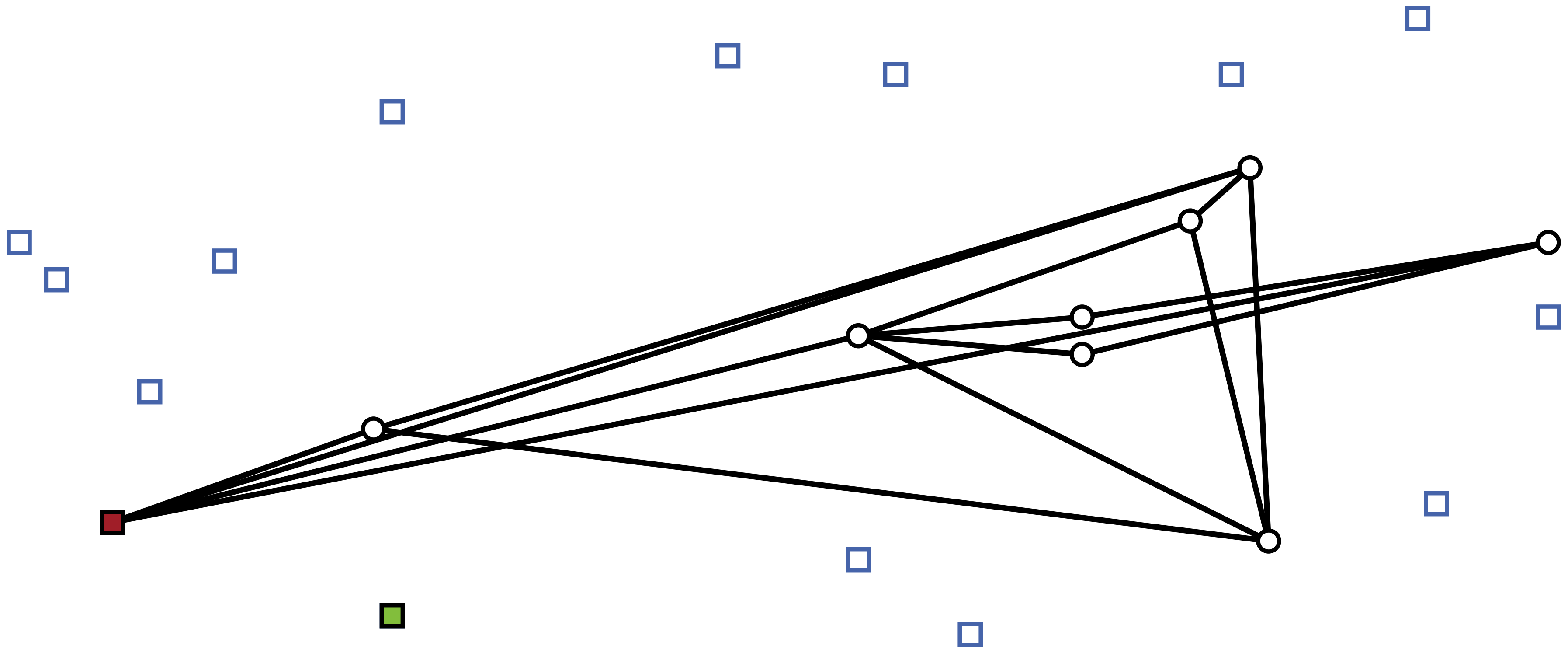

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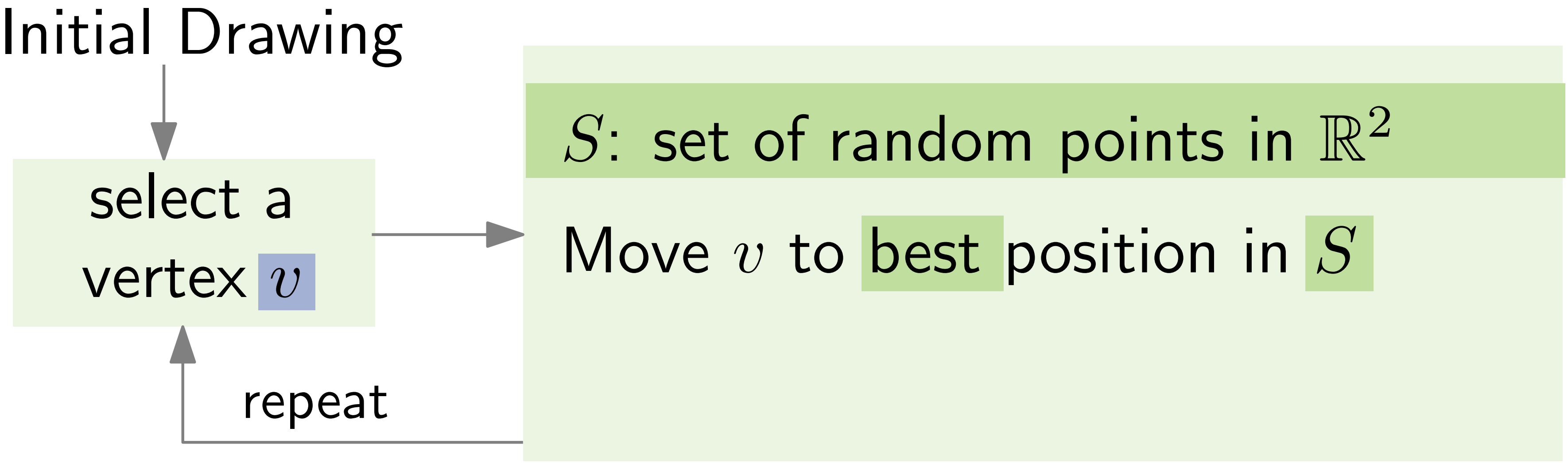

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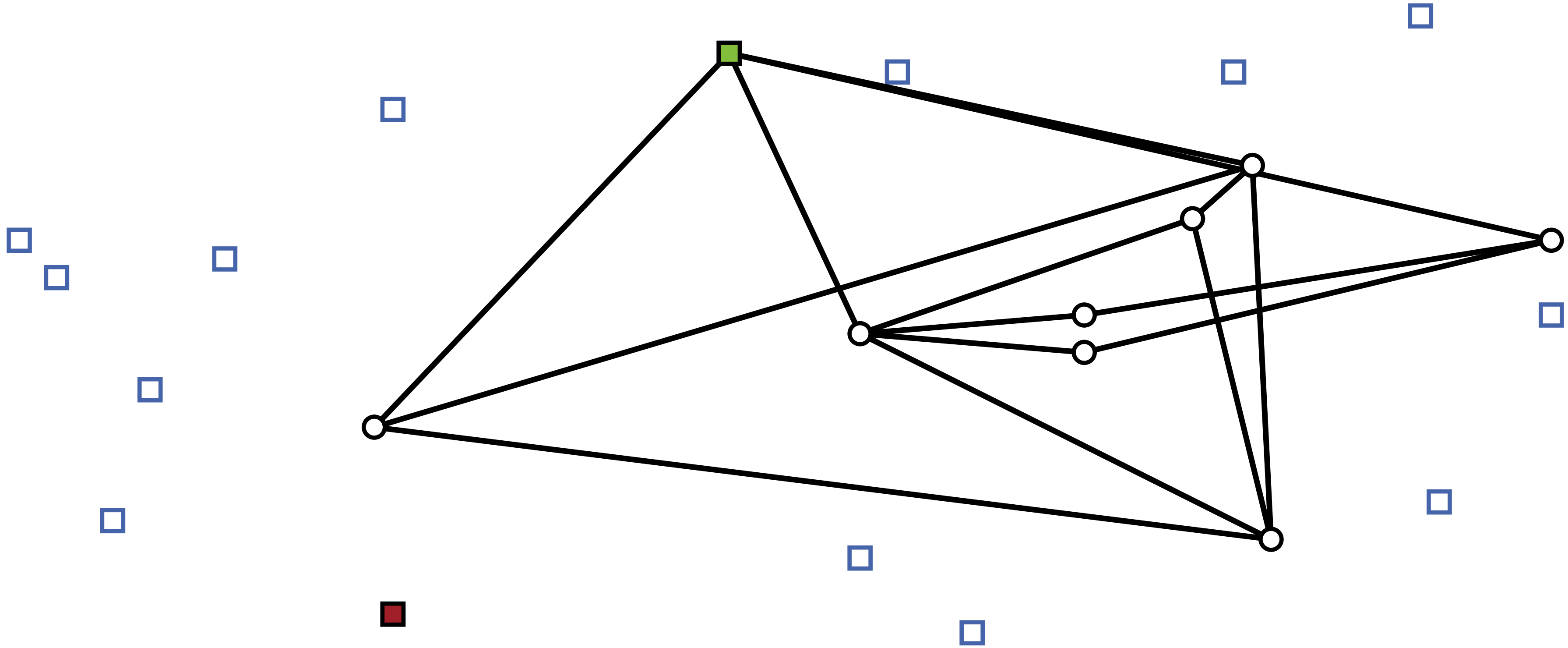

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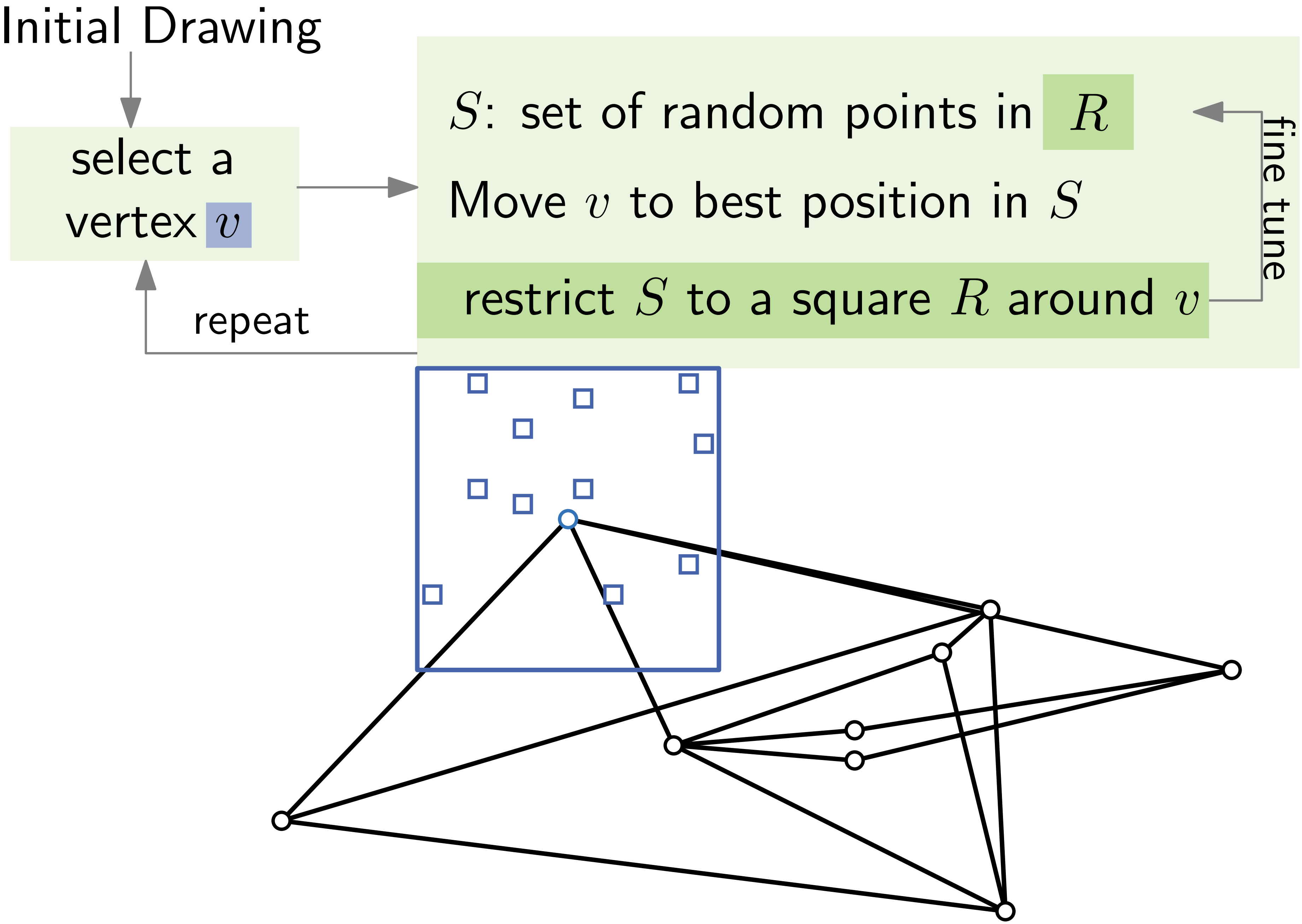


Initial Drawing

select a vertex v

S: set of random points in R

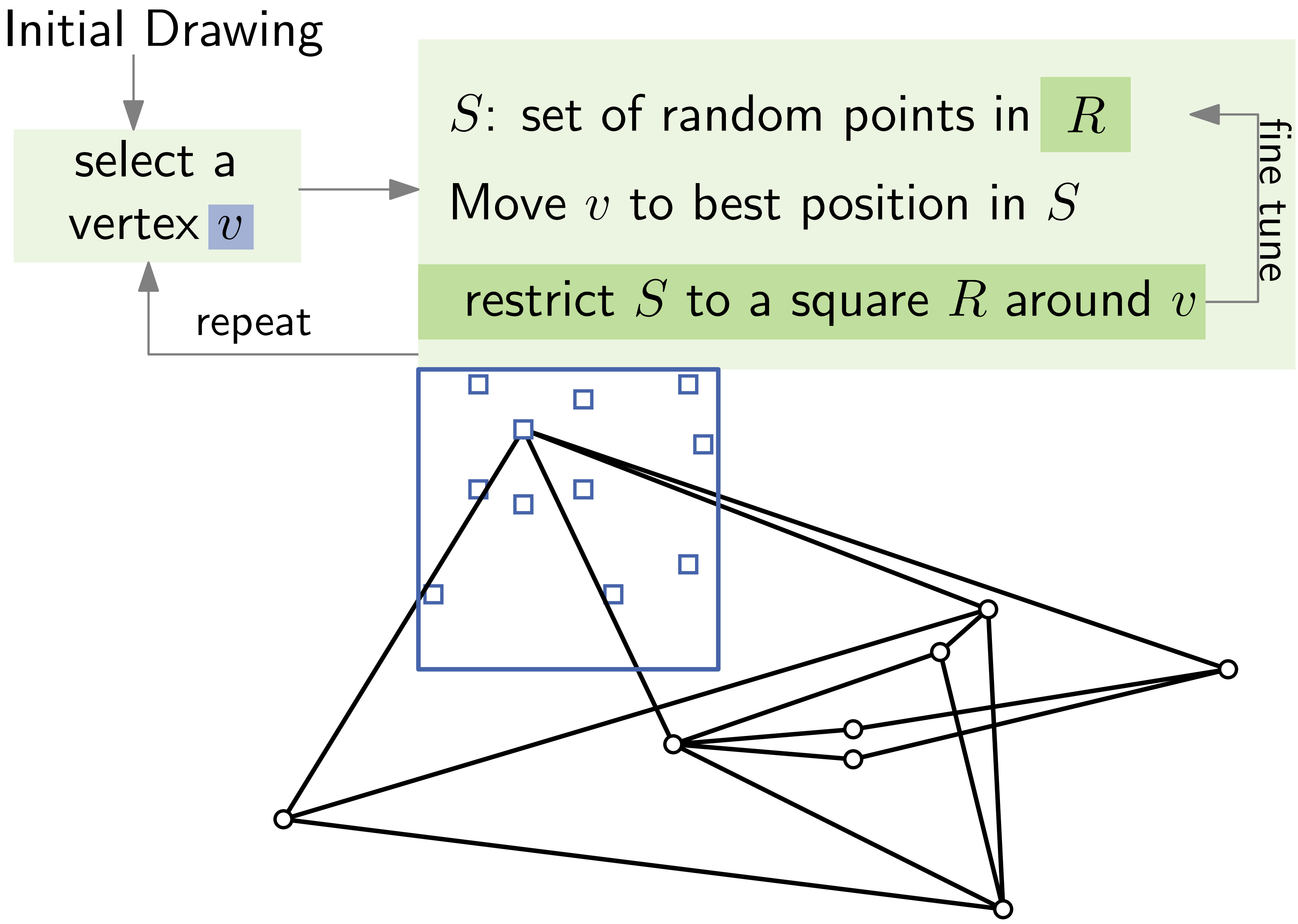
Move v to best position in S

restrict S to a square R around v

fine tune

repeat

Our Heuristic



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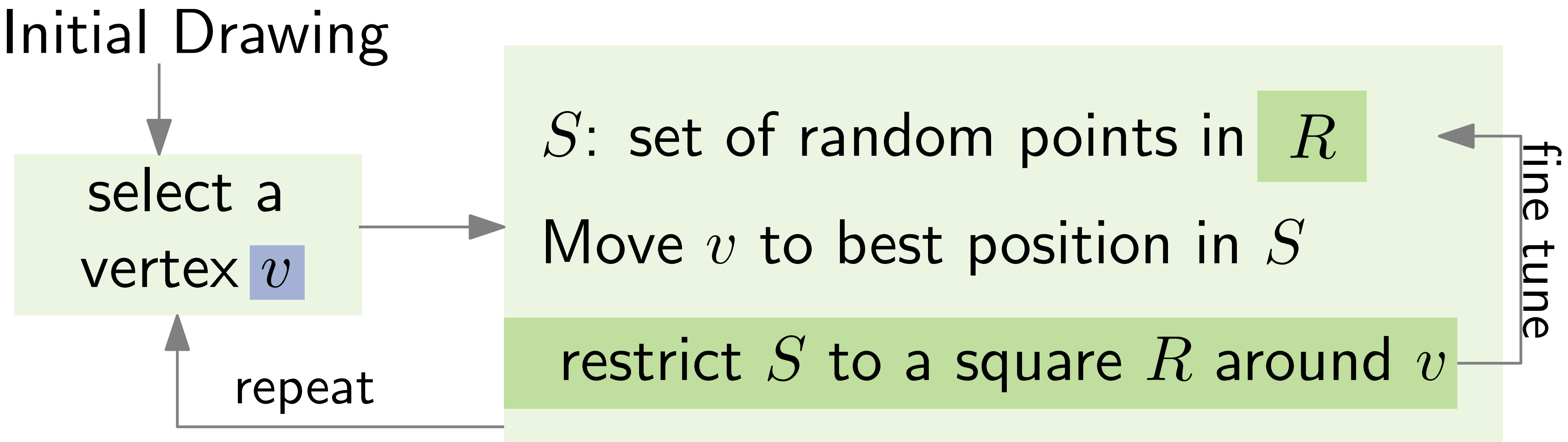

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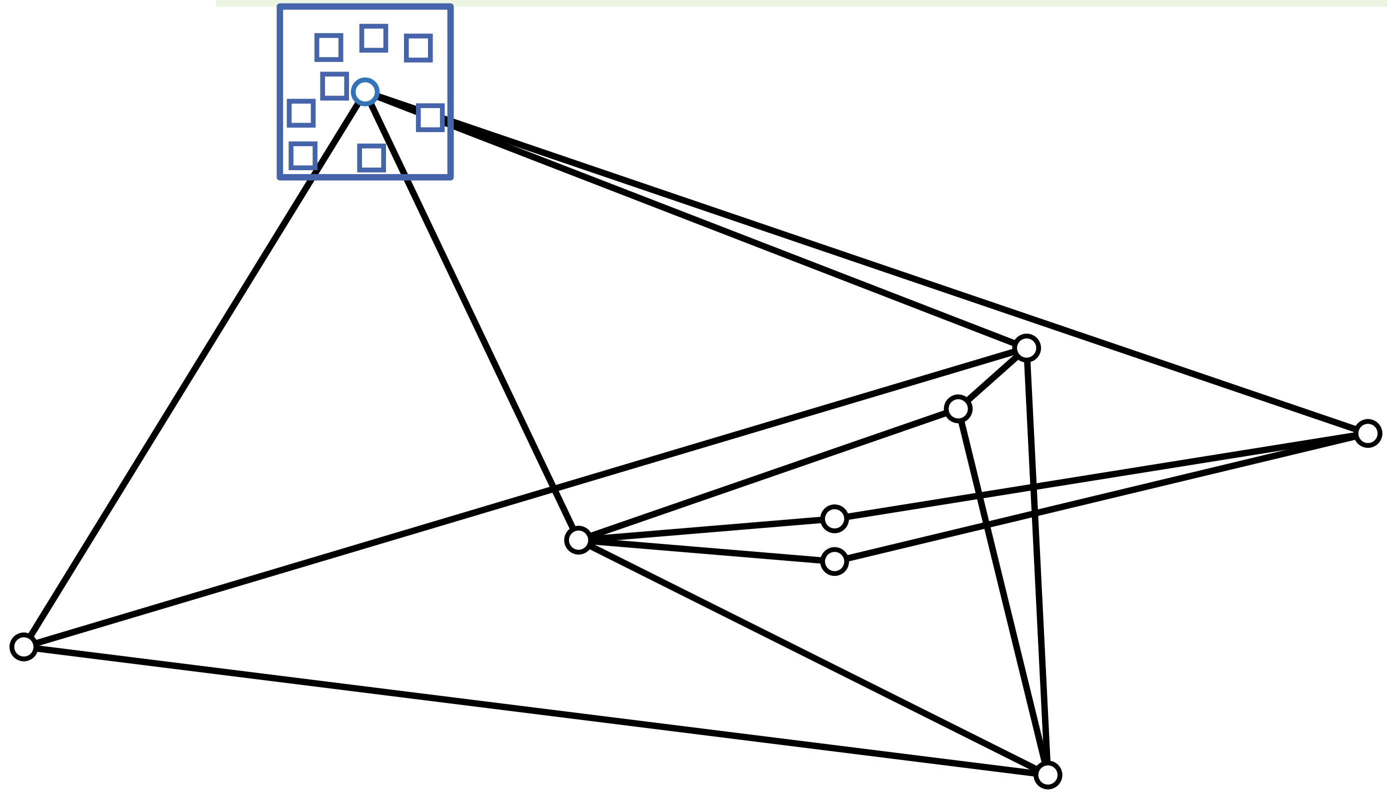

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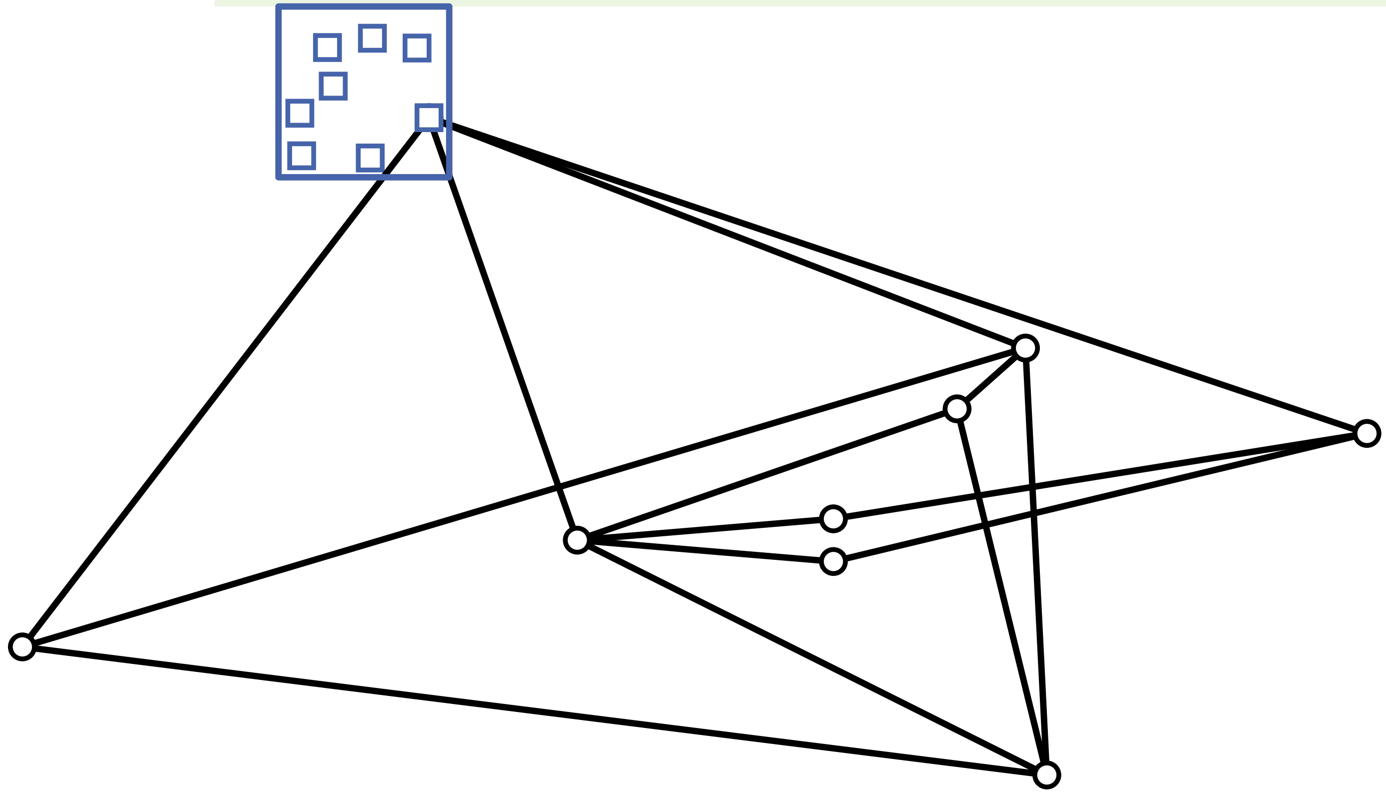

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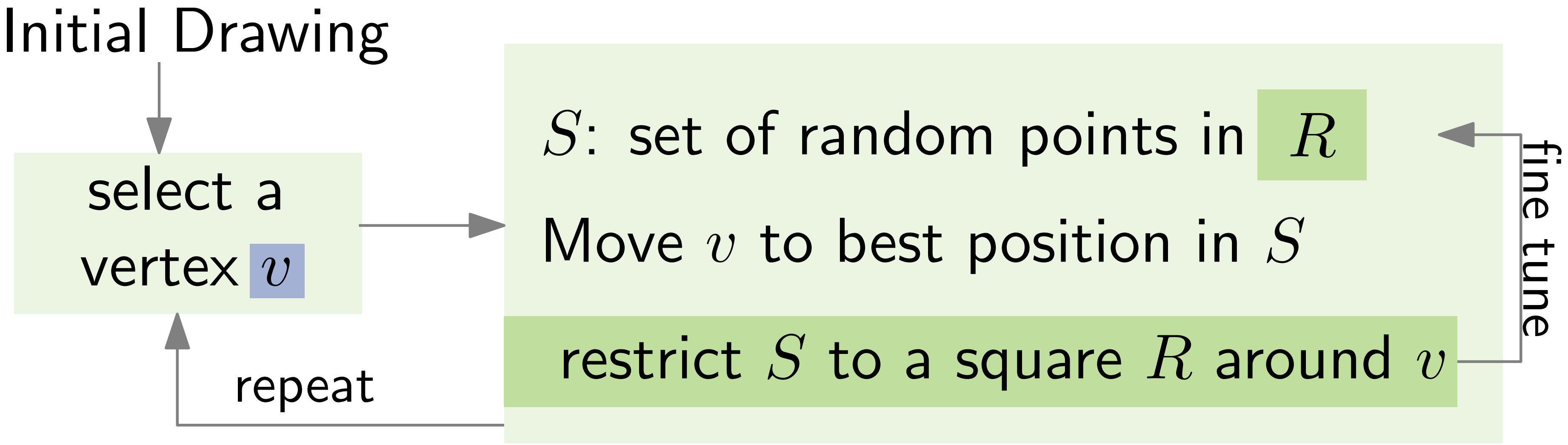

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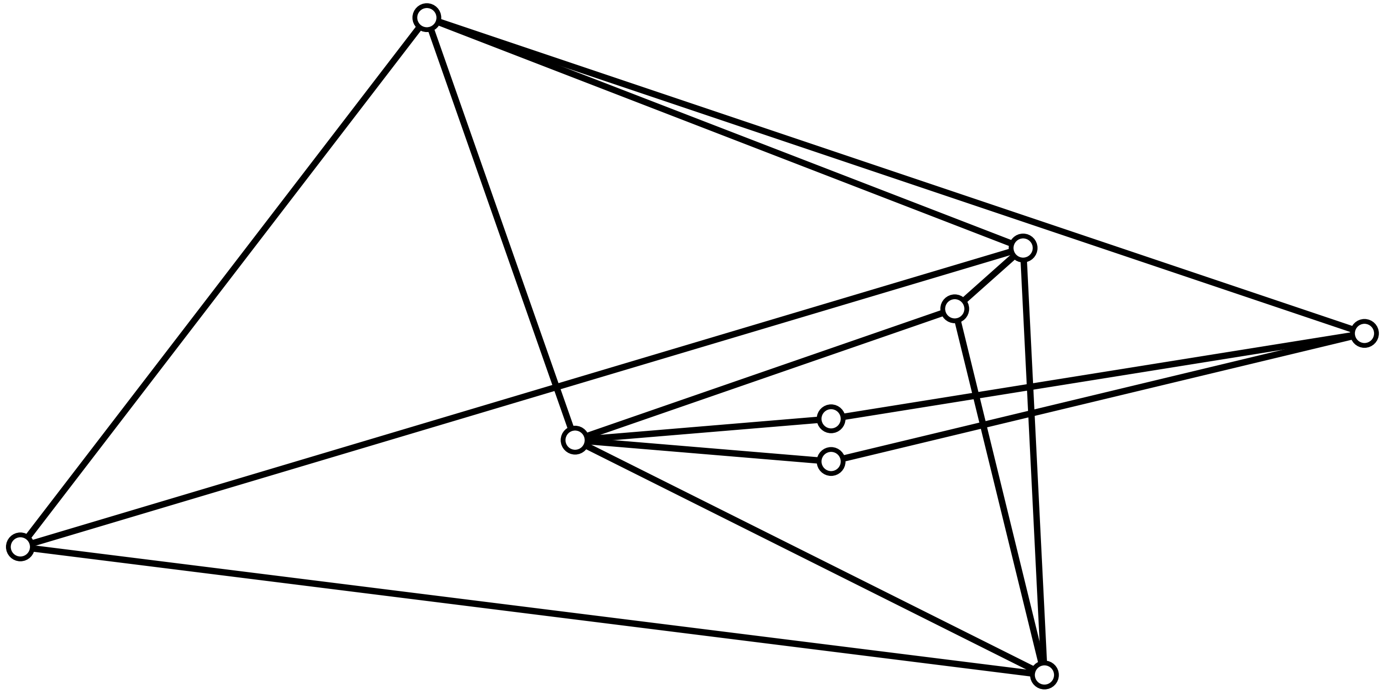
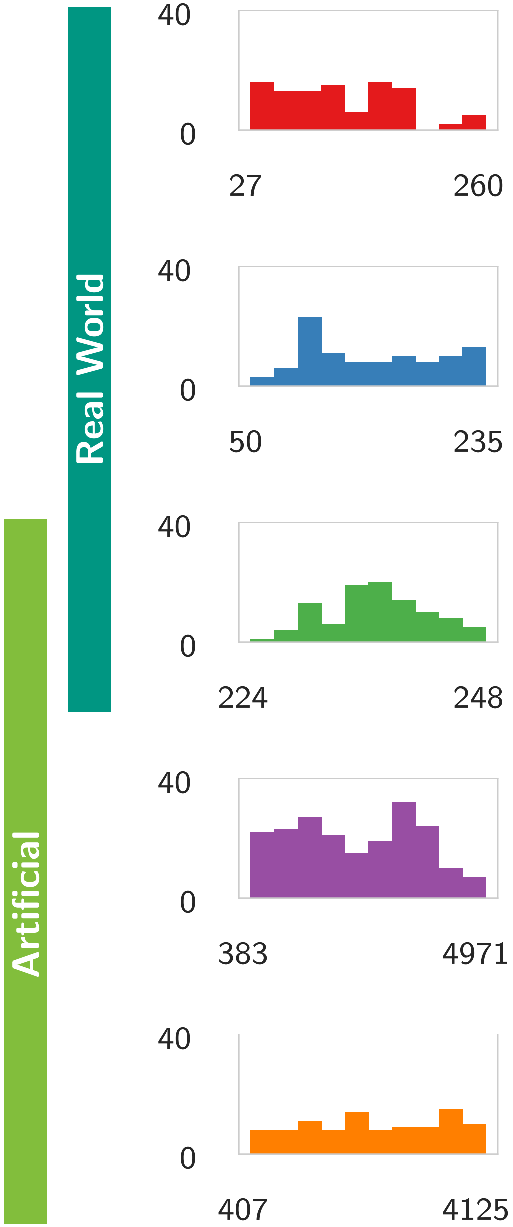


Evaluation

Test Instances



North

- collection of small real world graphs

Rome

- collection of small real world graphs

Community

- Resembles community structure

1-Planar

Triangulation + X

- Triangulation + set of random edges

100 randomly selected graphs per class

Research Questions

Q: What is good parametrization of our algorithm?

Q: What is a good choice for an initial drawing?

Q: Does our heuristic improve the crossing angle?

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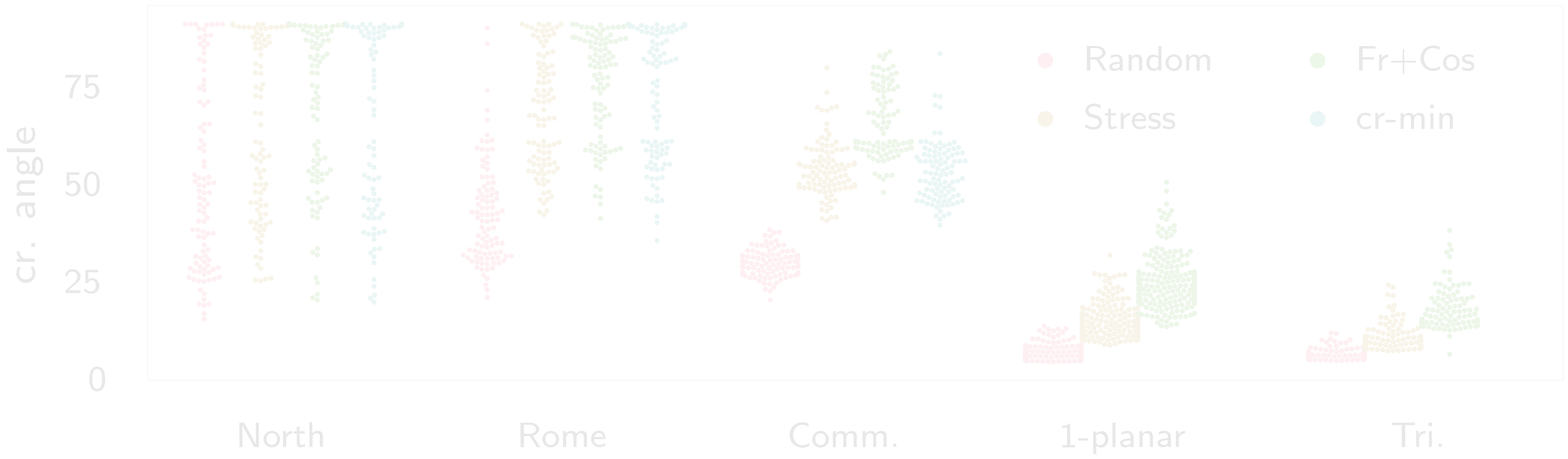
Good Initial Drawing

Initial Drawing Styles:

Random: random position per vertex
Fr+Cos: Force-Directed + Angle Max. Force [Huang et al. '14]
Stress: Stress Majorization [Gansner et al. '05, OGDF]
cr-small: Drawing with small number of crossings [R. et al.'18]



Observations

Random seems to be a bad choice

Tendence towards Fr+Cos

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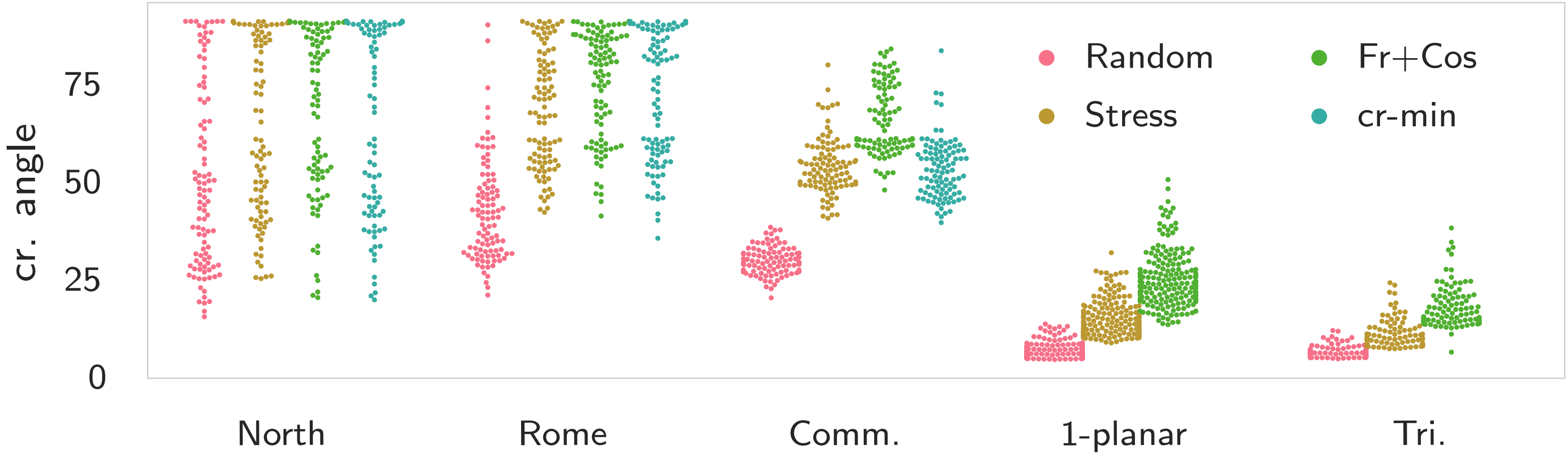

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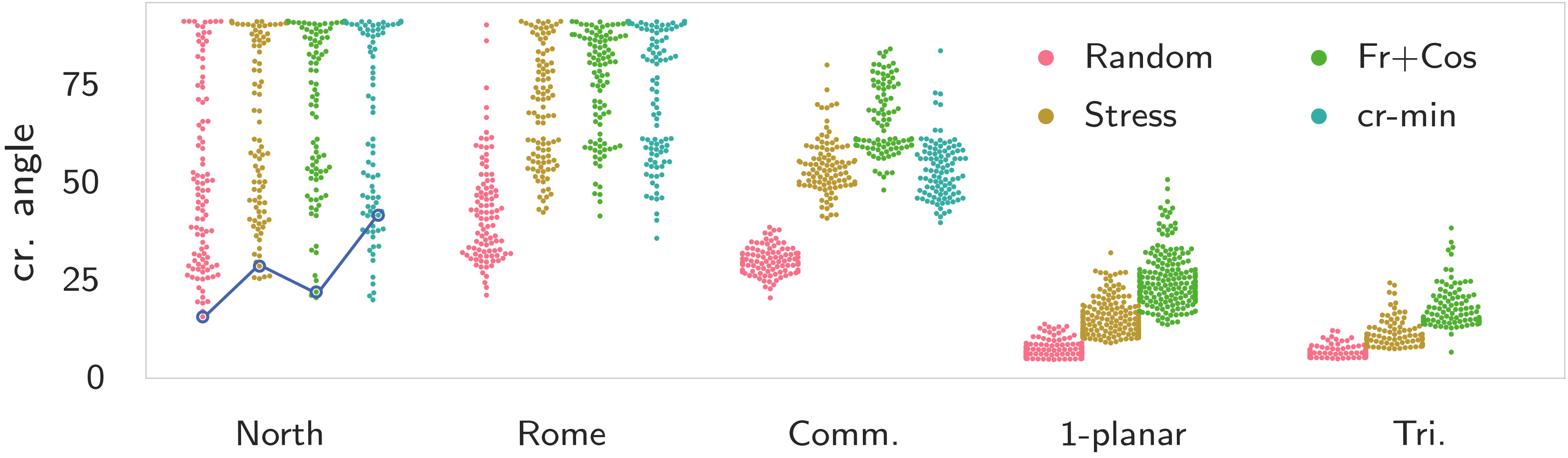

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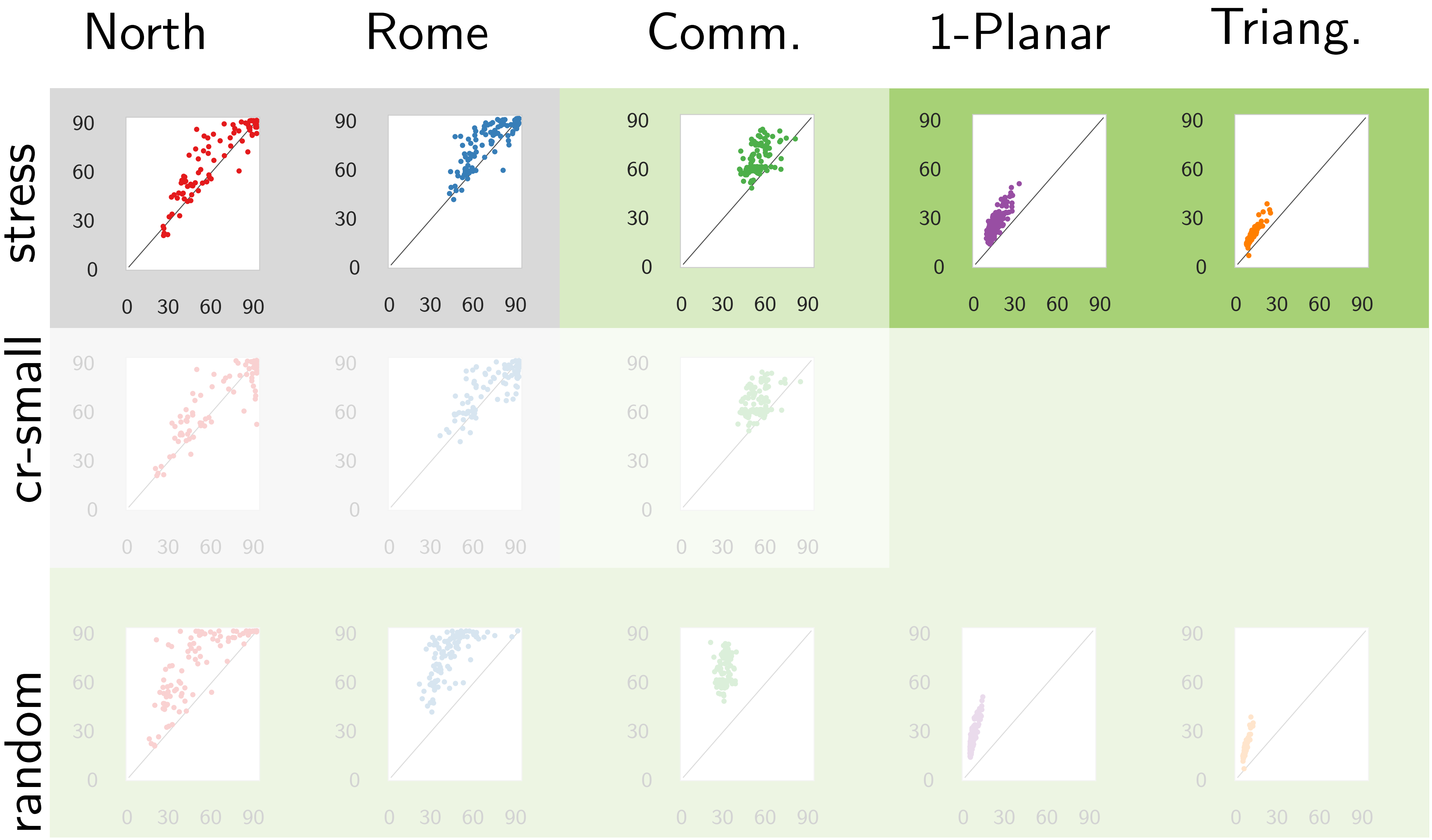

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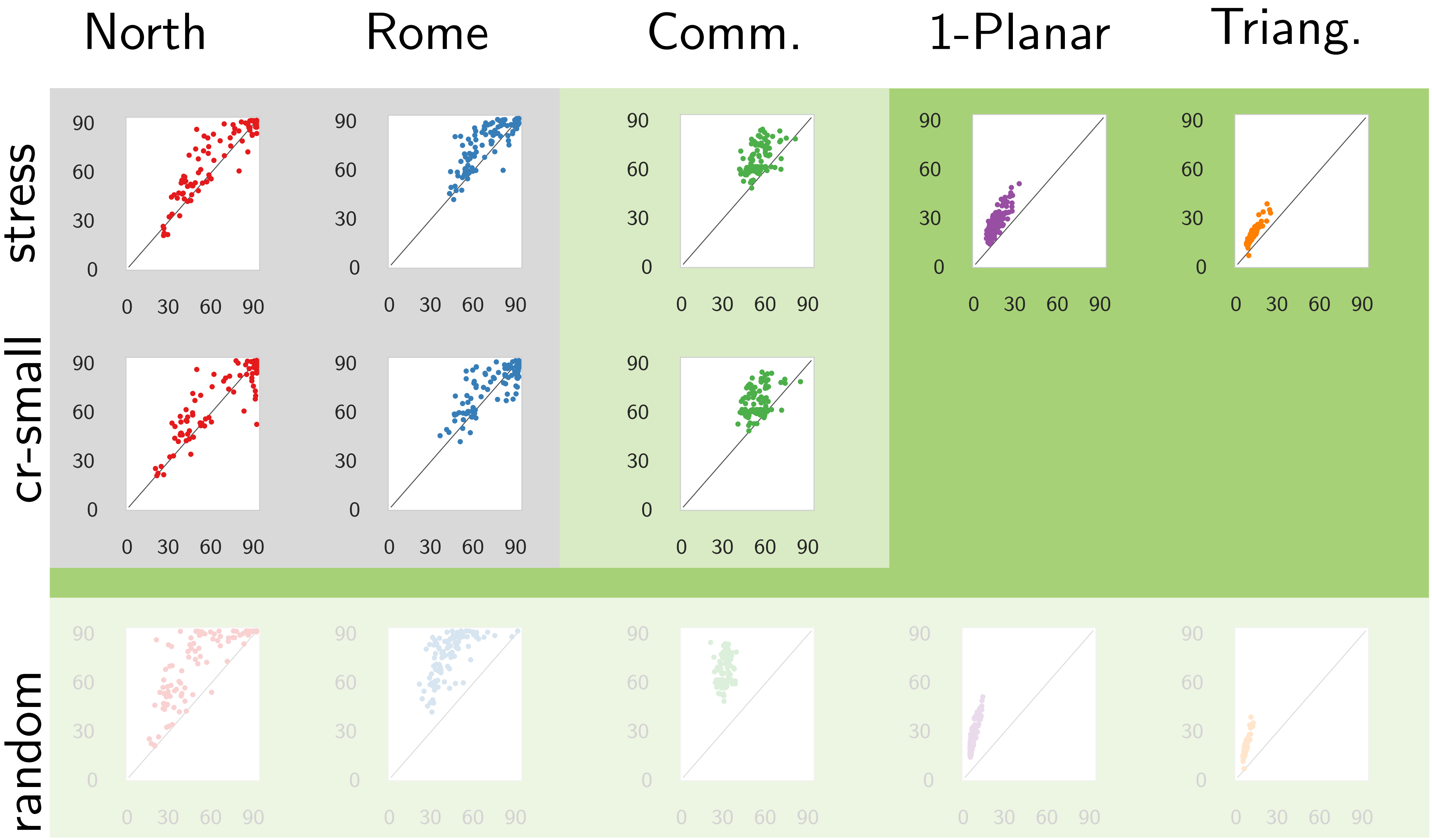

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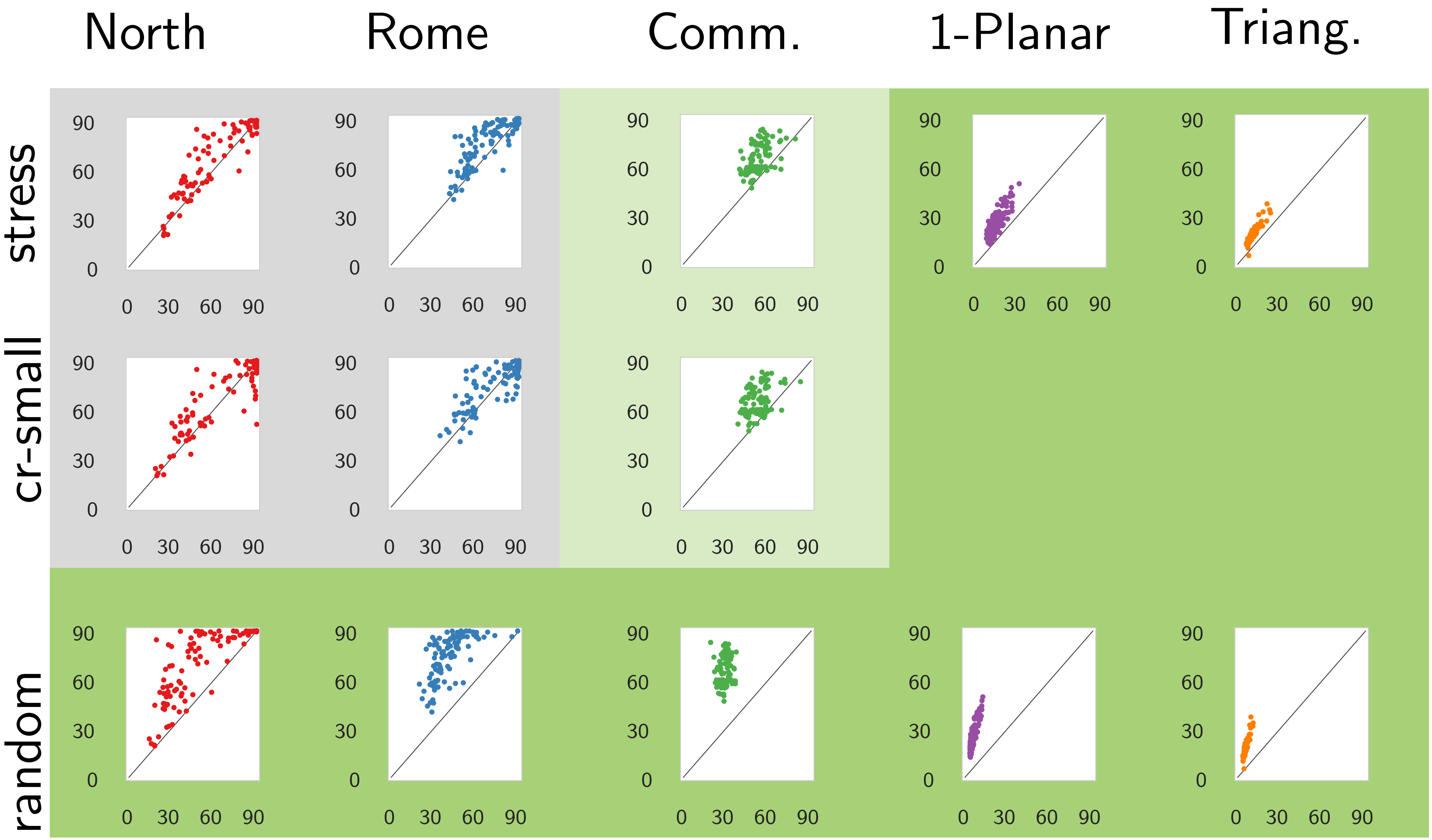

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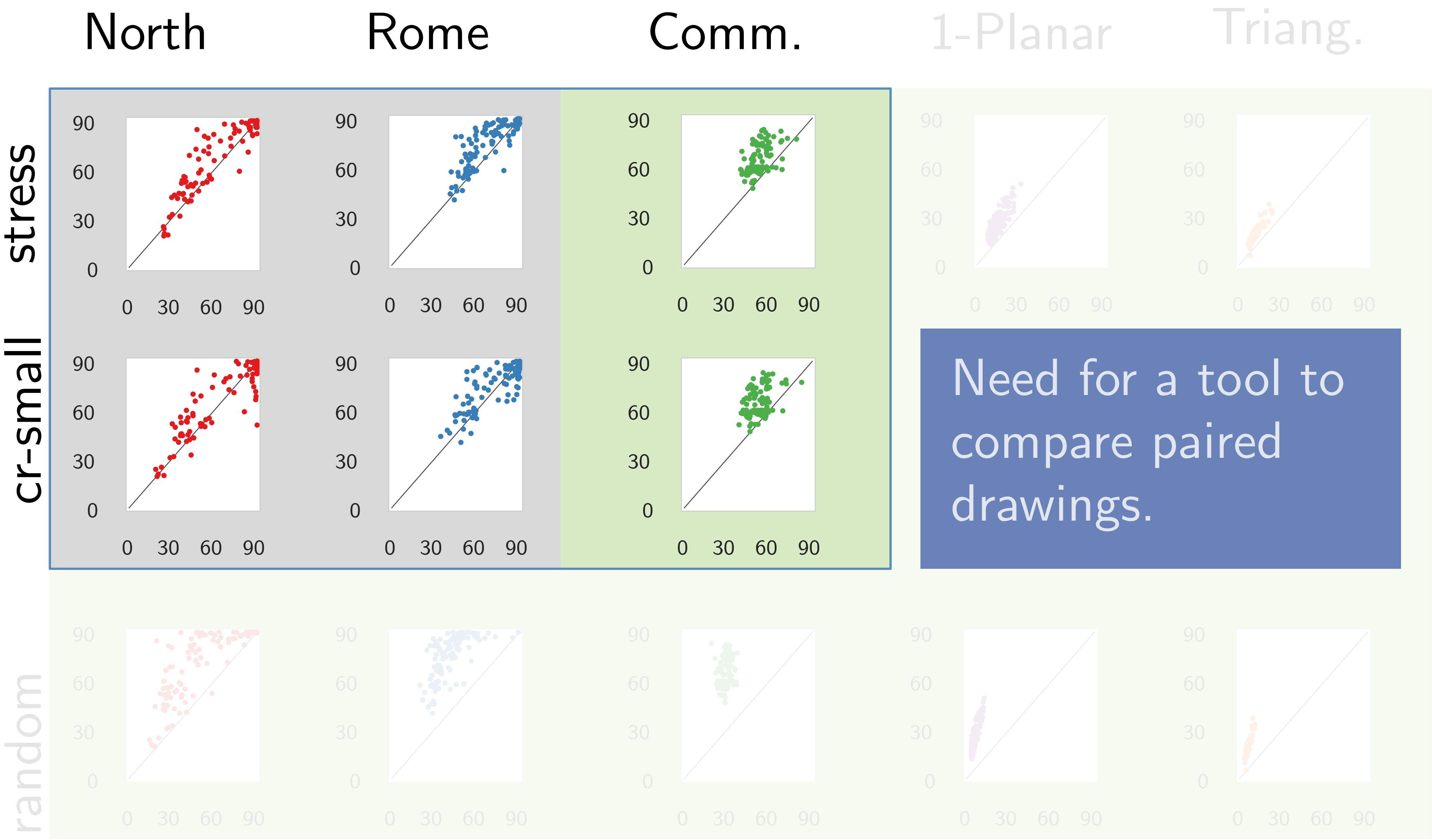
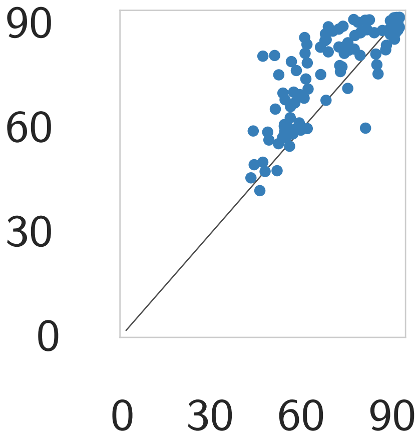


Need for a tool to compare paired drawings.

Tool to Compare Paired Drawings

Input: Ground set of Graphs $\mathcal{G} = \{G_1, G_2, \ldots, G_n\}$

Two sets of drawings of $\mathcal{G}$ $\{\Gamma[G_i] \mid G_i \in \mathcal{G}\}$ $\{\Pi[G_i] \mid G_i \in \mathcal{G}\}$



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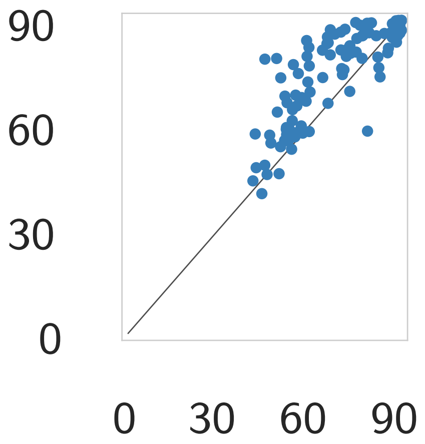
for all G_i $\quad$ $\text{cr-}\angle(\Gamma[G_i]) > \text{cr-}\angle(\Pi[G_i])$



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A number $p \in [0, 1]$

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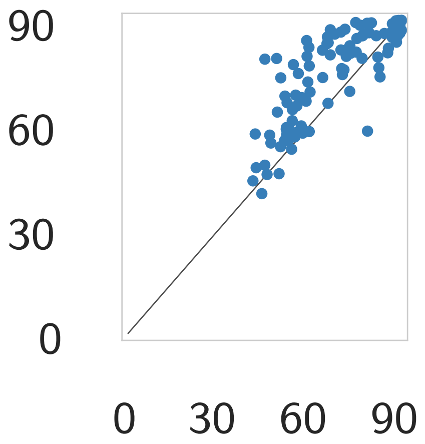

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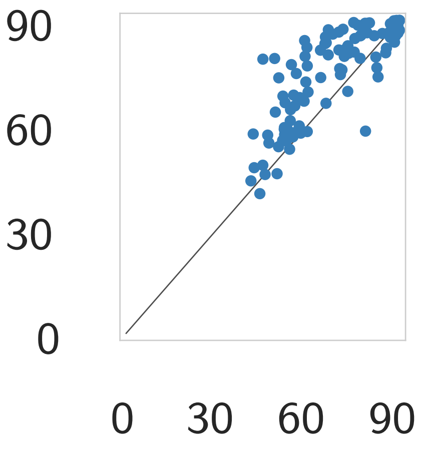

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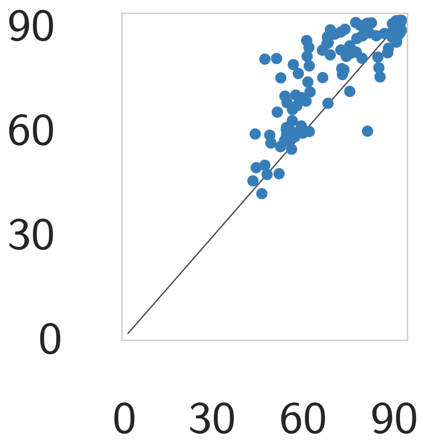

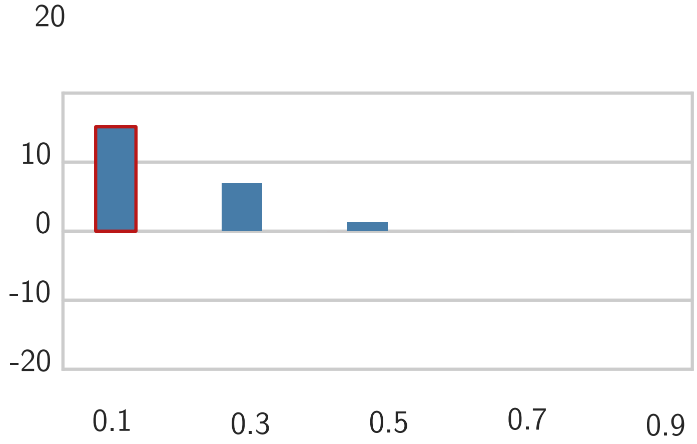

fr-cos cr-small

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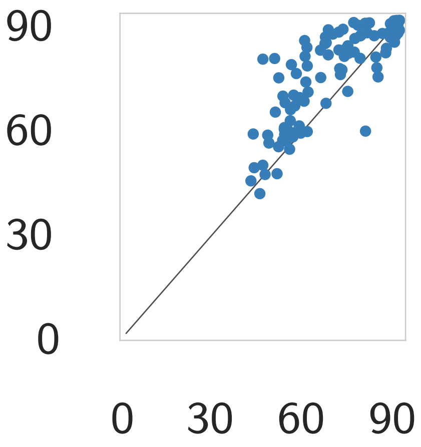

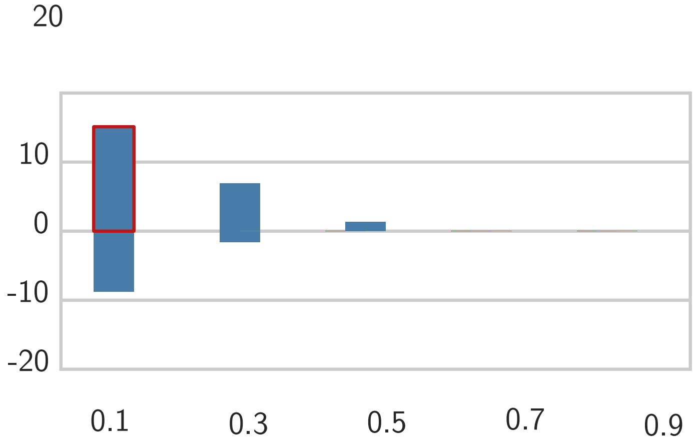

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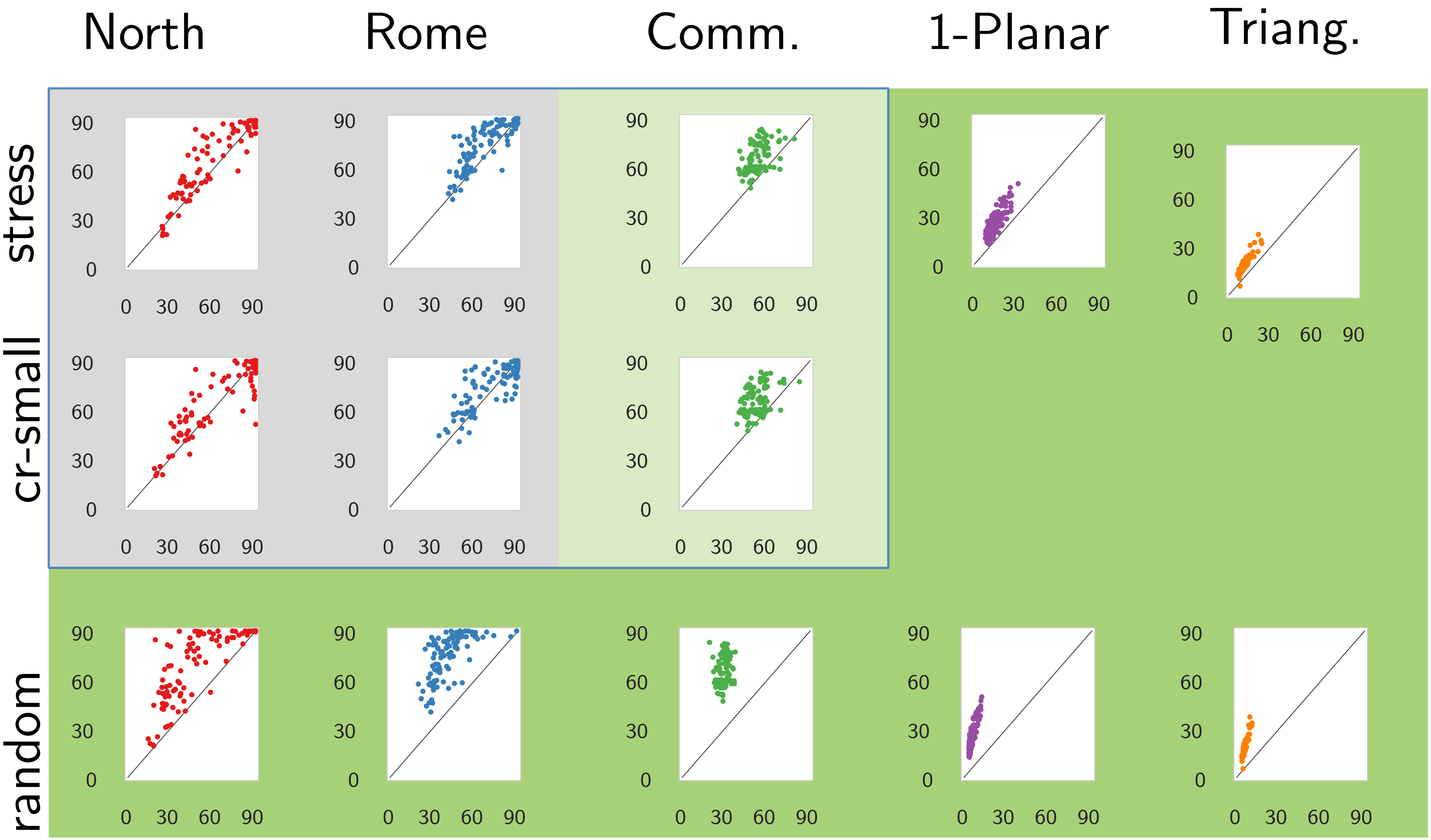

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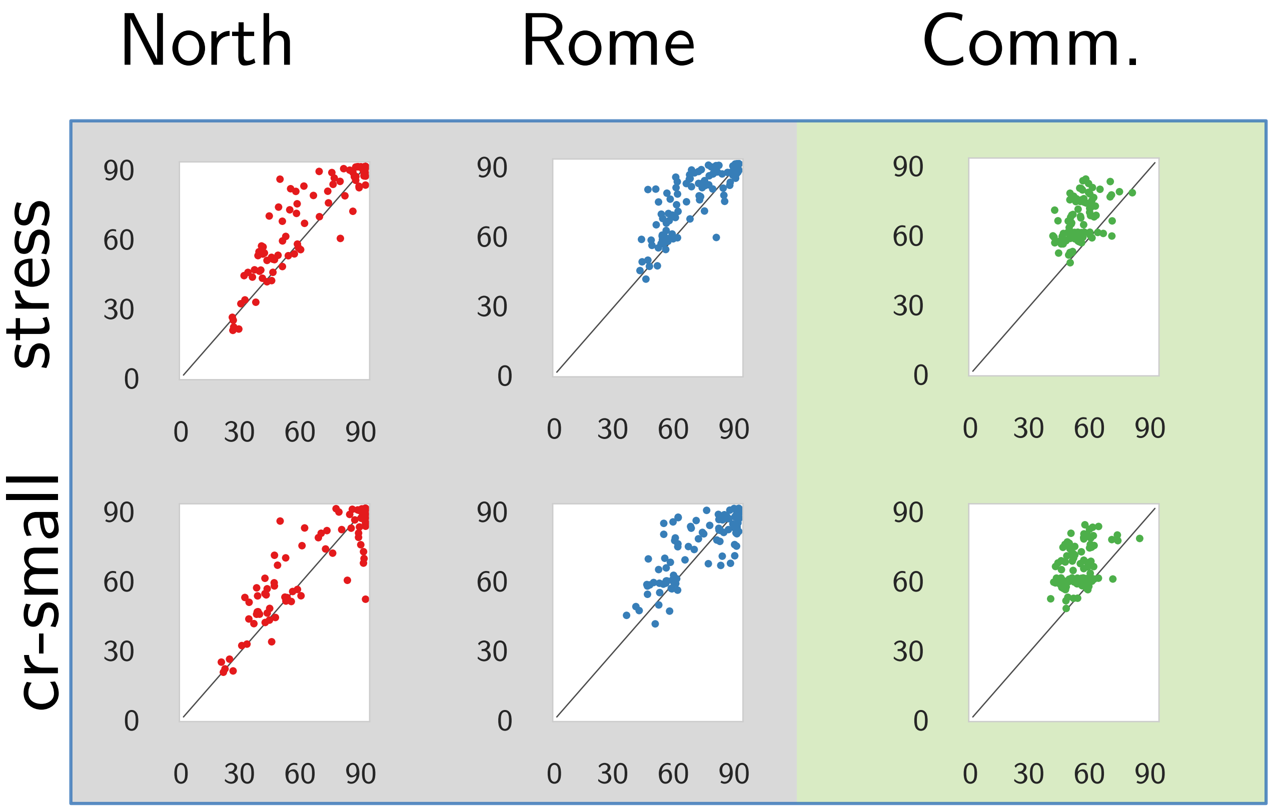
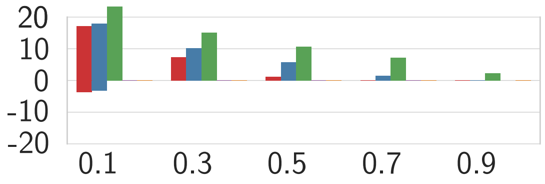
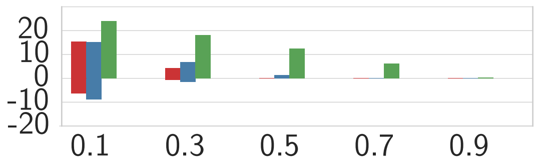


fr-cos vs stress





fr-cos vs cr-small

Research Questions

Q: What is good parametrization of our algorithm?

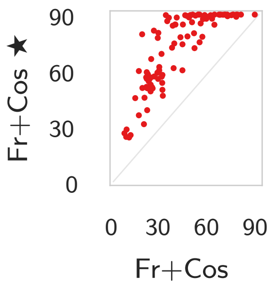
Q: What is a good choice for an initial drawing?

Q: Does our heuristic improve the crossing angle?

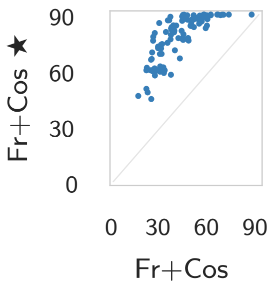
Improvement of the Crossing Angle

Q: Does our Heuristic improve the Crossing Angle?

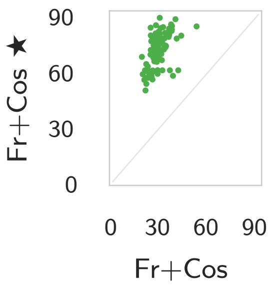
North



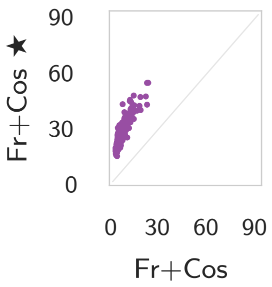
Rome



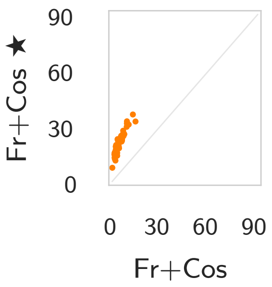
Comm.



1-Planar

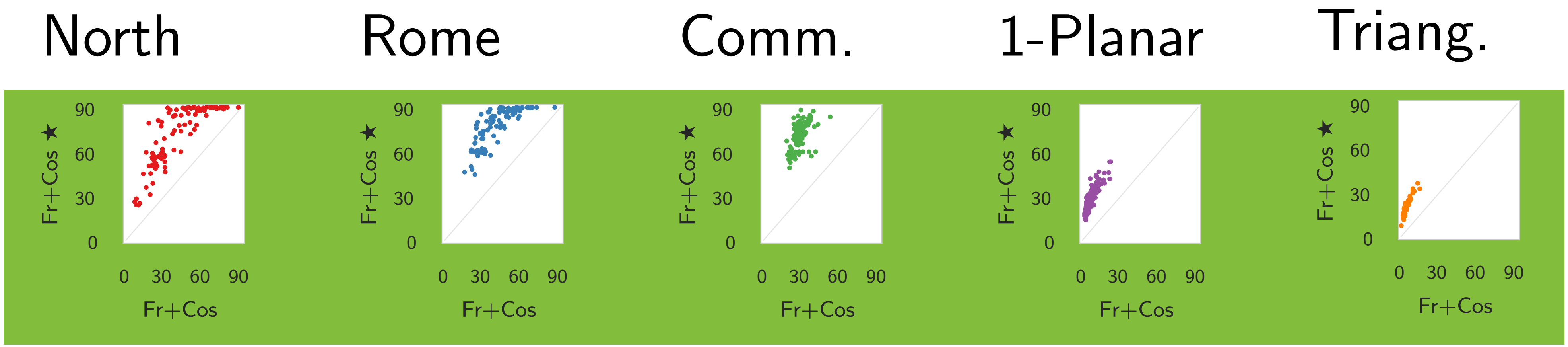


Triang.



Improvement of the Crossing Angle

Q: Does our Heuristic improve the Crossing Angle?



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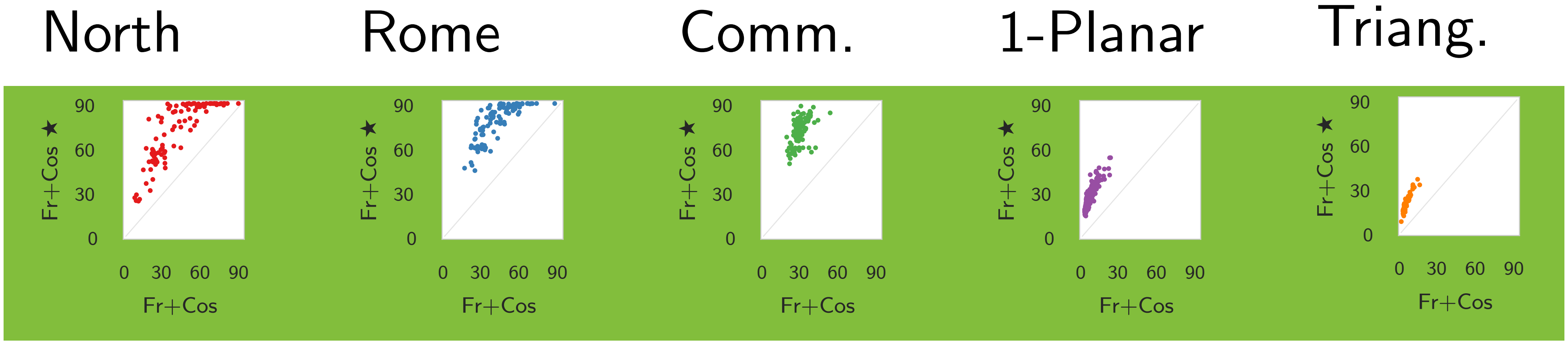
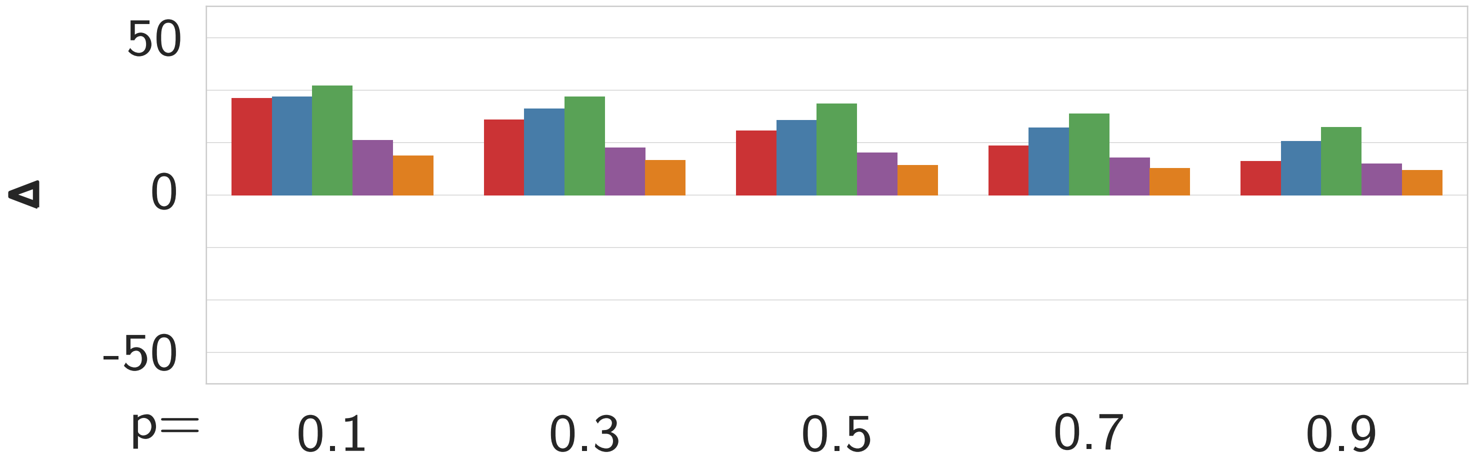
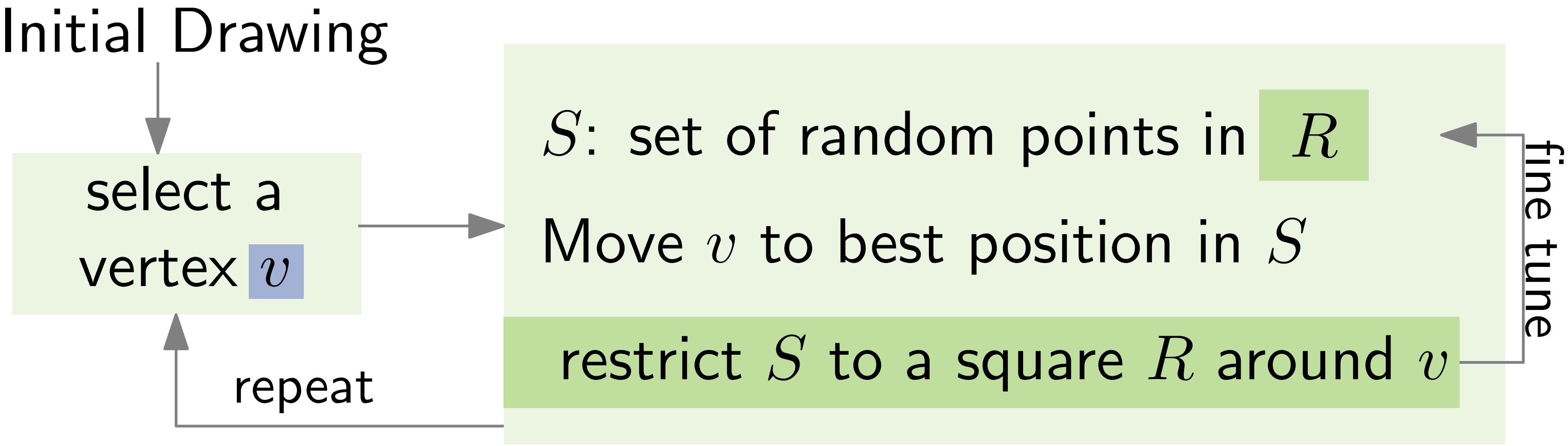




Our Heuristic



Initial Drawing

select a vertex v

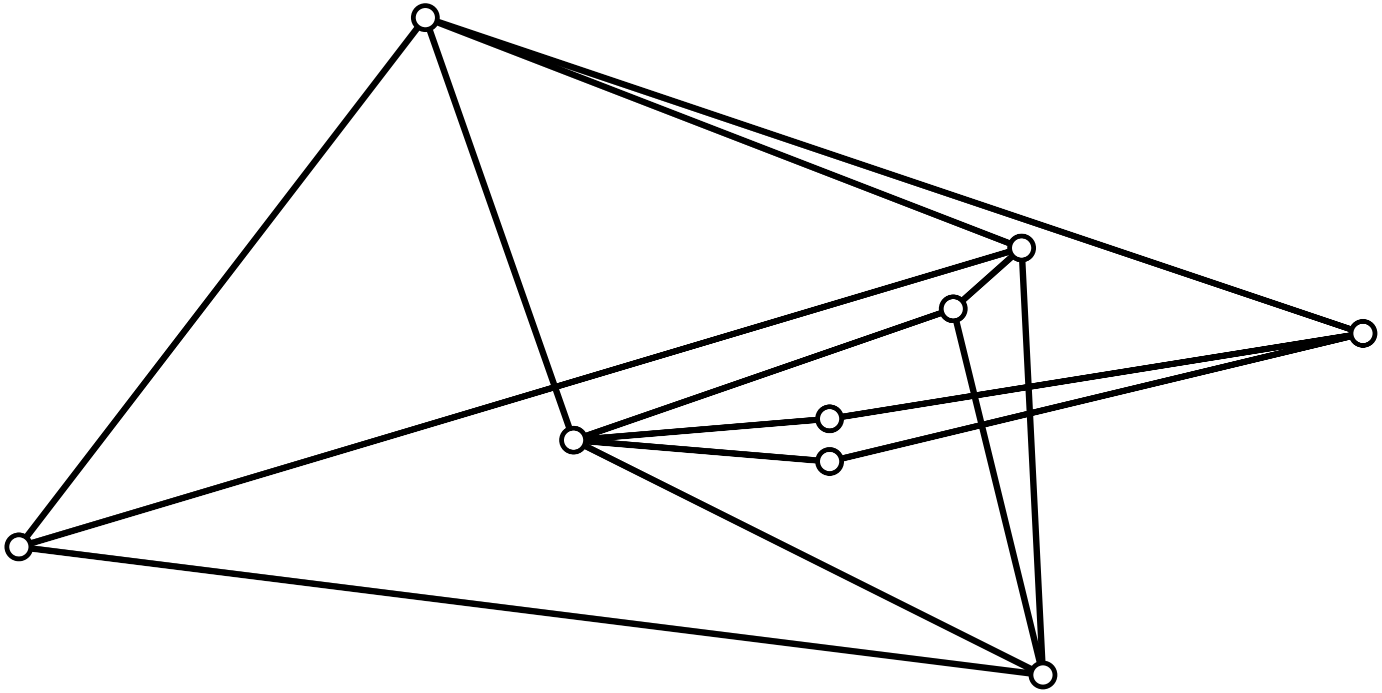
S: set of random points in R

Move v to best position in S

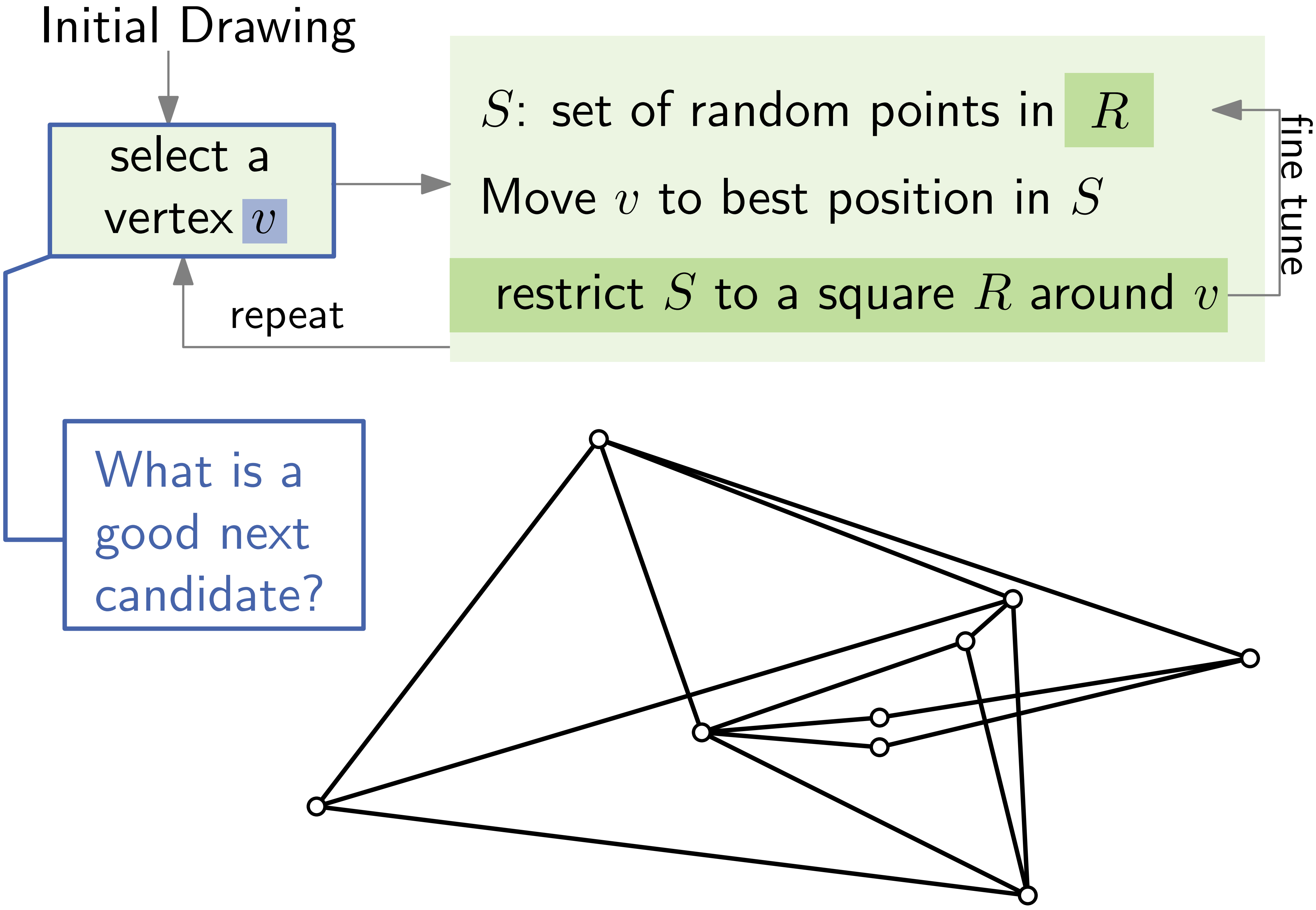
restrict S to a square R around v

fine tune

repeat



Our Heuristic



Initial Drawing

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What is a good next candidate?

Running Time

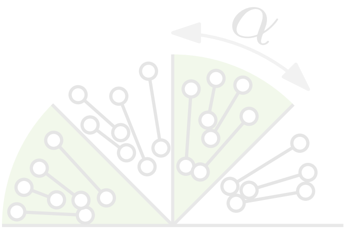
Task Find edges e,f s.t. $\text{cr-}\angle(\Gamma, e, f) = \text{cr-}\angle(\Gamma)|$

Possibility *Sweep*: Sweep-Line Algorithm

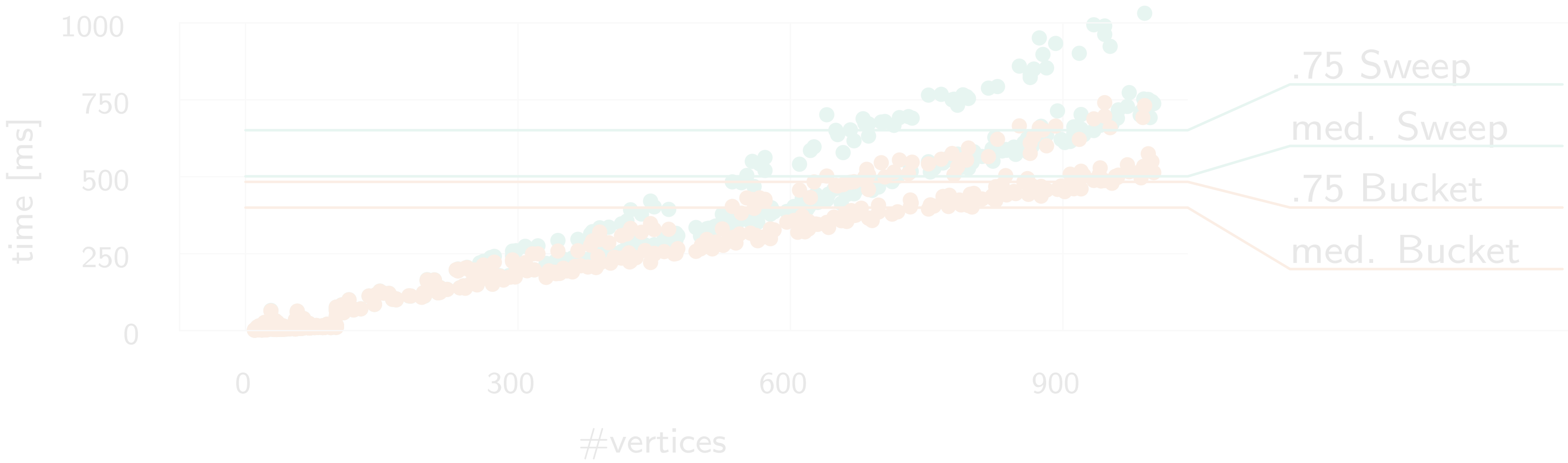
Possibility *Bucket*:

sort edges into buckets according to slopes

edges of adjacent buckets form $\text{cr-}\angle(\Gamma)$



Time to move a single vertex



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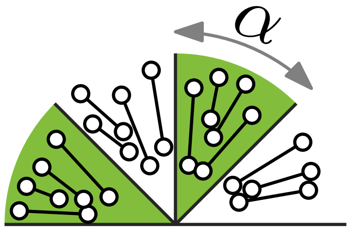

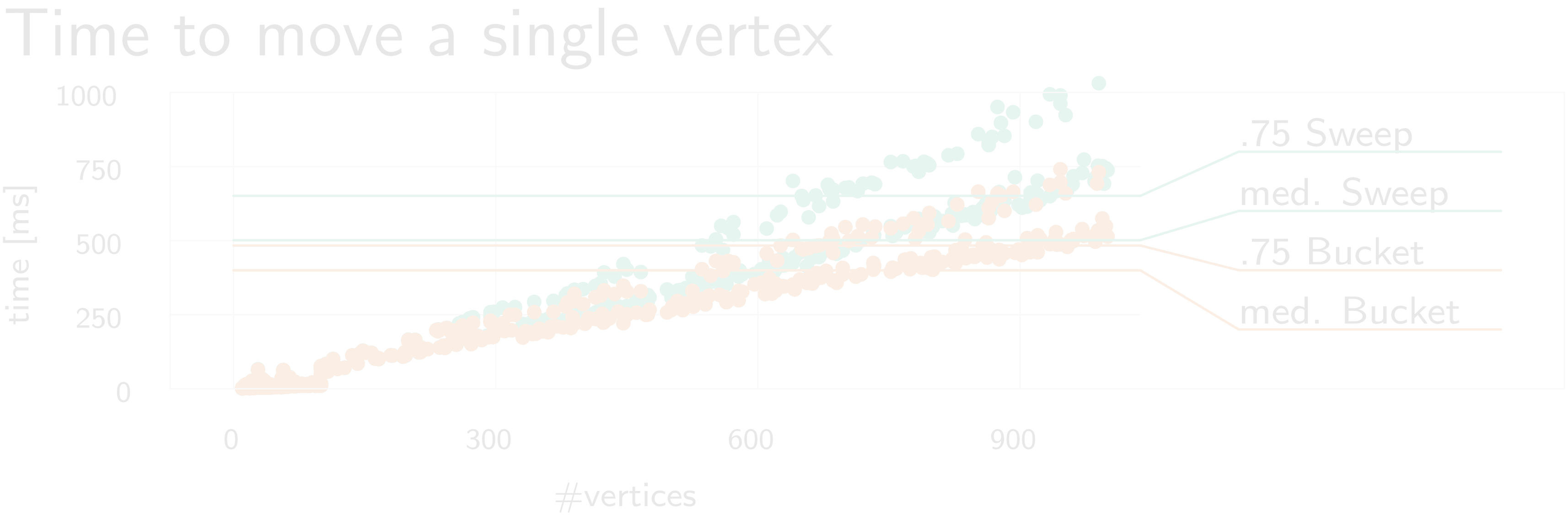

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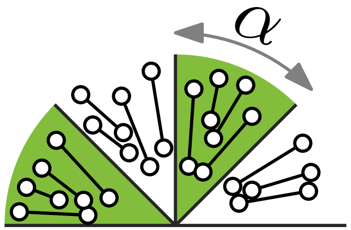

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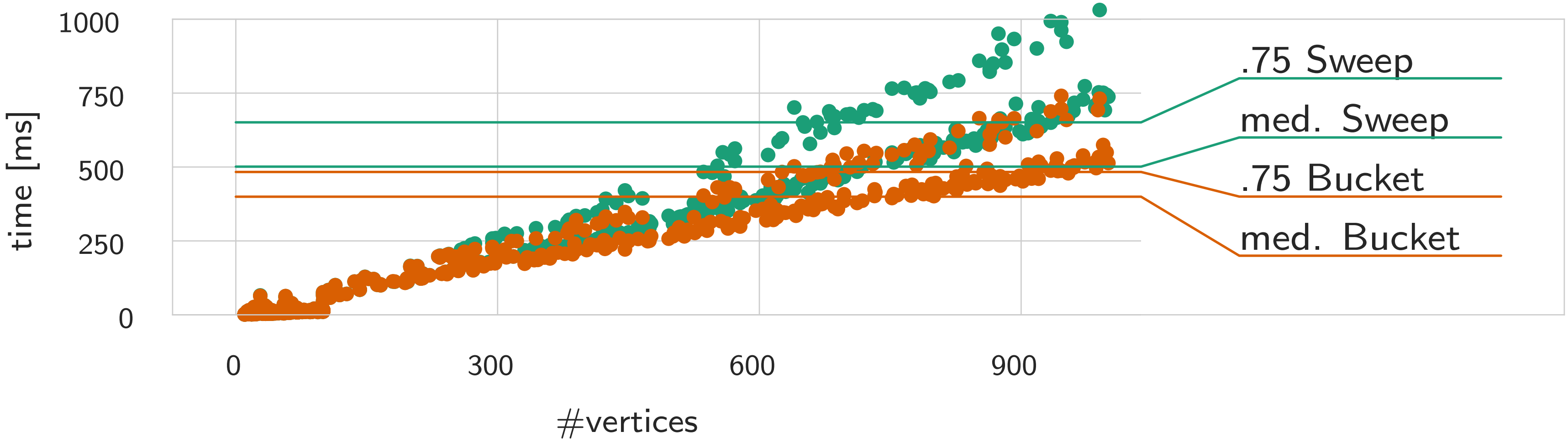
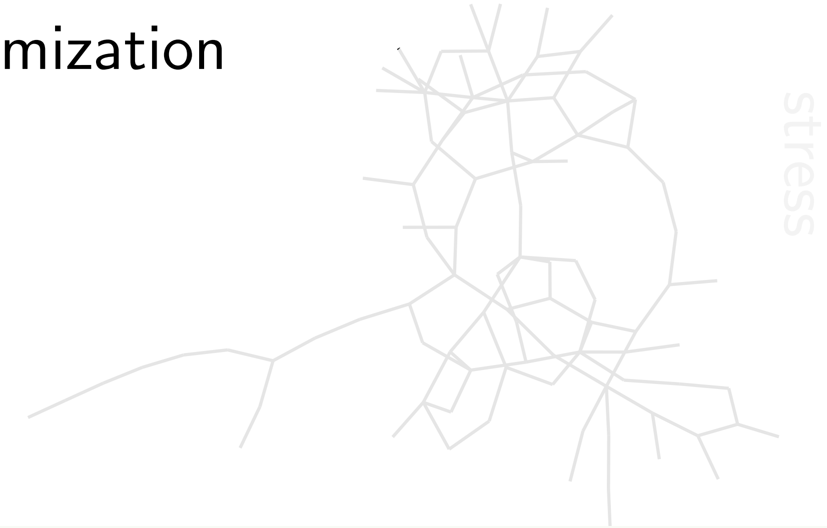


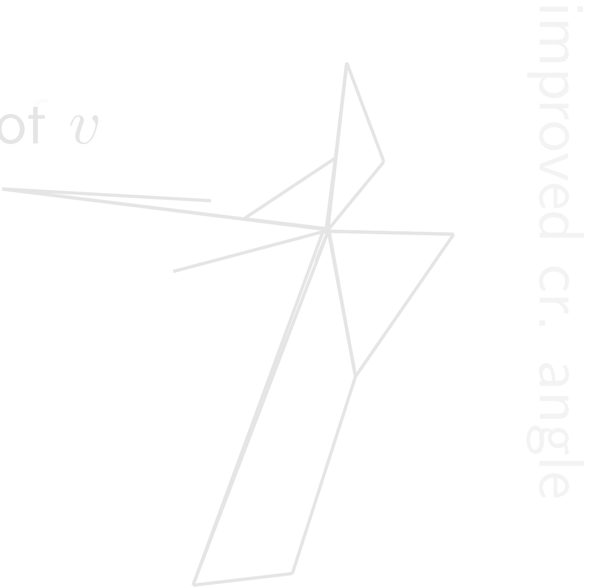
Conclusion

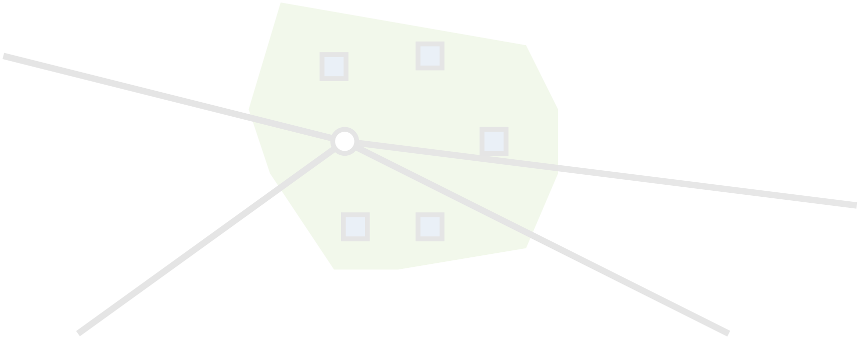
- a simple heuristic for Crossing-Angle Maximization
    - easy to implement
    - generic



Future Work

- Drawings are not necessarily *readable*
    - Let R be a region that ensure some properties of v
    - Optimize position of v within R


improved cr. angle



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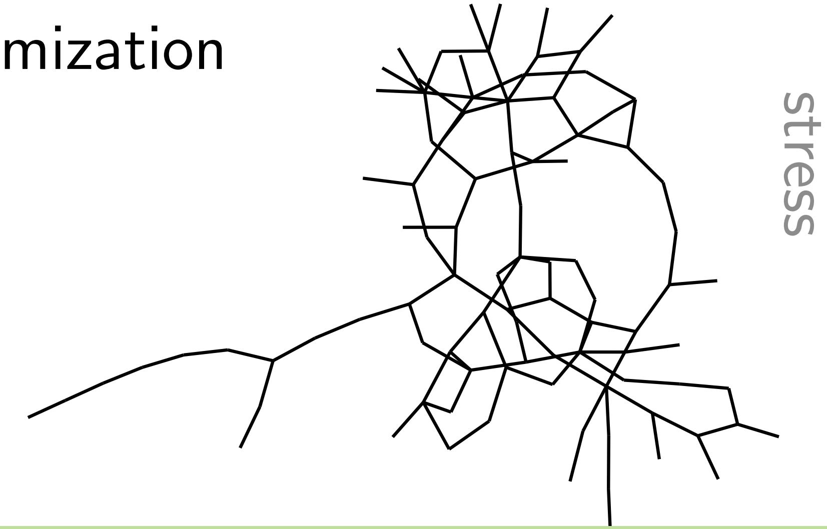

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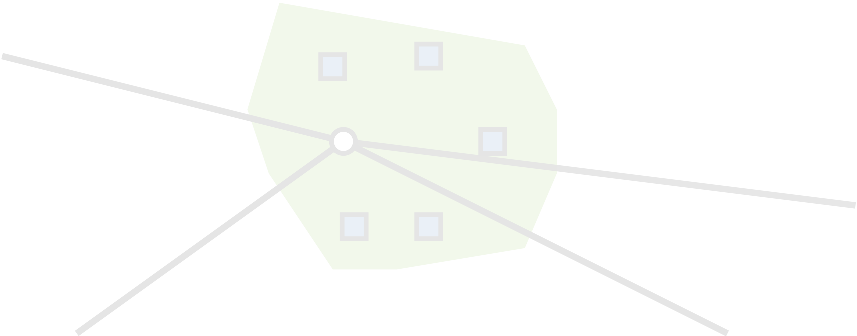

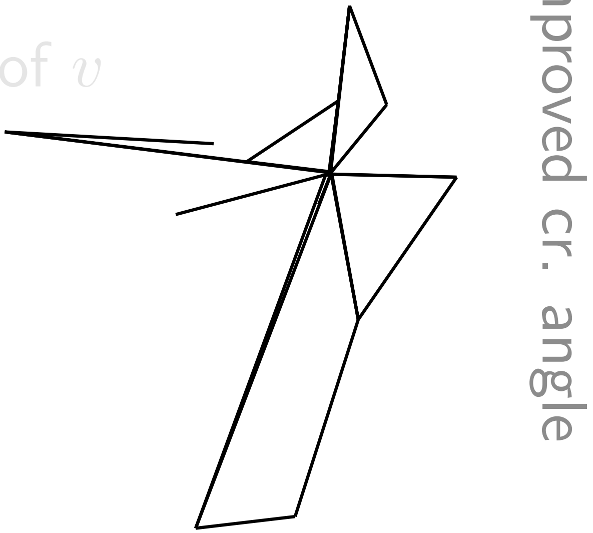
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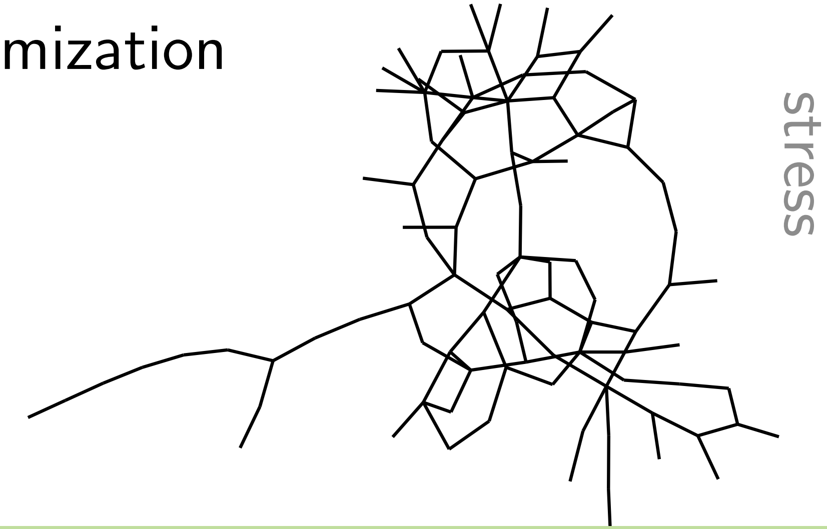

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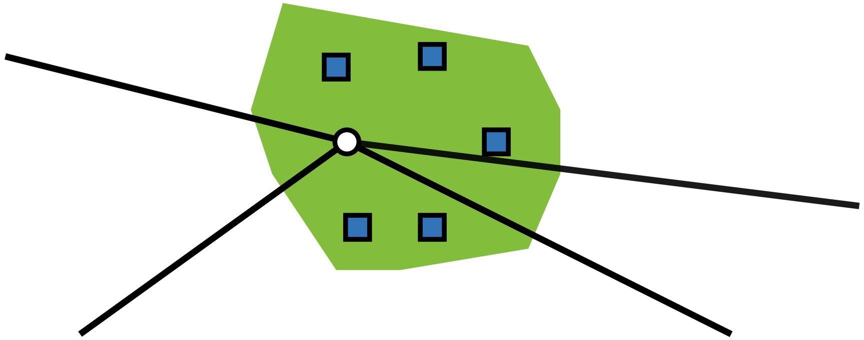

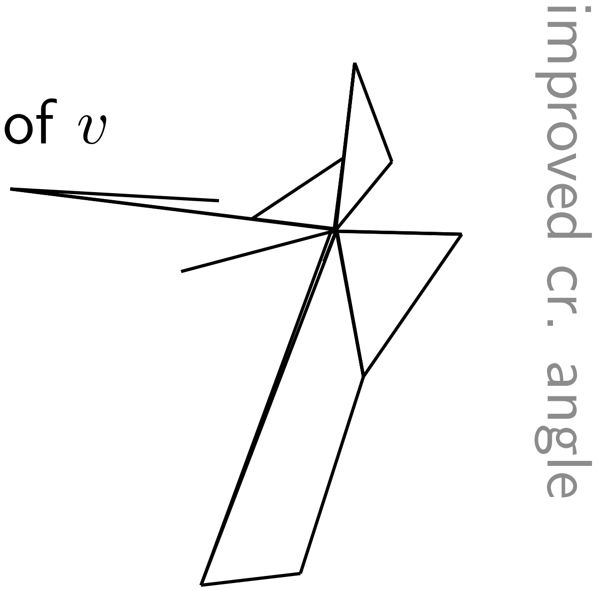
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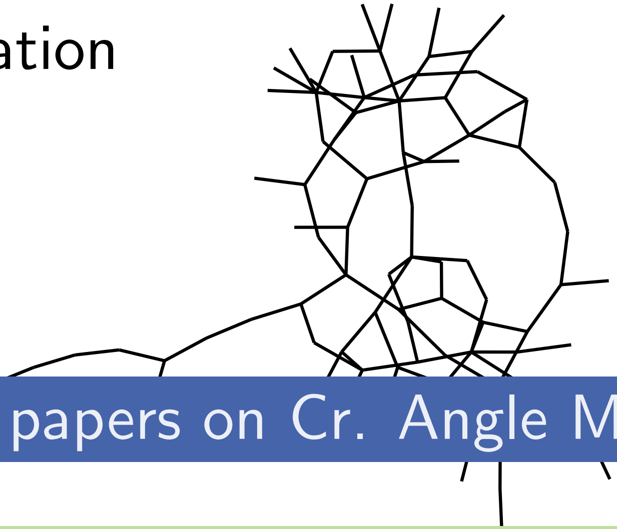

GD Contest duplicated the number of applied papers on Cr. Angle Max ;-)

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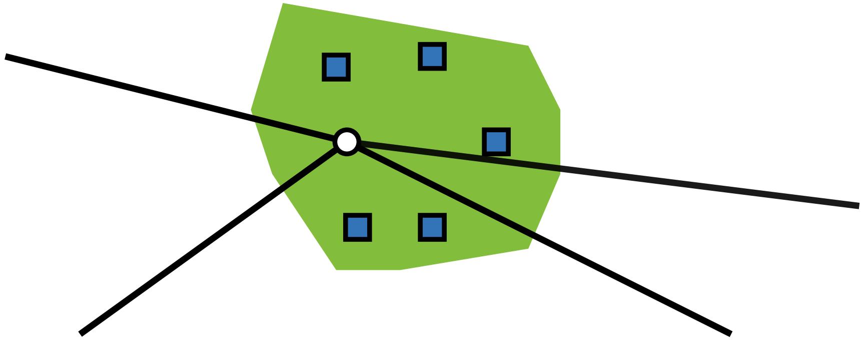
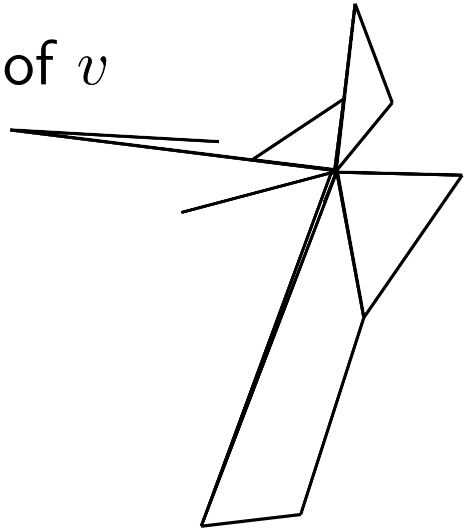




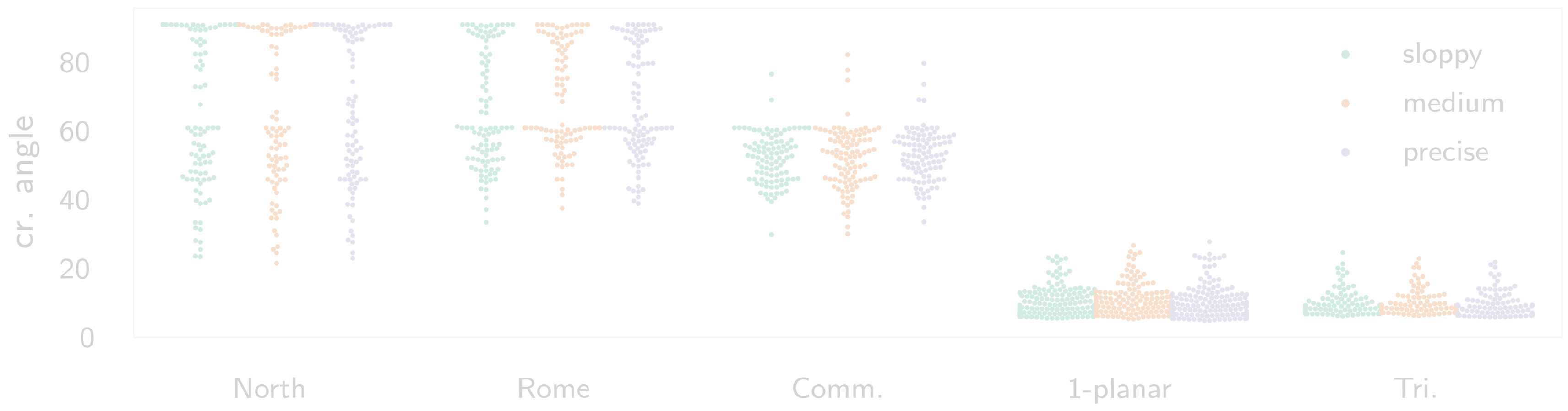
Thank you.

Good Parameter

Configurations:

Sloppy	Fast and *inaccurate*
Medium	Trade of between speed and accuracy
Precise	Slow and *accurate*

Time Limit: n seconds for an n-vertex graph



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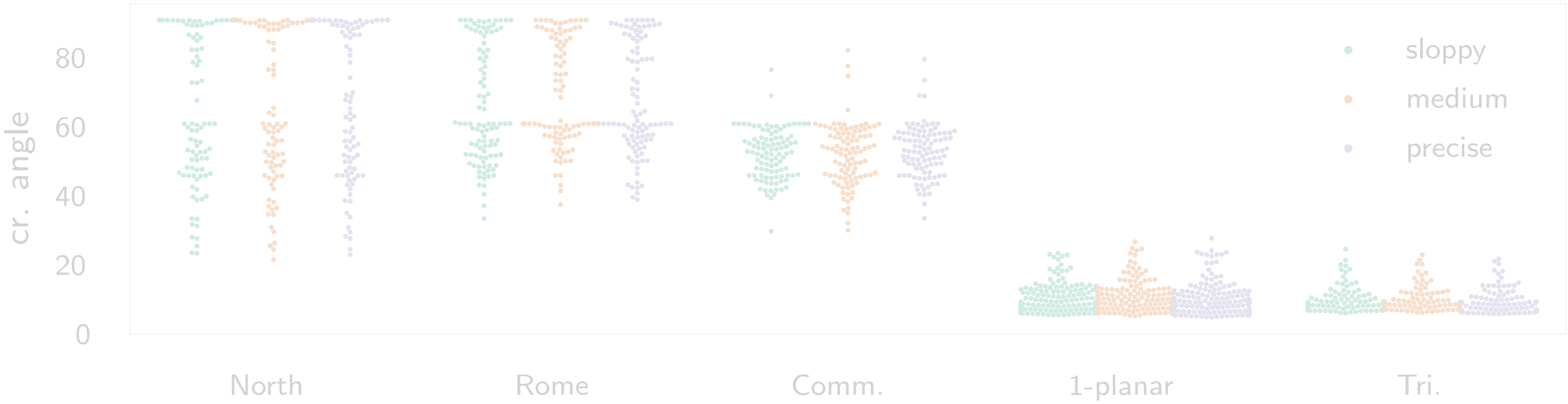

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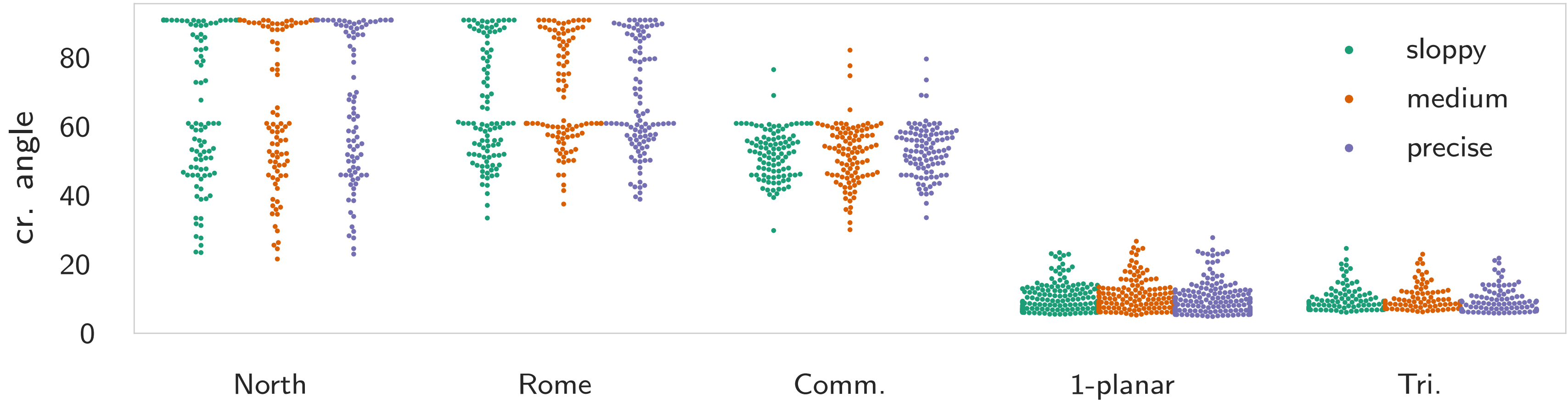

There is no obvious difference between the configurations

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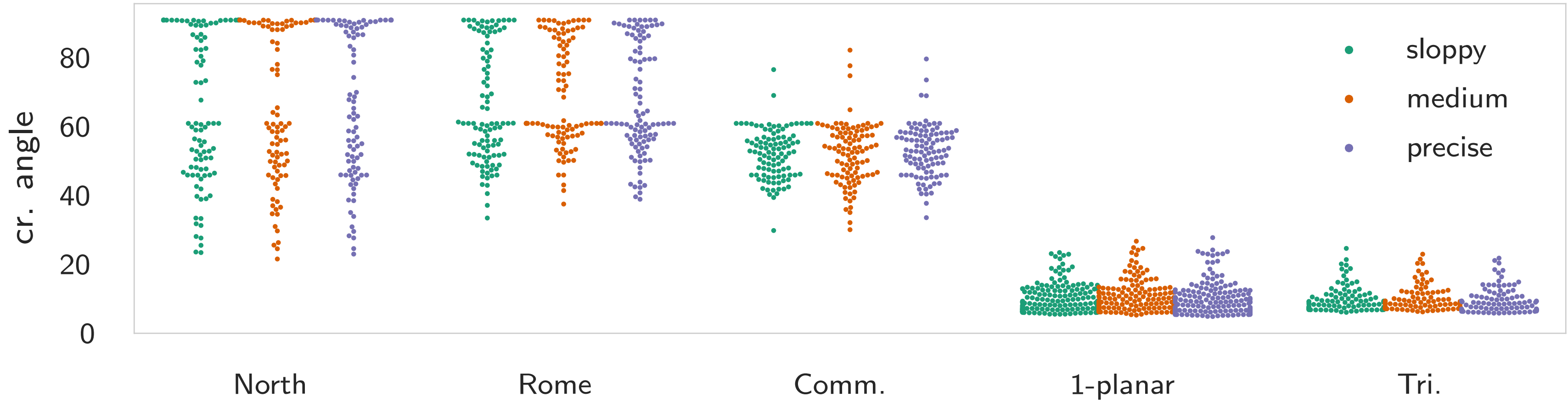

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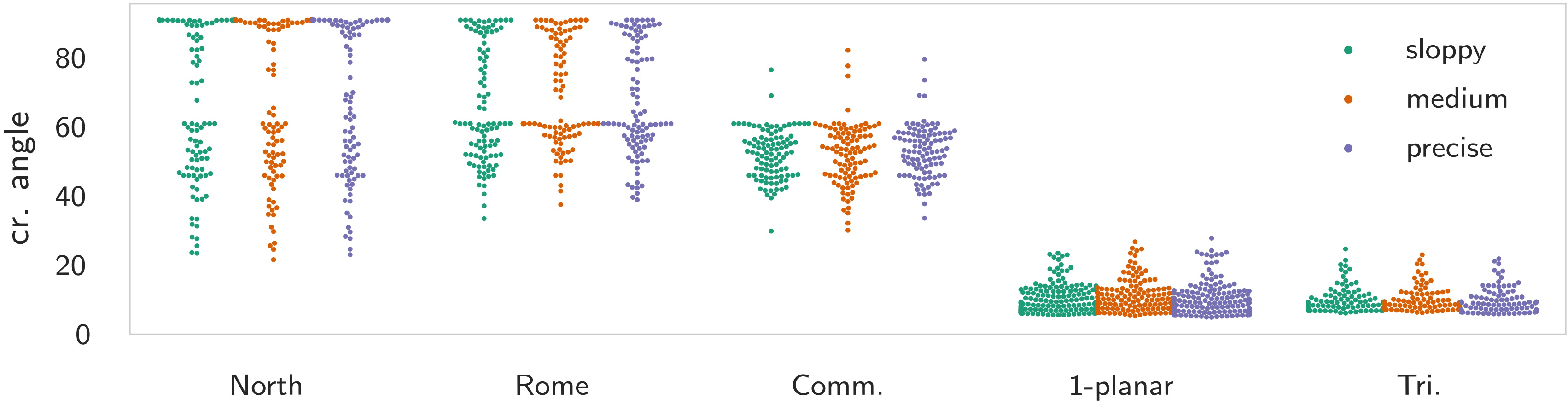

There is no obvious difference between the configurations?