A Flow Formulation for Horizontal Coordinate Assignment with Prescribed Width

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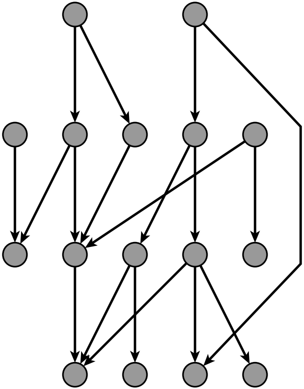
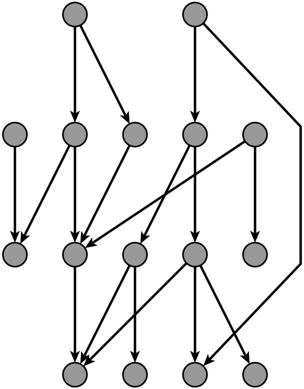
and Network Visualization 2018



Sugiyama framework

- (Cycle breaking)
- Layer assignment
- Crossing minimization
- Coordinate assignment
- (Edge routing)



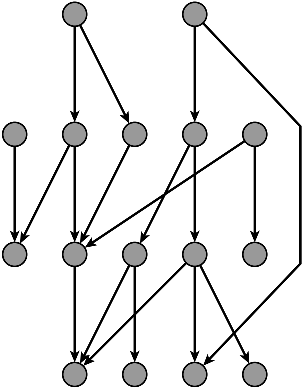
Coordinate assignment

Input:

- DAG G
- Layering $\mathcal{L}$
- Vertex ordering *ord*

Output:

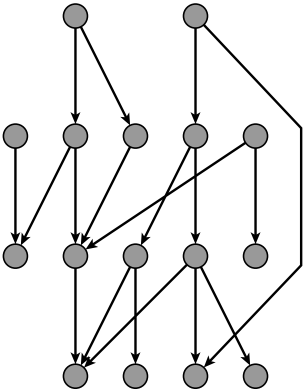
- Feasible x-coordinates, i.e. $x(u) + \delta \leq x(v)$, if $ord(u) < ord(v)$



Coordinate assignment

Goals:

- Short horizontal edge lengths
  →[Gansner et al., 1993]



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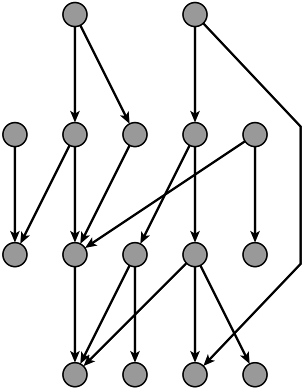
  →[Brandes and Köpf, 2001]

Introduction



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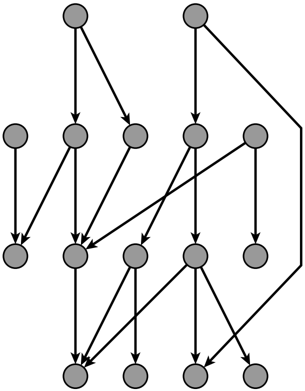
- Vertical edges



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What about the width?

Introduction

- Width of the layering:
  maximum number of nodes in a layer →[Coffman and Graham, 1972]

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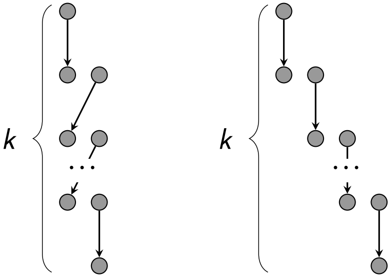
width $= 1$
edge length $= k-3$



width $= k-2$
edge length $= 0$

Our Contribution

Efficient algorithm for drawings with minimized total edge length under the restriction of bounded drawing width.

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In addition it realizes

- vertical inner segments
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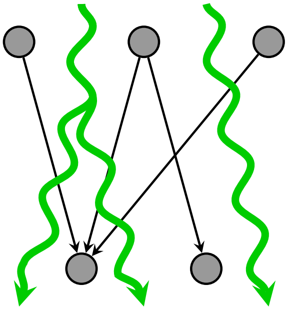
Method: Minimum cost flow formulation
for the coordinate assignment problem

Network Construction



Idea:

- Network with super source and super sink
- Send flow from top to bottom

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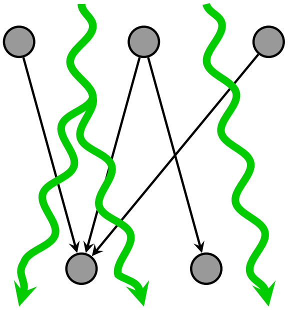
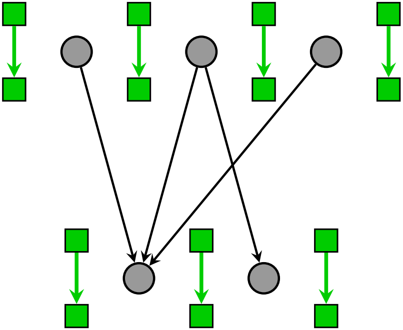


Idea:

- Network with super source and super sink
- Send flow from top to bottom
- Flow = horizontal distance
- Cost = edge length

Network Construction



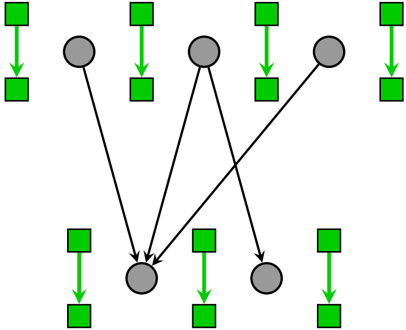
Minimum distance δ between nodes

$+$

Flow means distance

$\Rightarrow$ Network arcs between nodes with lower bound δ

Network Construction

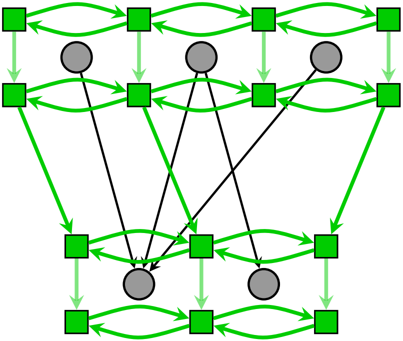


⇒ x-coordinate for node v:

$$x(v) = \sum_{a \text{ left of } v} f(a),$$

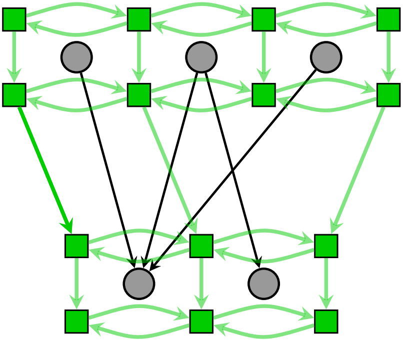
where $f(a)$ is the flow over network arc a

Network Construction



Send flow along layers and
over some inter-layer connections
without lower or upper bounds

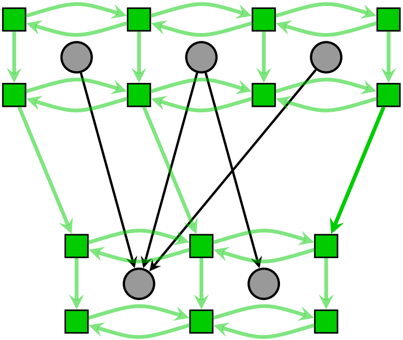
Network Construction



Connections between layers:

- arc from leftmost upper node to leftmost lower node

Network Construction



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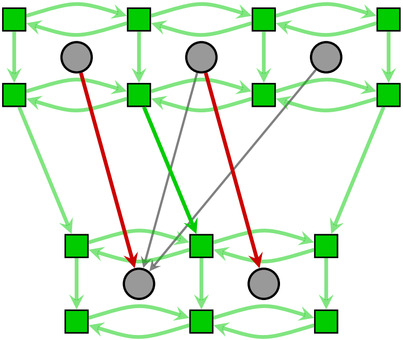

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- in *hug* situations: two network nodes enclosed by graph edges

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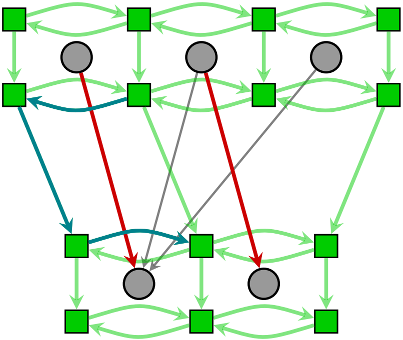

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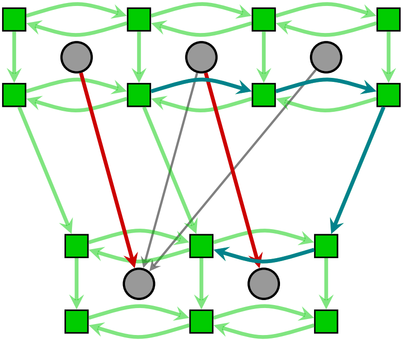

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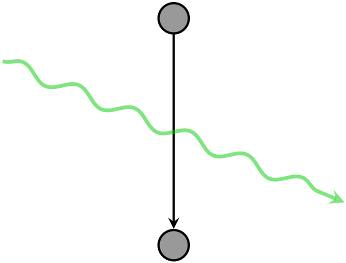
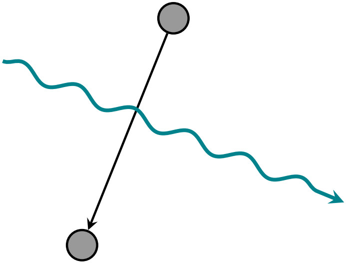


Cost corresponds to edge length
and flow corresponds to distance

Network Construction



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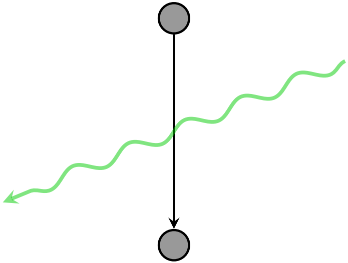

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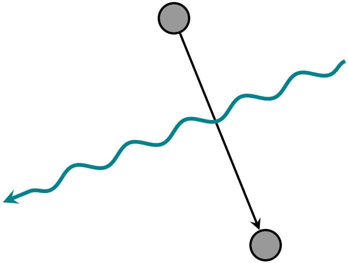

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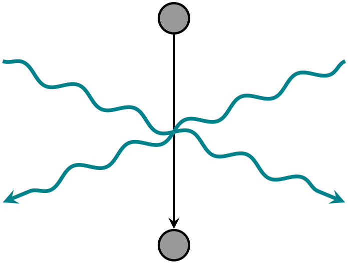
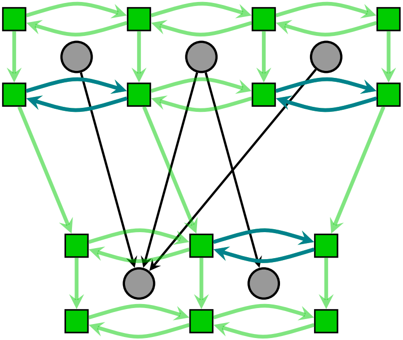


Cost corresponds to edge length
and flow corresponds to distance

$\Rightarrow$ cost = number of crossed edges

Network Construction

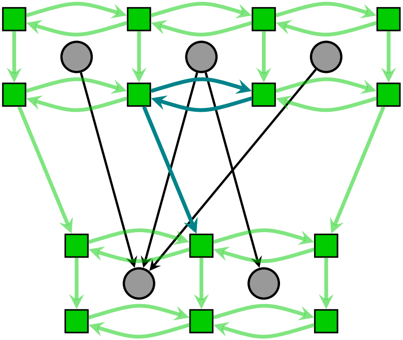


$\text{cost} = 1$

$\text{lower bound} = 0$

$\text{upper bound} = \infty$

Network Construction

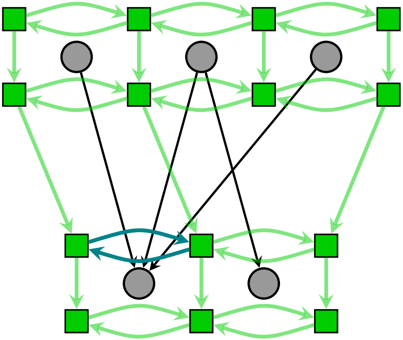


$\text{cost} = 2$

$\text{lower bound} = 0$

$\text{upper bound} = \infty$

Network Construction

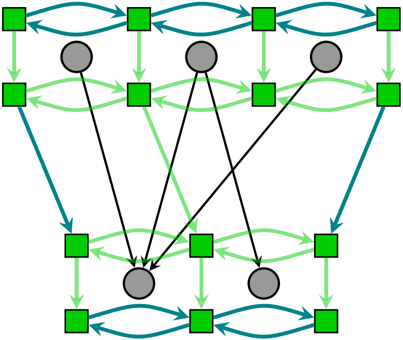


cost = 3

lower bound = 0

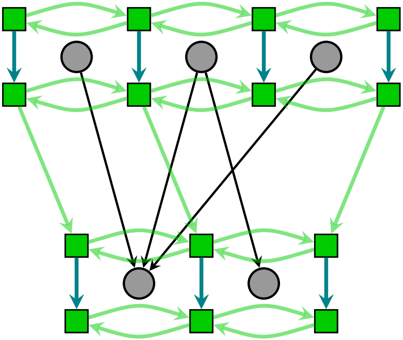
upper bound = ∞

Network Construction



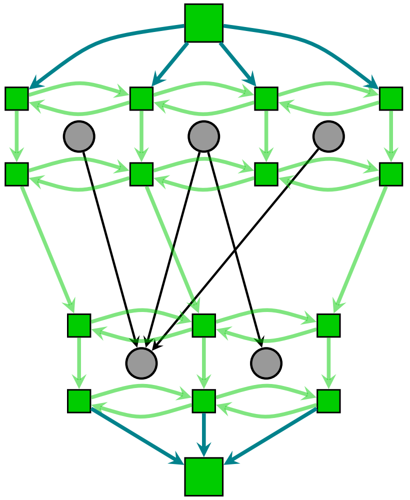
$$\text{cost} = 0$$
$$\text{lower bound} = 0$$
$$\text{upper bound} = \infty$$

Network Construction



$$\text{cost} = 0$$
$$\text{lower bound} = \delta \text{ (or 0)}$$
$$\text{upper bound} = \infty$$

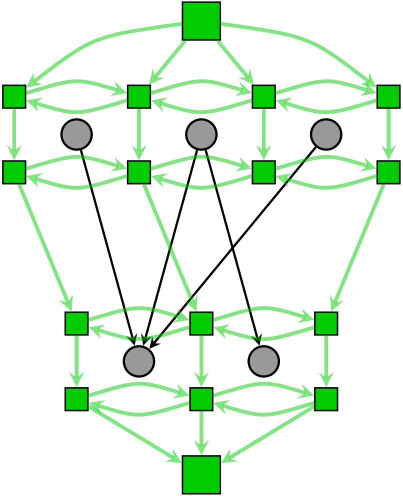
Network Construction



super source and super sink

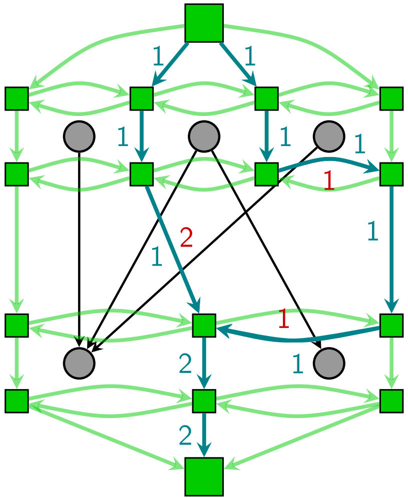
$$\text{cost} = 0$$
$$\text{lower bound} = 0$$
$$\text{upper bound} = \infty$$

Example Flows and Corresponding Drawings



$\delta = 1$
flow in blue
arc cost in red

Example Flows and Corresponding Drawings



$$\sum \text{cost} \cdot \text{flow} = 2 \cdot 1 + 1 \cdot 1 + 1 \cdot 1$$
$$= 4$$

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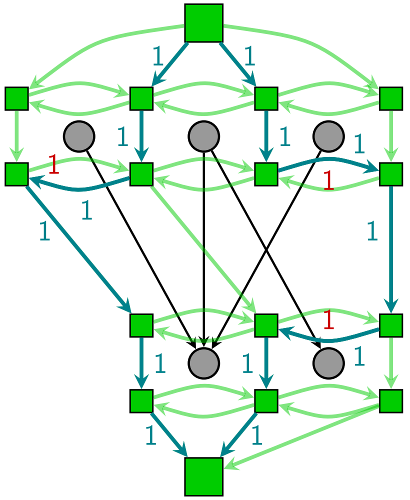

$$\sum \text{cost} \cdot \text{flow} = 2 \cdot 1 + 1 \cdot 1$$
$$= 3$$

Results

Main Theorem

Every minimum cost flow in network described above defines feasible x-coordinates and minimum total horizontal edge length.

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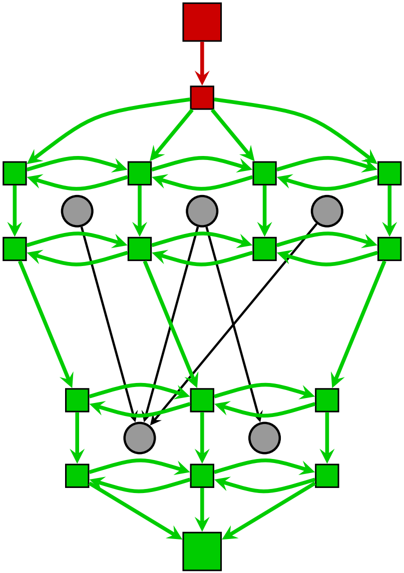
What about other contraints?

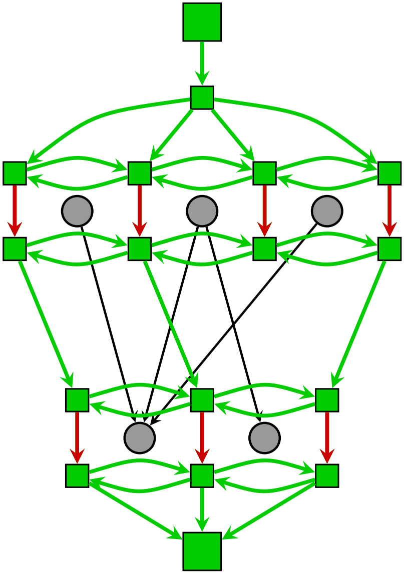
Further constraints



Width:

split super source and
set upper bound of new arc
to maximum width

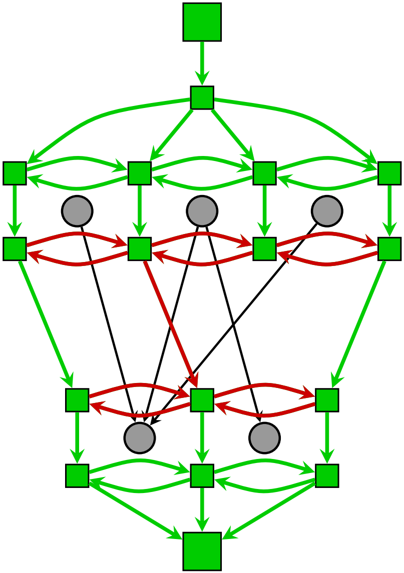
Further constraints



Maximum/exact distances:

set bounds to
appropriate values

Further constraints

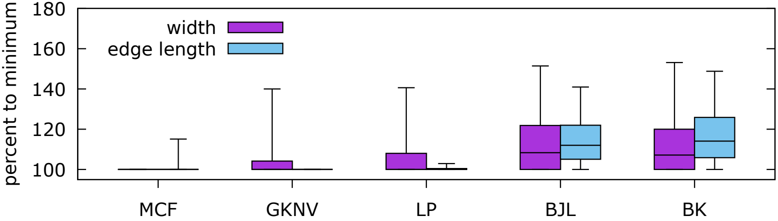


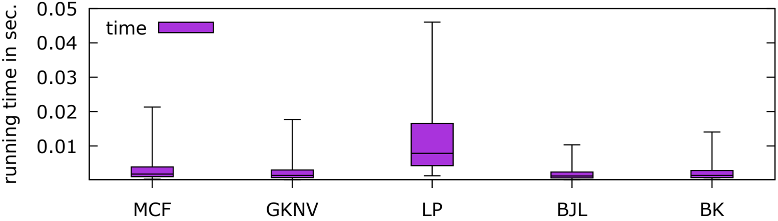
Vertical edges:

delete appropriate arcs

Computational results

Test set: 1277 graphs, |V| = 10-100 (subset of AT&T)
Implementation: OGDF, network simplex algorithm





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- Minimize edge length while respecting further constraints
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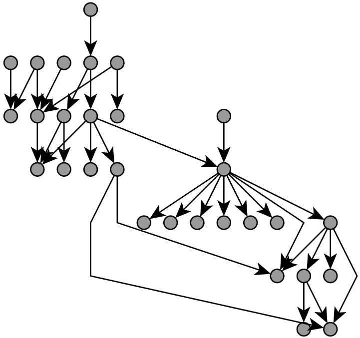
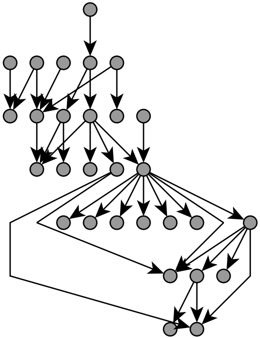


GKNV
width: 13, edge length: 54



MCF
width: 9, edge length: 58

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