Queue Layouts of Planar 3-Trees

Jawaherul Alam   Michalis Bekos   Martin Gronemann

Michael Kaufmann   **Sergey Pupyrev**

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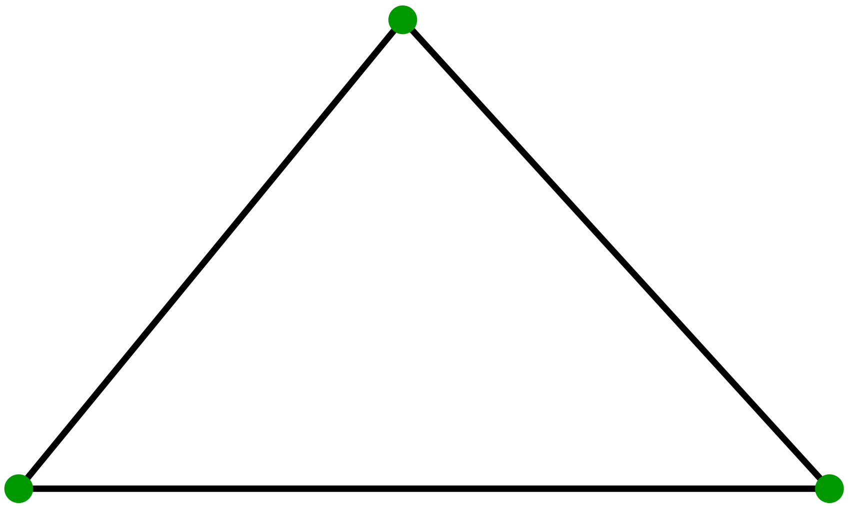
Def. planar 3-tree



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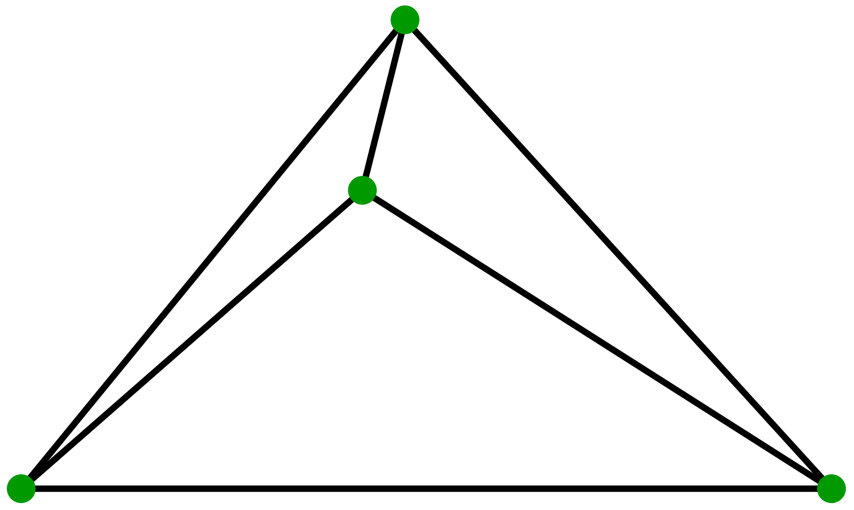

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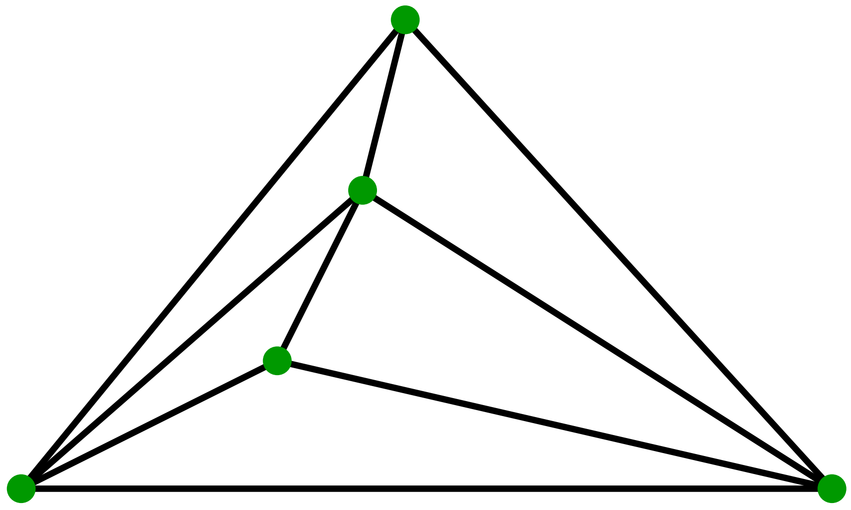

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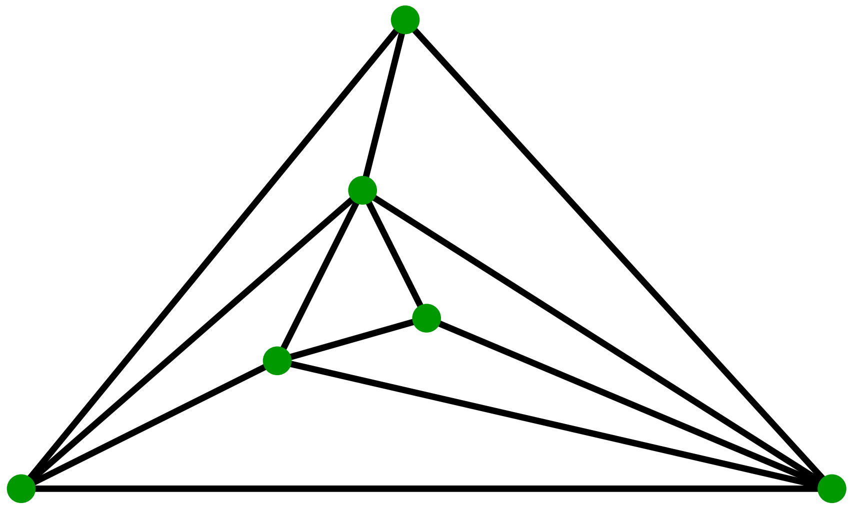

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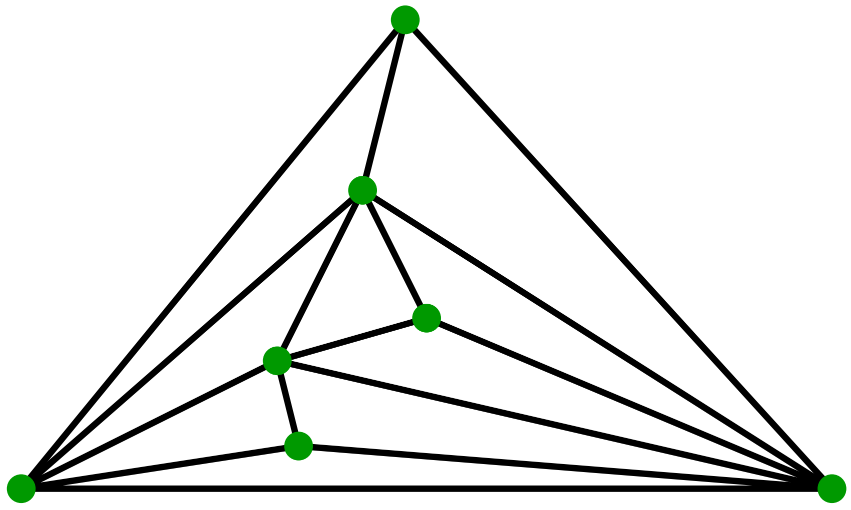

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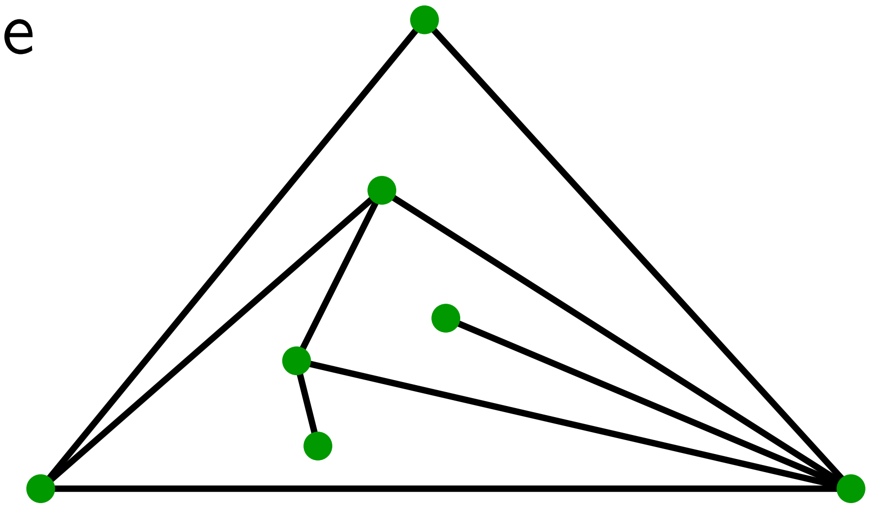

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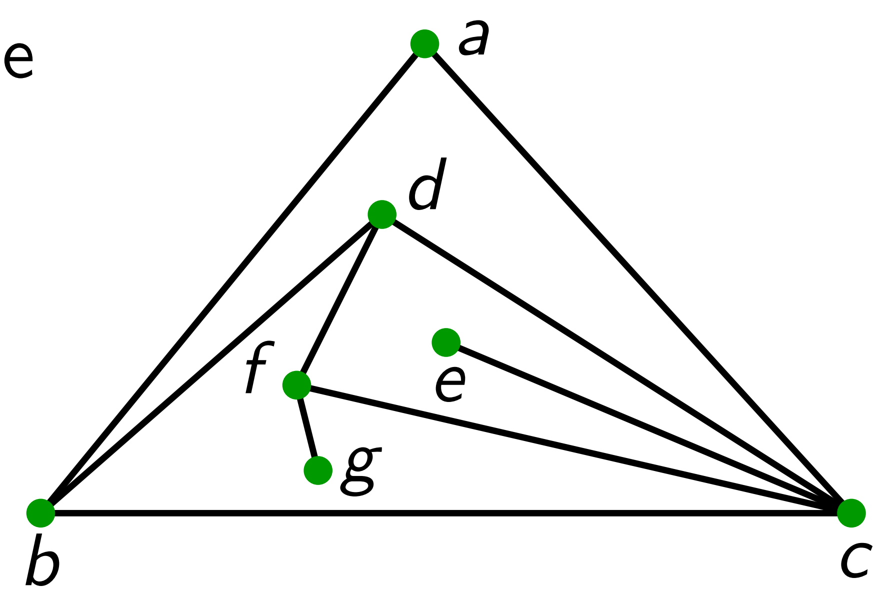

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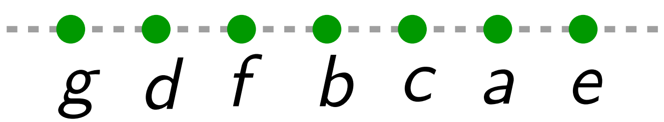

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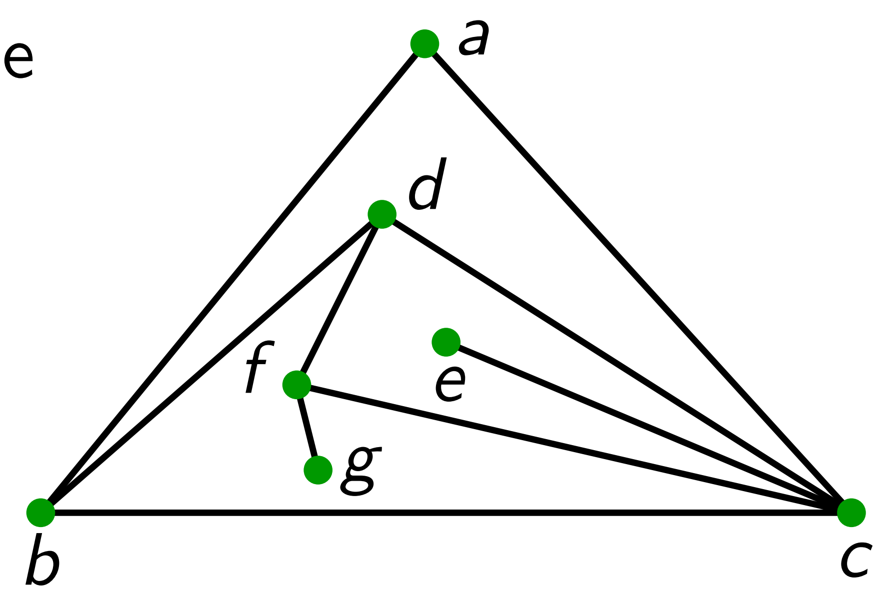

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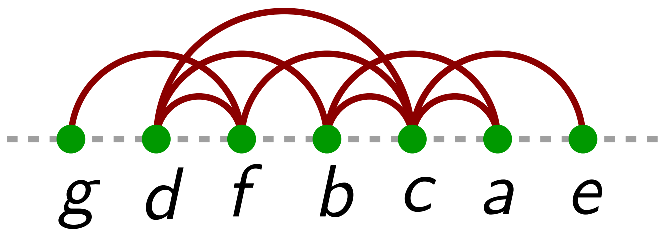

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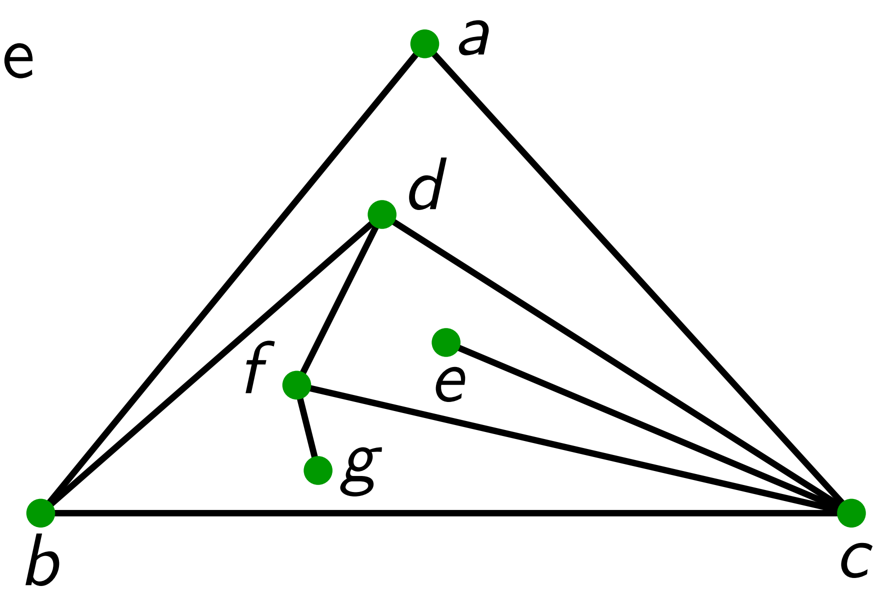

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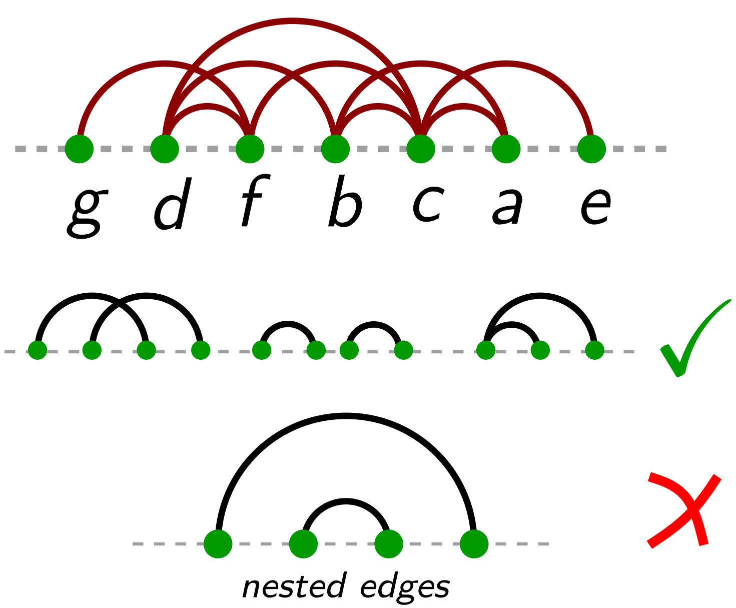

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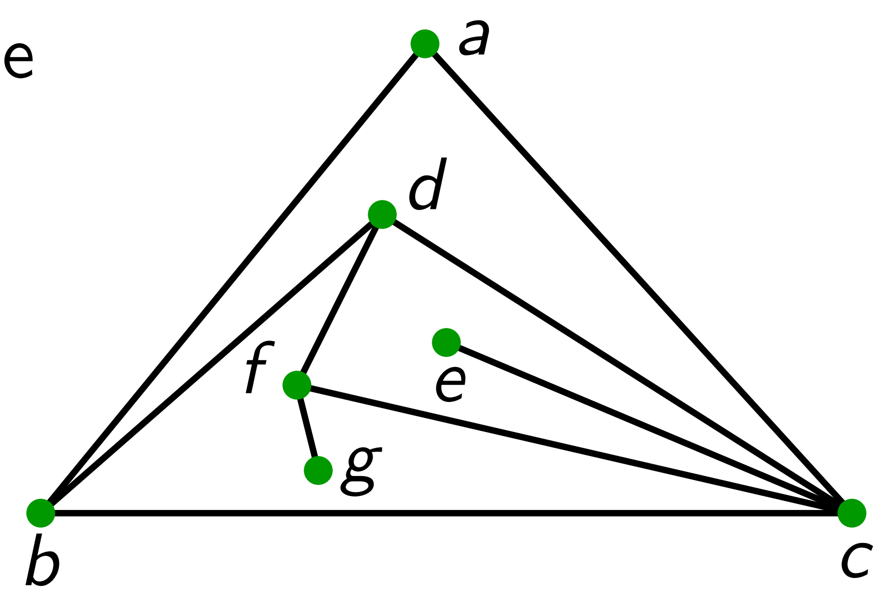

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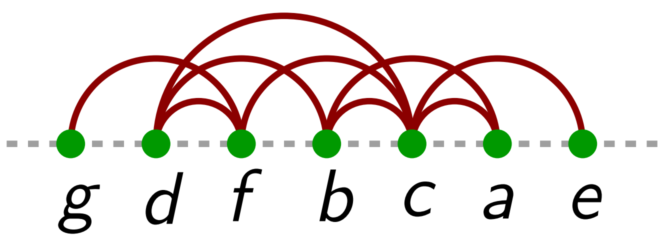

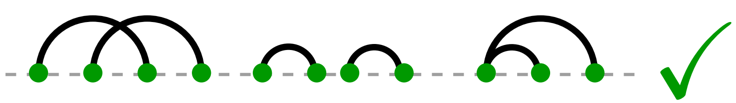

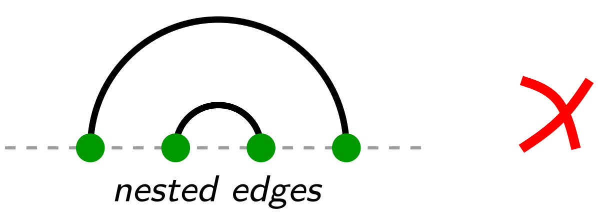

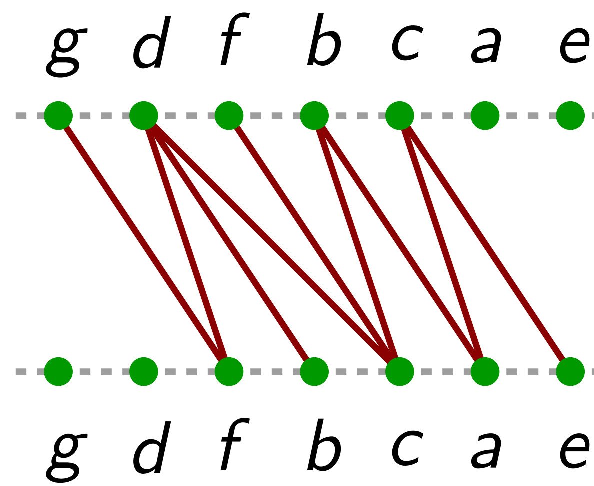

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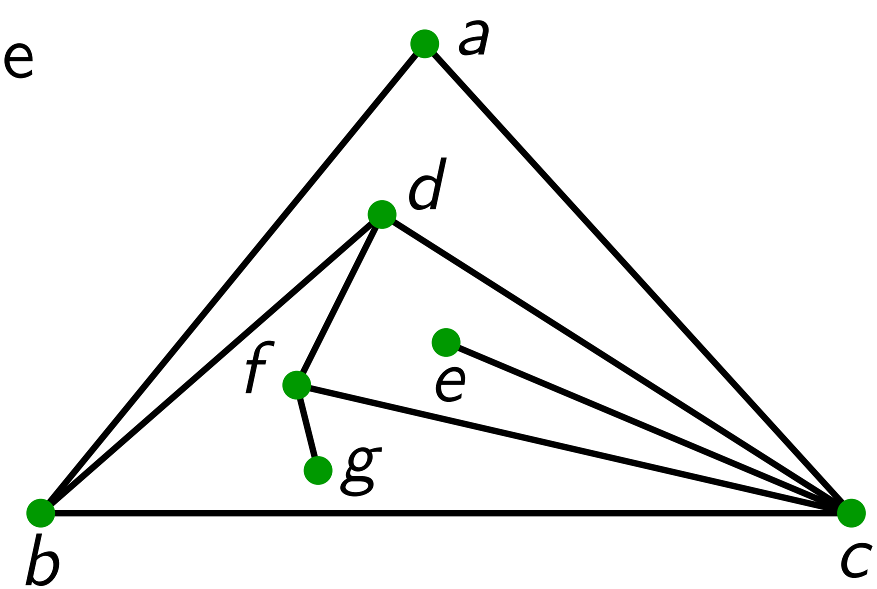

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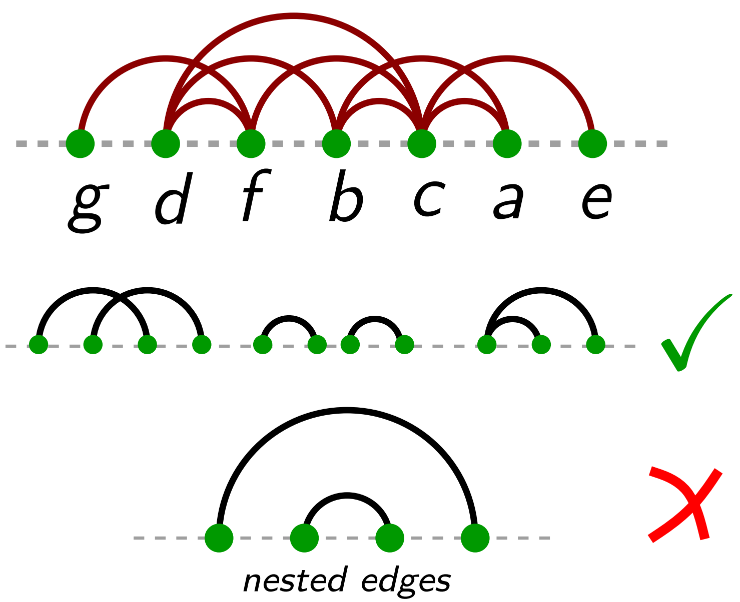

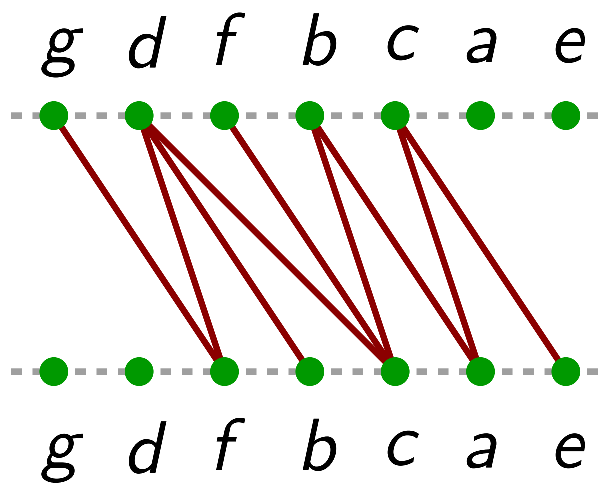

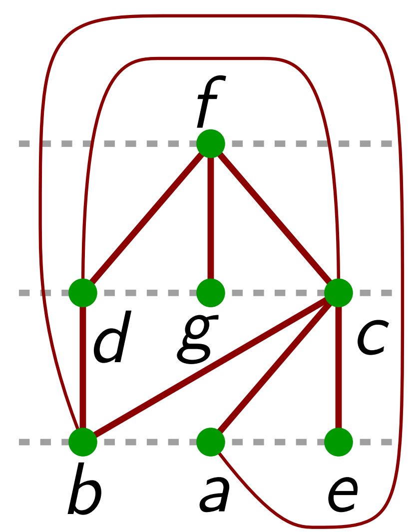

Prior Work

graph class	lower bound	upper bound
tree		
outerplanar		
planar 2-tree		
planar 3-tree		
planar		

Prior Work

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tree	$\mathbf{1}$	$\mathbf{1}$ [Heath Rosenberg 1992]
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planar 2-tree	**3**	[Wiechert 2017]	**3**	[Rengarajan Madhavan 1995]
planar 3-tree				
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Our Results

1. Every planar 3-tree admits a **5**-queue layout
2. There exist planar 3-trees with queue number **4**

Lower bound

Theorem 1

There exist planar 3-trees with queue number **4**

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What does **not** work

- manually testing (small) instances

Lower bound

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- manually testing (small) instances
- computer-assisted exhaustive search of *all* 57,949,430 maximal planar 3-trees with $n \leq 18$

Lower bound

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- computer-assisted exhaustive search of 1,000,000,000 maximal planar 3-trees with $20 \leq n \leq 200$

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What **does** work

- a fully combinatorial proof for a graph with $\approx 10.5 \times 10^{12}$ vertices
- a computer-assisted verified example with 1,038 vertices

Upper bound: Overview

Theorem 2

Every planar 3-tree admits a **5**-queue layout

Upper bound: Overview

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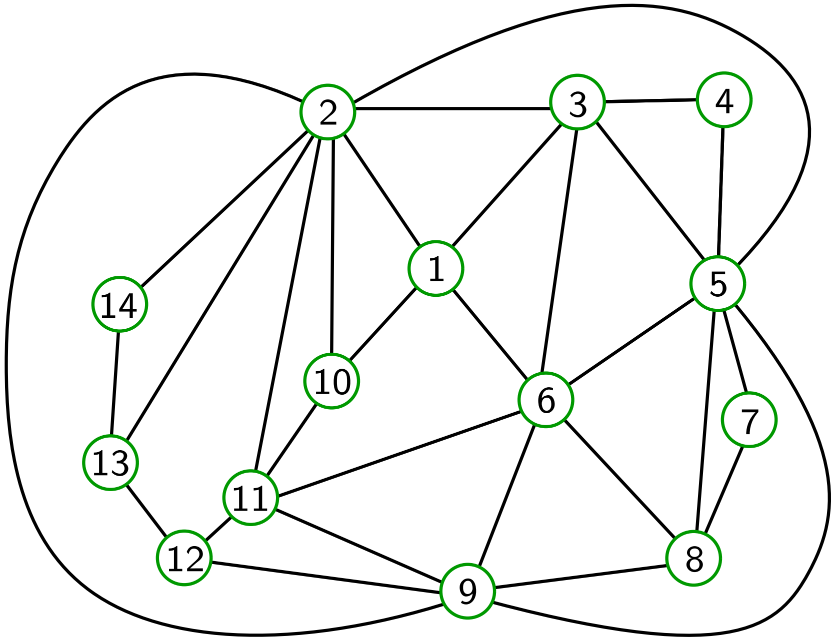

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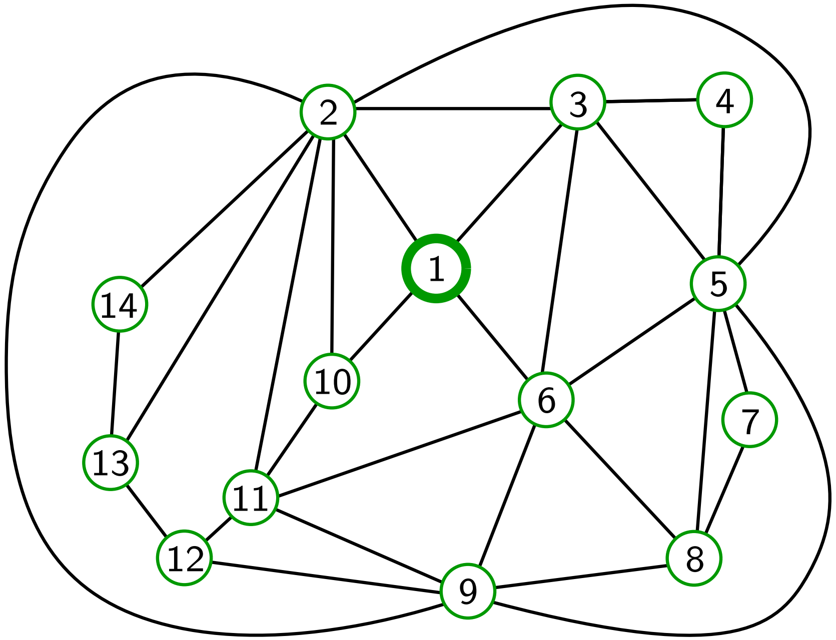

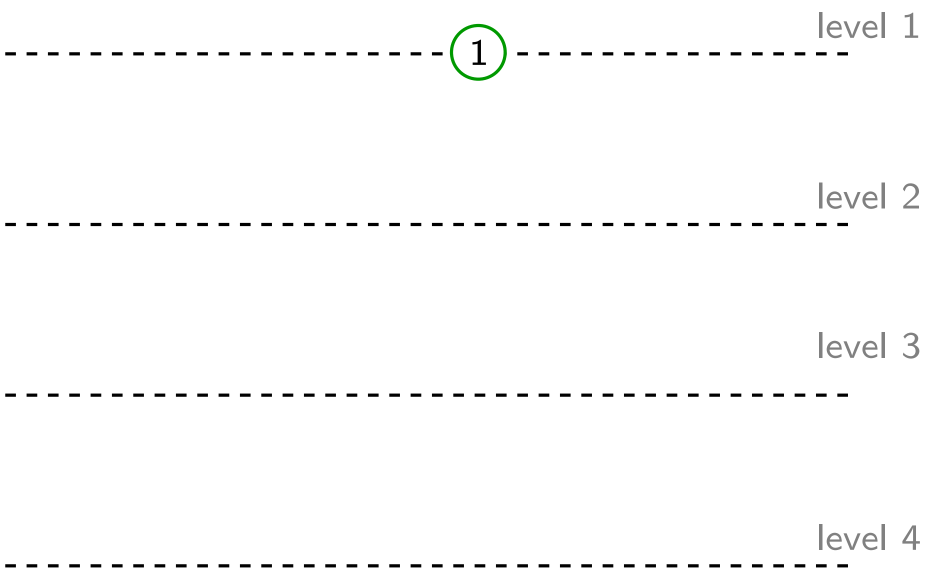

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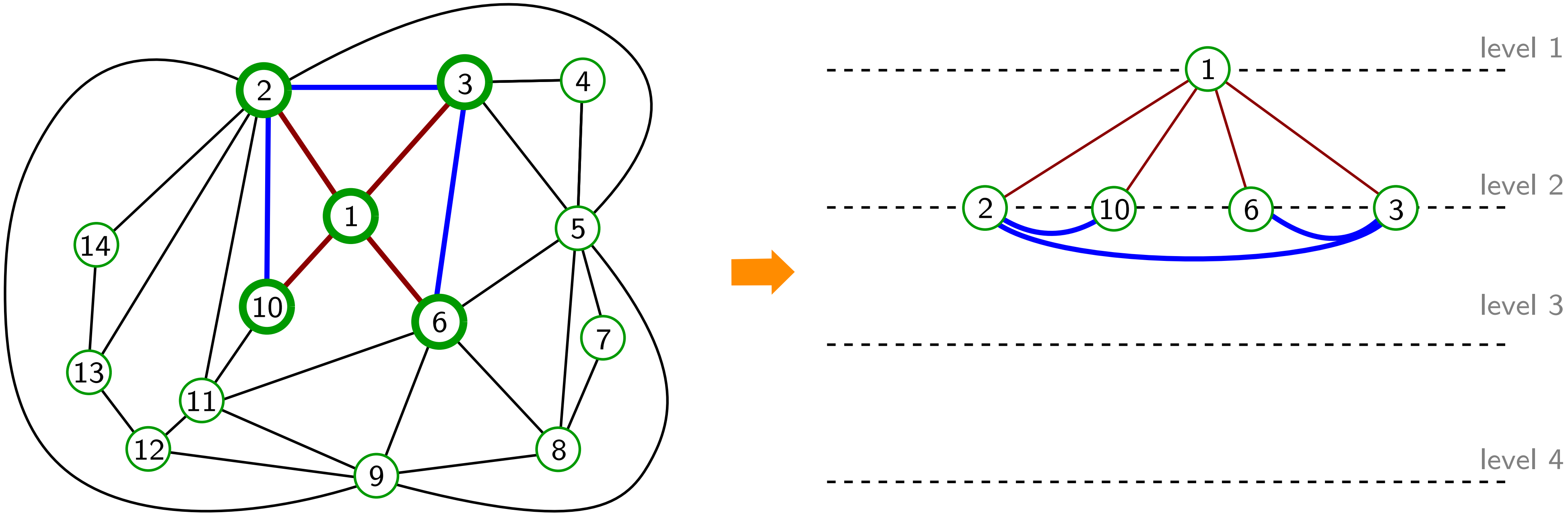

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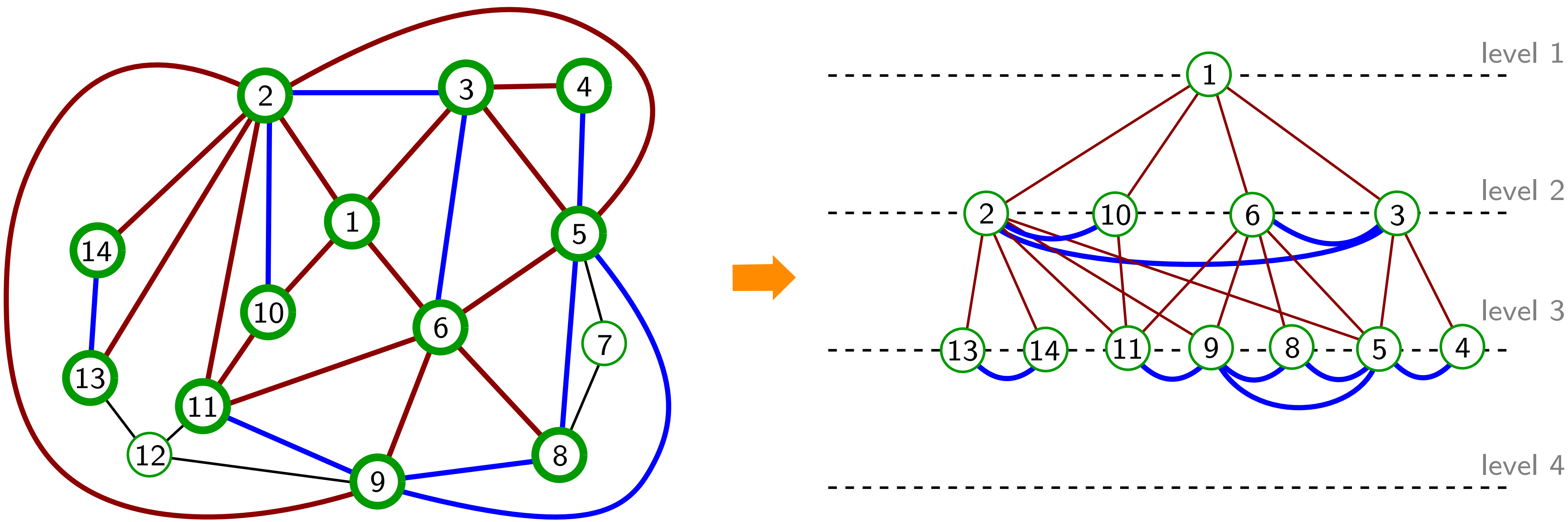

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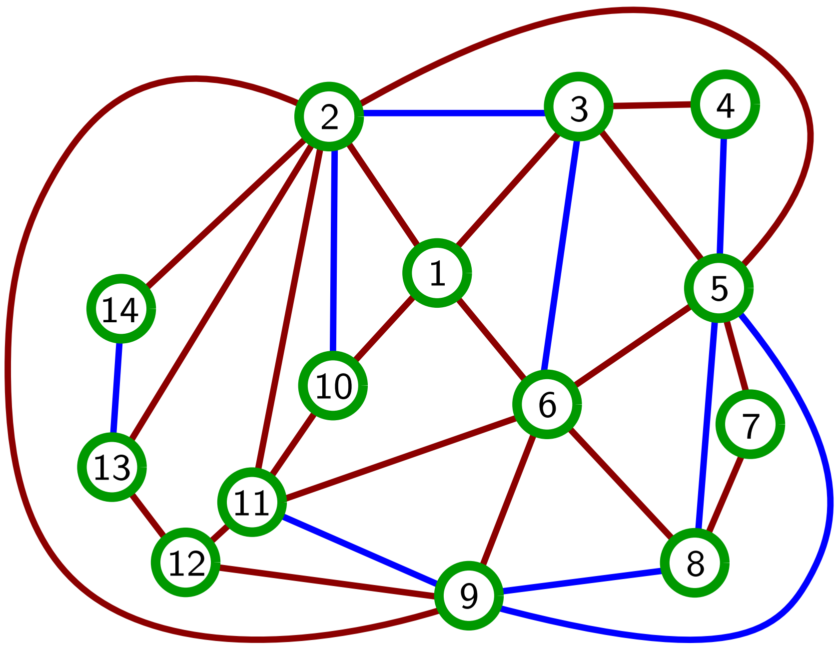

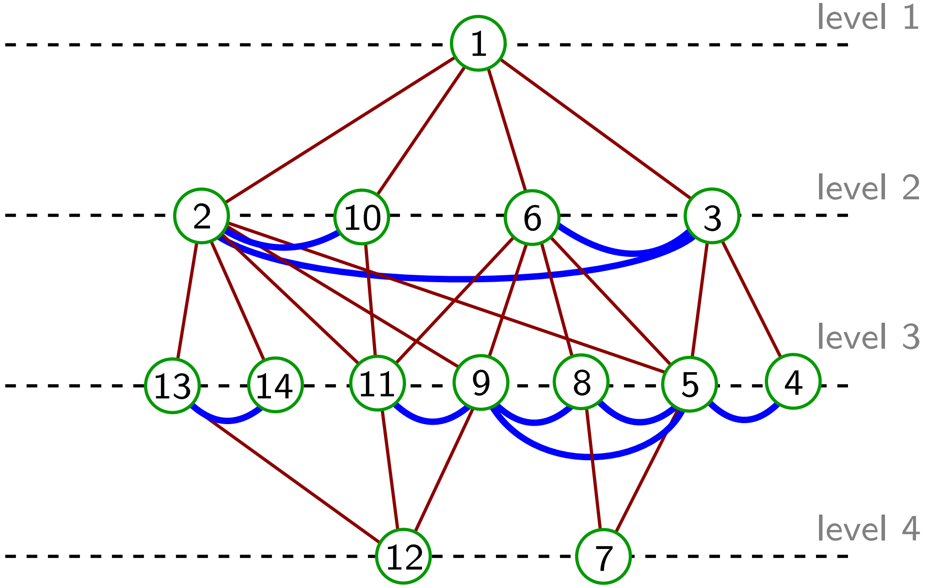

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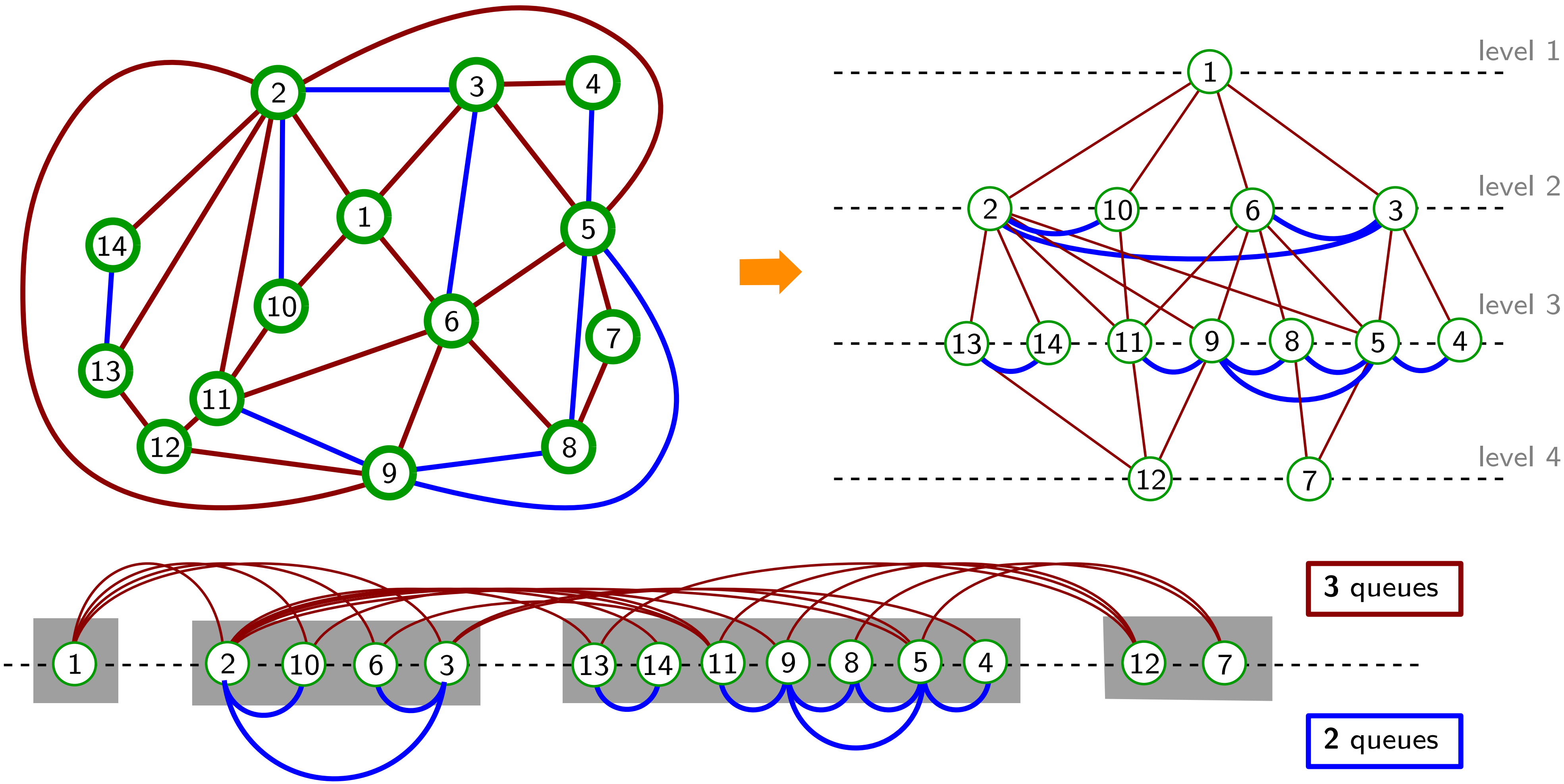
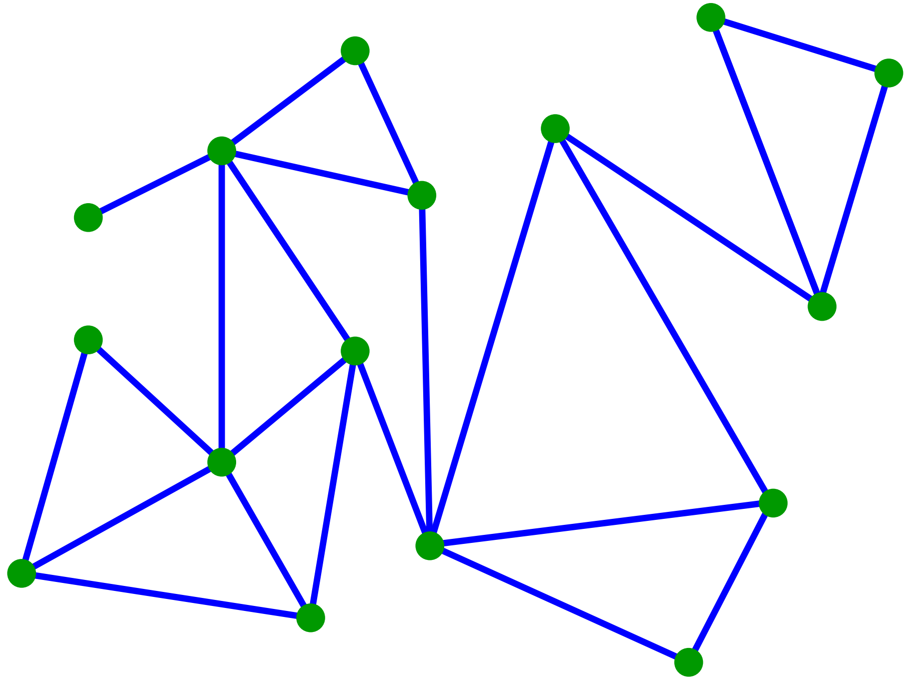


Upper bound: Two levels

Key observations:

- every level is outerplanar



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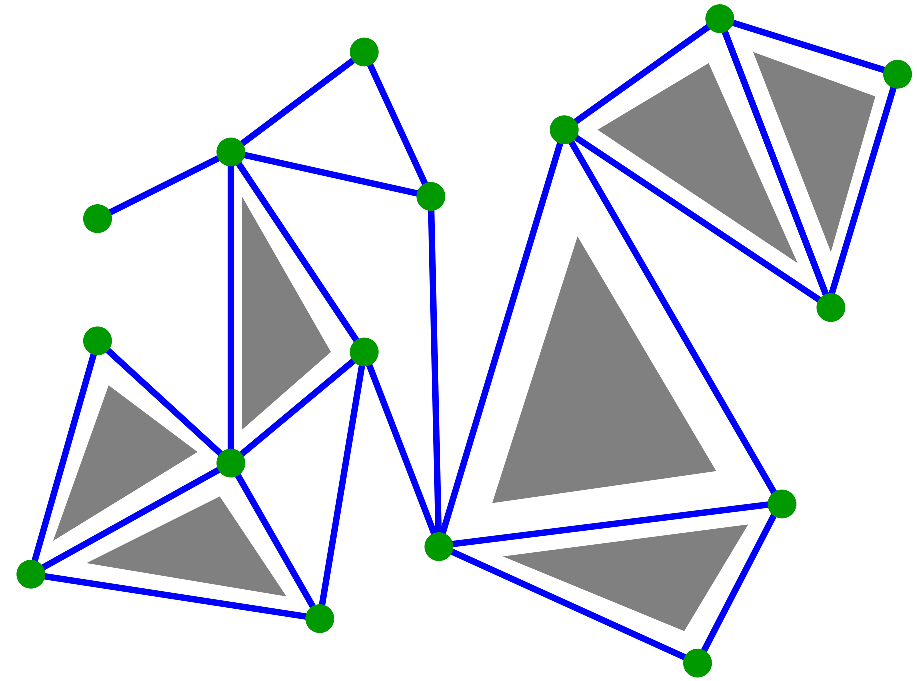

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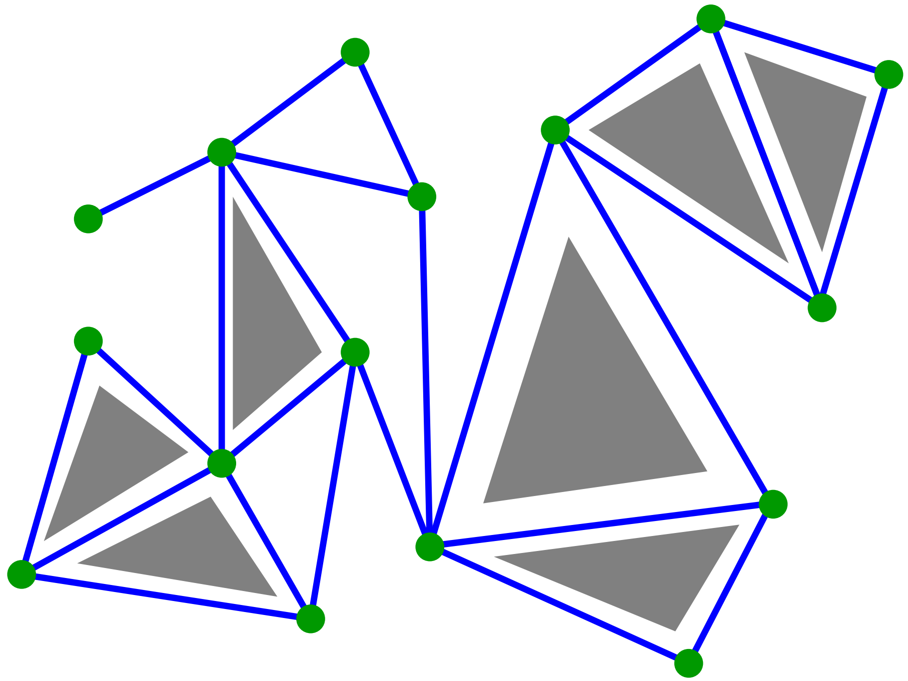

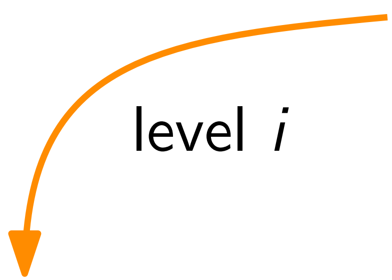

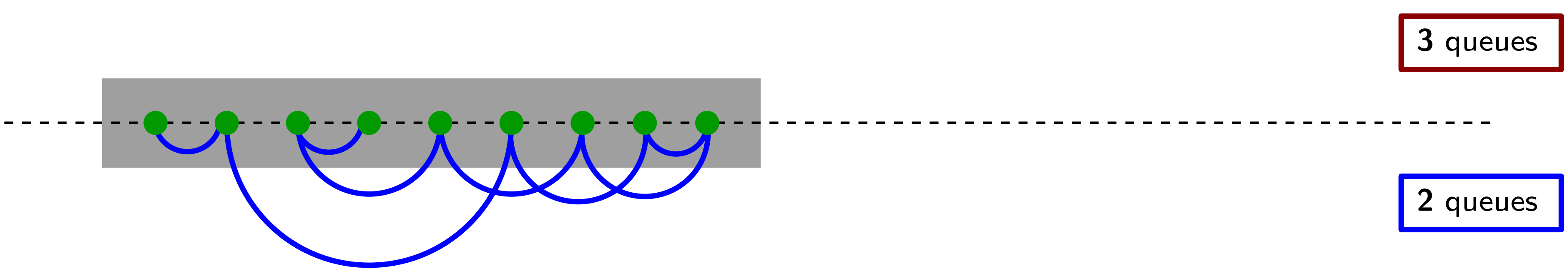

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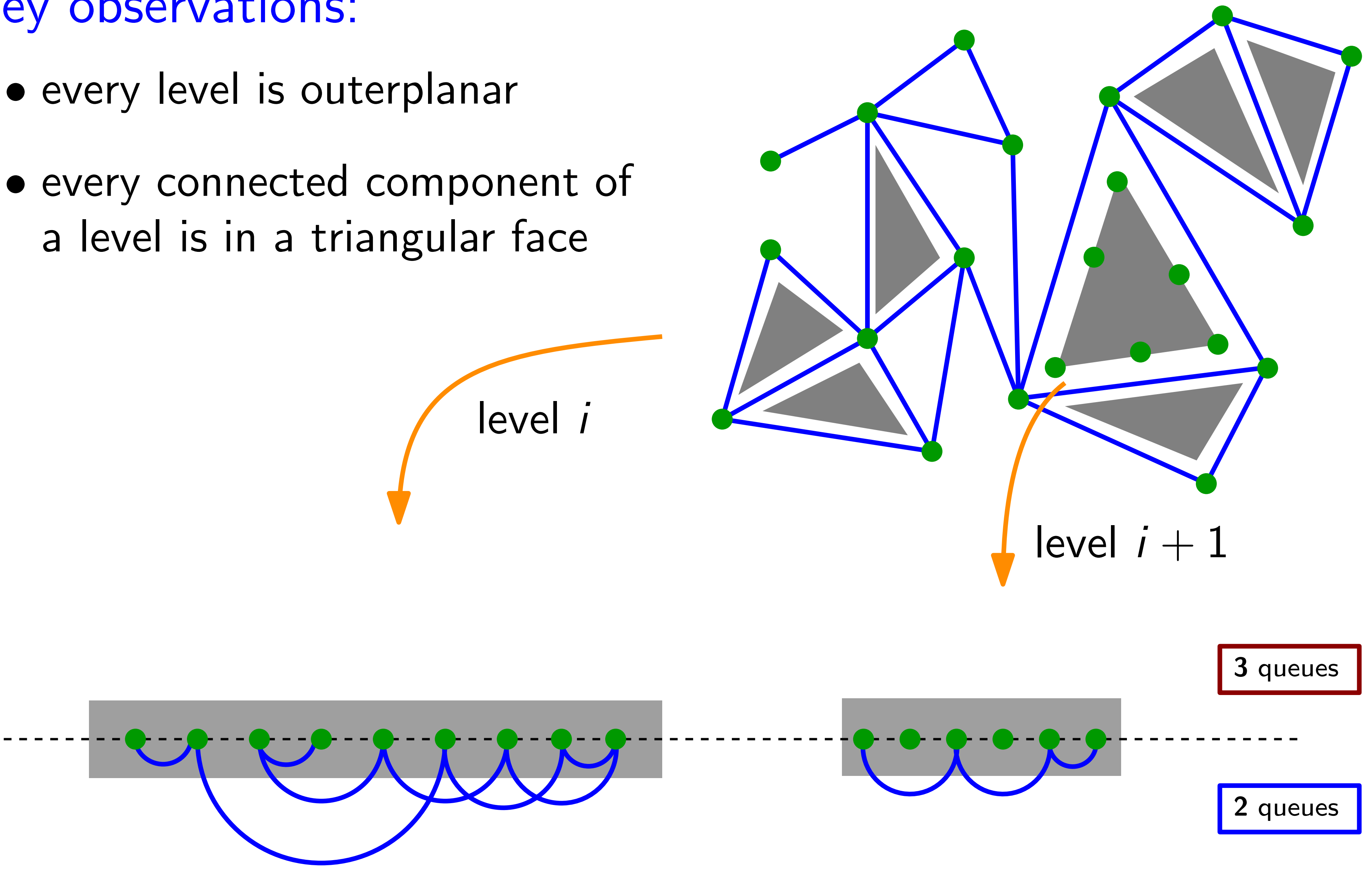

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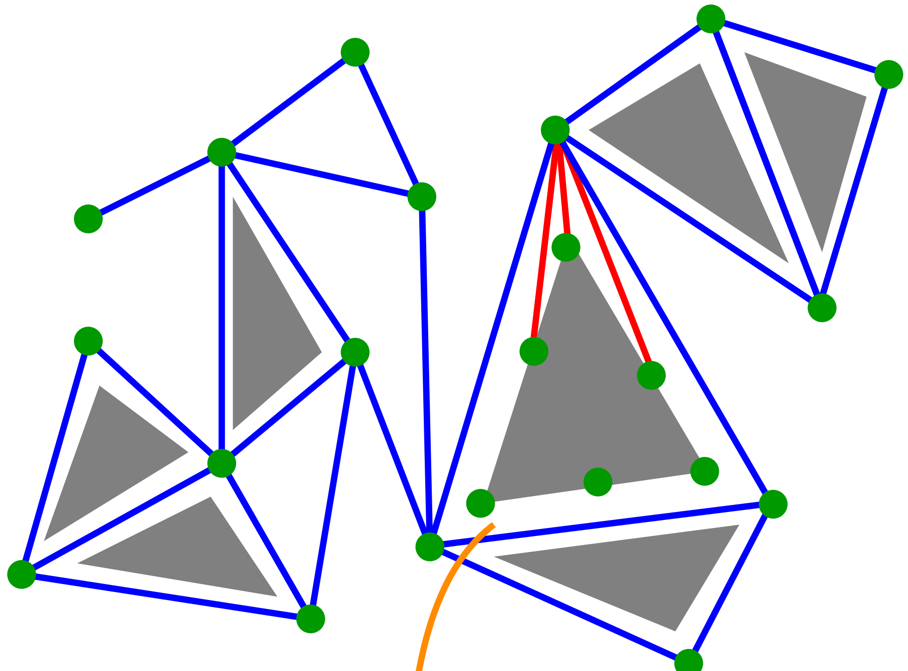

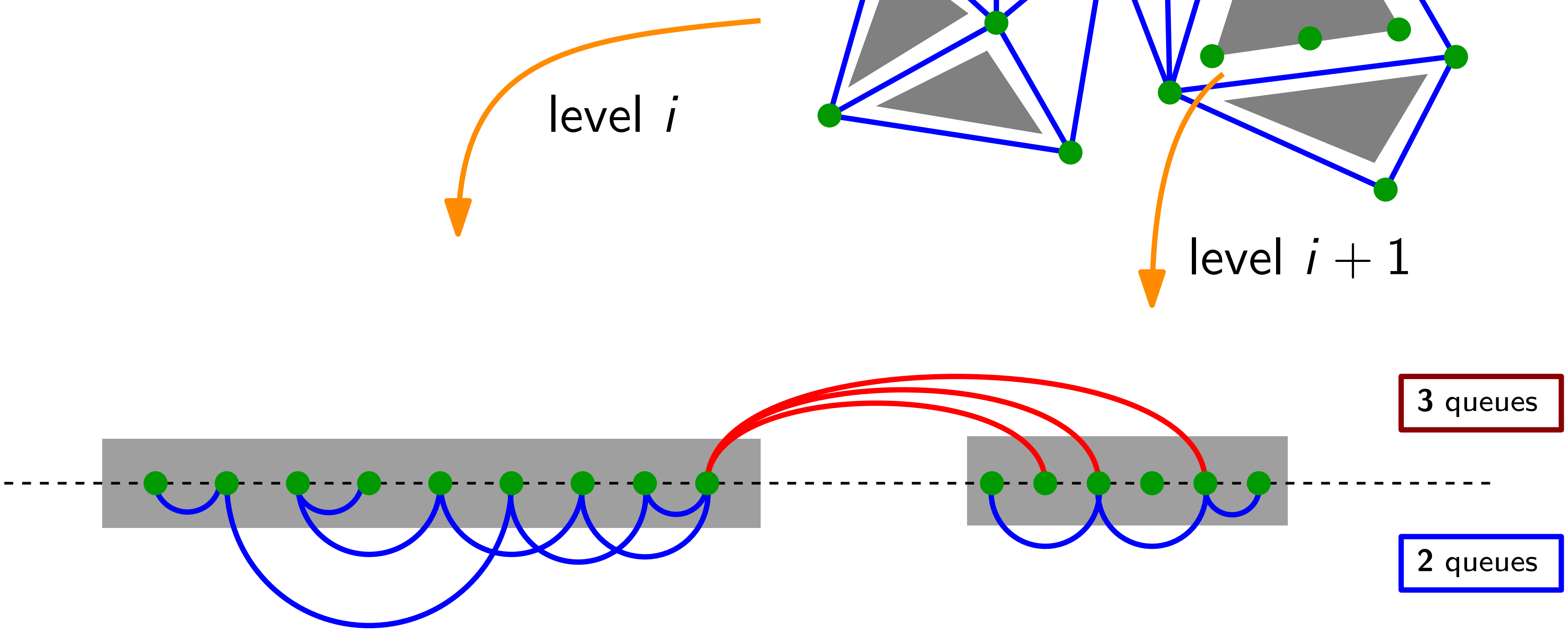

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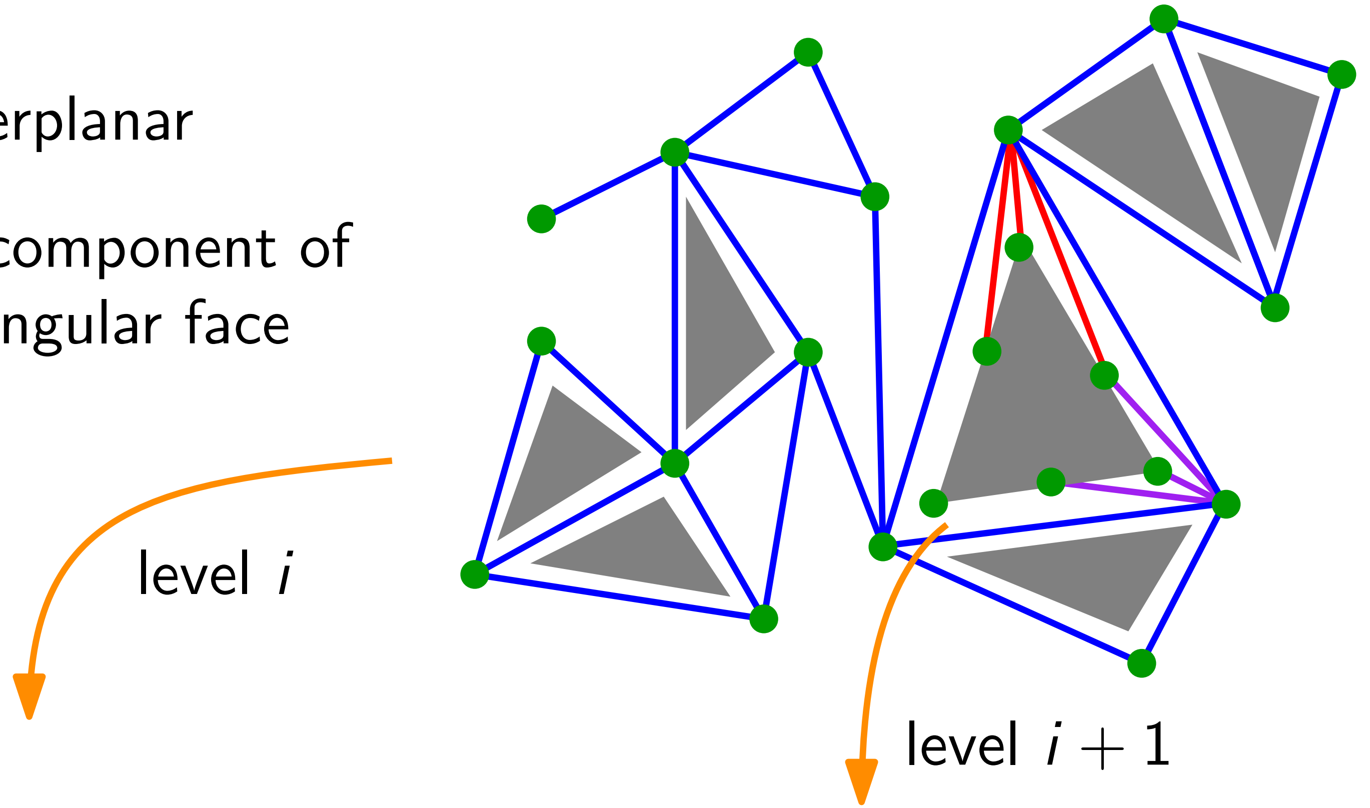

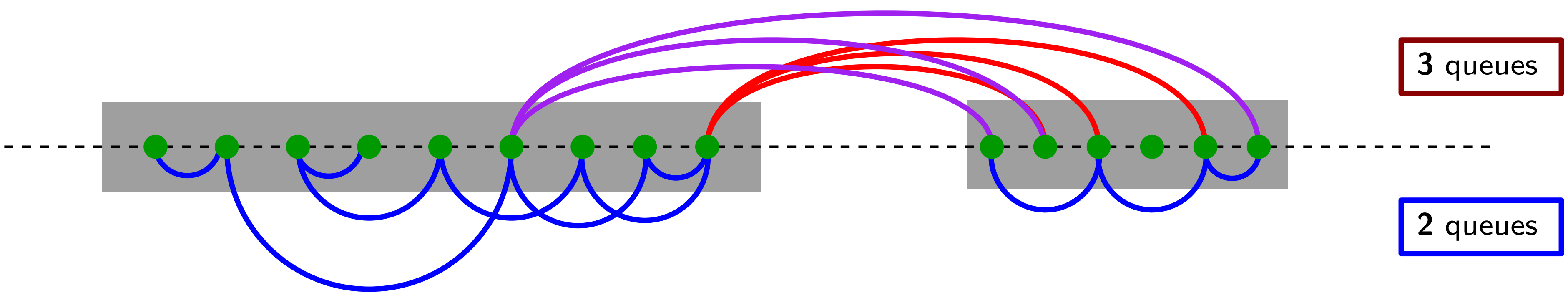

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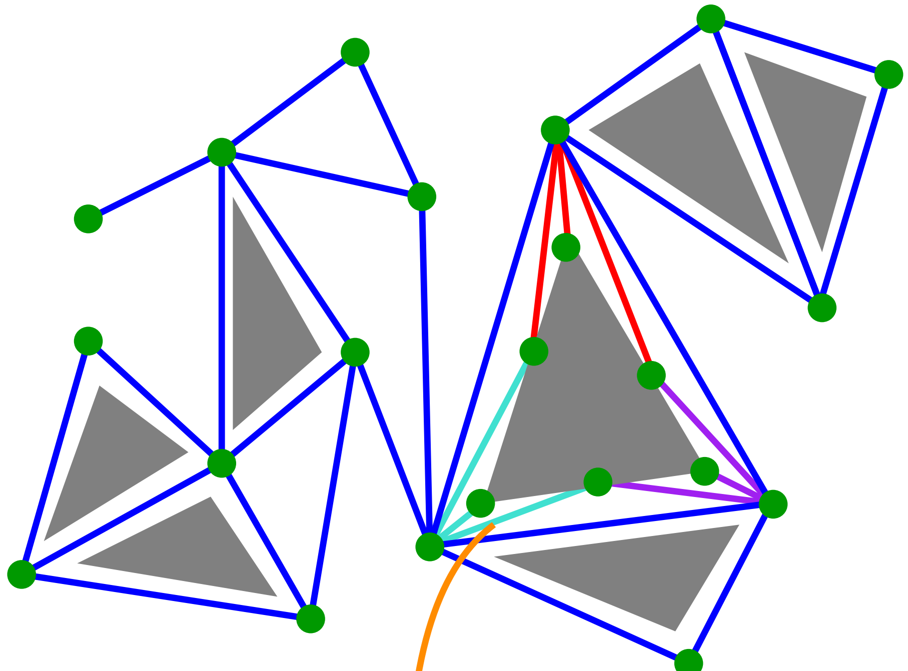
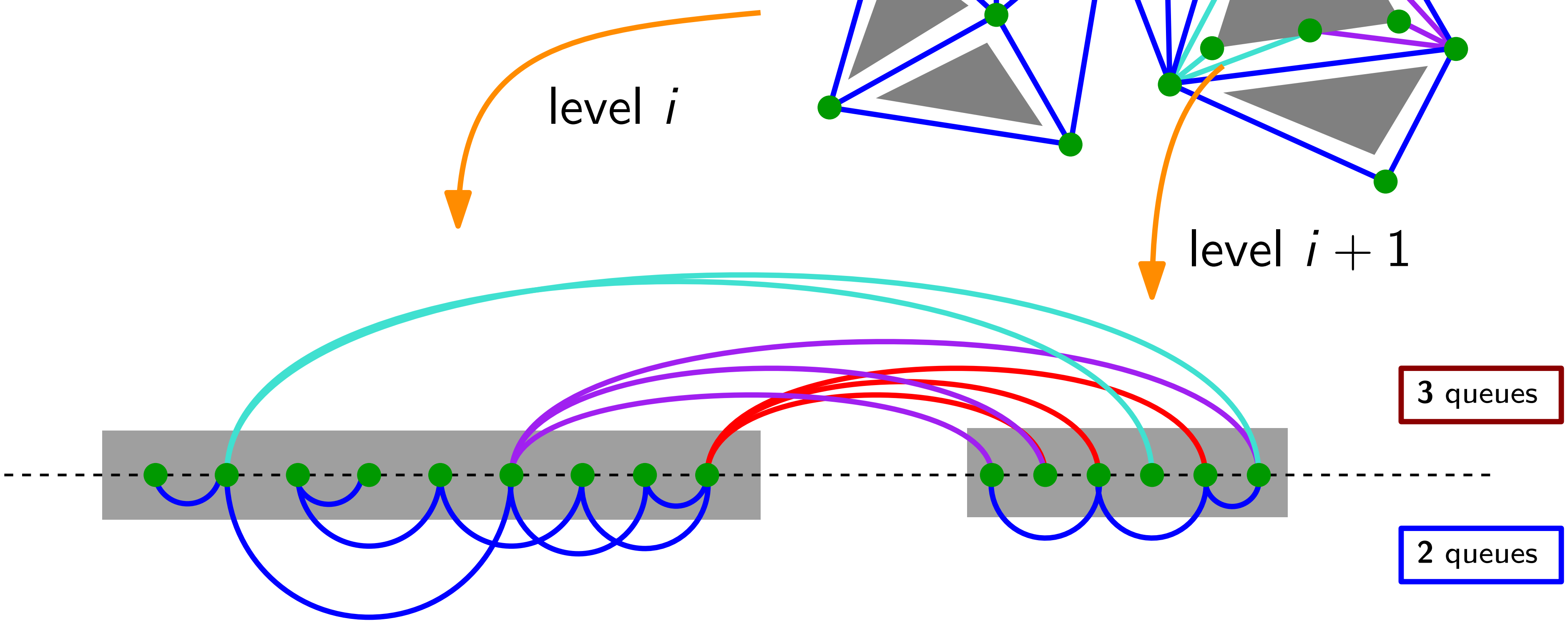
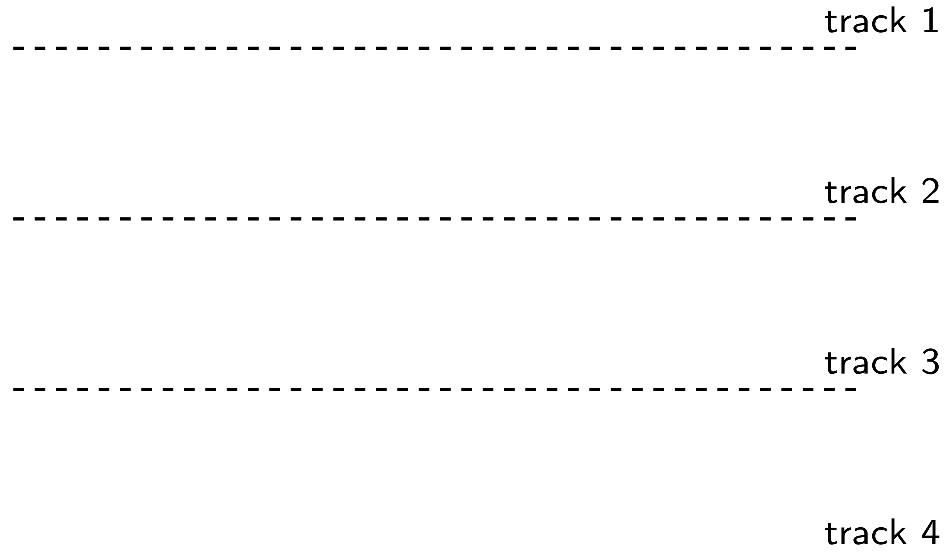




Track number of planar 3-trees

Def. track layout



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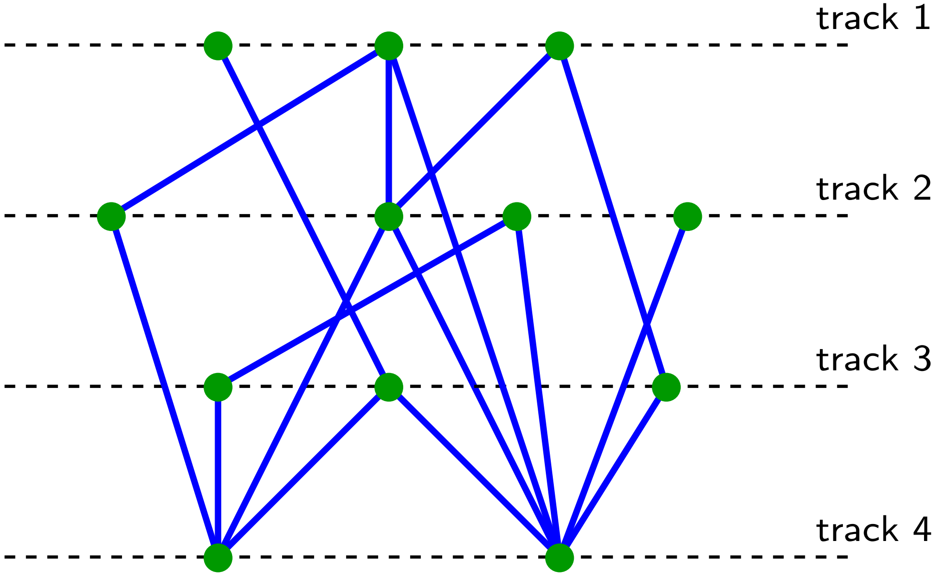

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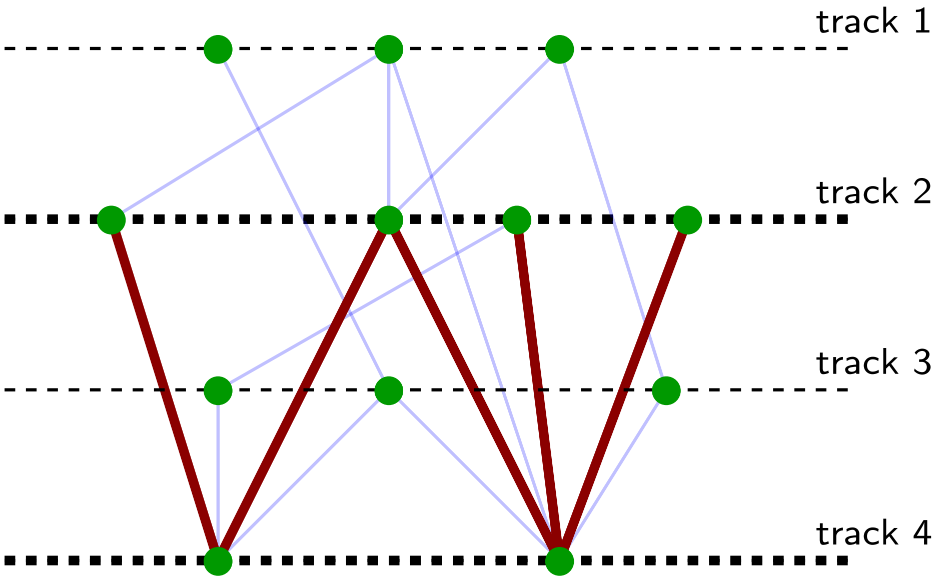

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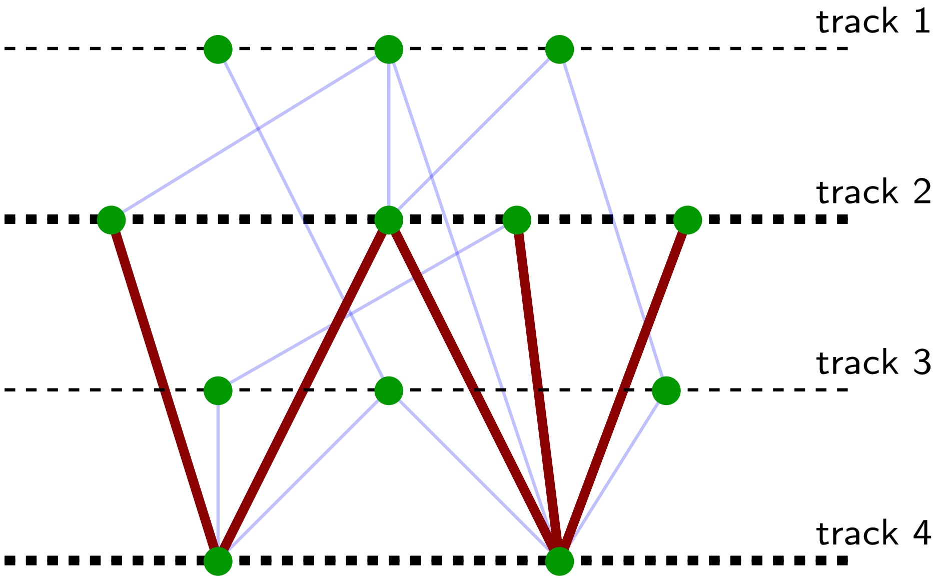

[Dujmović Pór Morin Wood 2004]

- *Every* graph has a $\mathcal{O}(1)$-queue layout iff it has $\mathcal{O}(1)$-track layout
- *Every* graph with track number t has $\mathcal{O}(t)\times\mathcal{O}(t)\times\mathcal{O}(n)$ drawing

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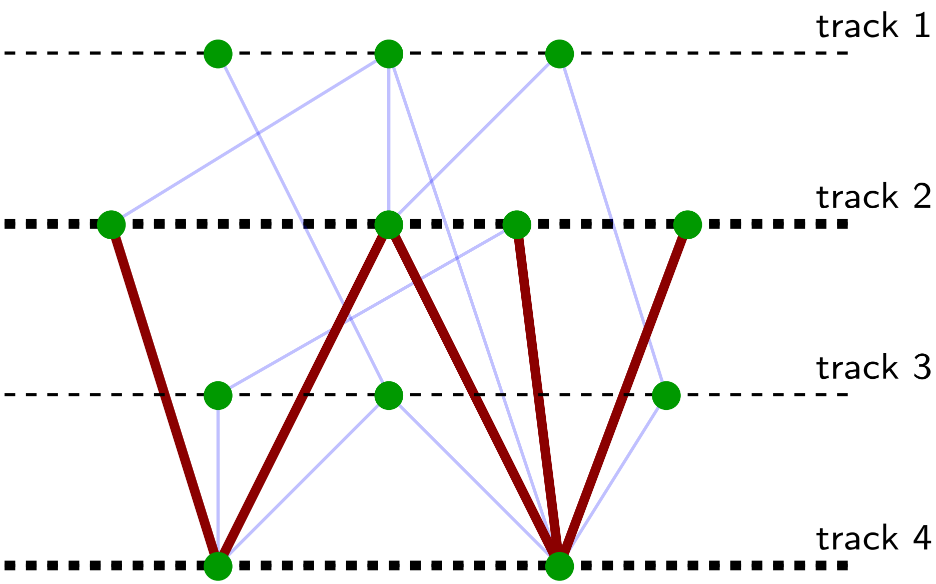

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Theorem 3

The track number of a planar 3-tree is at most 15*

Open Problems

graph class	lower bound	upper bound
tree	**1**	**1**
outerplanar	**2**	**2**
planar 2-tree	**3**	**3**
planar 3-tree	**4**	**5**
planar	**4**	**O**(log n)

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Open Problem 1

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Open Problem 2

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planar	**4**	$\mathbf{O}(\log n)$	Open Problem 2
(non-planar) k-tree	$\mathbf{k}+1$	$2^{\mathbf{k}}-1$	Open Problem 3

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(non-planar) k-tree	$\mathbf{k}+1$	$2^{\mathbf{k}}-1$	Open Problem 3
cubic planar	**2**	$\mathbf{O}(\log n)$	Open Problem 4
bipartite planar	**2**	$\mathbf{O}(\log n)$	Open Problem 5

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bipartite planar	2	$O(\log n)$	Open Problem 5

Questions?