

Drawing Subcubic 1-Planar Graphs with Few Bends, Few Slopes, and Large Angles



Philipp Kindermann
Cheriton School of Computer Science
University of Waterloo



Joint work with
Fabrizio Montecchiani,
Lena Schlipf, and André Schulz

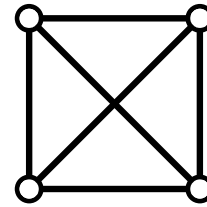


Slope Number

“How many slopes do I need to draw a graph straight-line?”

Slope Number

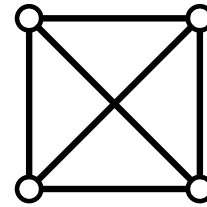
“How many slopes do I need to draw a graph straight-line?”



4 slopes

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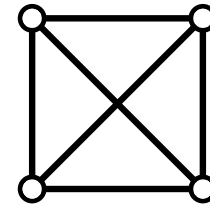


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“How many slopes do I need to draw a graph **planar** straight-line?”

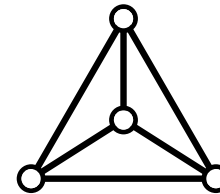
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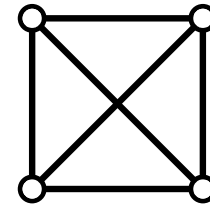
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6 slopes

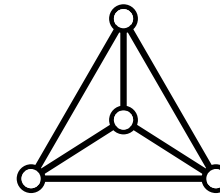
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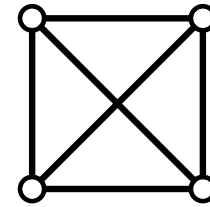


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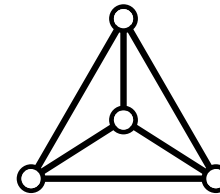
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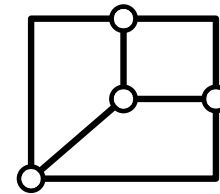
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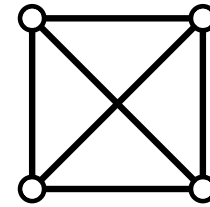
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1 bend,
3 slopes

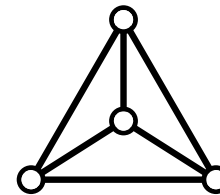
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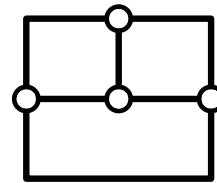
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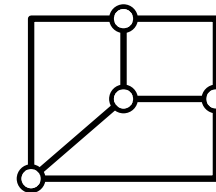


6 slopes

“How many slopes do I need to draw a graph **planar with k bends?**”



2 bend,
2 slopes

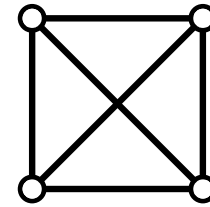


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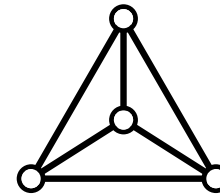
maximum degree Δ

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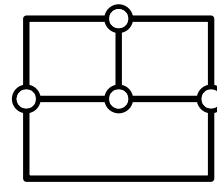
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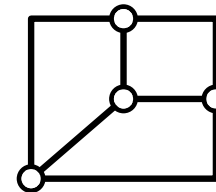


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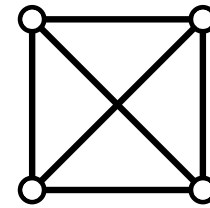
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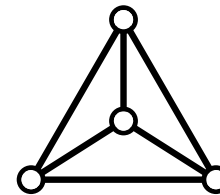
“How many slopes do I need to draw a graph straight-line?”

[Barát, Matousek, Wood 2006]
[Pach, Pálvölgyi 2006]
unbounded w.r.t. Δ



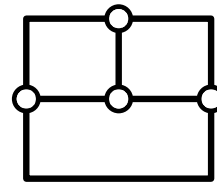
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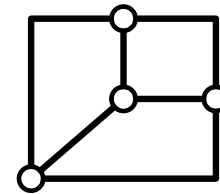


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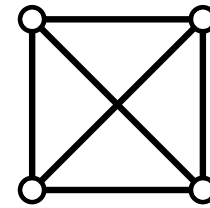
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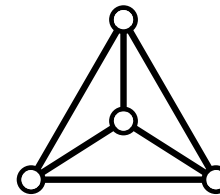
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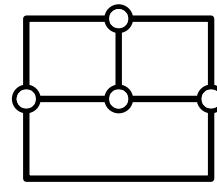
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[Keszegh, Pach, Pálvölgyi 2013]
bounded by $2^{O(\Delta)}$

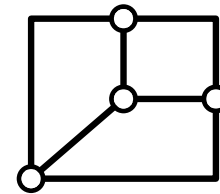


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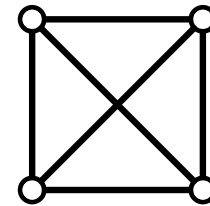
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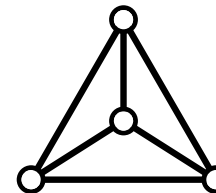
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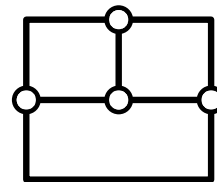
[Keszegh, Pach, Pálvölgyi 2013]
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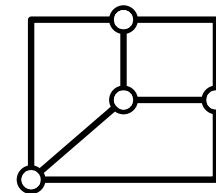
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“How many slopes do I need to draw a graph **planar with k bends?**”

[Keszegh, Pach, Pálvölgyi 2013]
2 bends: $\leq \lceil \Delta/2 \rceil$



2 bend,
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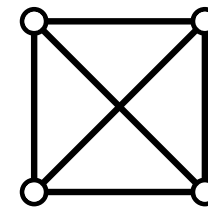
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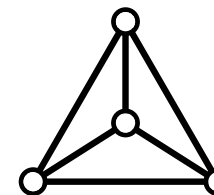
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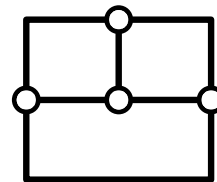
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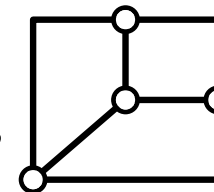
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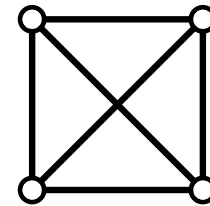
[Angelini, Bekos, Liotta, Montecchiani 2017]
1 bend: $\leq \Delta - 1$

Slope Number

maximum degree Δ

“How many slopes do I need to draw a graph straight-line?”

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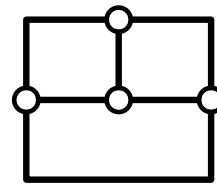
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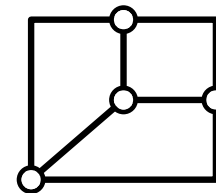
[Di Giacomo, Liotta, Montec. 2018]
subcubic: ≤ 4

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2 bends: $\leq \lceil \Delta/2 \rceil$



2 bend,
2 slopes



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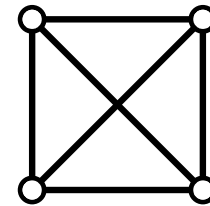
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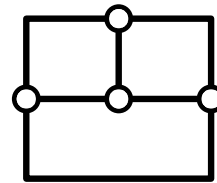
“How many slopes do I need to draw a graph^{1-?}planar straight-line?”

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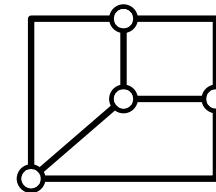
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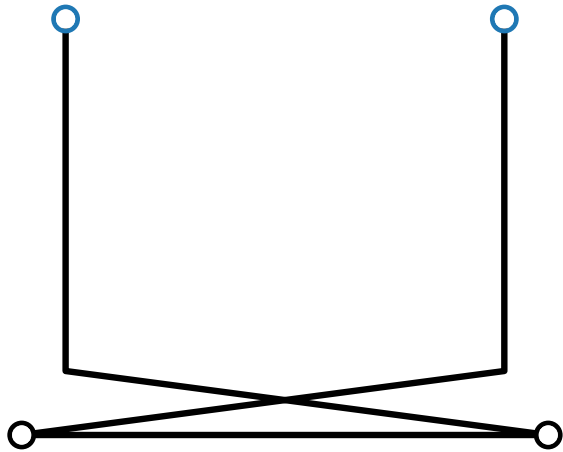
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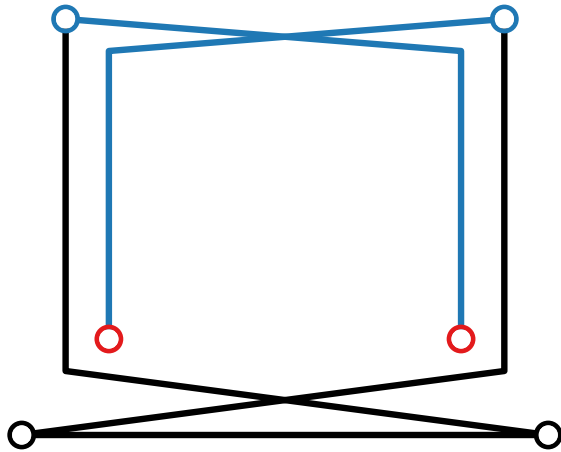
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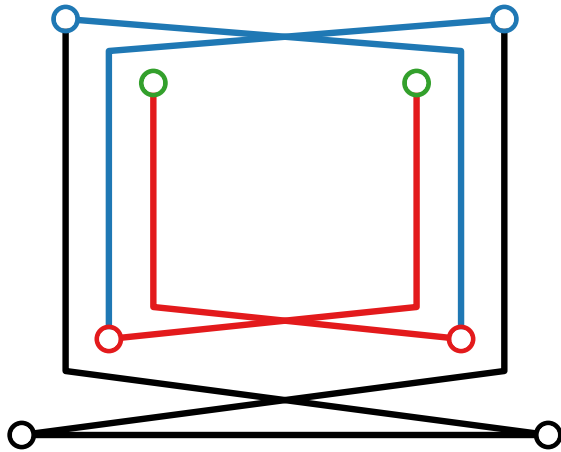
0 Bends, Lower Bounds



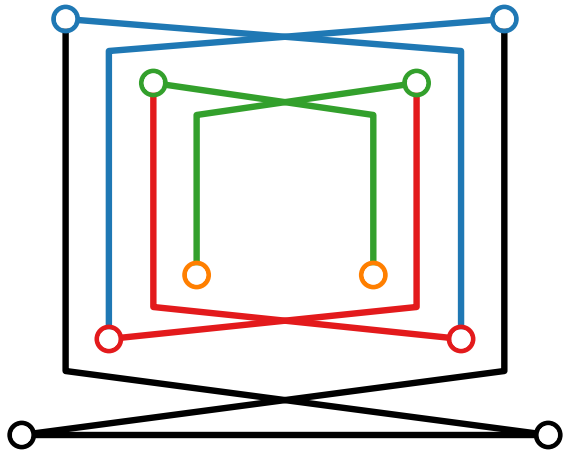
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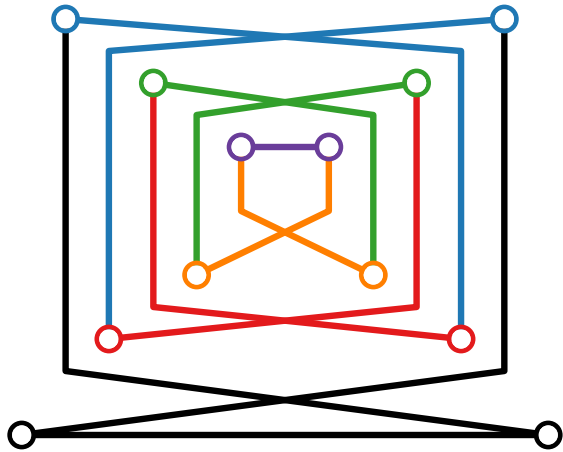
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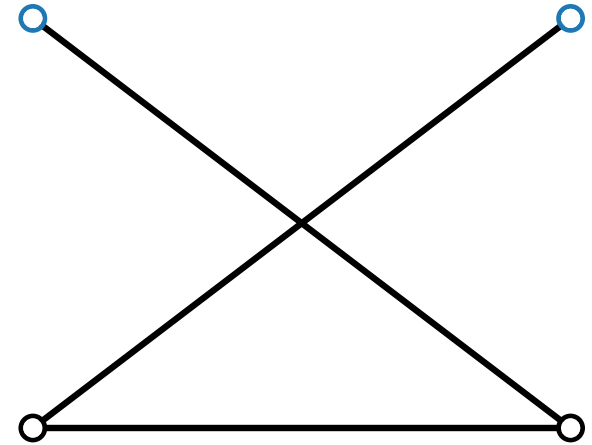
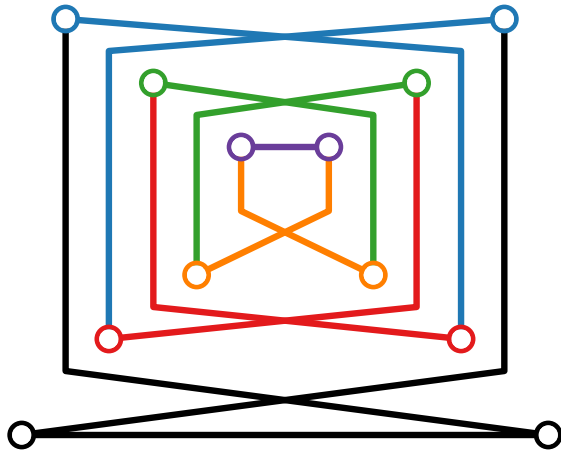
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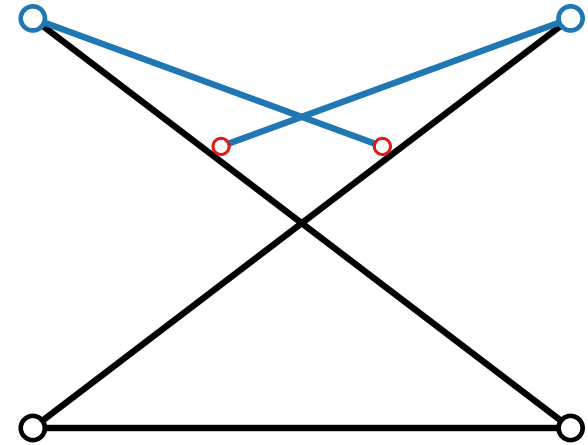
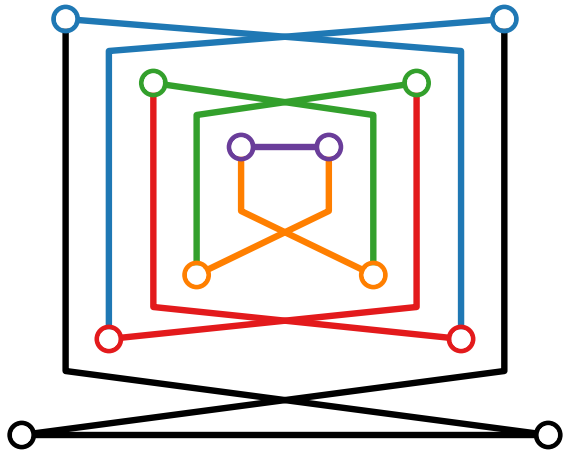
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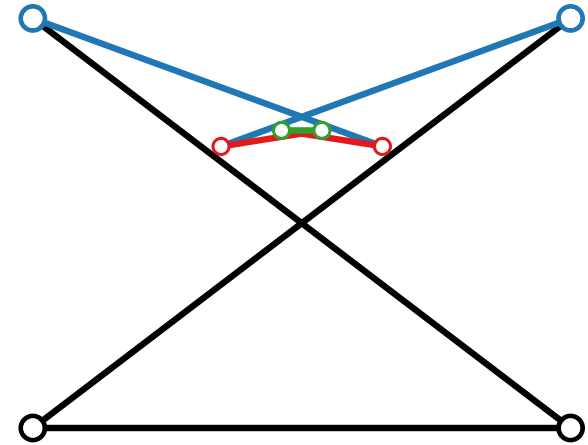
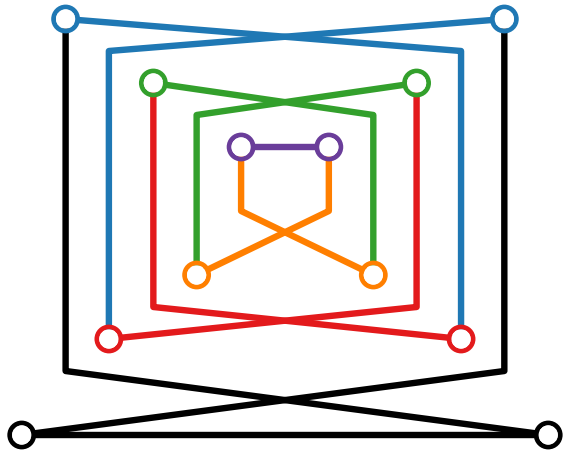
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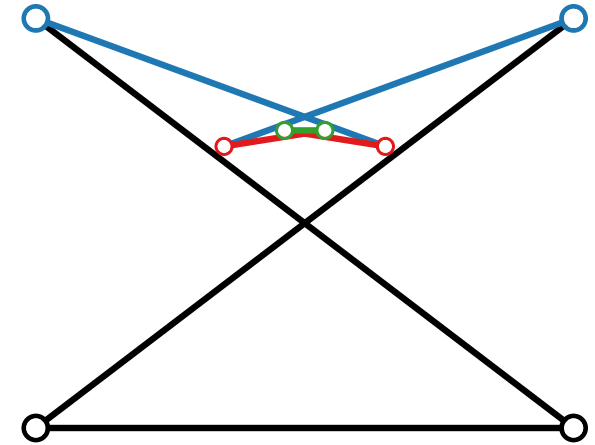
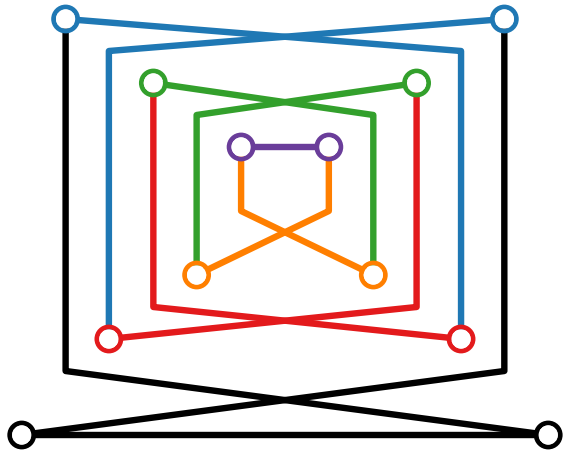
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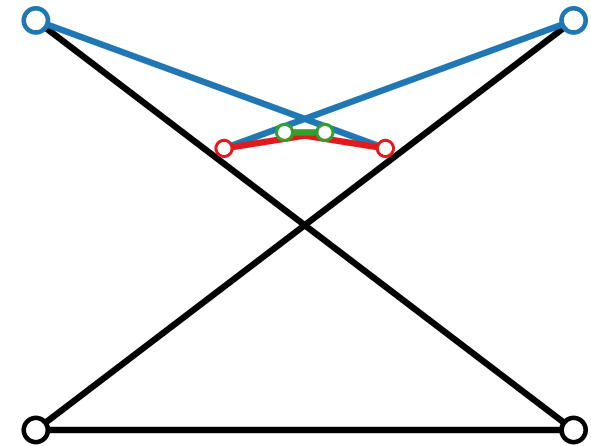
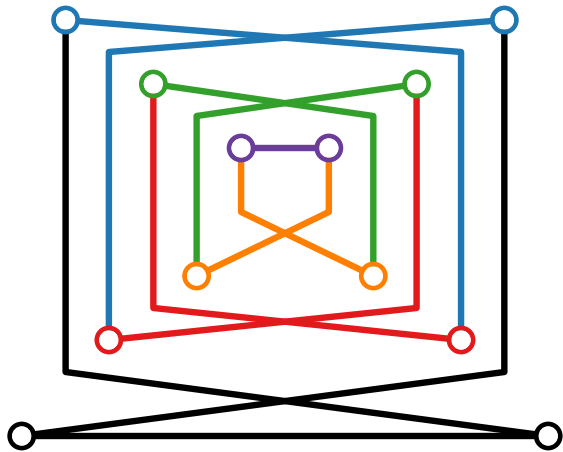


0 Bends, Lower Bounds

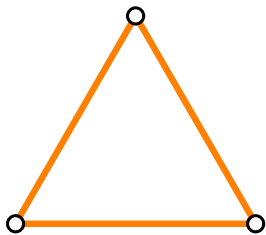


2-regular 2-connected 1-plane $\Rightarrow \Omega(n)$ slopes

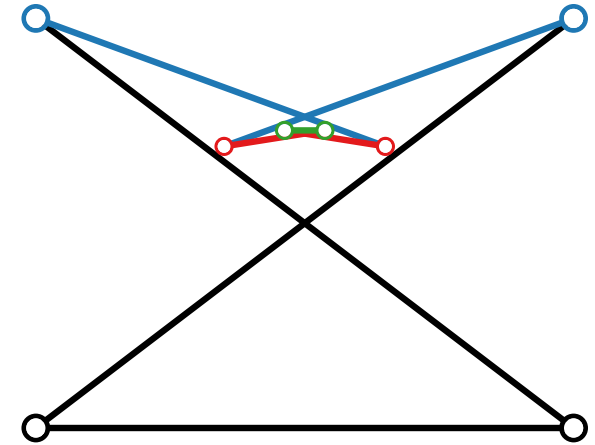
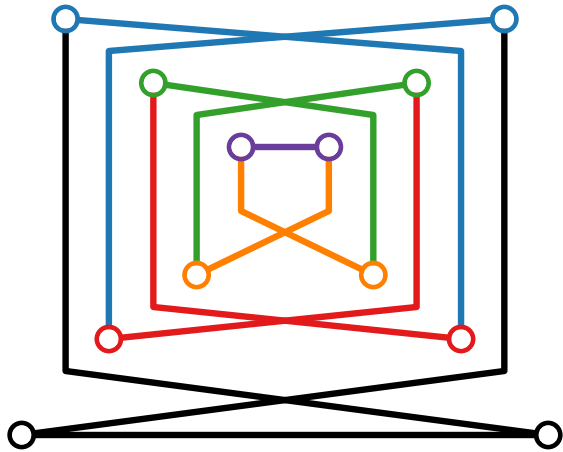
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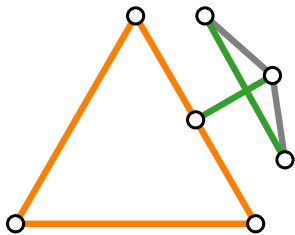
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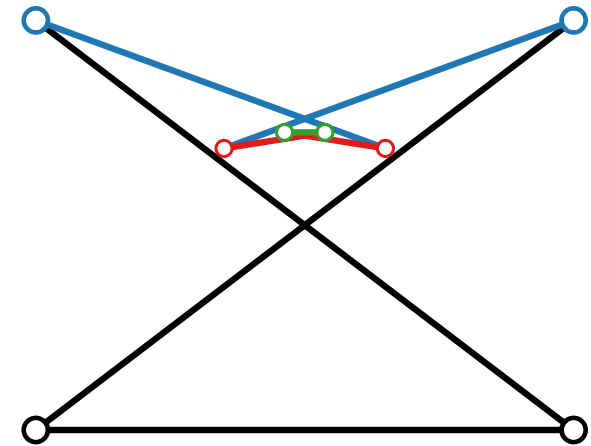
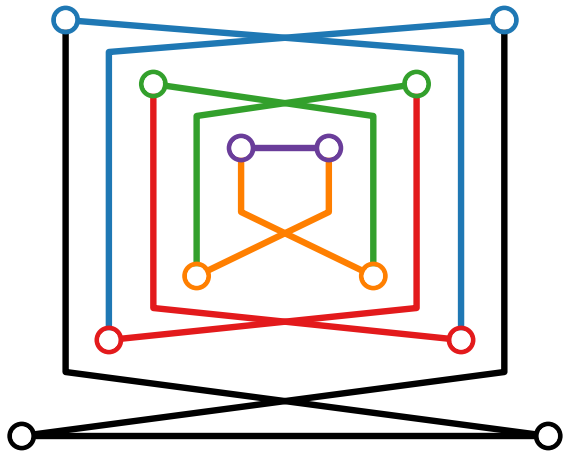
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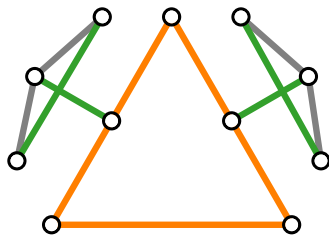
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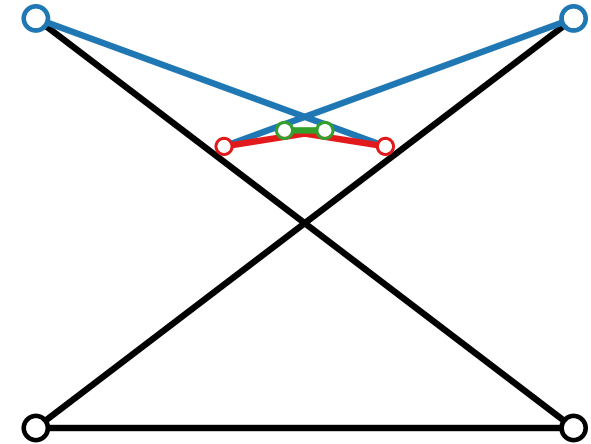
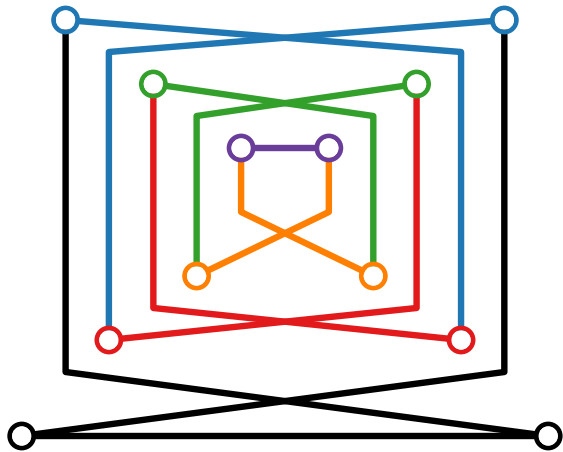
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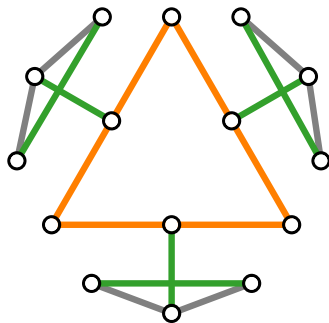
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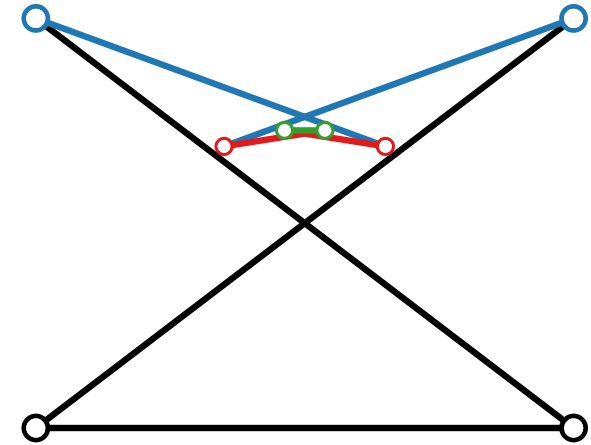
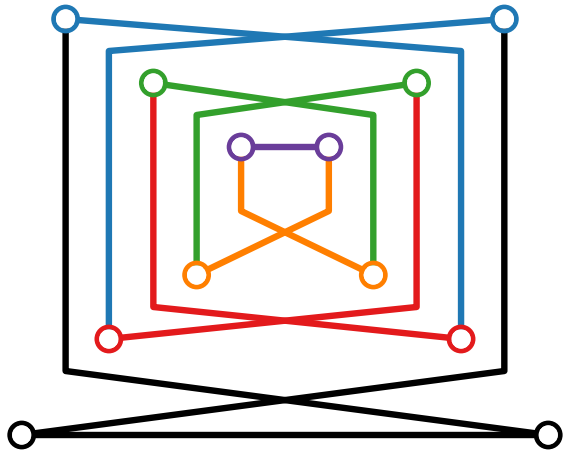
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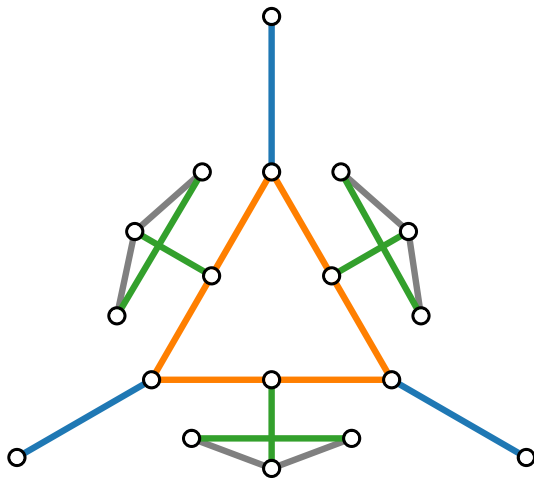
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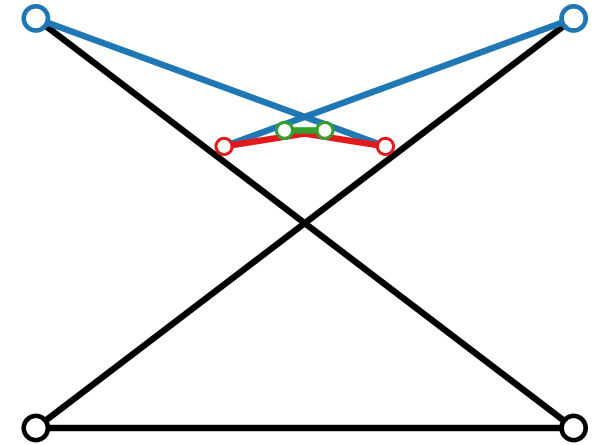
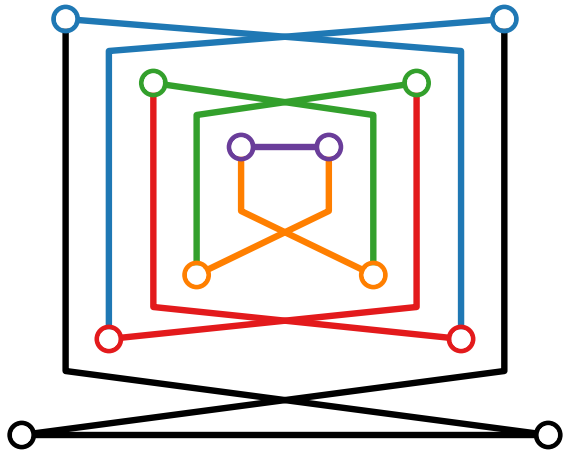
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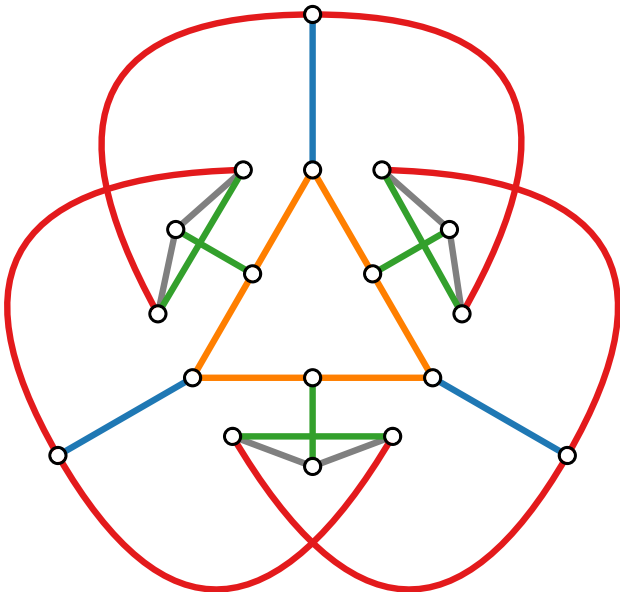
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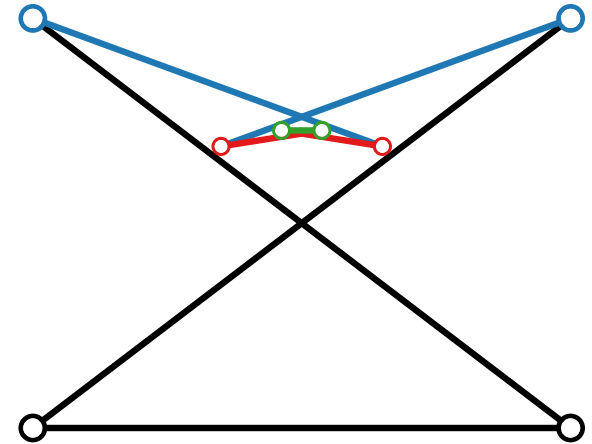
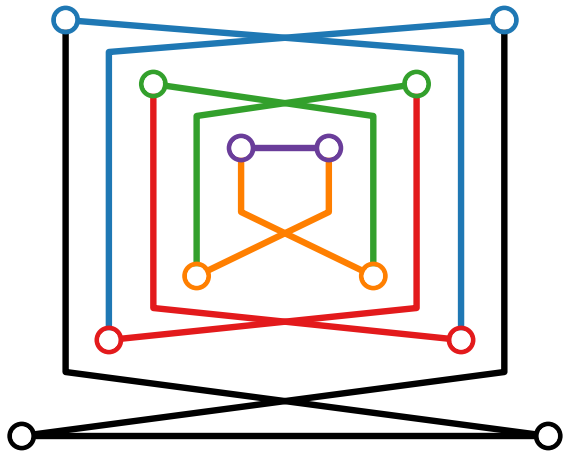
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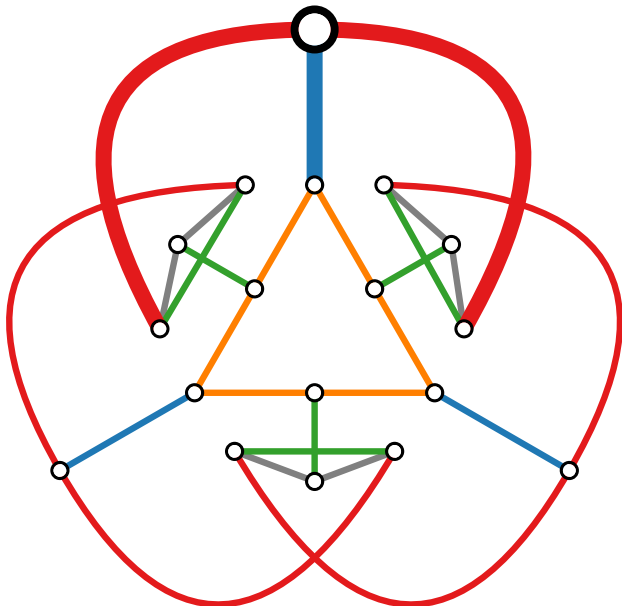
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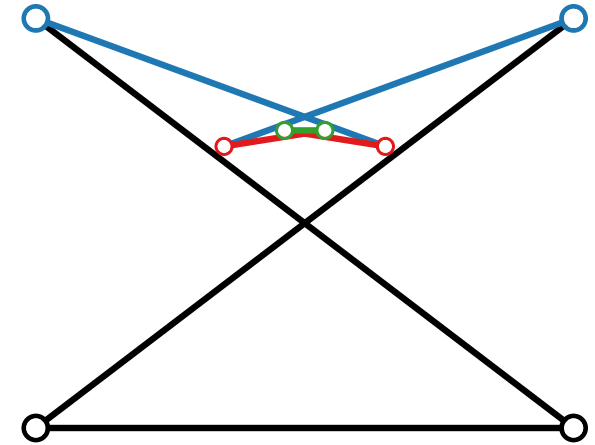
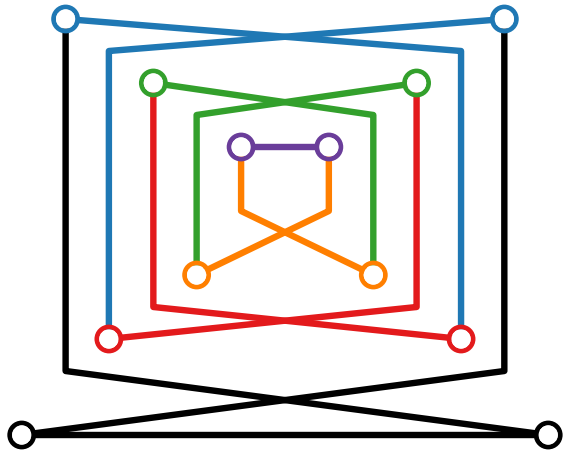
0 Bends, Lower Bounds



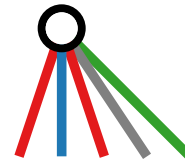
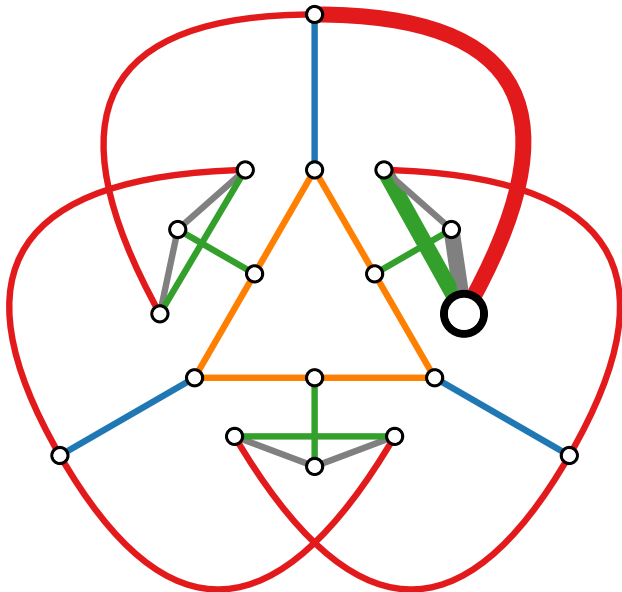
2-regular 2-connected 1-plane $\Rightarrow \Omega(n)$ slopes



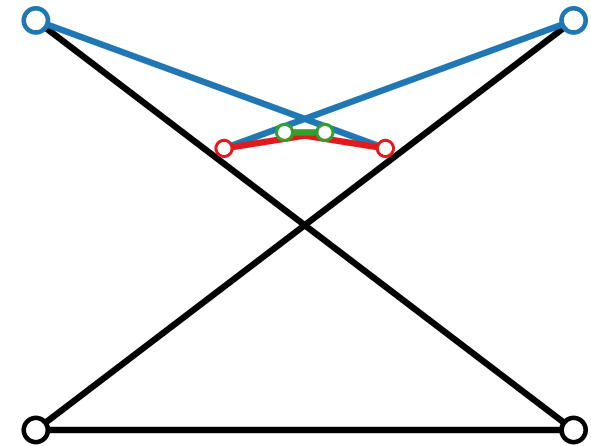
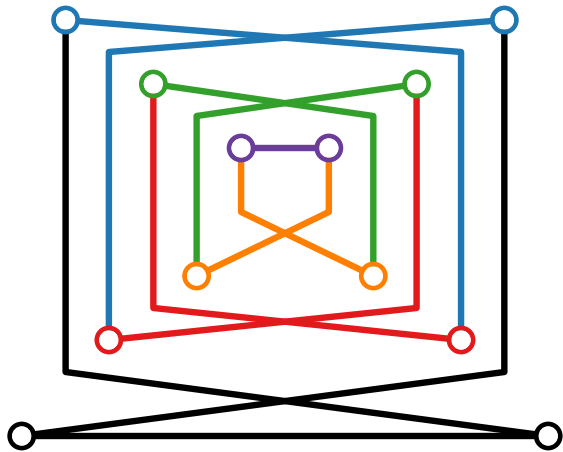
0 Bends, Lower Bounds



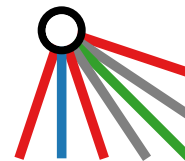
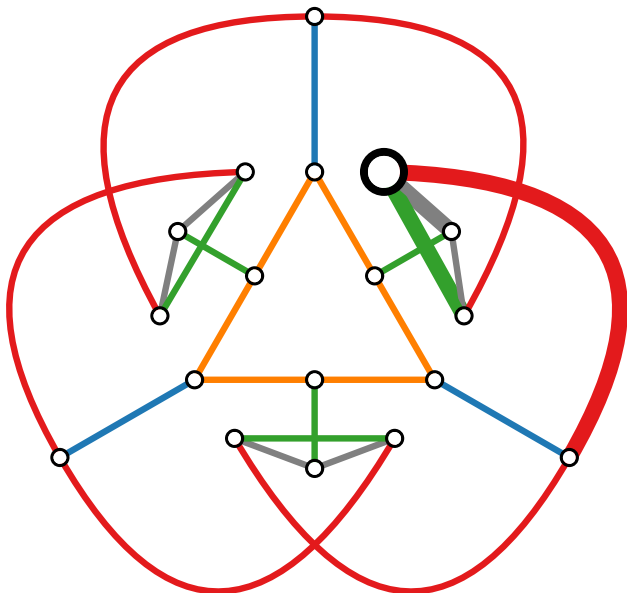
2-regular 2-connected 1-plane $\Rightarrow \Omega(n)$ slopes



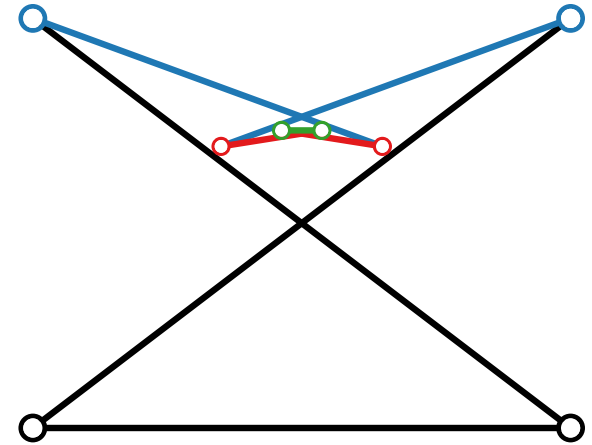
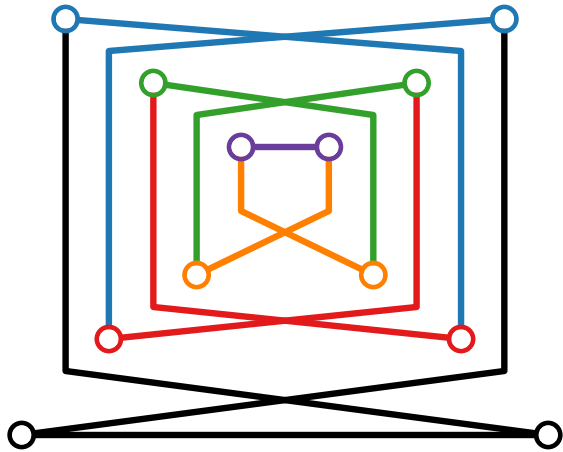
0 Bends, Lower Bounds



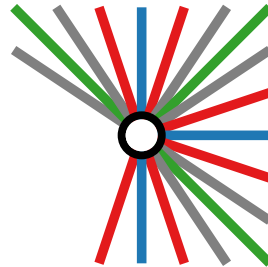
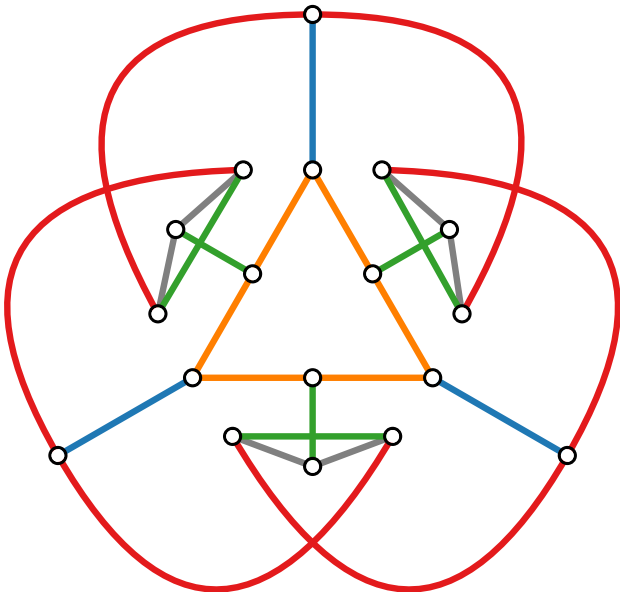
2-regular 2-connected 1-plane $\Rightarrow \Omega(n)$ slopes



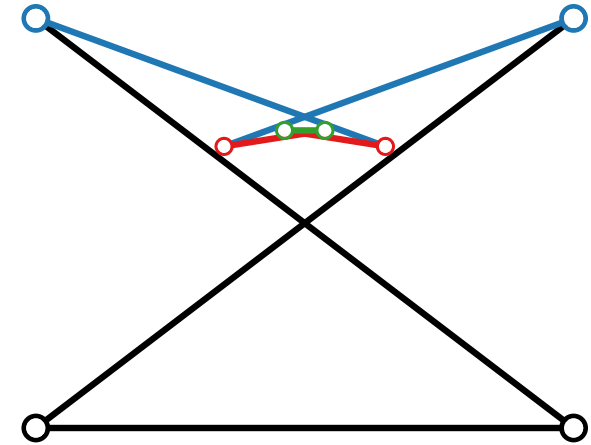
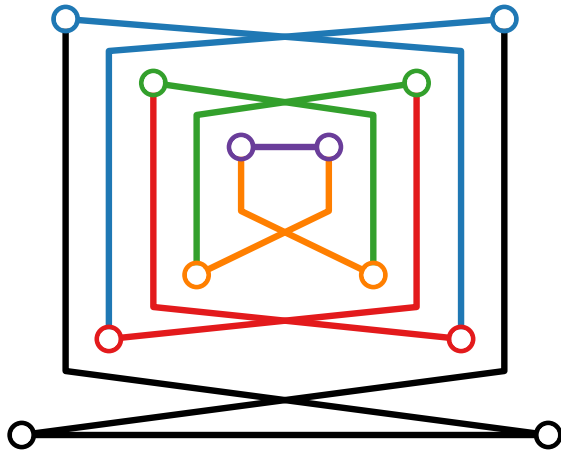
0 Bends, Lower Bounds



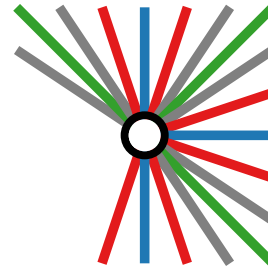
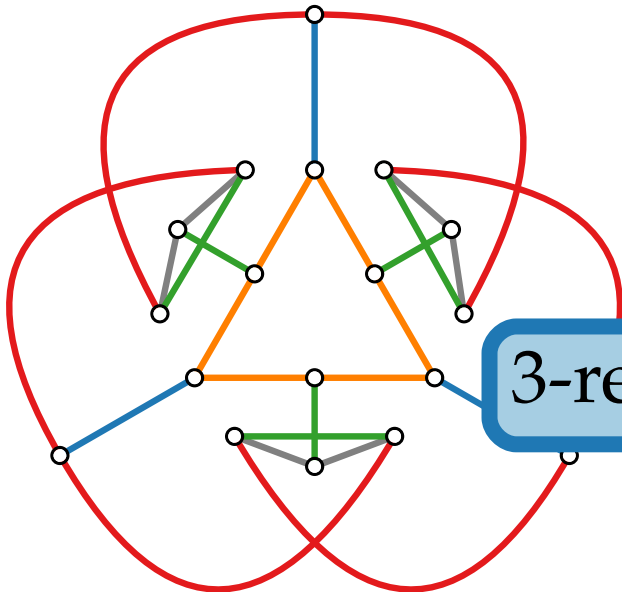
2-regular 2-connected 1-plane $\Rightarrow \Omega(n)$ slopes



0 Bends, Lower Bounds

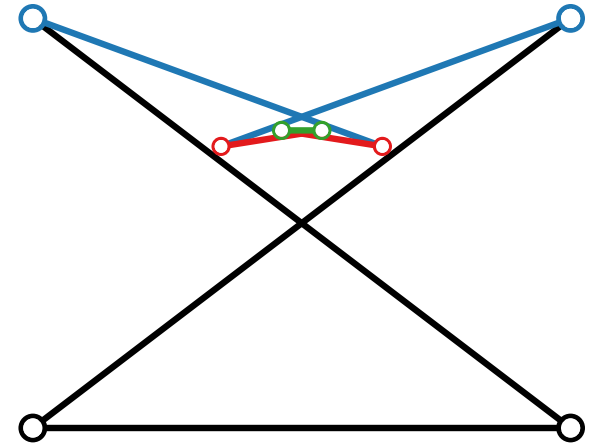
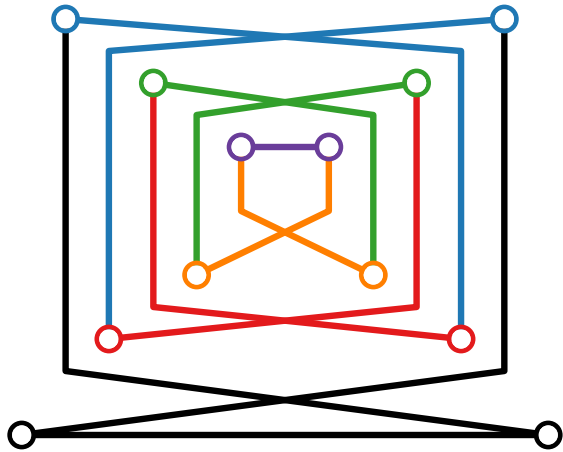


2-regular 2-connected 1-plane $\Rightarrow \Omega(n)$ slopes

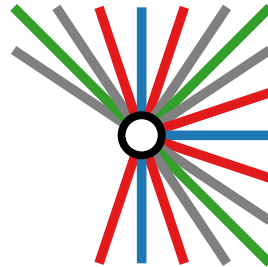
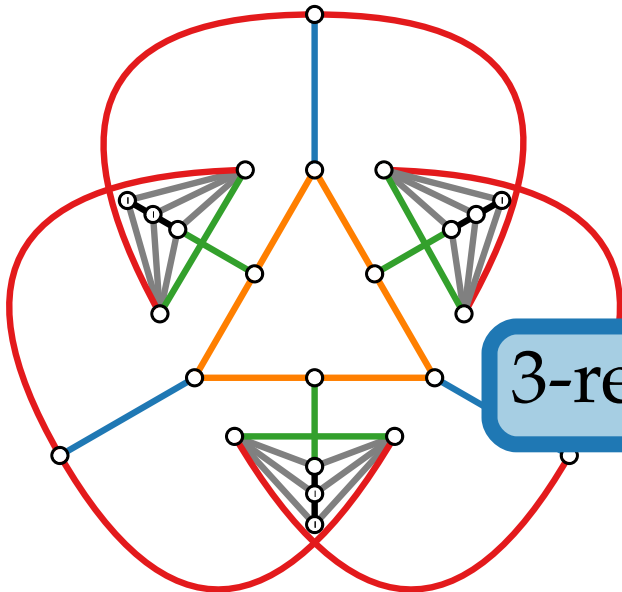


3-regular 3-connected 1-plane $\Rightarrow \geq 18$ slopes

0 Bends, Lower Bounds

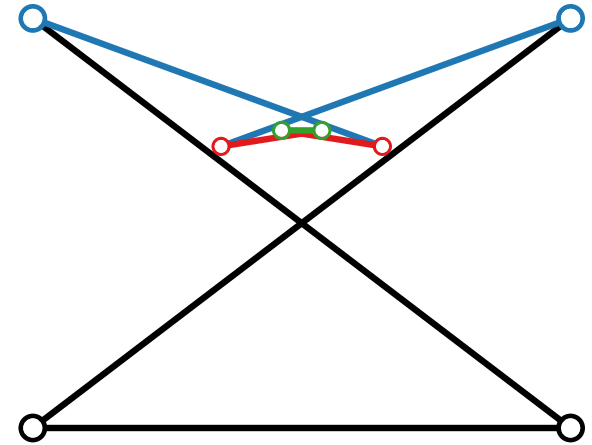
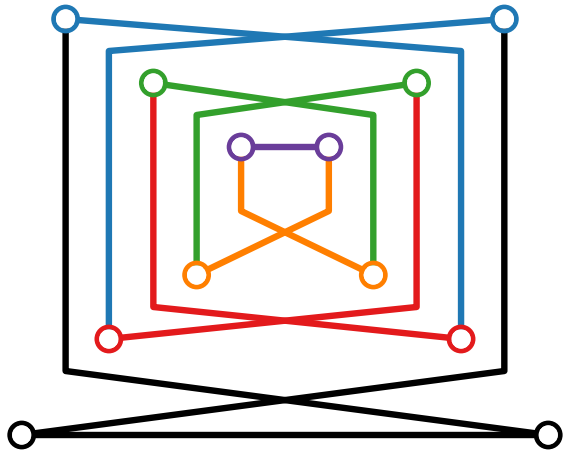


2-regular 2-connected 1-plane $\Rightarrow \Omega(n)$ slopes

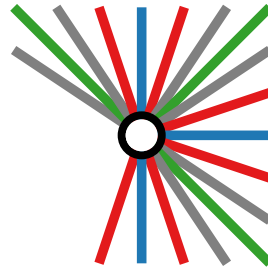
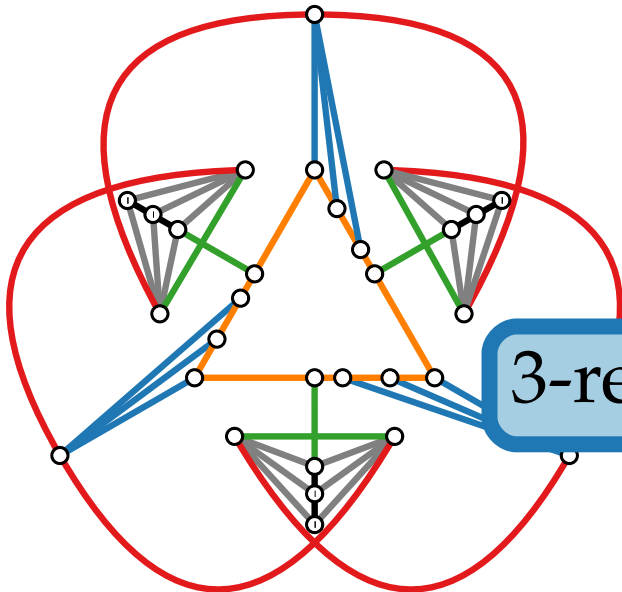


3-regular 3-connected 1-plane $\Rightarrow \geq 18$ slopes

0 Bends, Lower Bounds

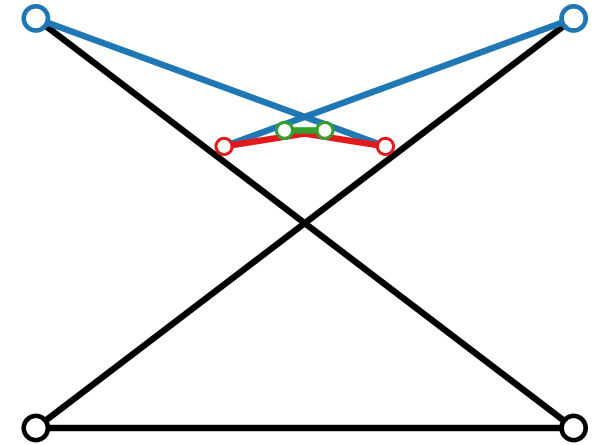
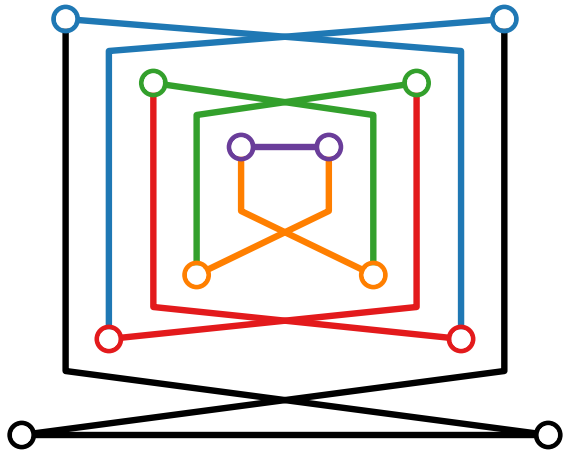


2-regular 2-connected 1-plane $\Rightarrow \Omega(n)$ slopes

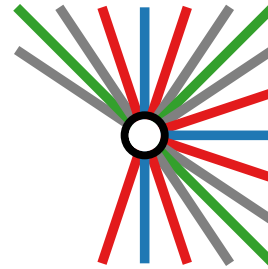
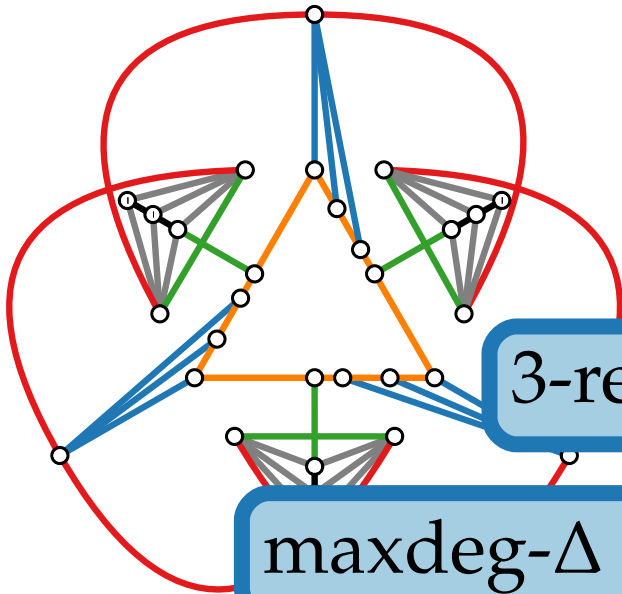


3-regular 3-connected 1-plane $\Rightarrow \geq 18$ slopes

0 Bends, Lower Bounds



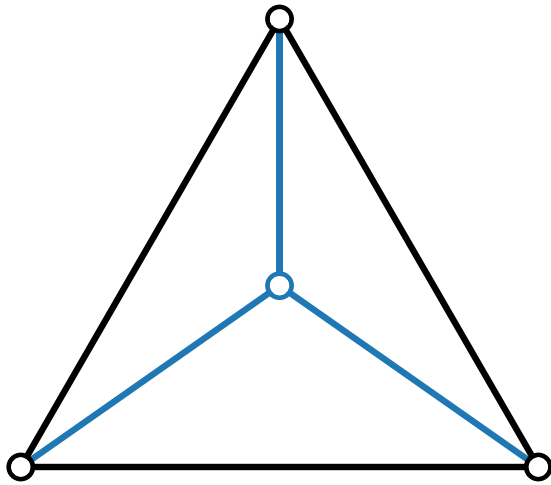
2-regular 2-connected 1-plane $\Rightarrow \Omega(n)$ slopes



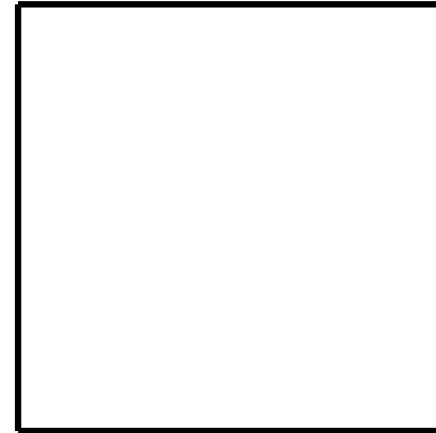
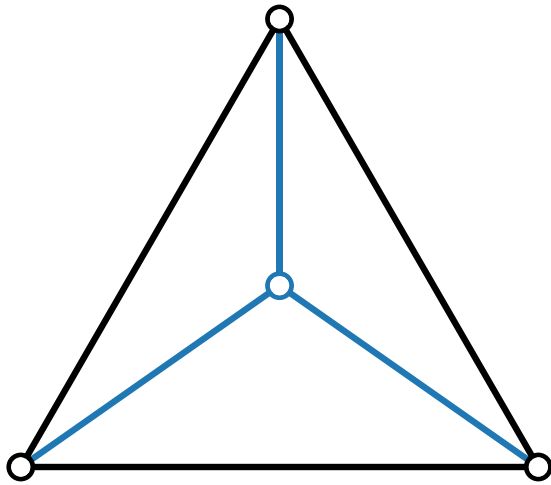
3-regular 3-connected 1-plane $\Rightarrow \geq 18$ slopes

maxdeg- Δ 3-connected 1-plane $\Rightarrow \geq 9(\Delta - 1)$ slopes

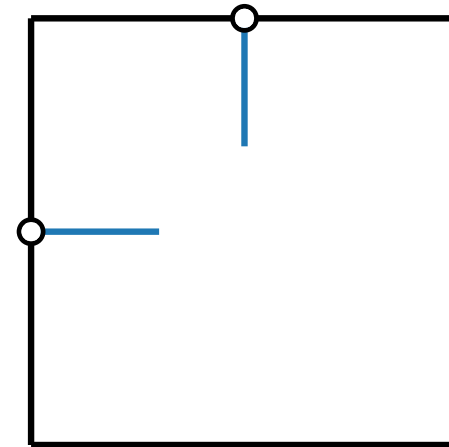
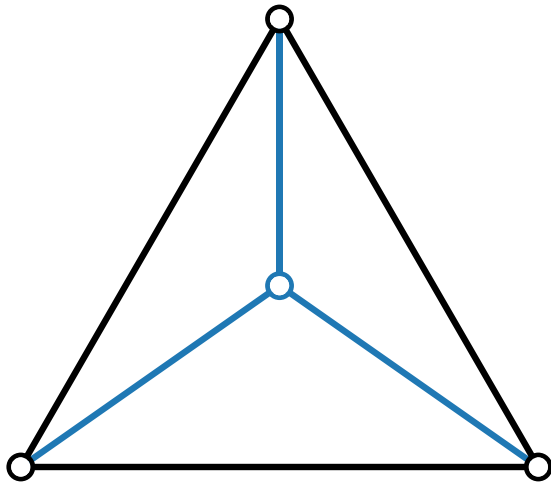
1 Bend, Lower Bound



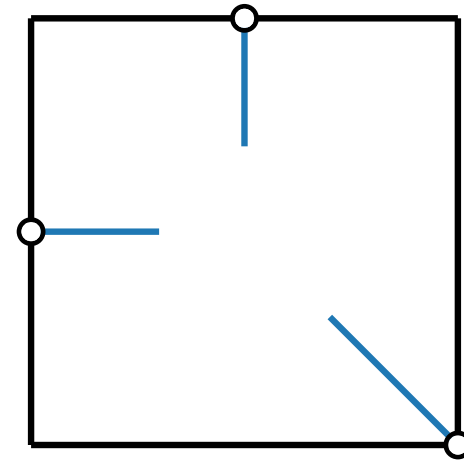
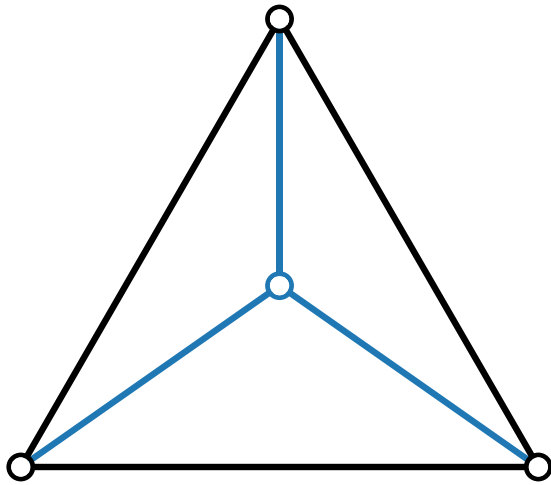
1 Bend, Lower Bound



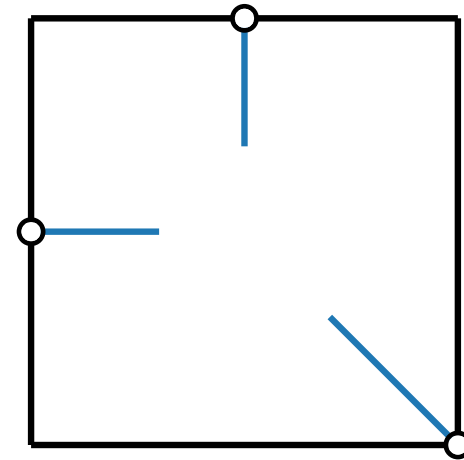
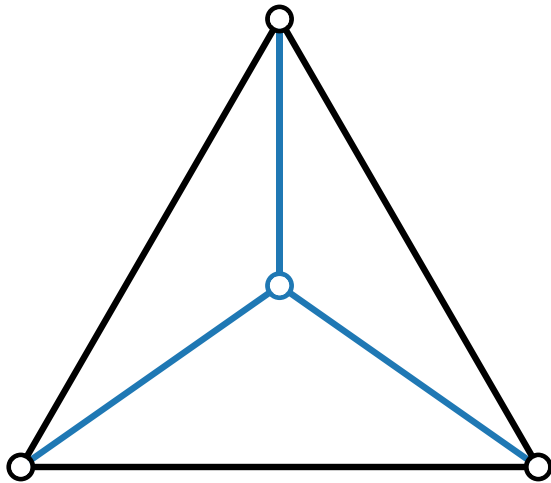
1 Bend, Lower Bound



1 Bend, Lower Bound



1 Bend, Lower Bound



cubic 3-connected 1-plane 1-bend \Rightarrow 3 slopes

Reembedding 1-Planar Graphs

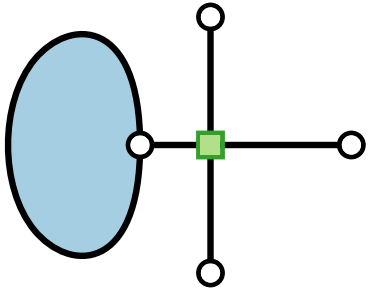
G 1-plane, G^* planarization

\Rightarrow re-embed G 1-plane s.t. no cutvertex of G^* is dummy

Reembedding 1-Planar Graphs

G 1-plane, G^* planarization

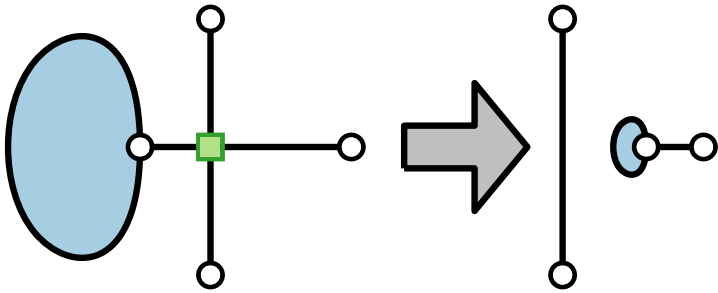
\Rightarrow re-embed G 1-plane s.t. no cutvertex of G^* is dummy



Reembedding 1-Planar Graphs

G 1-plane, G^* planarization

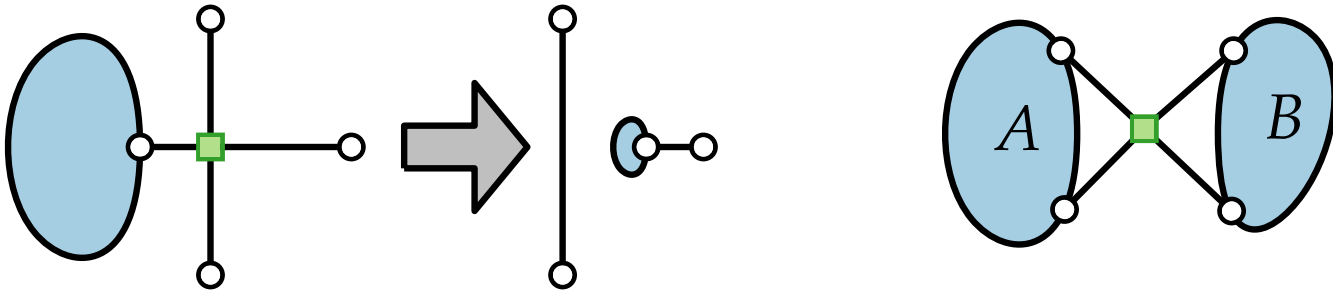
\Rightarrow re-embed G 1-plane s.t. no cutvertex of G^* is dummy



Reembedding 1-Planar Graphs

G 1-plane, G^* planarization

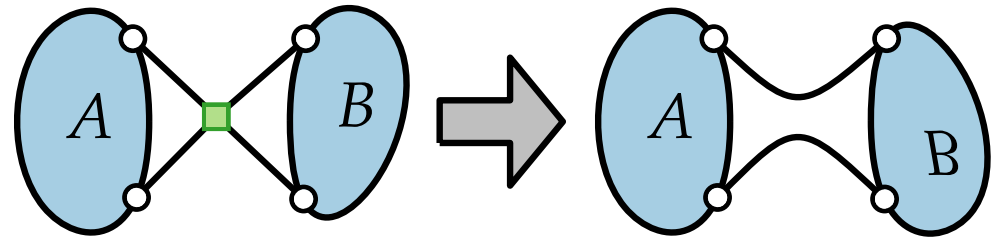
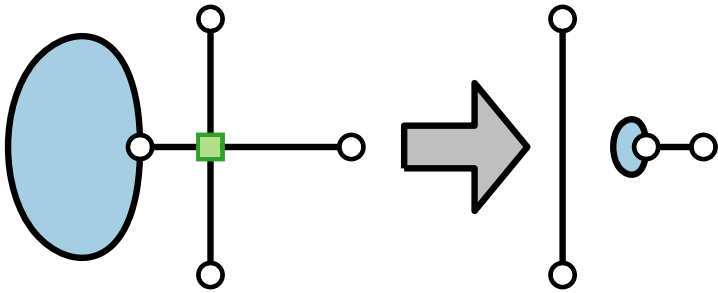
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Reembedding 1-Planar Graphs

G 1-plane, G^* planarization

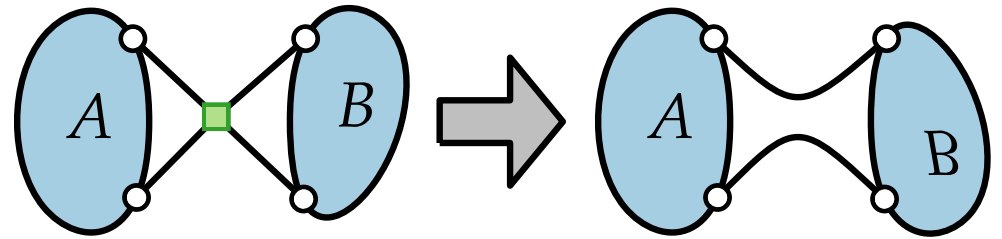
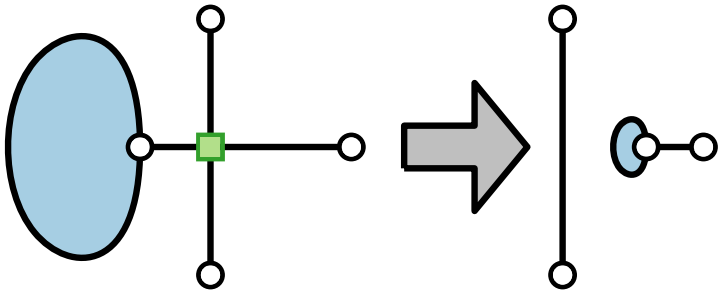
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Reembedding 1-Planar Graphs

G 1-plane, G^* planarization

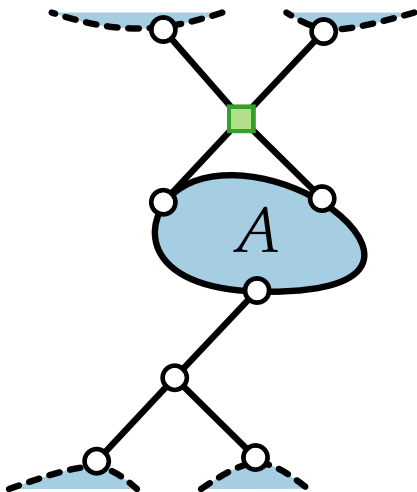
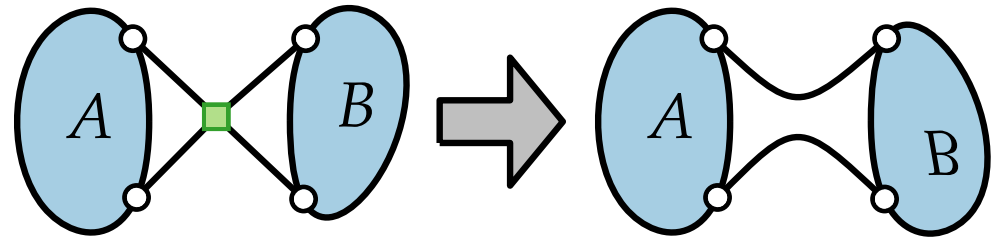
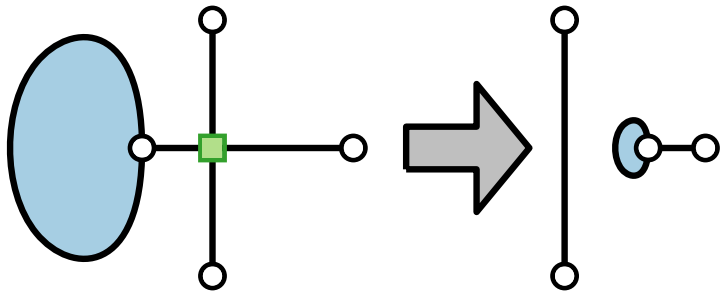
\Rightarrow re-embed G 1-plane s.t. no cutvertex of G^* is dummy
and if G 3-connected, then G^* 3-connected.



Reembedding 1-Planar Graphs

G 1-plane, G^* planarization

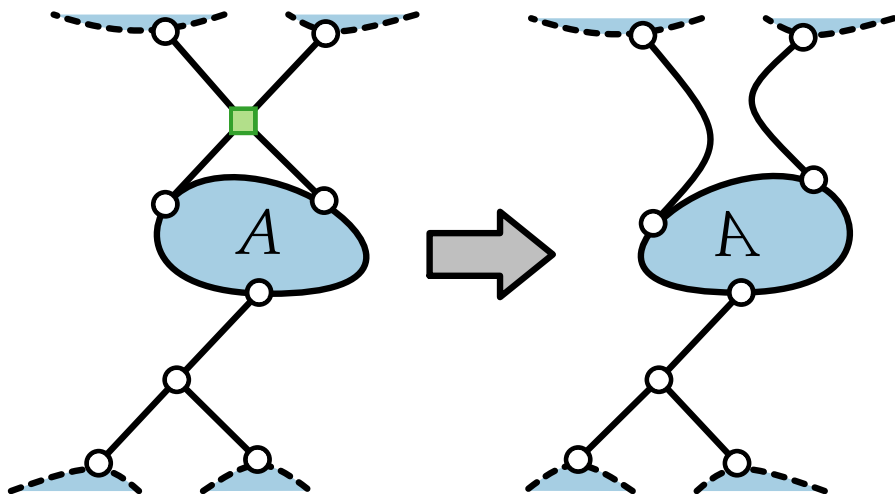
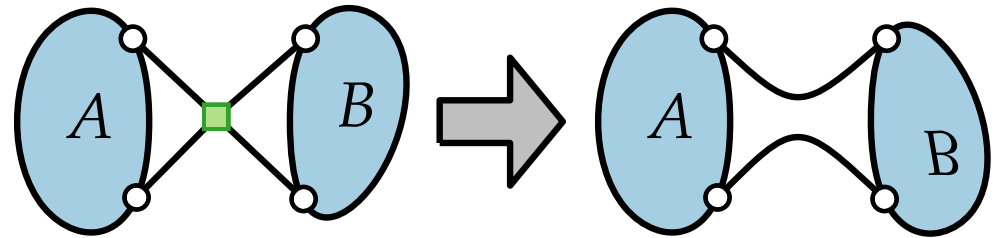
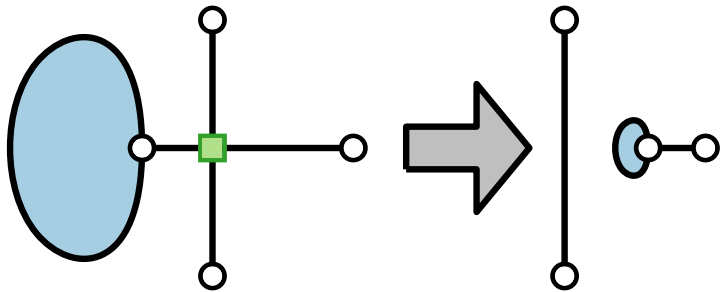
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Reembedding 1-Planar Graphs

G 1-plane, G^* planarization

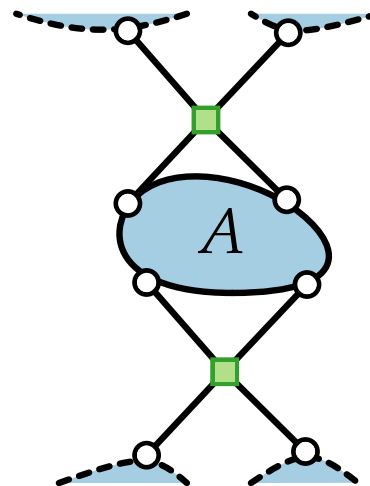
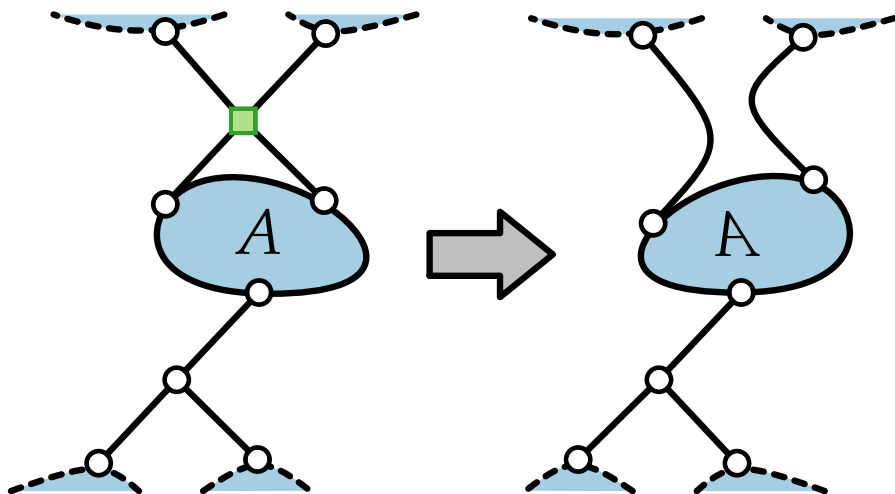
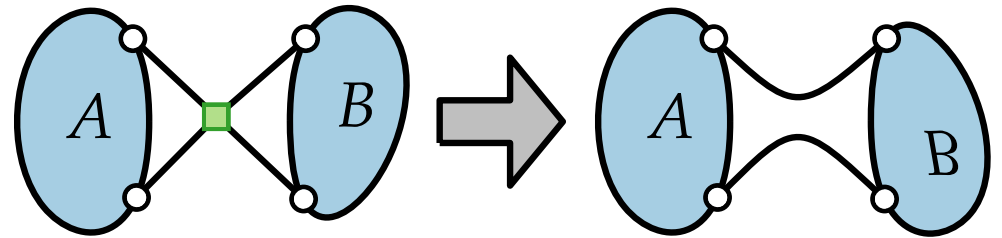
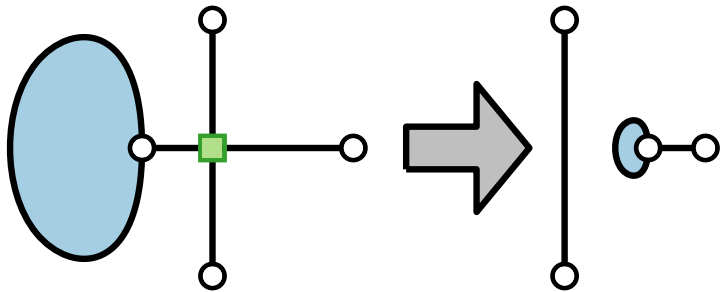
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Reembedding 1-Planar Graphs

G 1-plane, G^* planarization

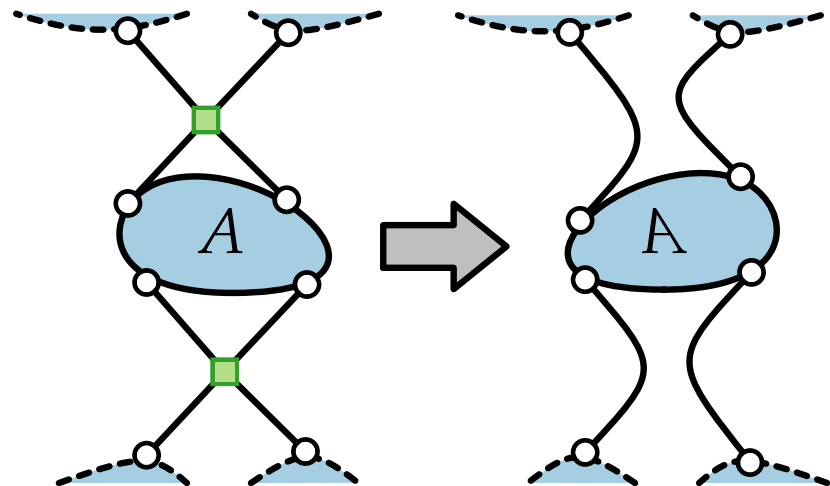
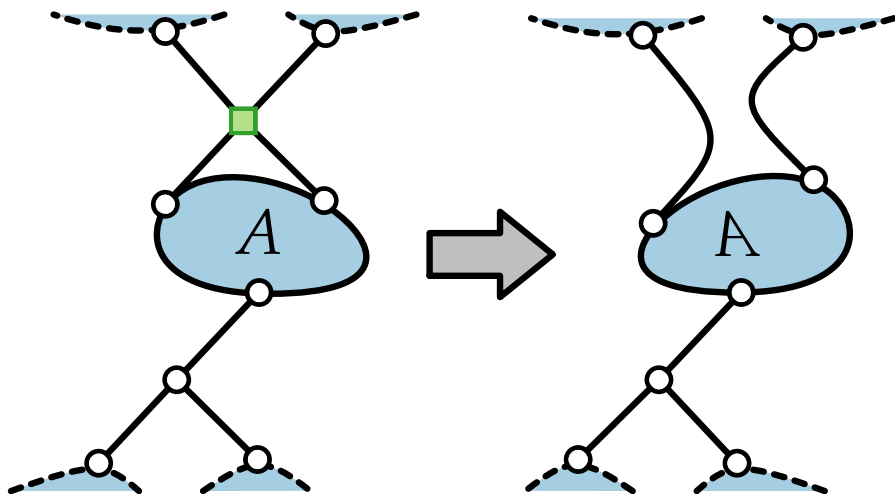
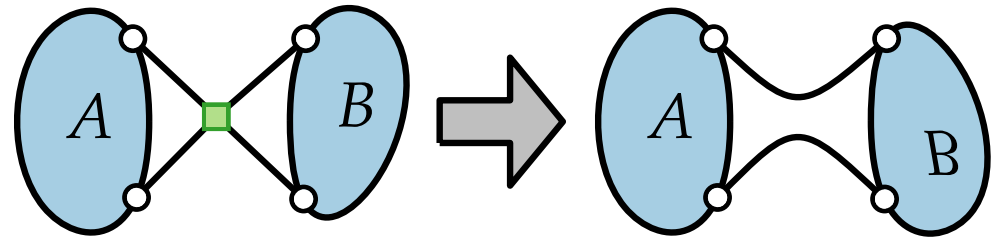
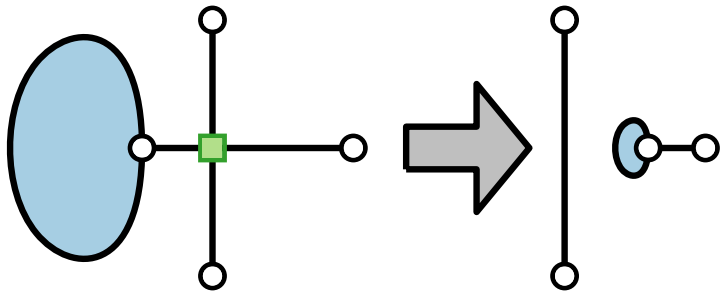
\Rightarrow re-embed G 1-plane s.t. no cutvertex of G^* is dummy
and if G 3-connected, then G^* 3-connected.



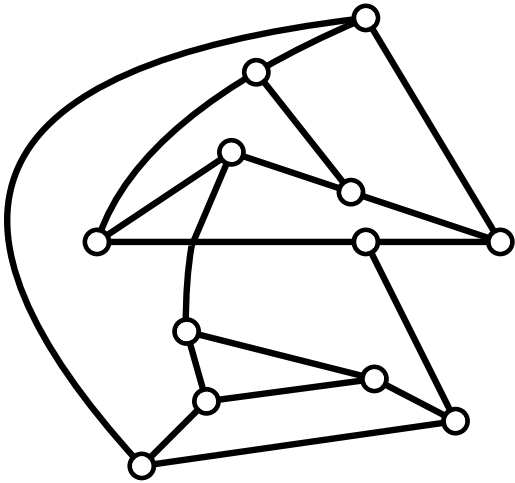
Reembedding 1-Planar Graphs

G 1-plane, G^* planarization

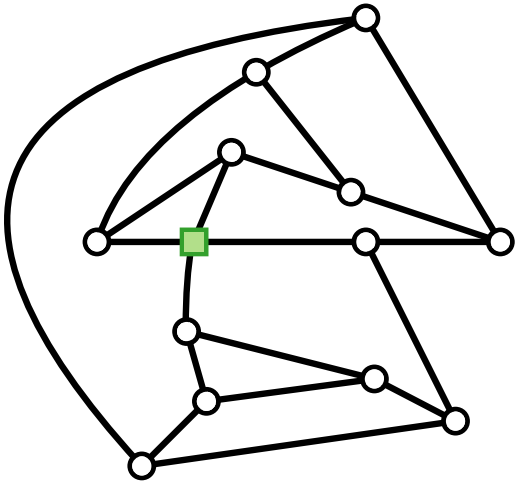
\Rightarrow re-embed G 1-plane s.t. no cutvertex of G^* is dummy
and if G 3-connected, then G^* 3-connected.



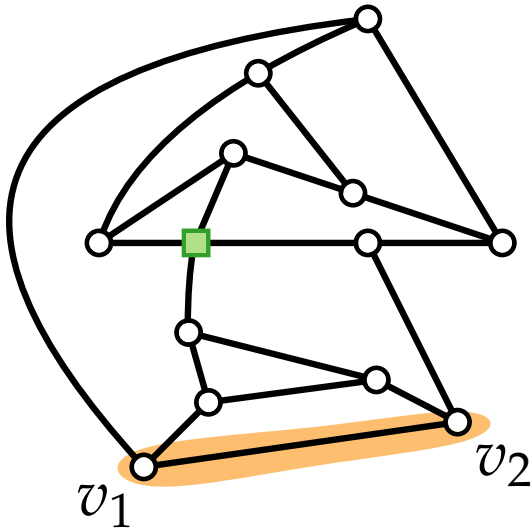
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



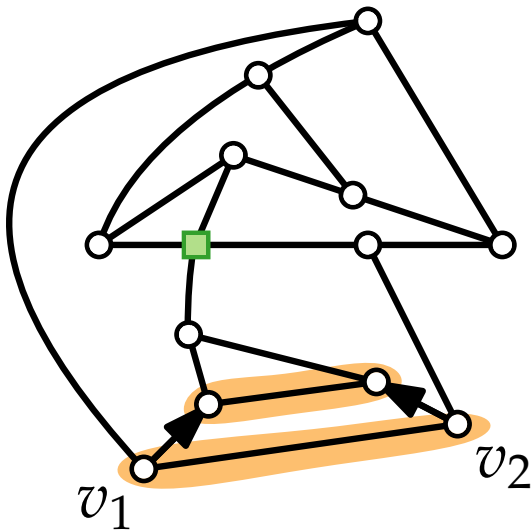
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



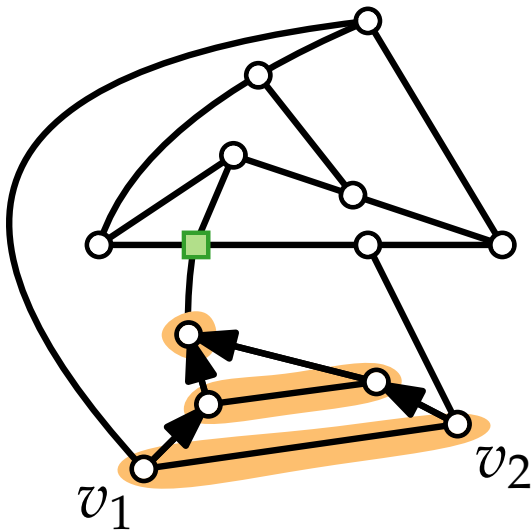
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



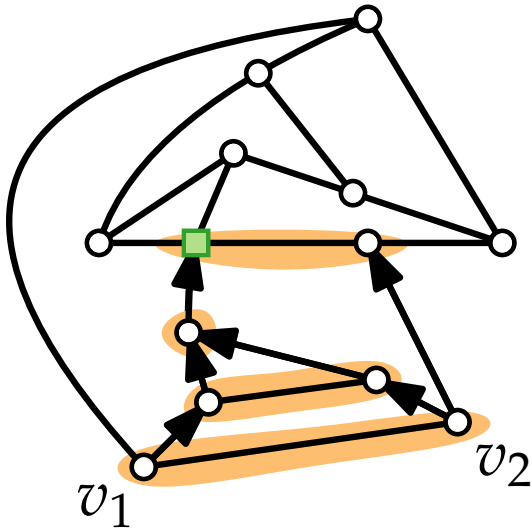
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



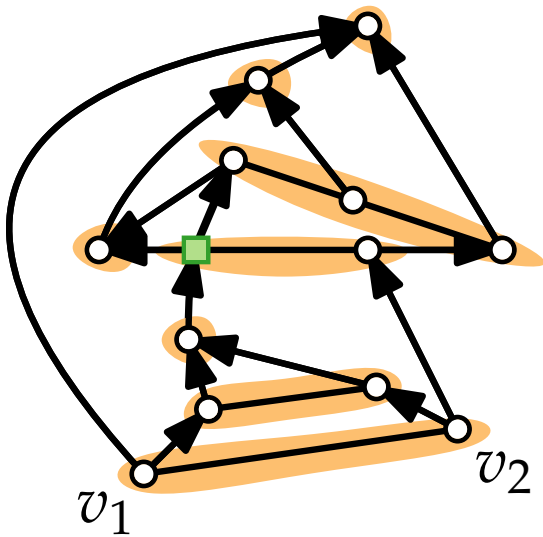
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



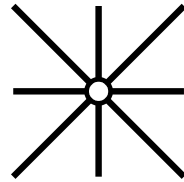
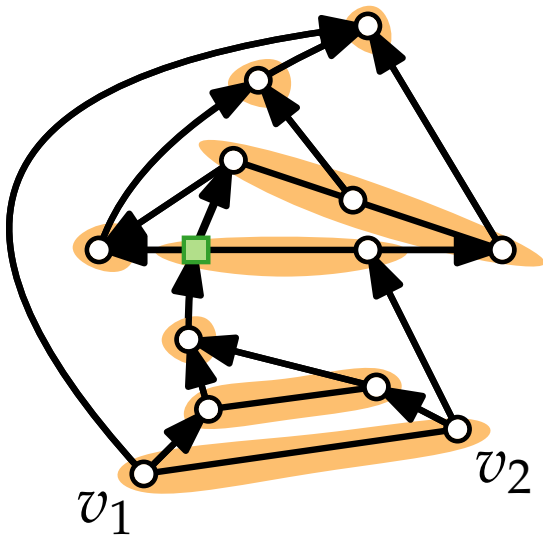
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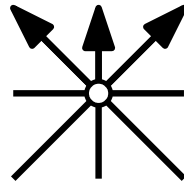
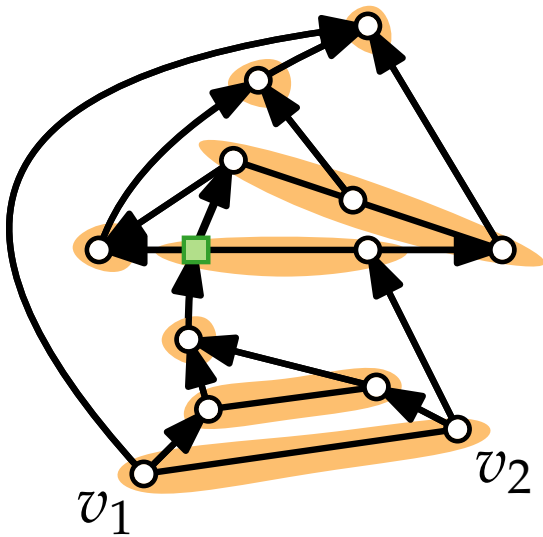
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



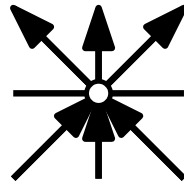
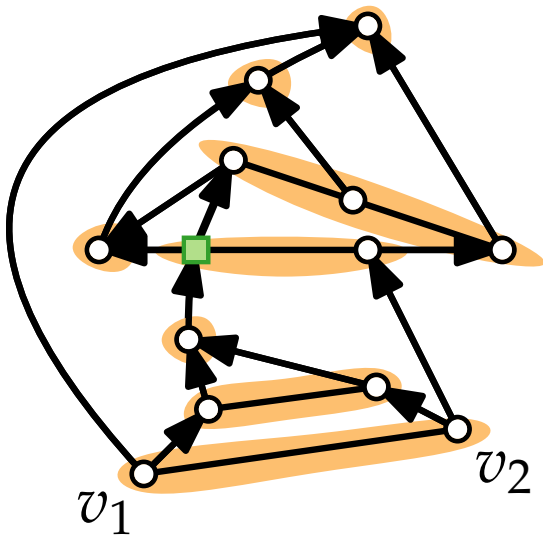
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



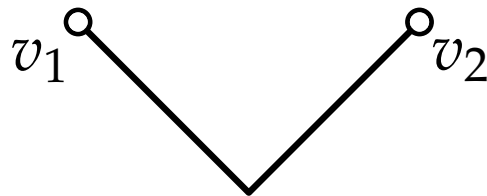
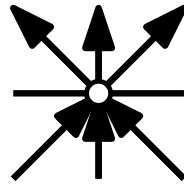
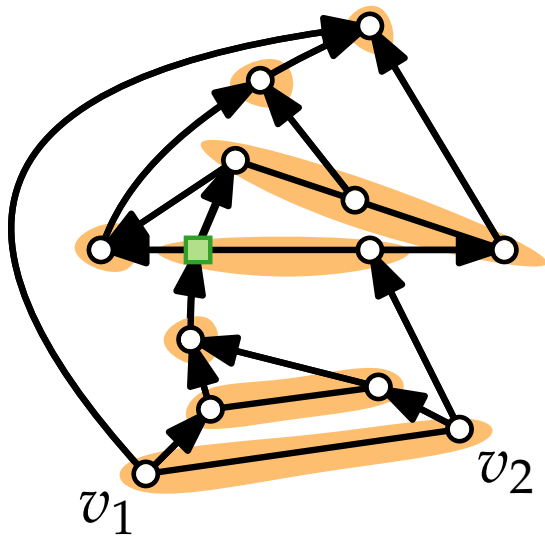
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



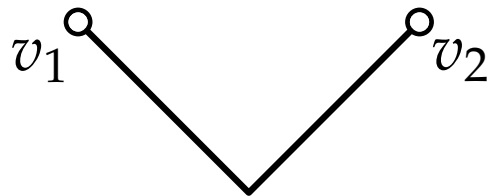
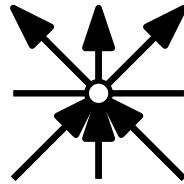
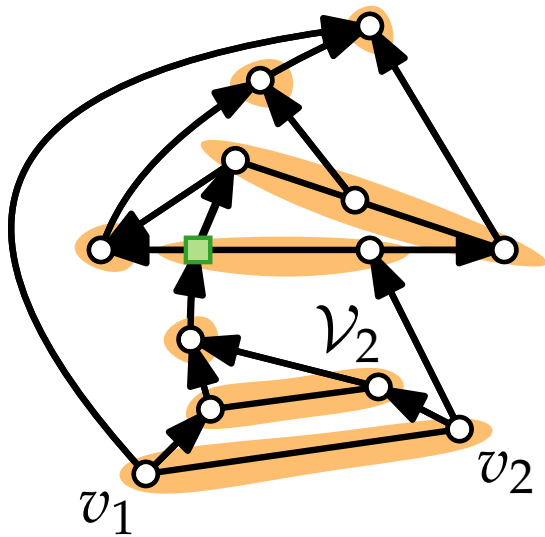
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



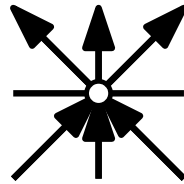
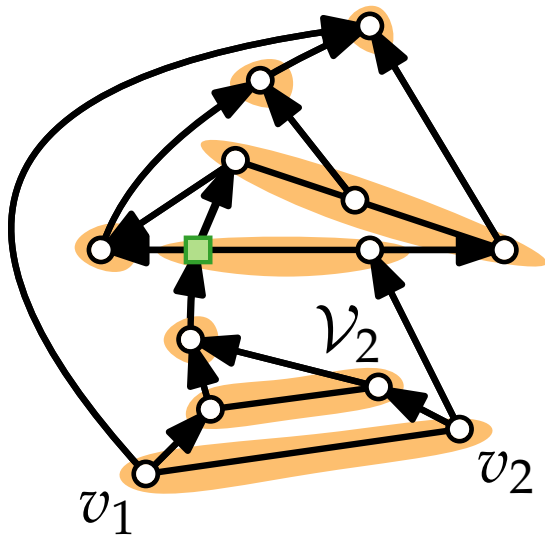
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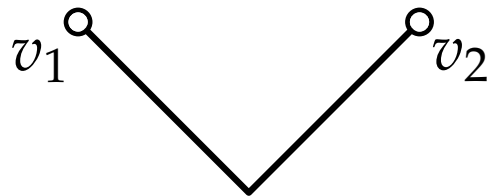
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



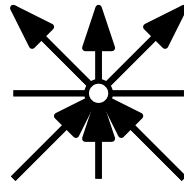
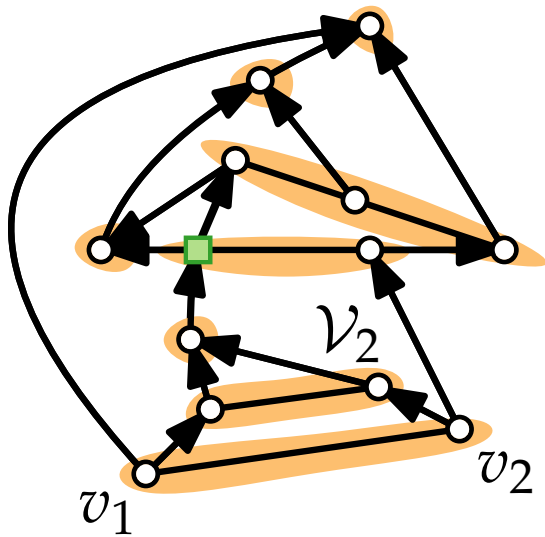
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



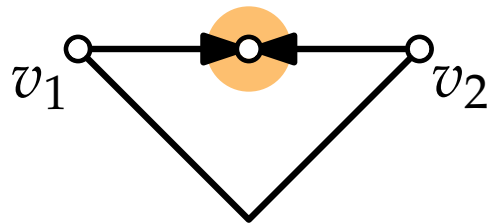
v_2 : real vertex



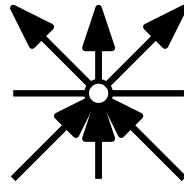
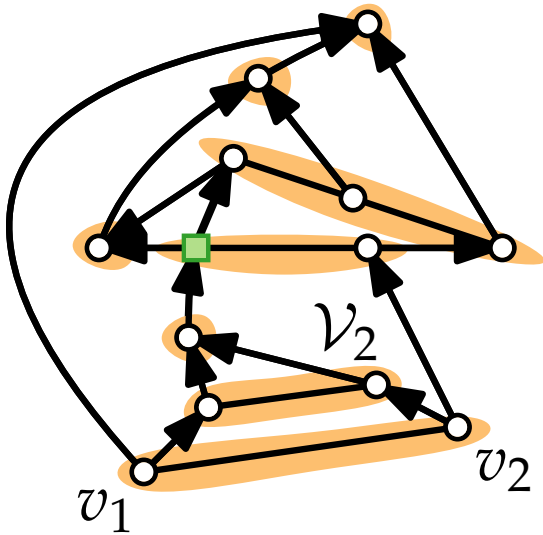
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



v_2 : real vertex

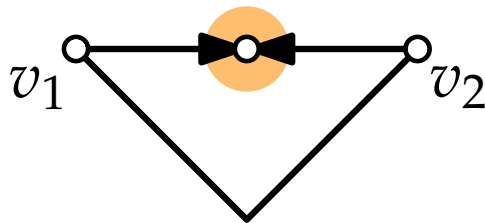


Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes

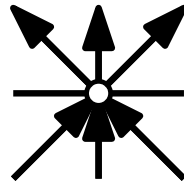
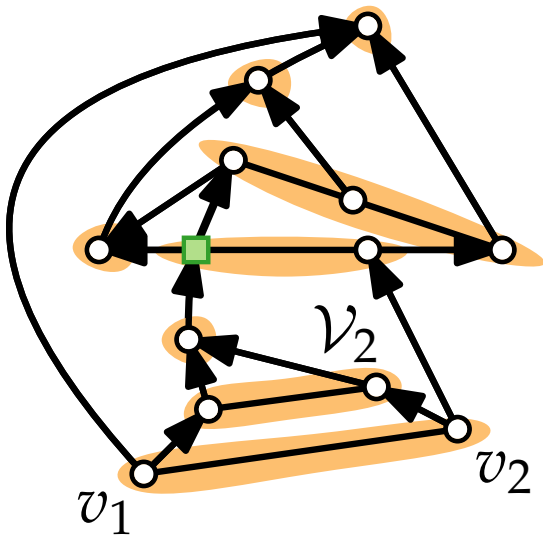


v_2 : real vertex

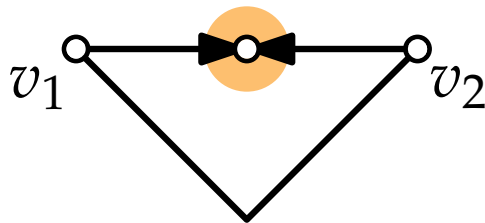
dummy vertex



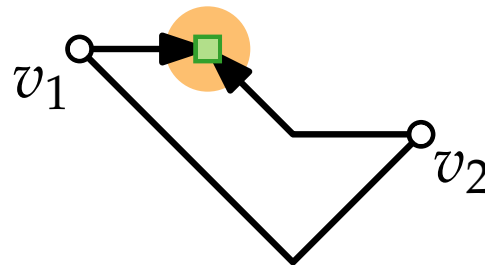
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



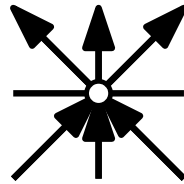
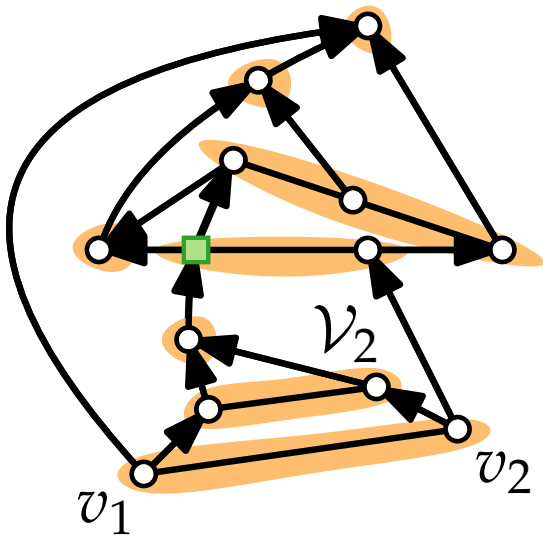
v_2 : real vertex



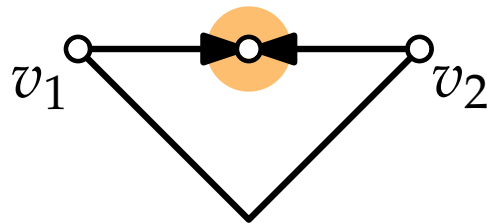
dummy vertex



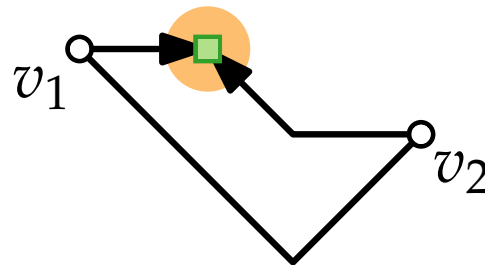
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



\mathcal{V}_2 : real vertex

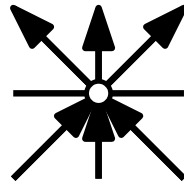
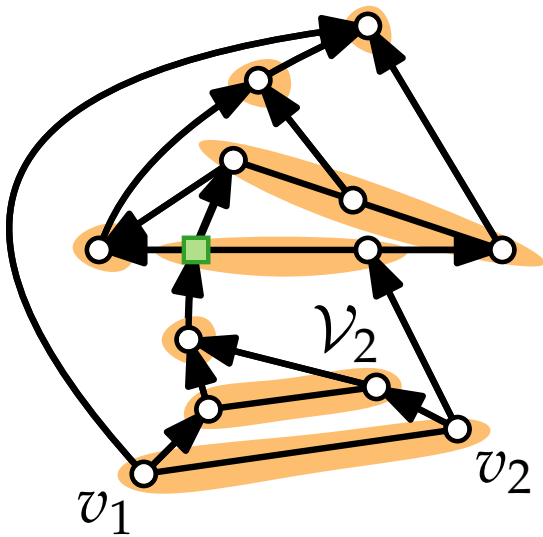


dummy vertex

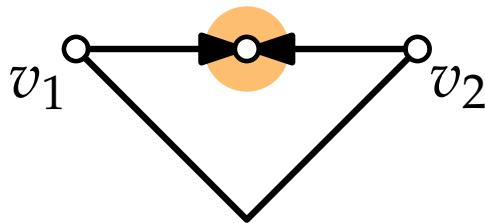


chain

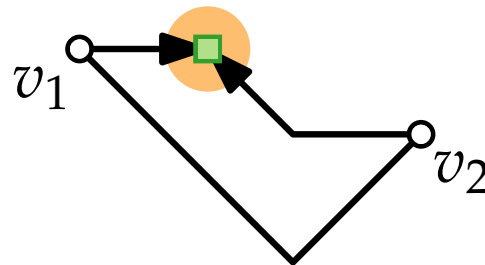
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



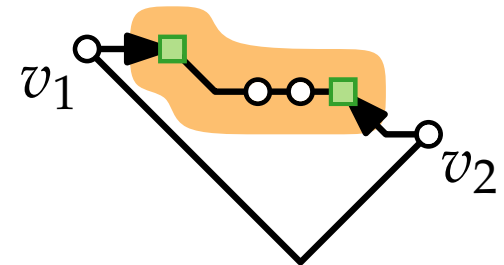
\mathcal{V}_2 : real vertex



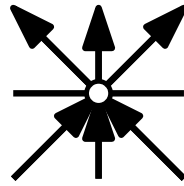
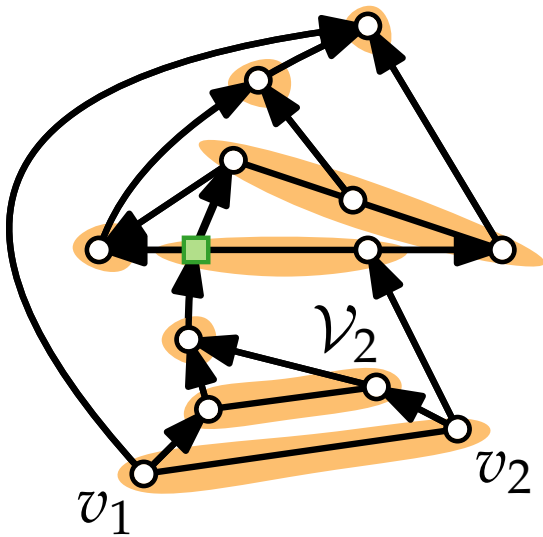
dummy vertex



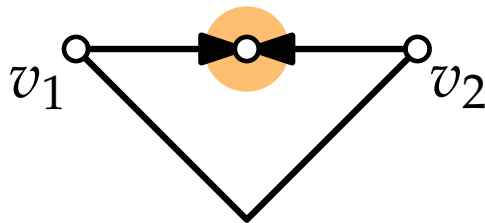
chain



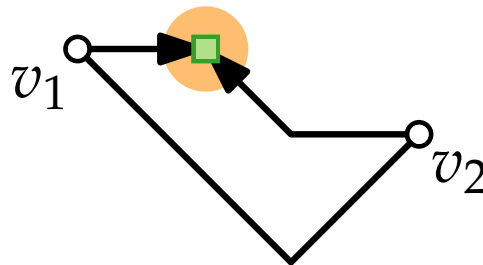
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



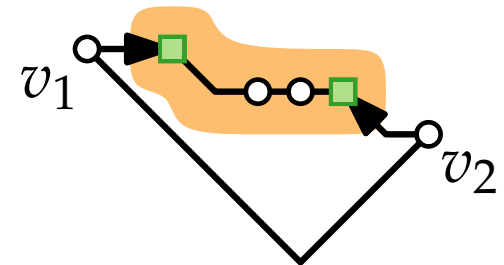
\mathcal{V}_2 : real vertex



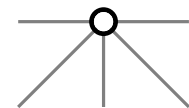
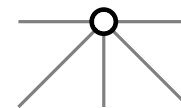
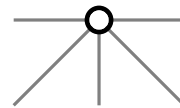
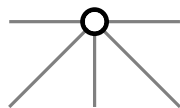
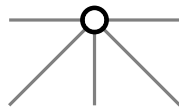
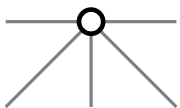
dummy vertex



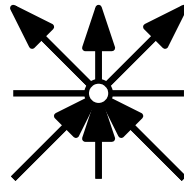
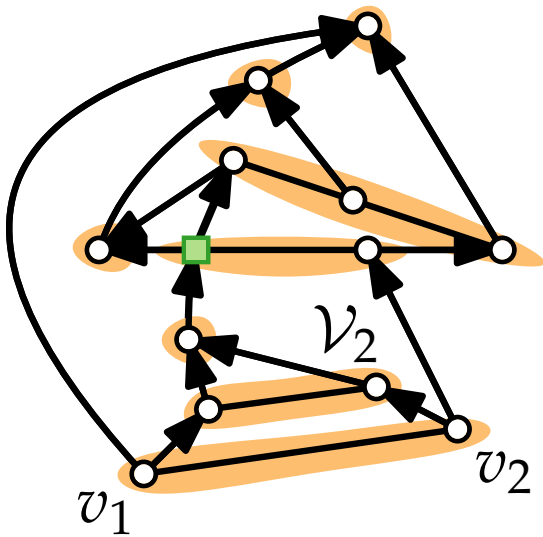
chain



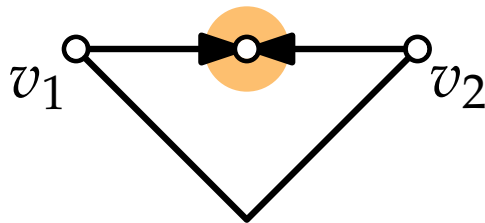
\mathcal{V}_i :



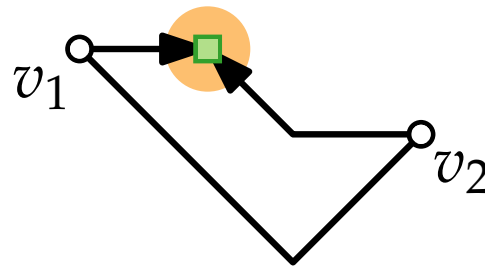
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



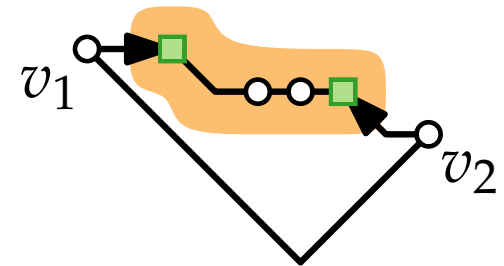
\mathcal{V}_2 : real vertex



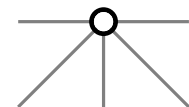
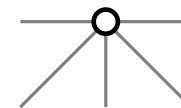
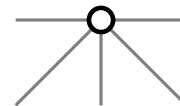
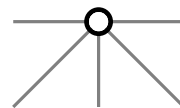
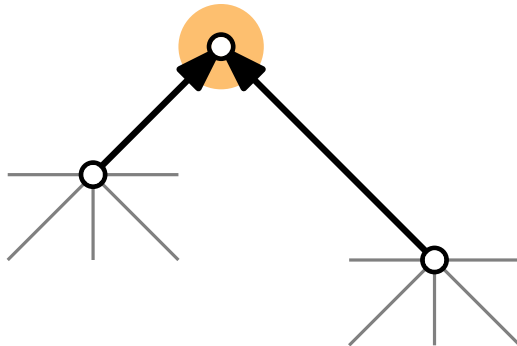
dummy vertex



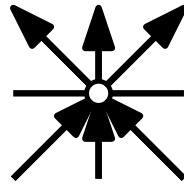
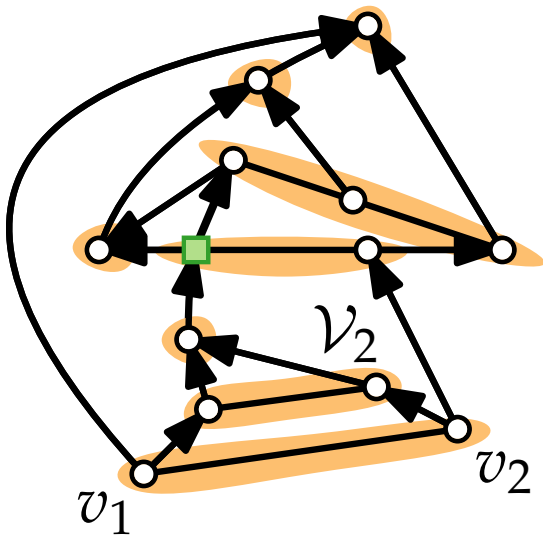
chain



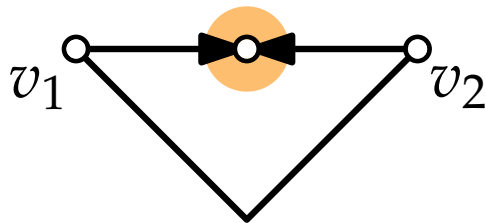
\mathcal{V}_i :



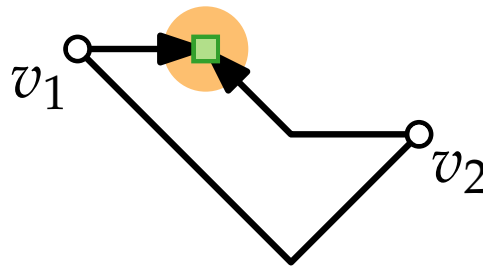
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



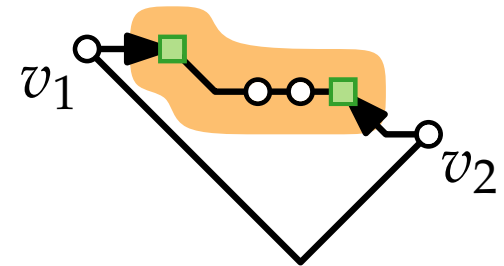
\mathcal{V}_2 : real vertex



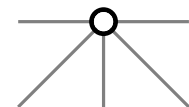
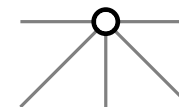
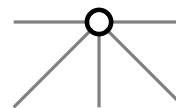
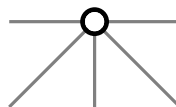
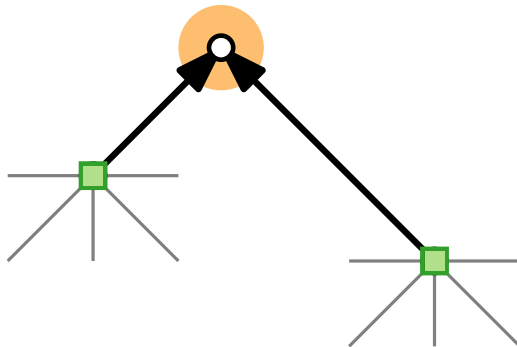
dummy vertex



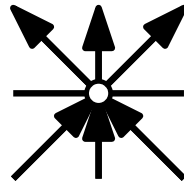
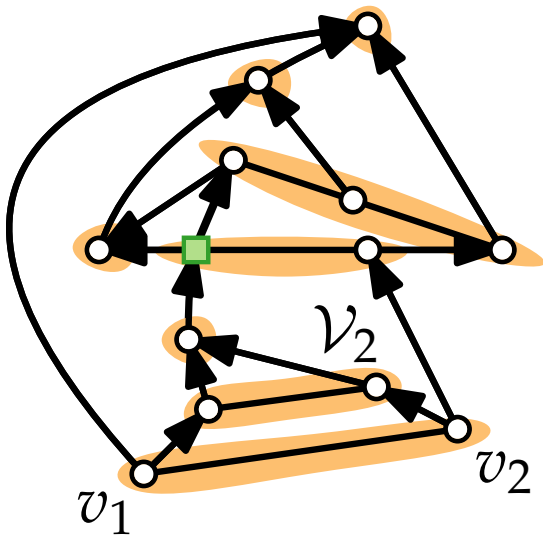
chain



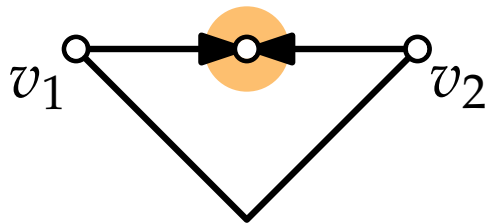
\mathcal{V}_i :



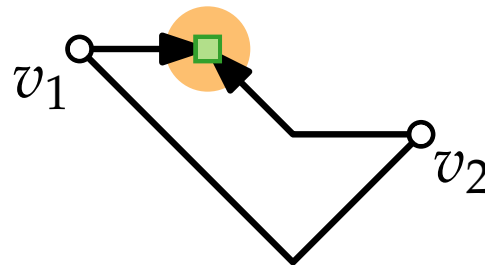
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



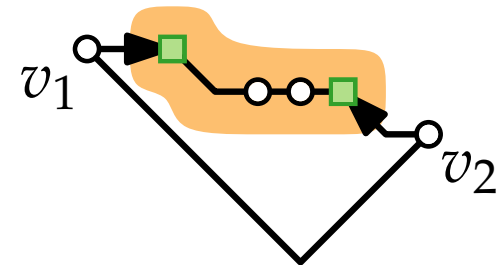
\mathcal{V}_2 : real vertex



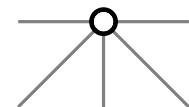
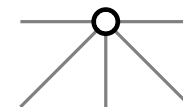
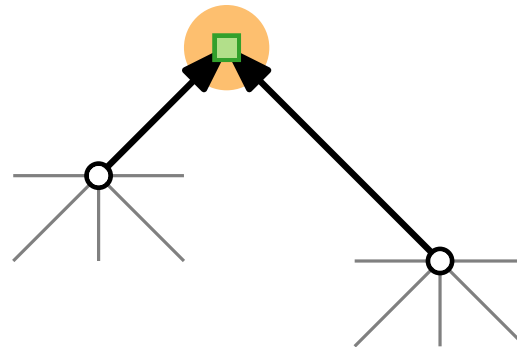
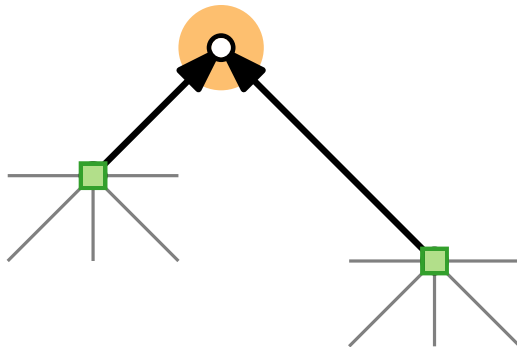
dummy vertex



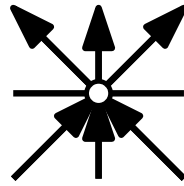
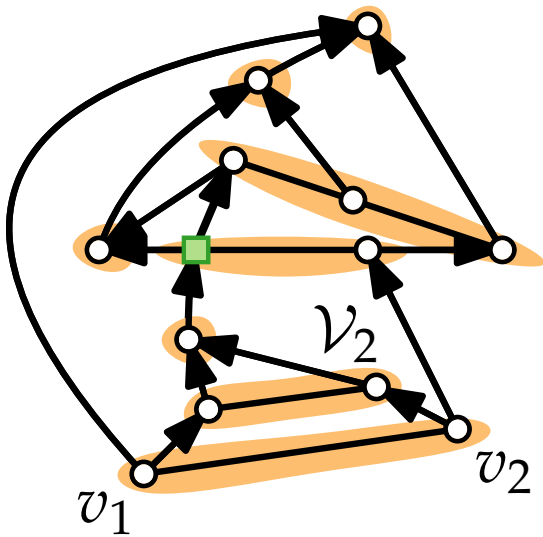
chain



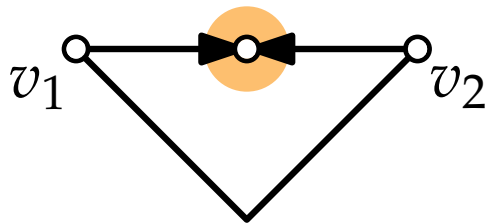
\mathcal{V}_i :



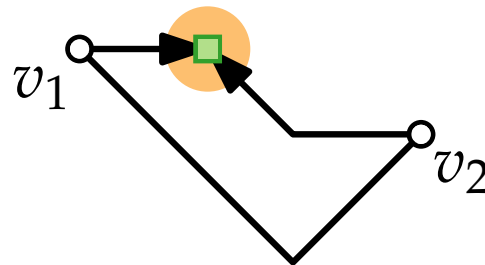
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



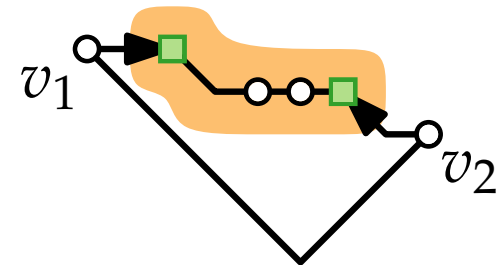
\mathcal{V}_2 : real vertex



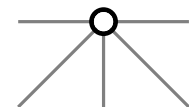
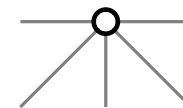
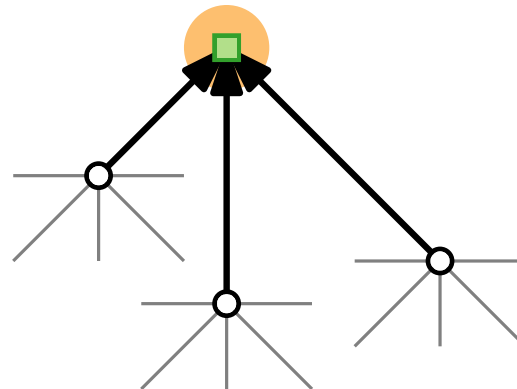
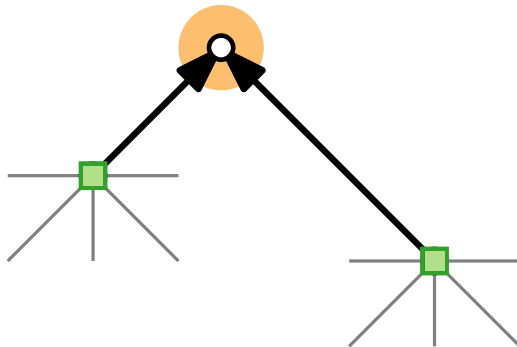
dummy vertex



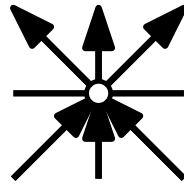
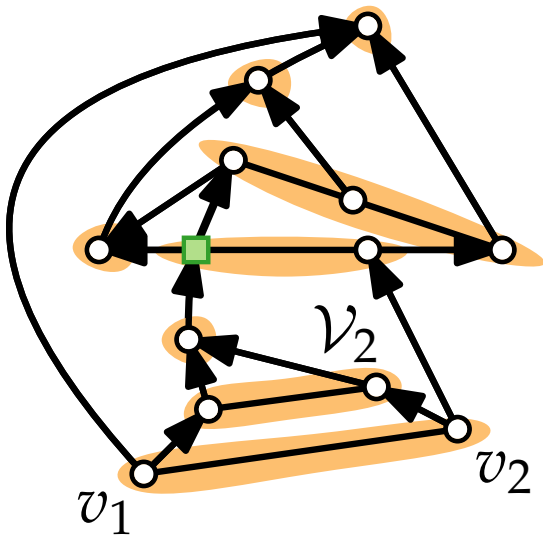
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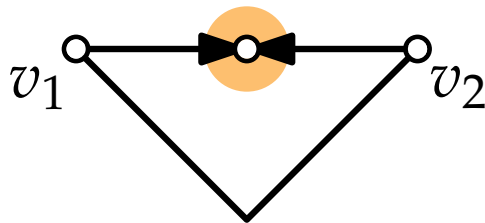
\mathcal{V}_i :



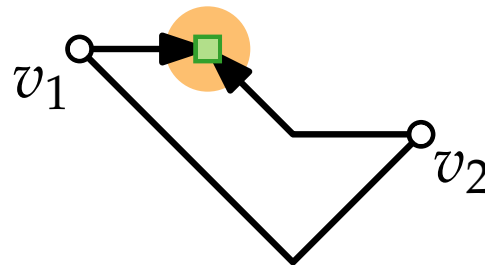
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



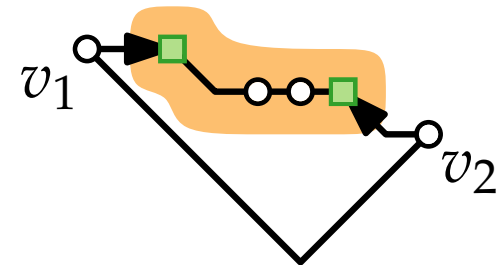
\mathcal{V}_2 : real vertex



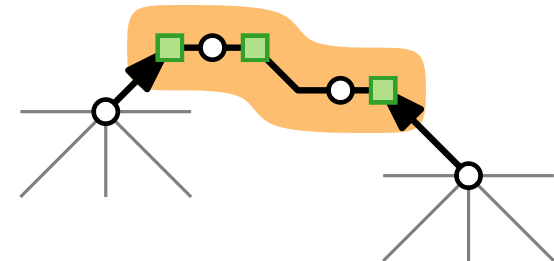
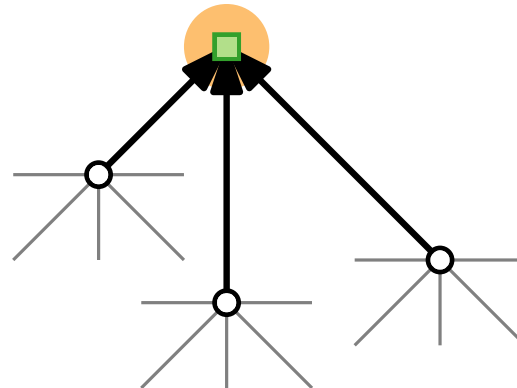
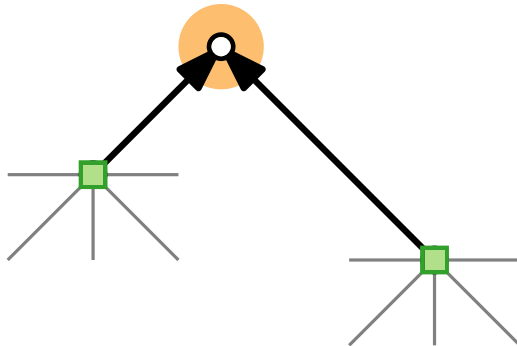
dummy vertex



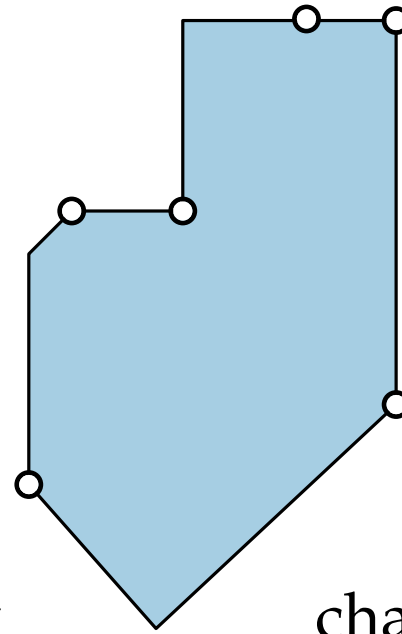
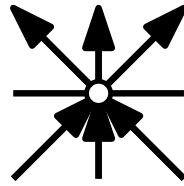
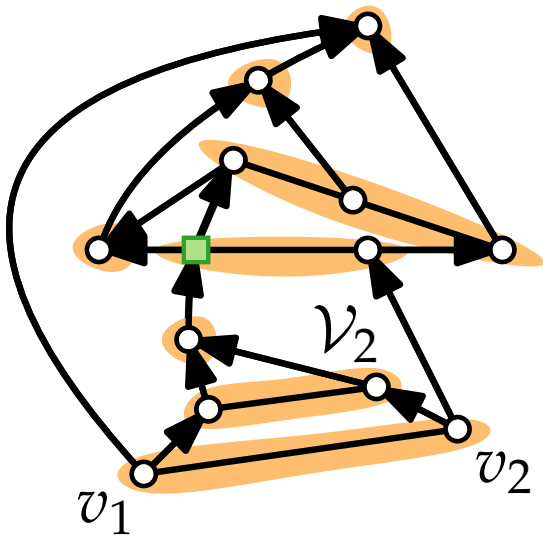
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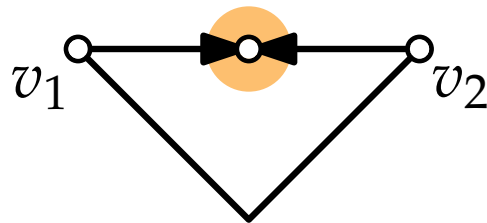
\mathcal{V}_i :



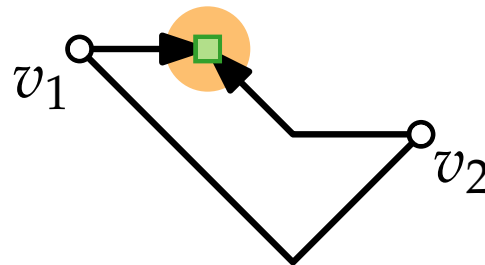
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



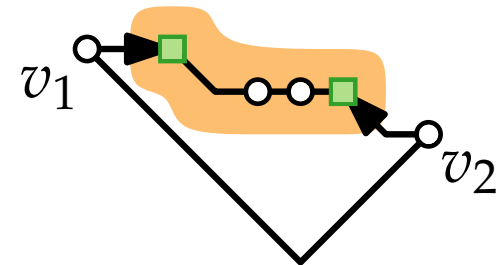
\mathcal{V}_2 : real vertex



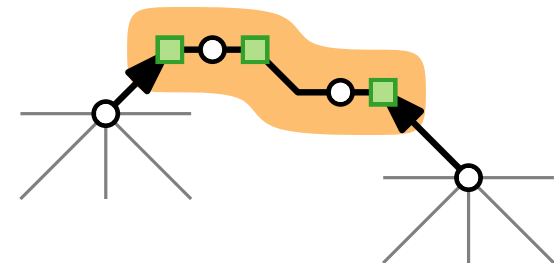
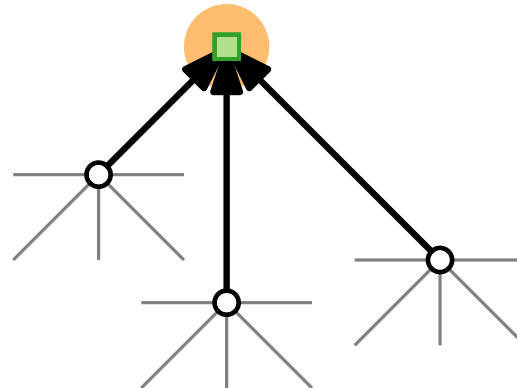
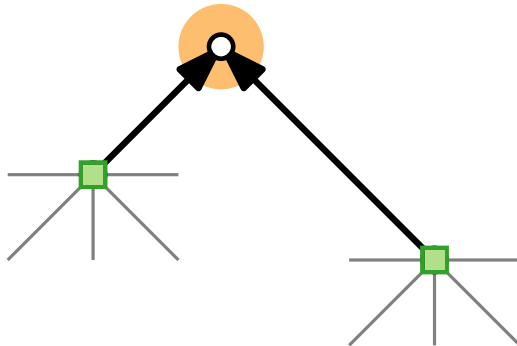
dummy vertex



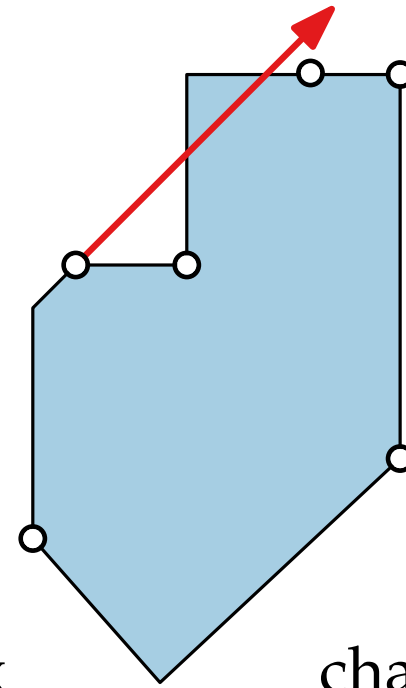
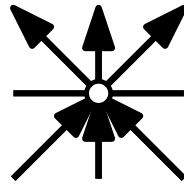
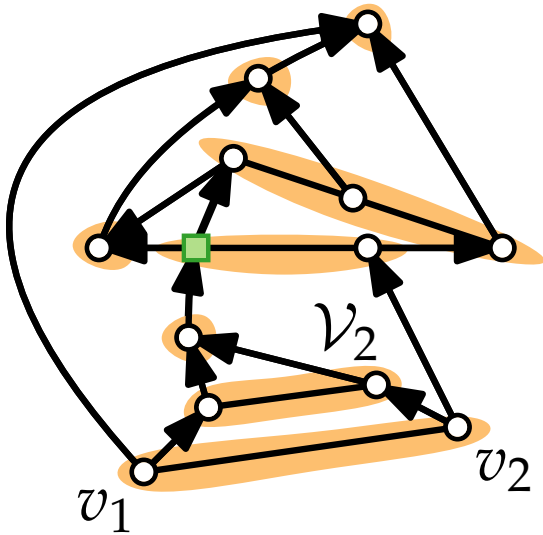
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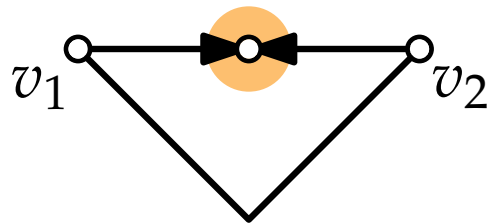
\mathcal{V}_i :



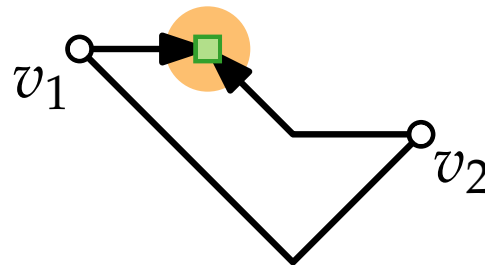
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



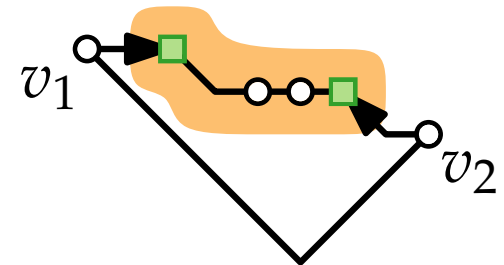
\mathcal{V}_2 : real vertex



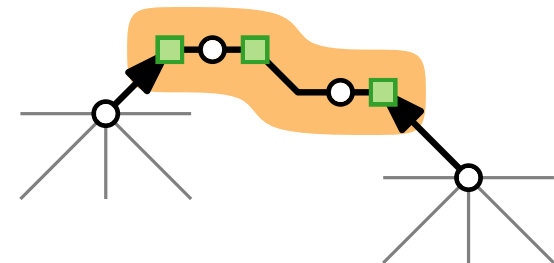
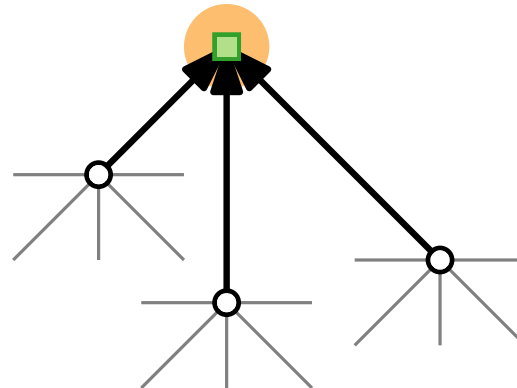
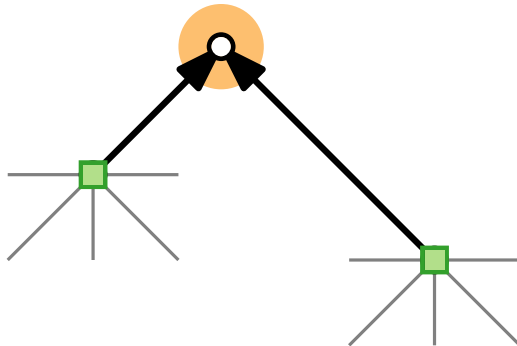
dummy vertex



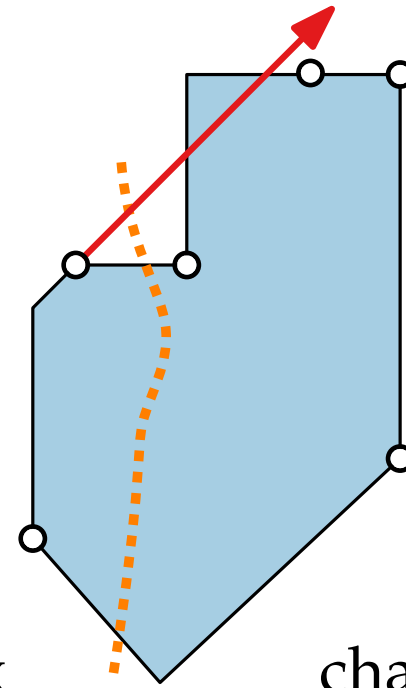
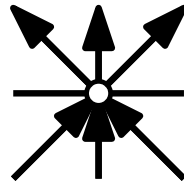
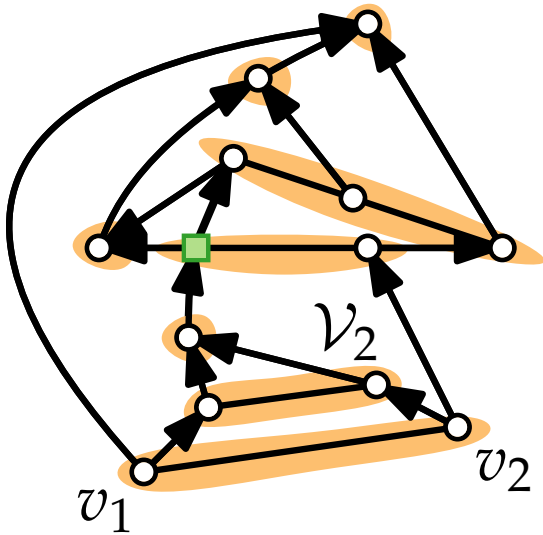
chain



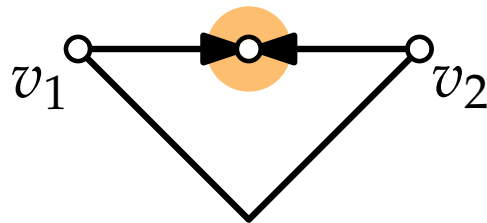
\mathcal{V}_i :



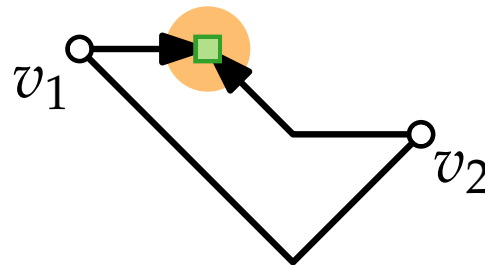
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



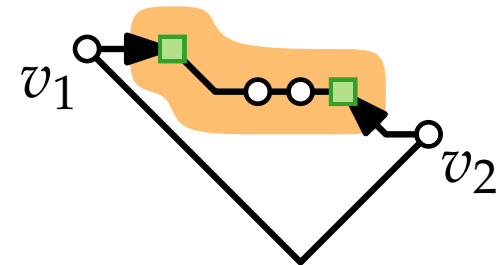
\mathcal{V}_2 : real vertex



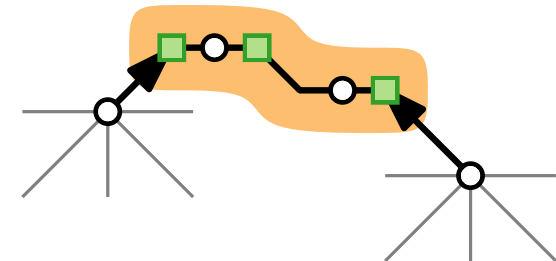
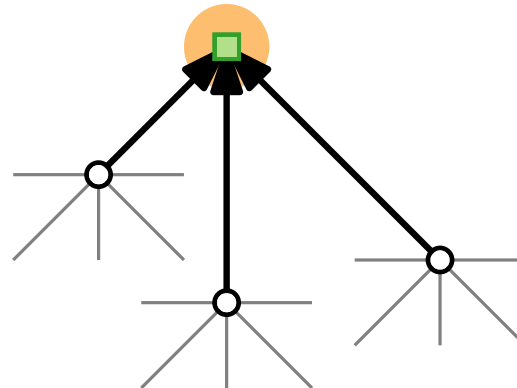
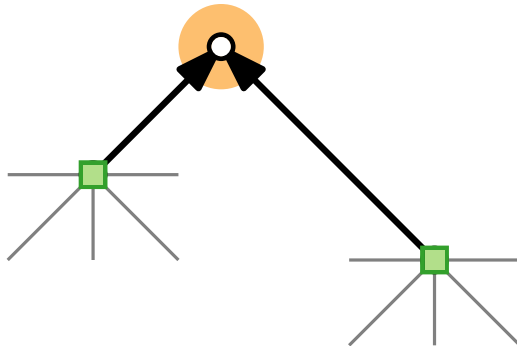
dummy vertex



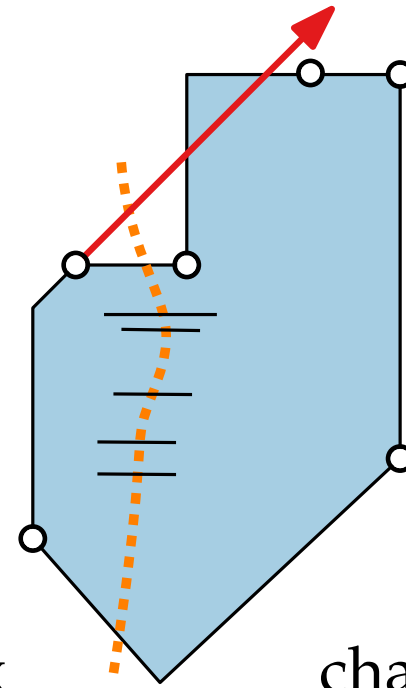
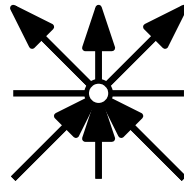
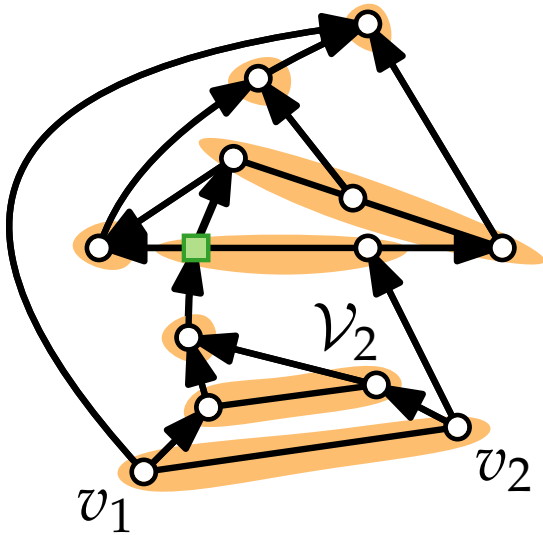
chain



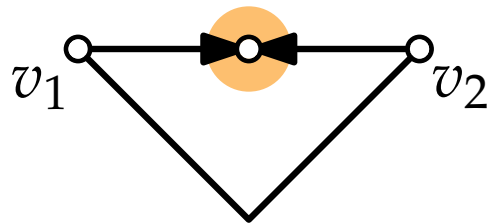
\mathcal{V}_i :



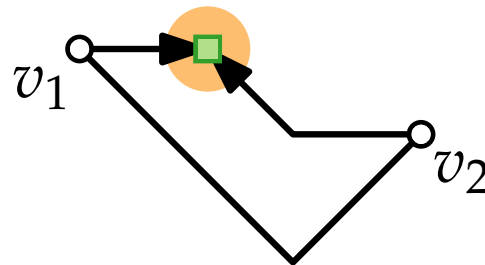
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



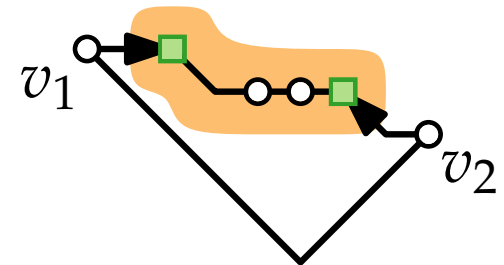
\mathcal{V}_2 : real vertex



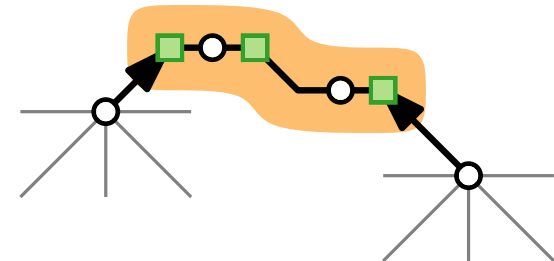
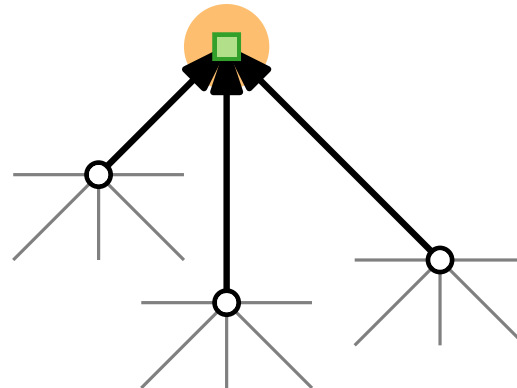
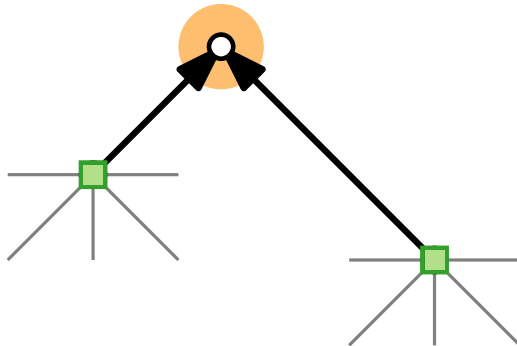
dummy vertex



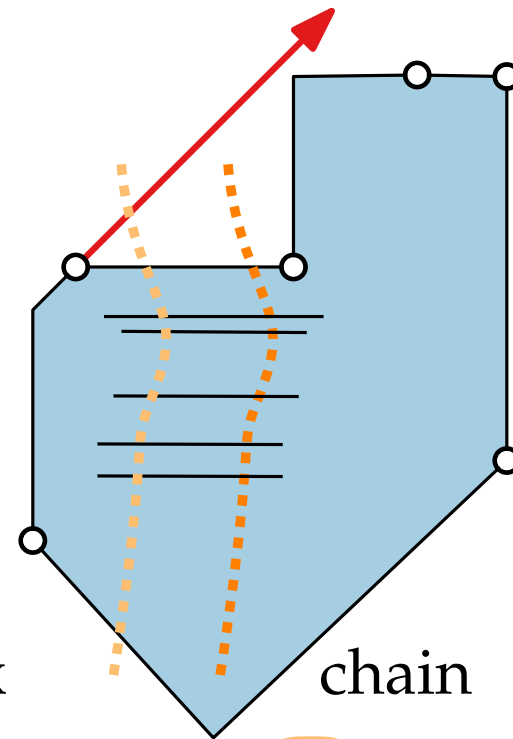
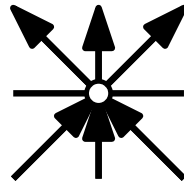
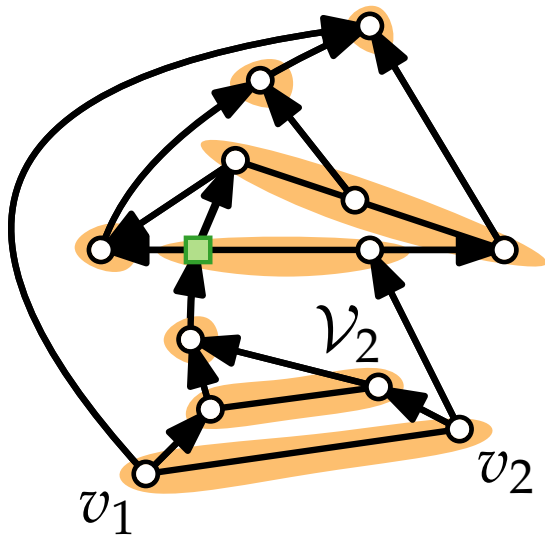
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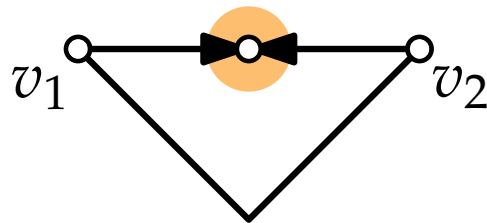
\mathcal{V}_i :



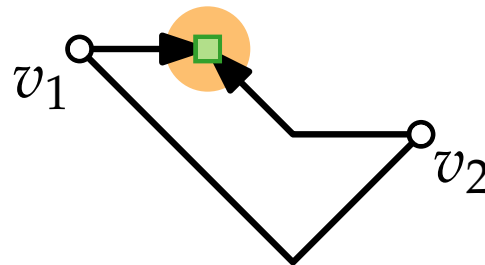
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



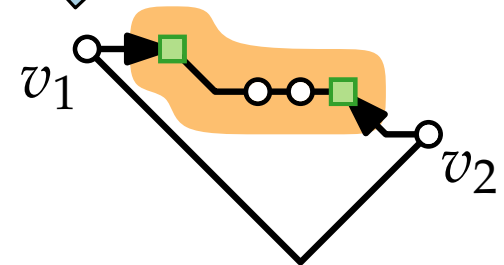
\mathcal{V}_2 : real vertex



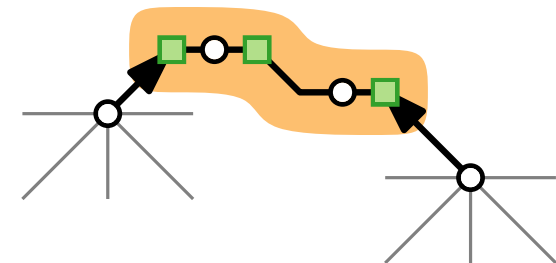
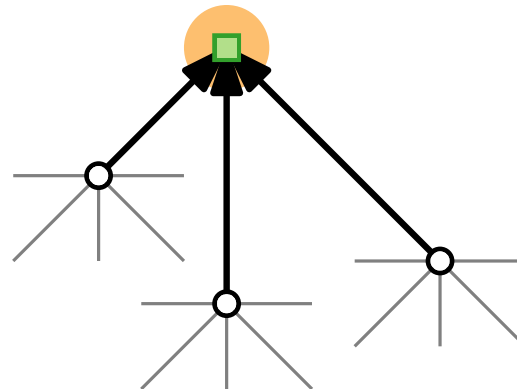
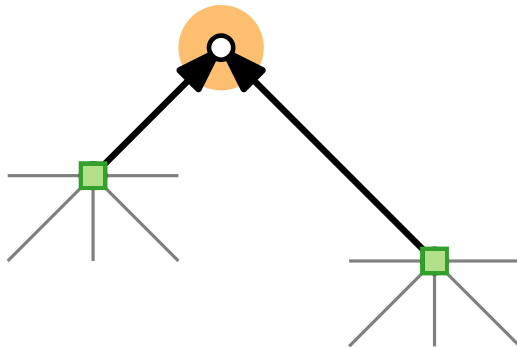
dummy vertex



chain

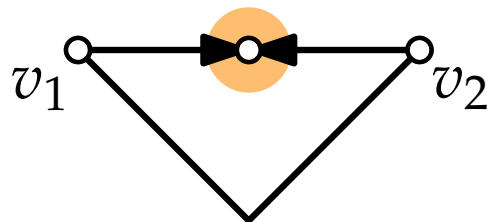


\mathcal{V}_i :

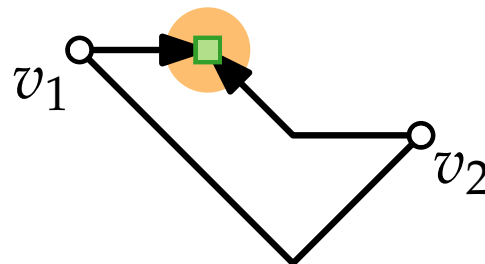


Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes

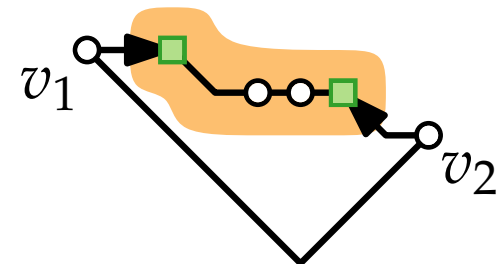
\mathcal{V}_2 : real vertex



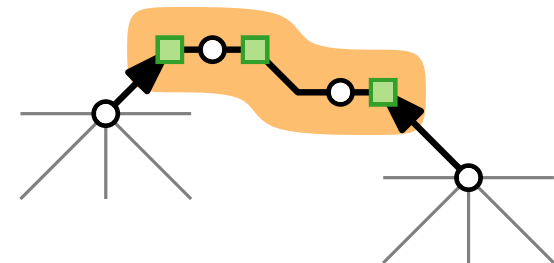
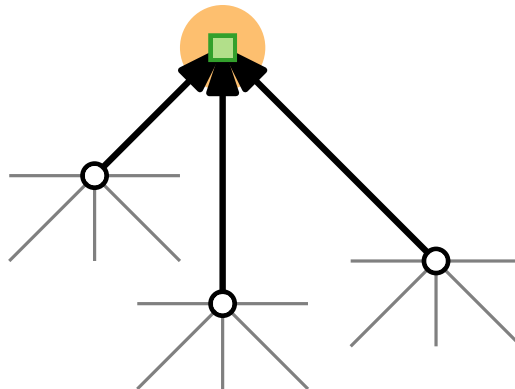
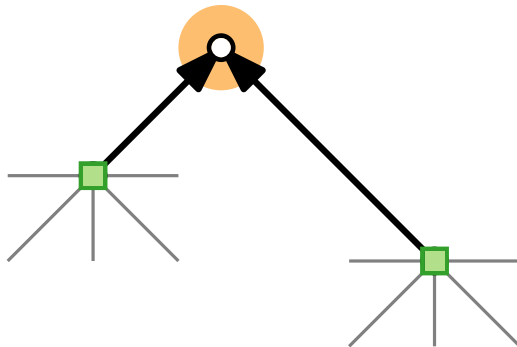
dummy vertex



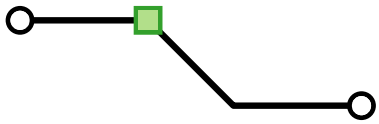
chain



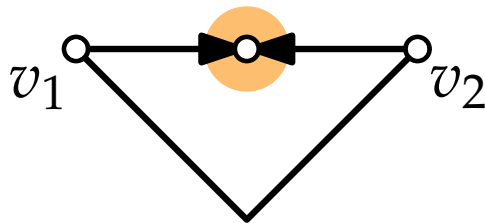
\mathcal{V}_i :



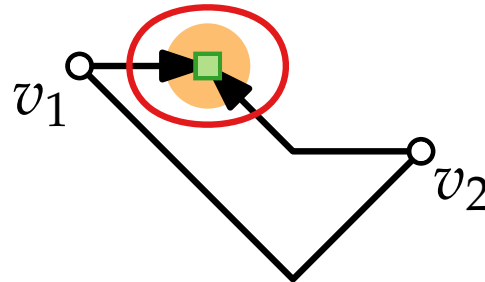
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



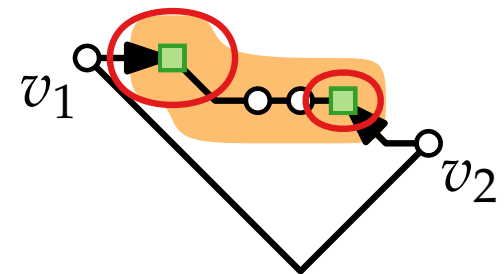
\mathcal{V}_2 : real vertex



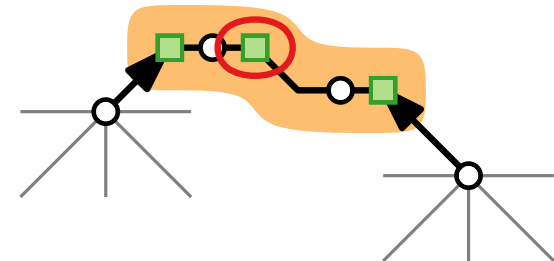
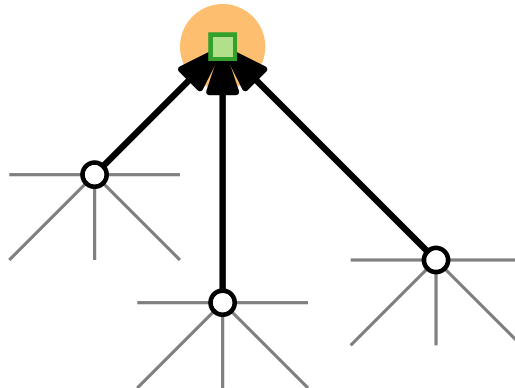
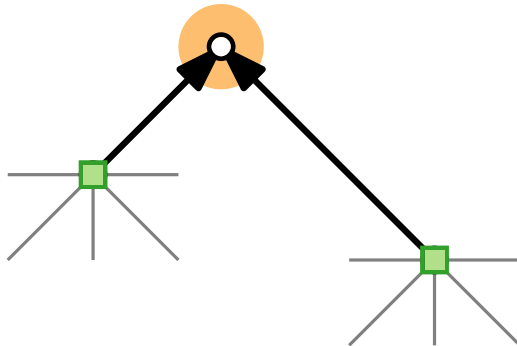
dummy vertex



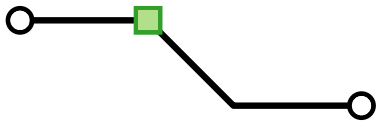
chain



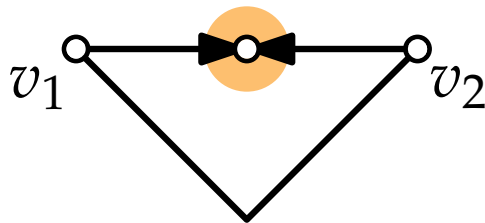
\mathcal{V}_i :



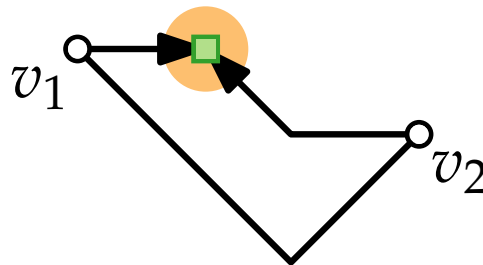
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



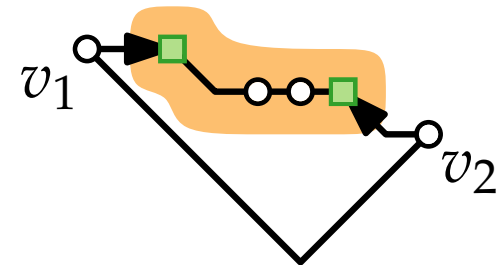
\mathcal{V}_2 : real vertex



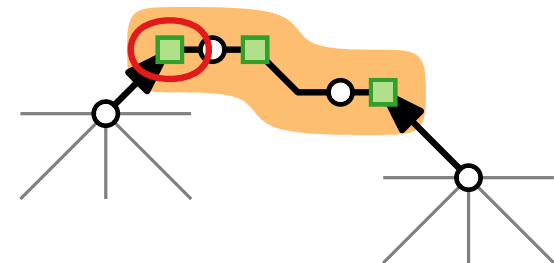
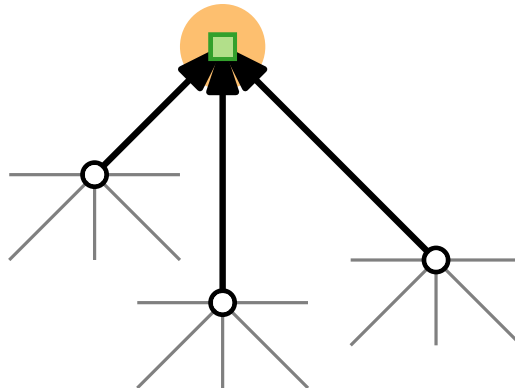
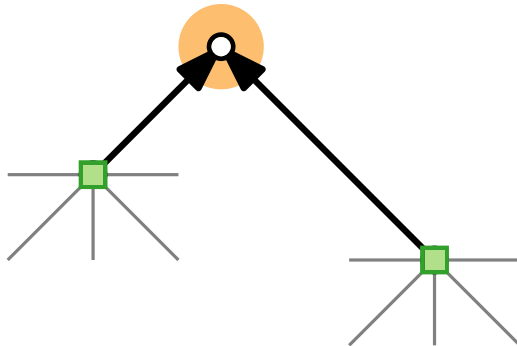
dummy vertex



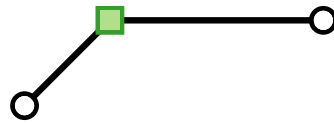
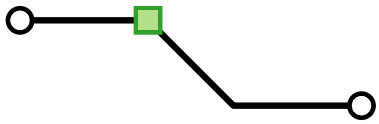
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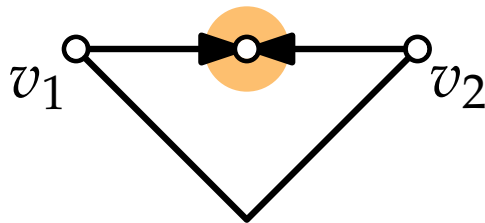
\mathcal{V}_i :



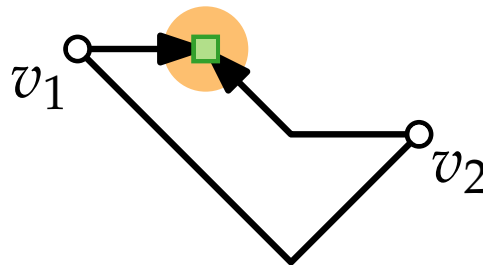
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



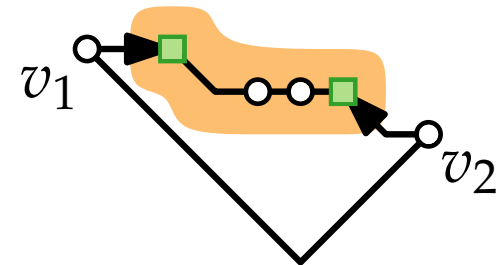
\mathcal{V}_2 : real vertex



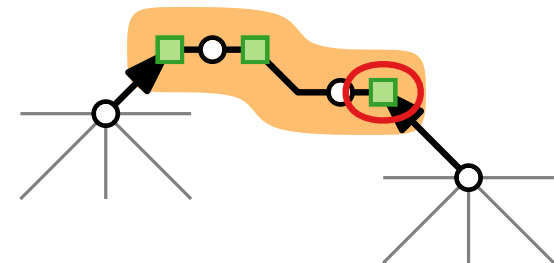
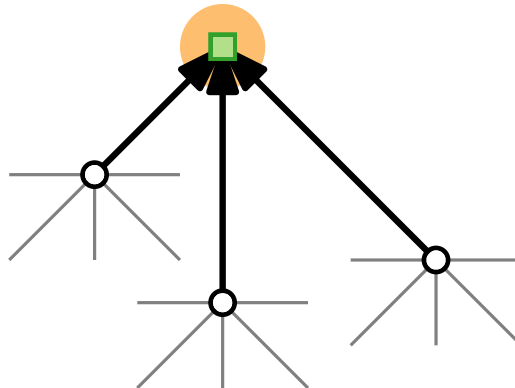
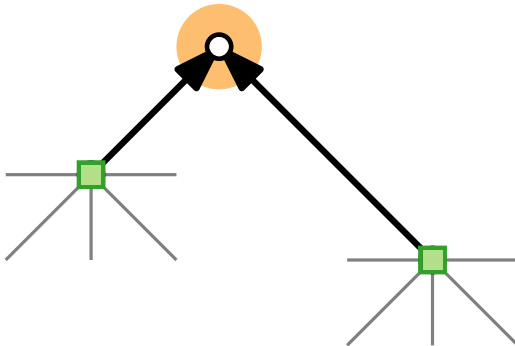
dummy vertex



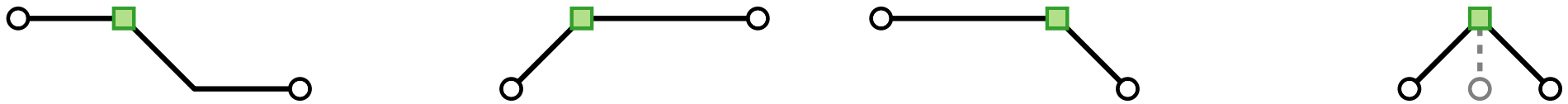
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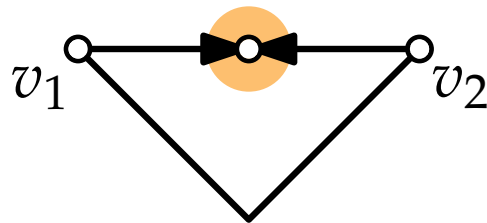
\mathcal{V}_i :



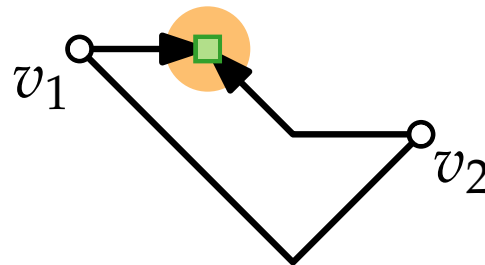
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



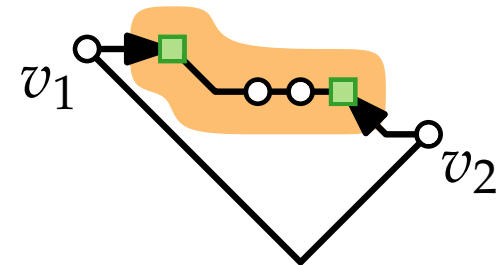
\mathcal{V}_2 : real vertex



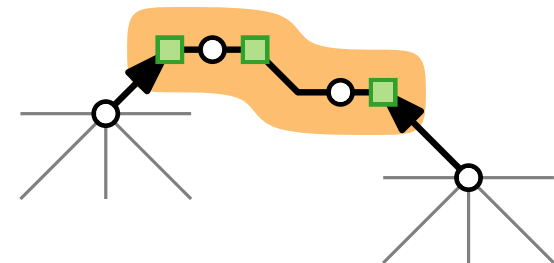
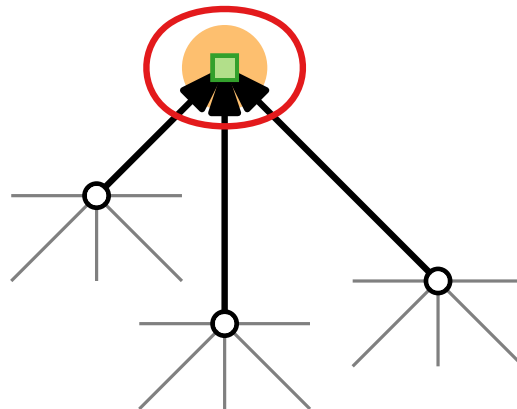
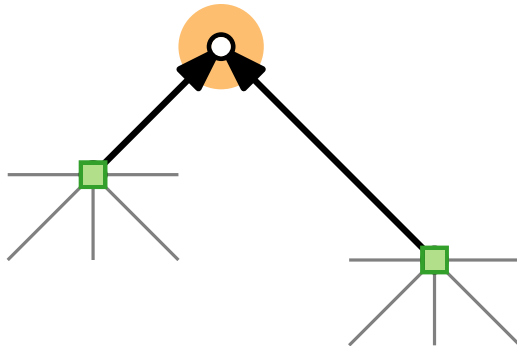
dummy vertex



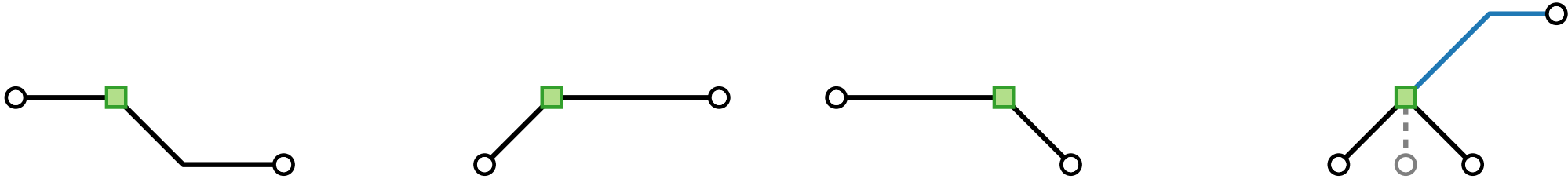
chain



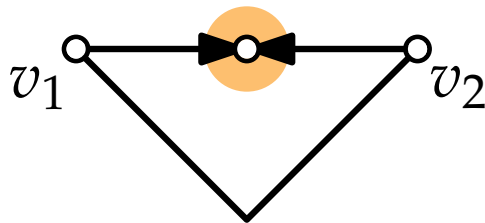
\mathcal{V}_i :



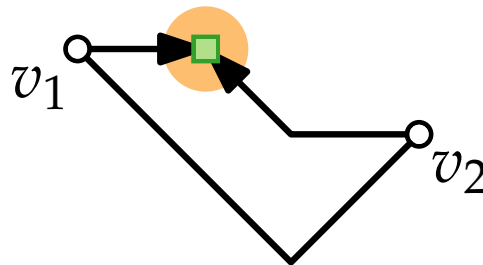
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



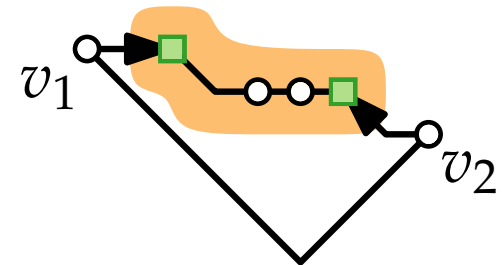
\mathcal{V}_2 : real vertex



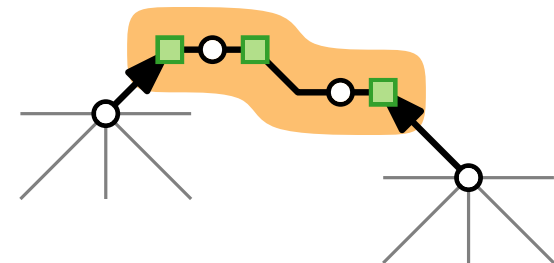
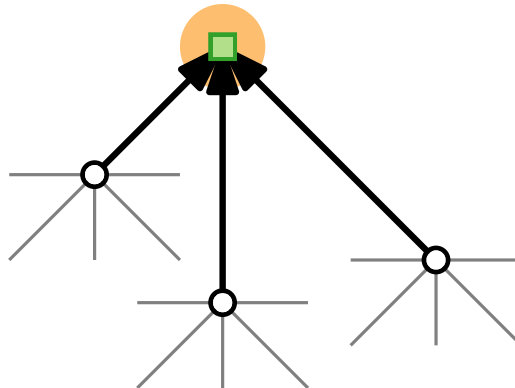
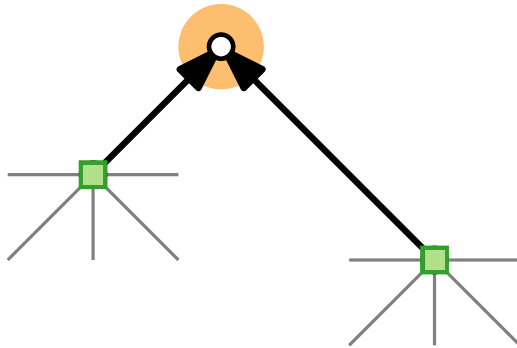
dummy vertex



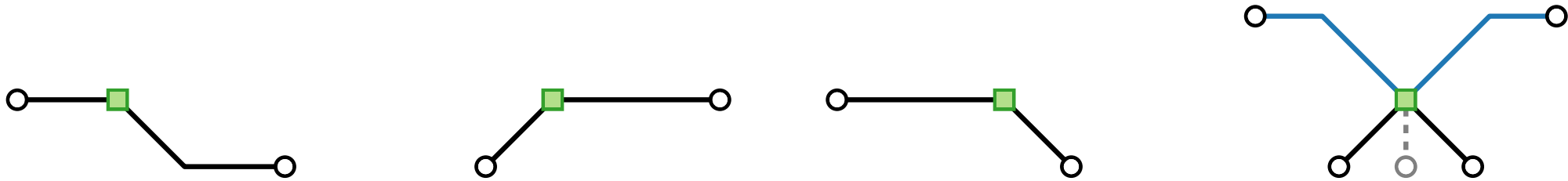
chain



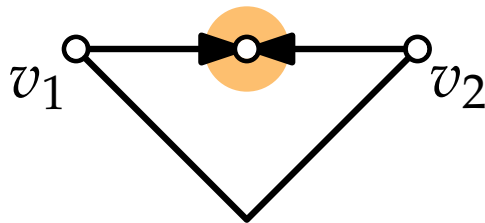
\mathcal{V}_i :



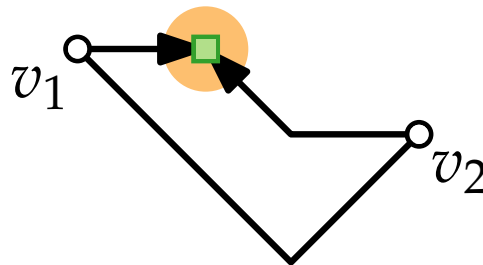
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



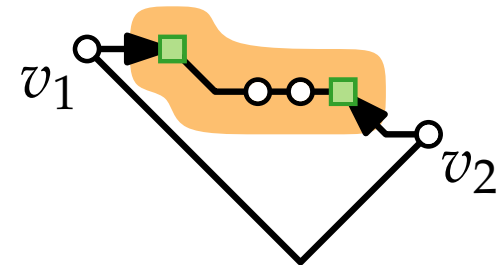
\mathcal{V}_2 : real vertex



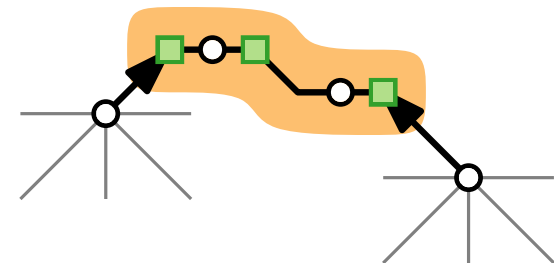
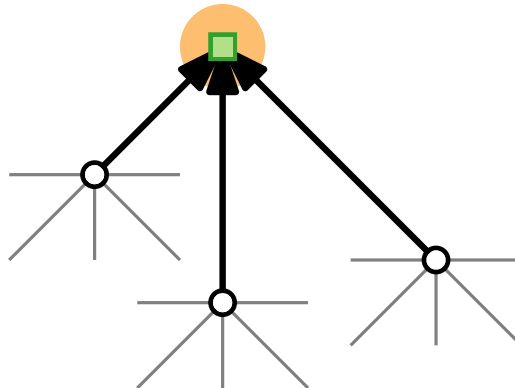
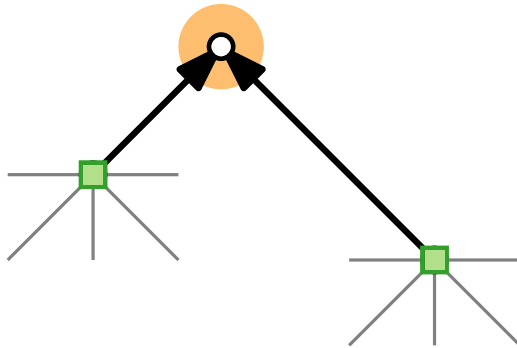
dummy vertex



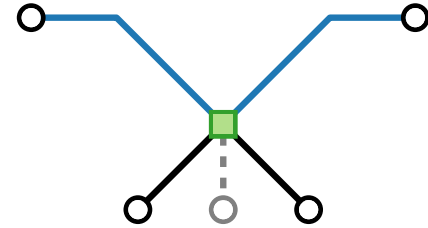
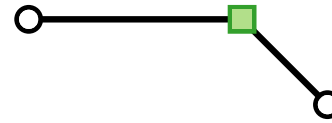
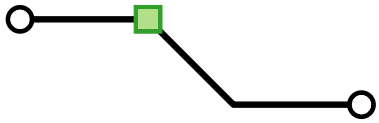
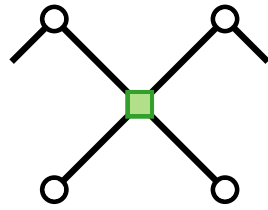
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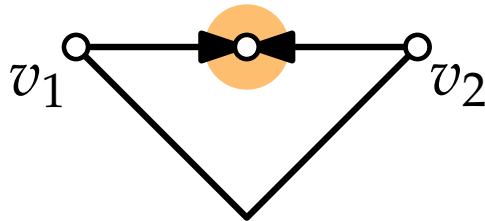
\mathcal{V}_i :



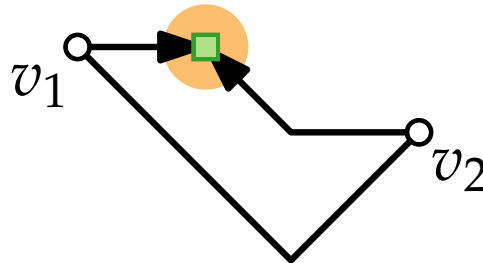
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



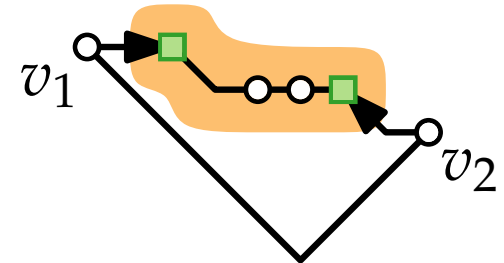
\mathcal{V}_2 : real vertex



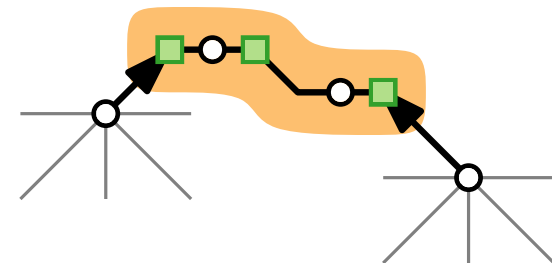
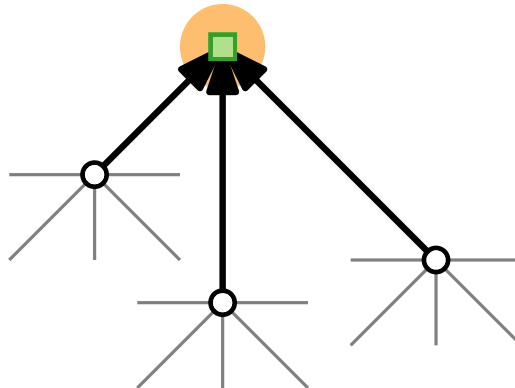
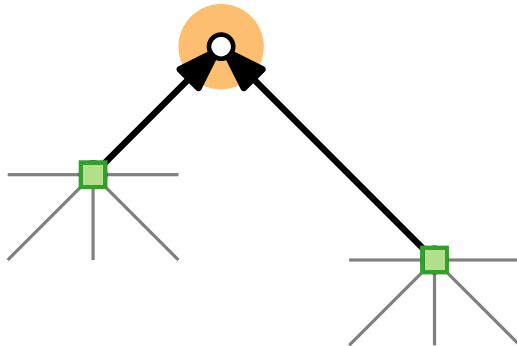
dummy vertex



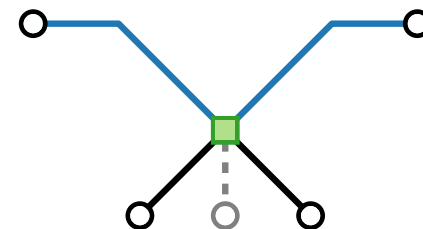
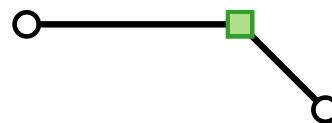
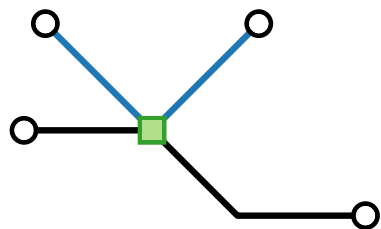
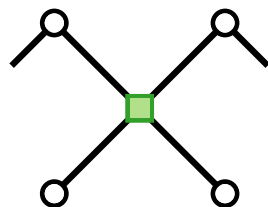
chain



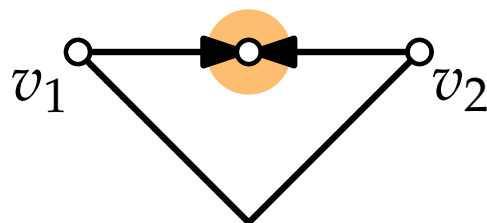
\mathcal{V}_i :



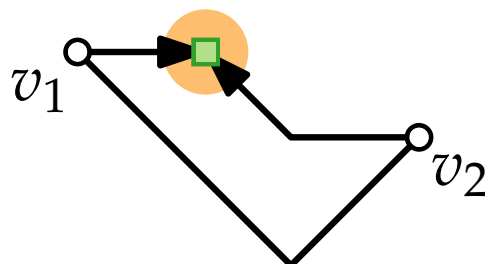
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



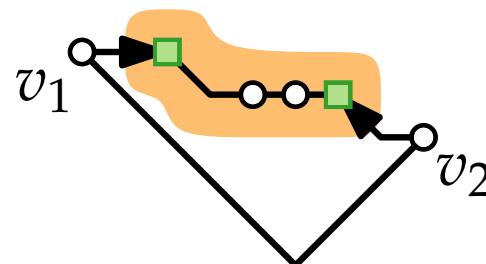
\mathcal{V}_2 : real vertex



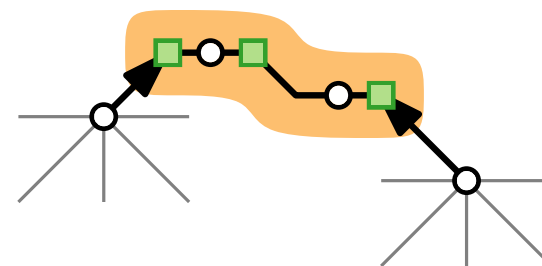
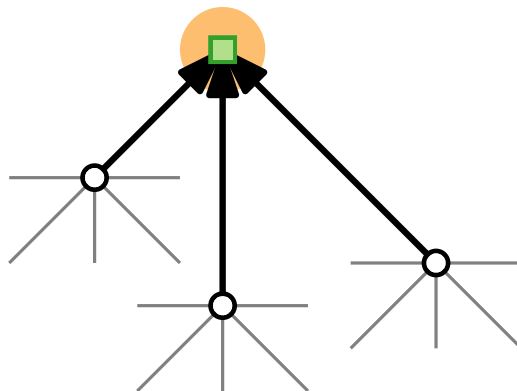
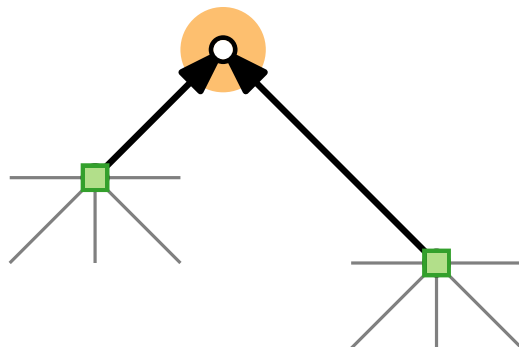
dummy vertex



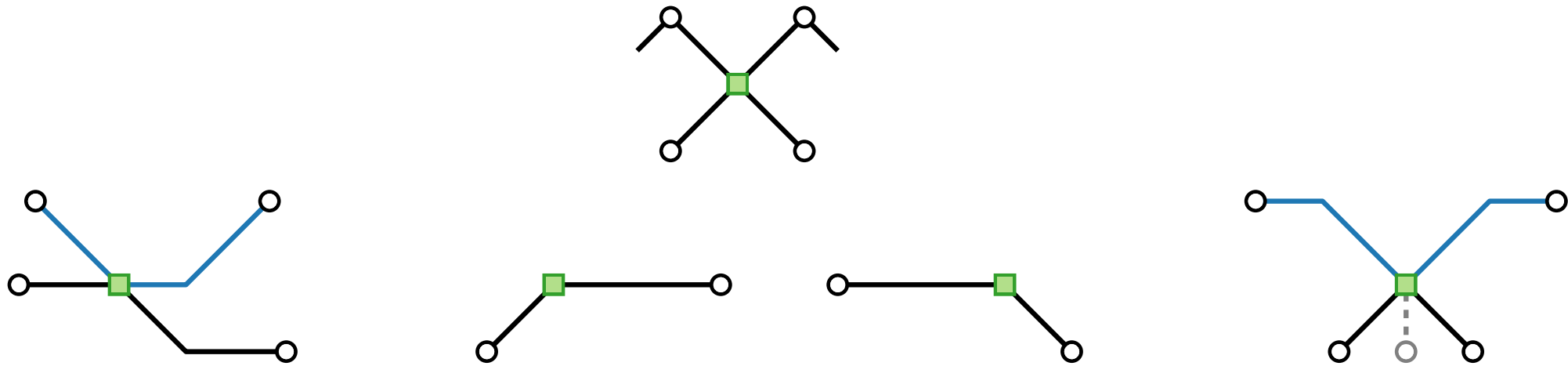
chain



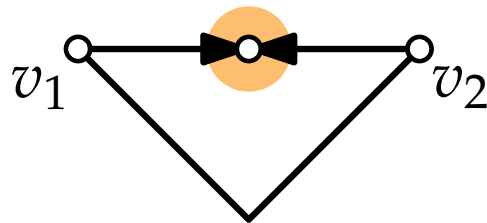
\mathcal{V}_i :



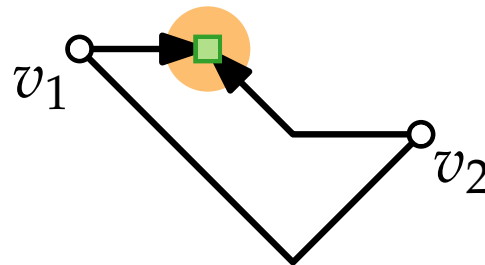
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



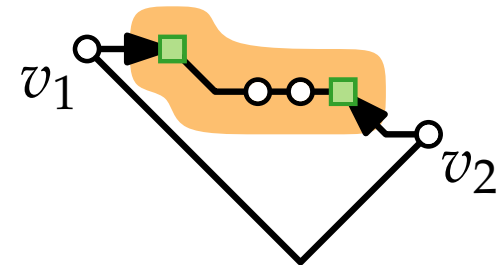
\mathcal{V}_2 : real vertex



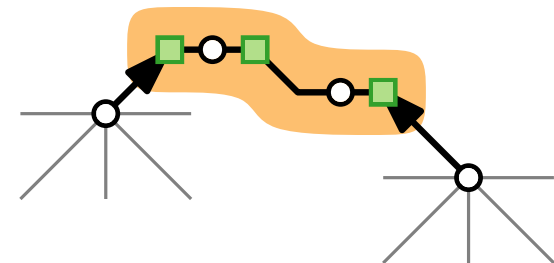
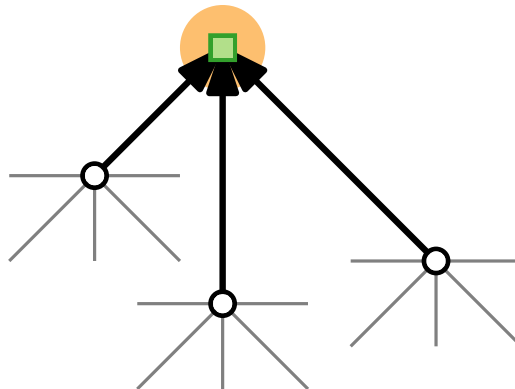
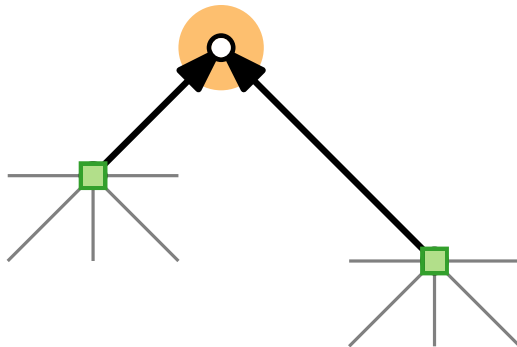
dummy vertex



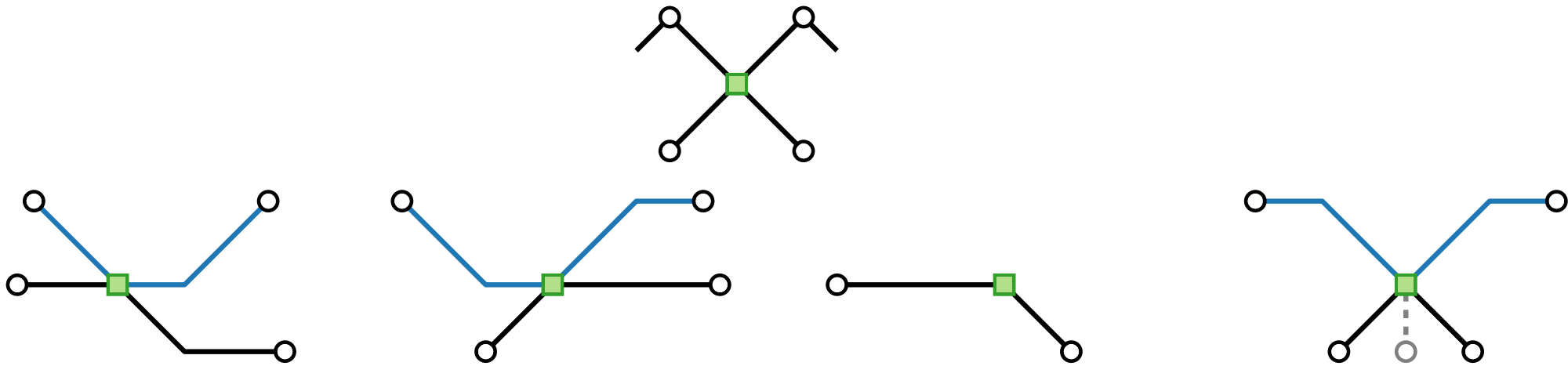
chain



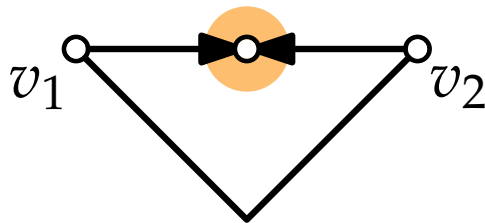
\mathcal{V}_i :



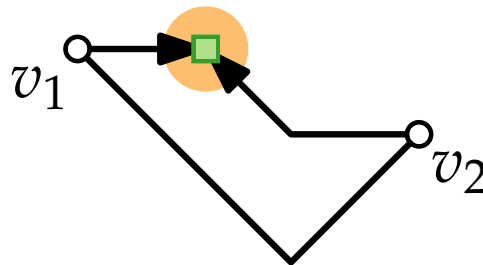
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



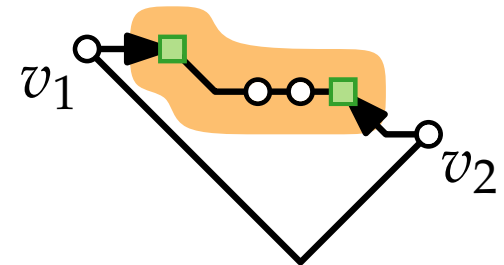
\mathcal{V}_2 : real vertex



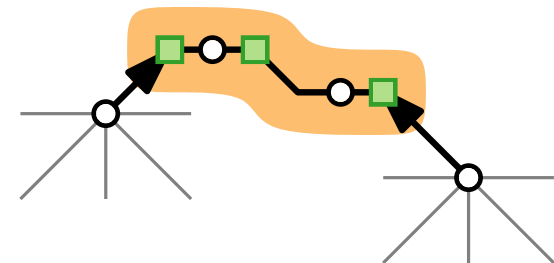
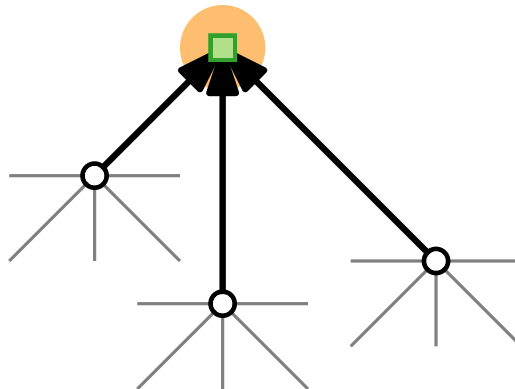
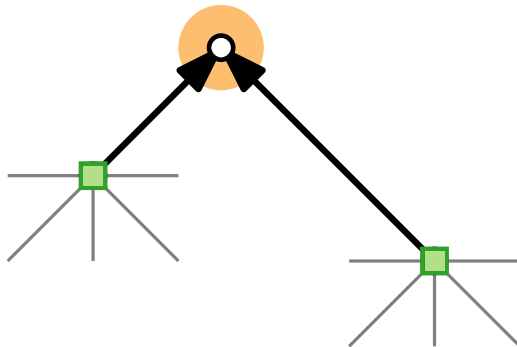
dummy vertex



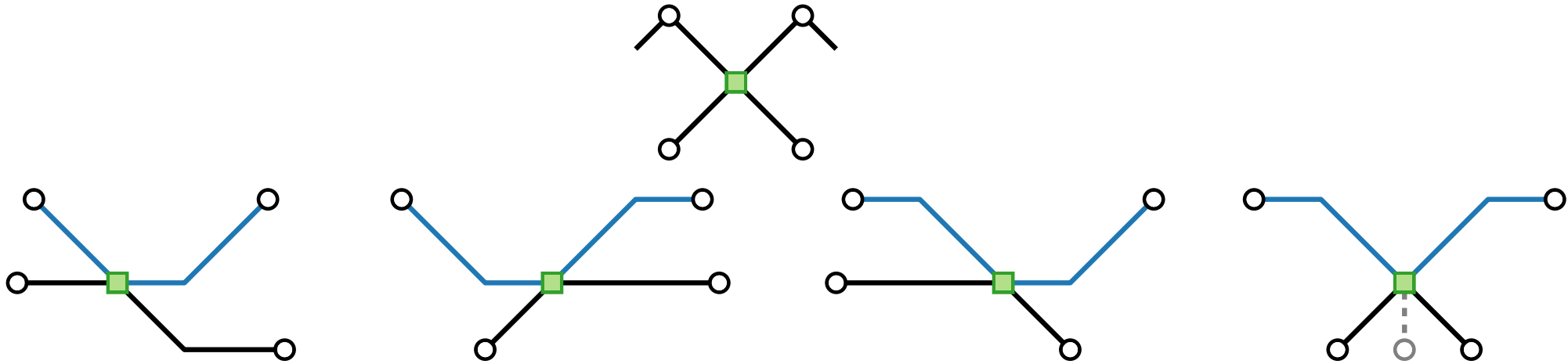
chain



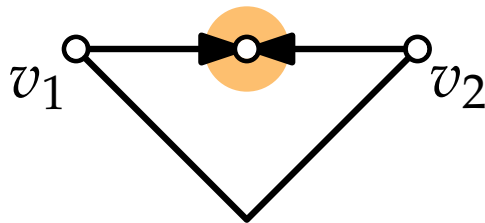
\mathcal{V}_i :



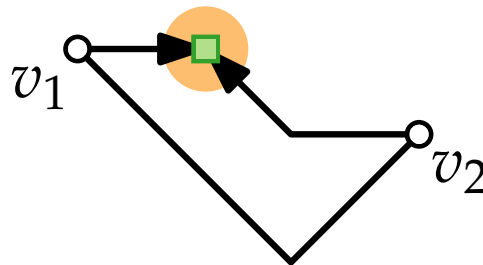
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



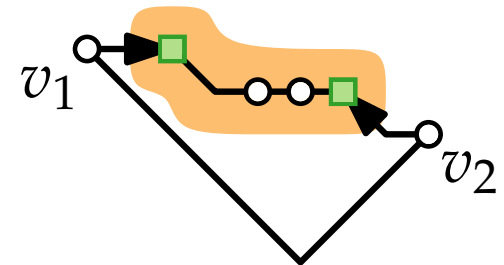
\mathcal{V}_2 : real vertex



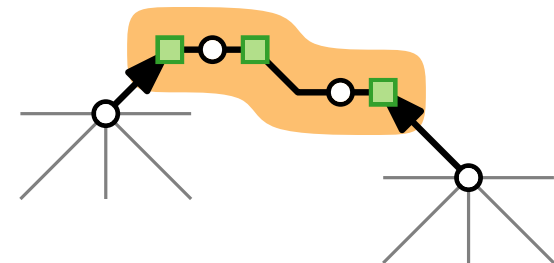
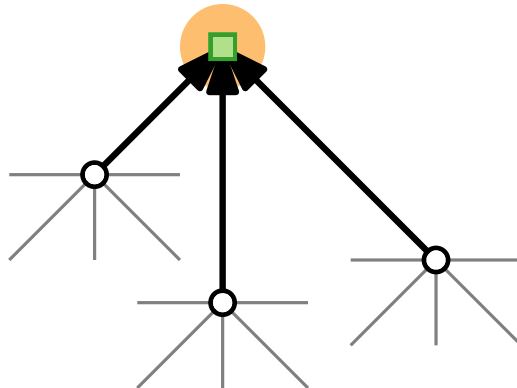
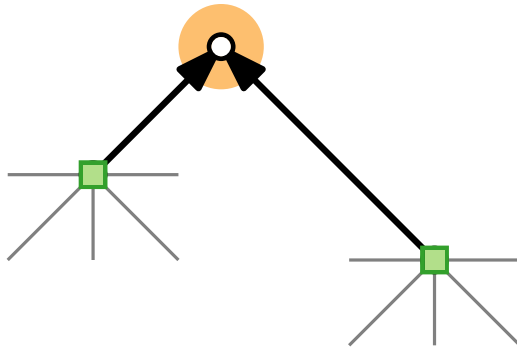
dummy vertex



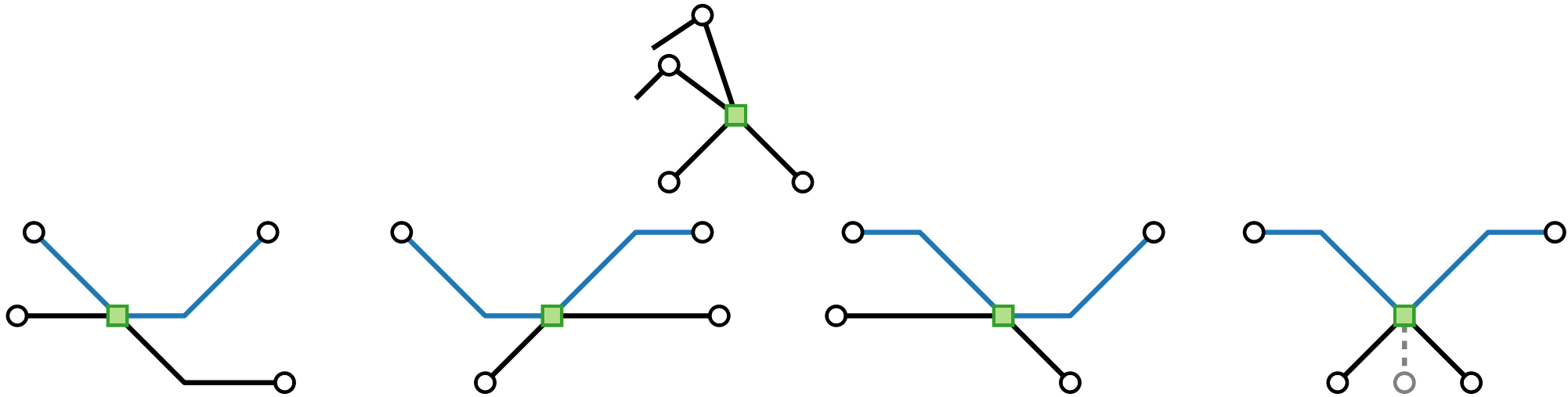
chain



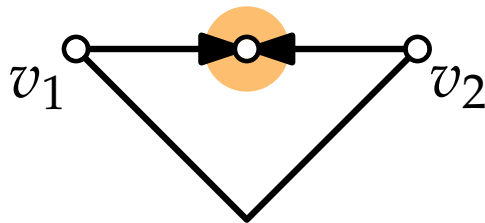
\mathcal{V}_i :



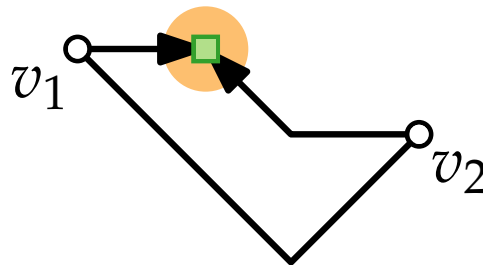
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



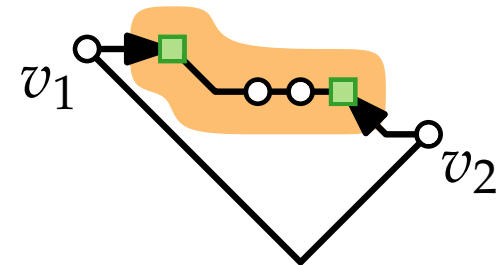
\mathcal{V}_2 : real vertex



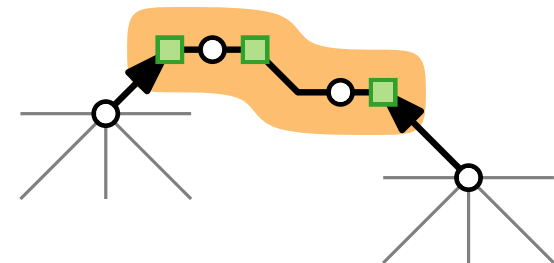
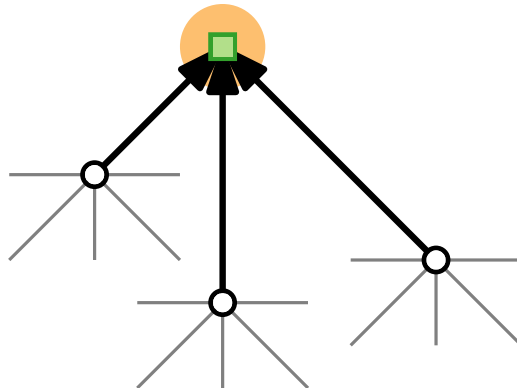
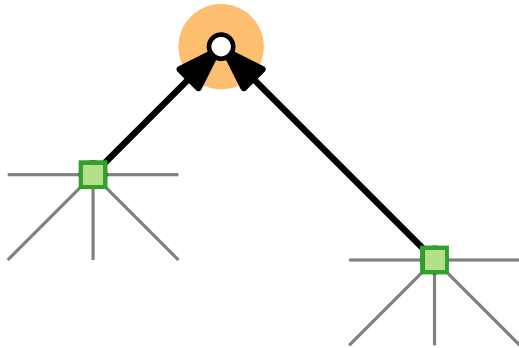
dummy vertex



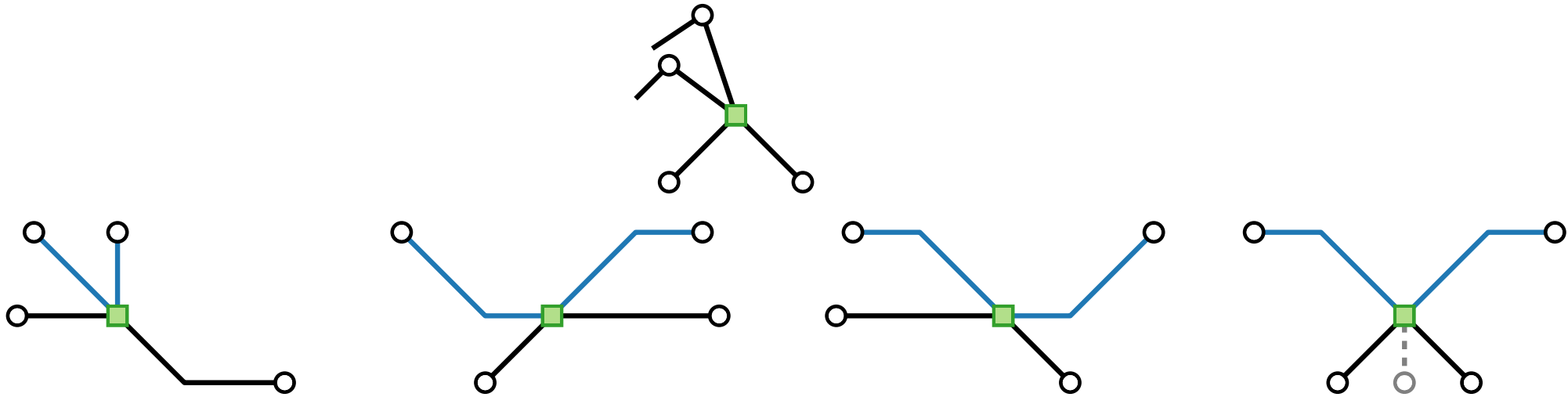
chain



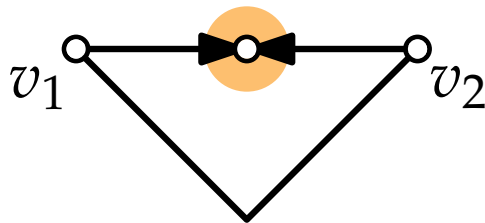
\mathcal{V}_i :



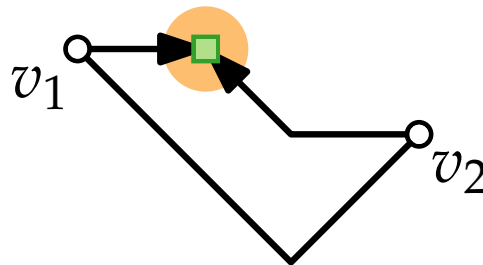
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



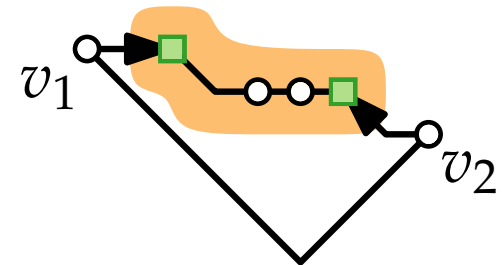
\mathcal{V}_2 : real vertex



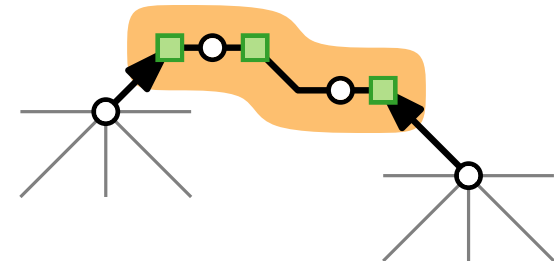
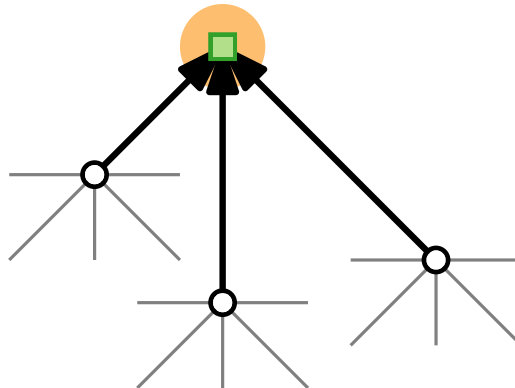
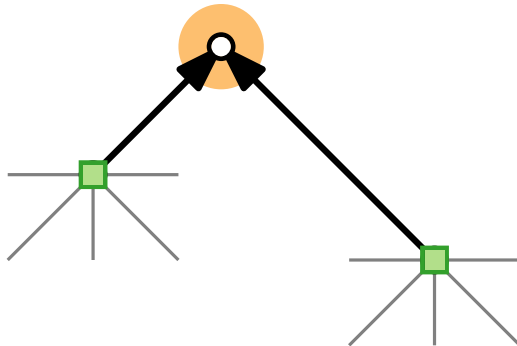
dummy vertex



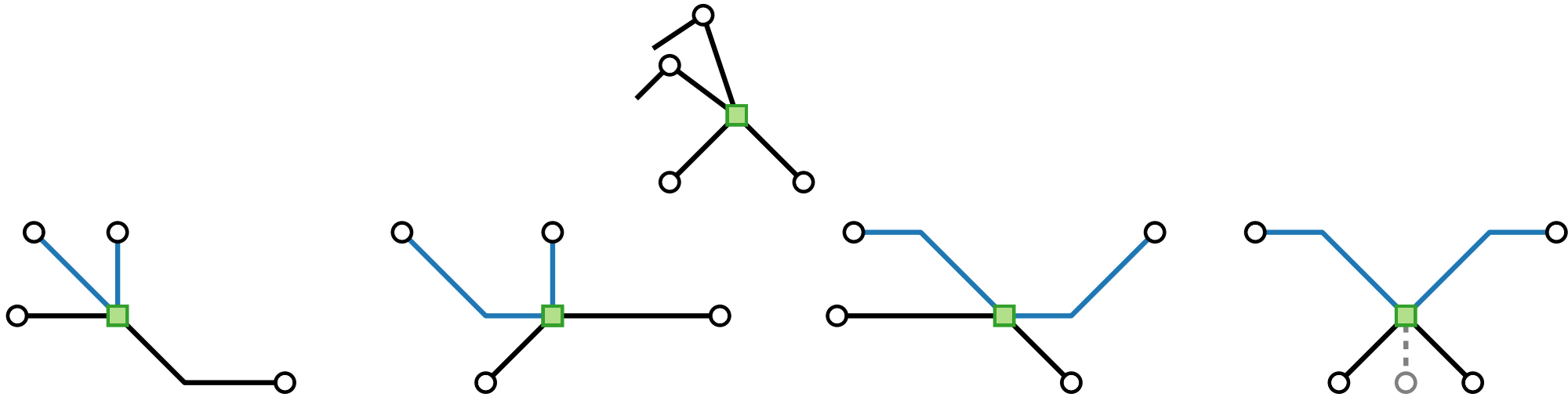
chain



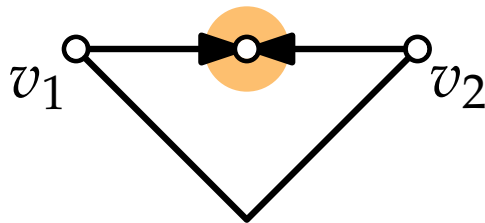
\mathcal{V}_i :



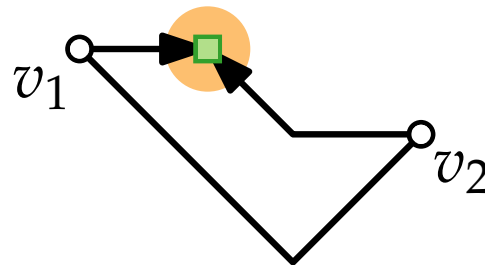
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



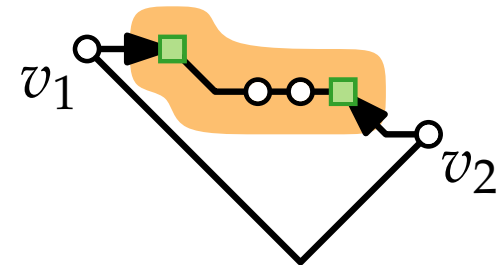
\mathcal{V}_2 : real vertex



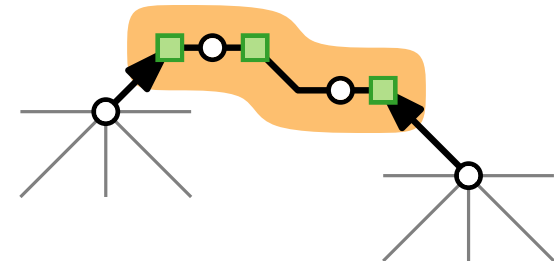
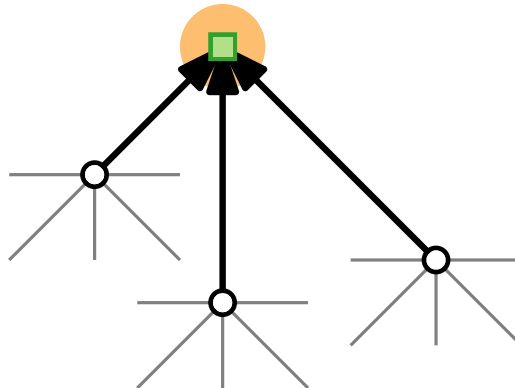
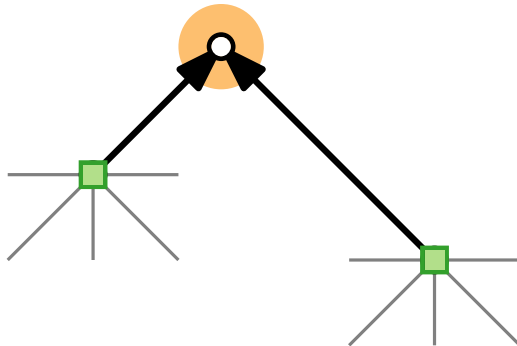
dummy vertex



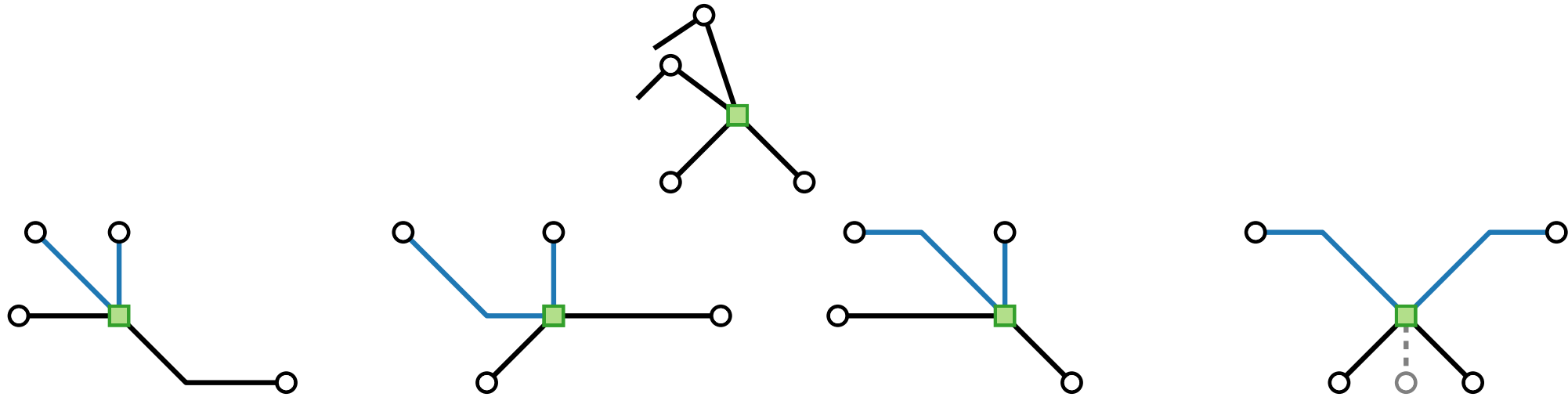
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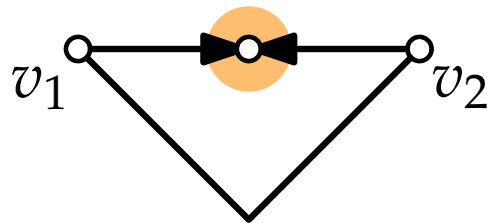
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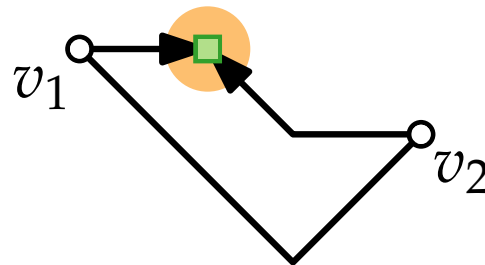
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



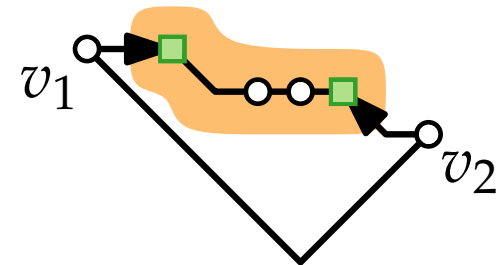
\mathcal{V}_2 : real vertex



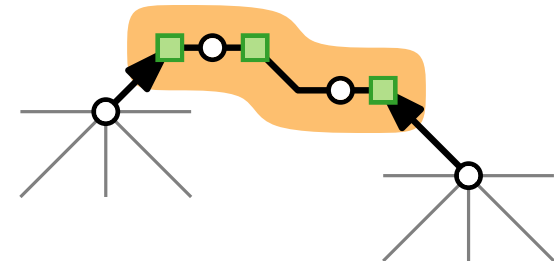
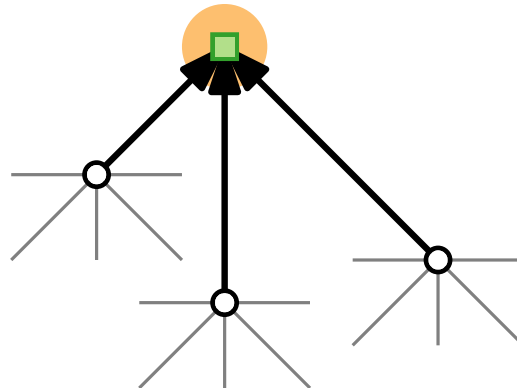
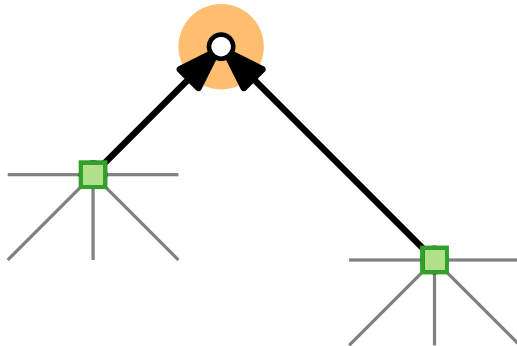
dummy vertex



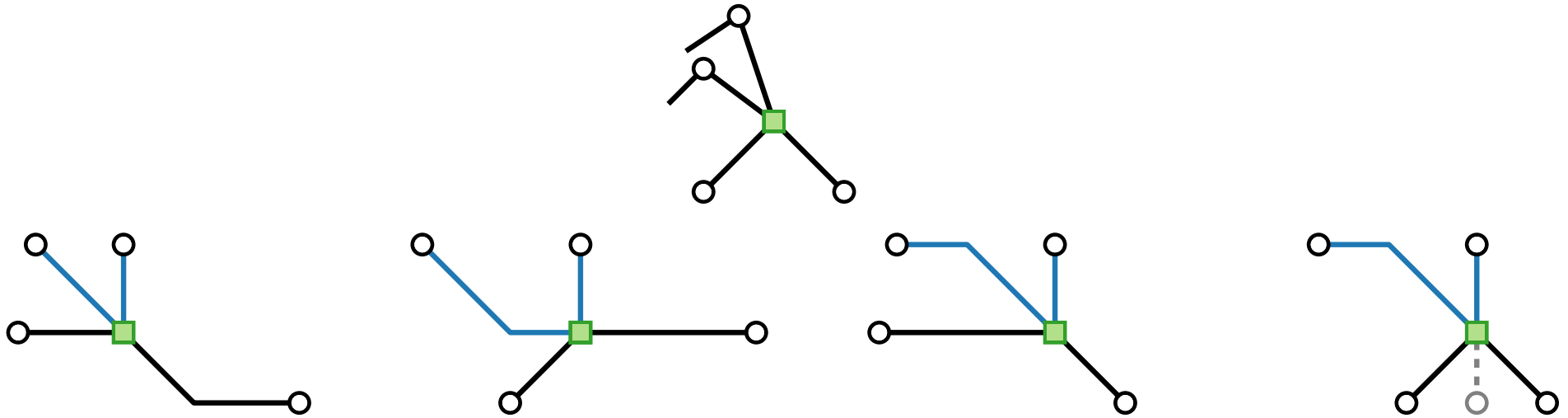
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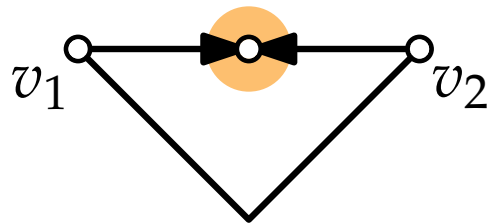
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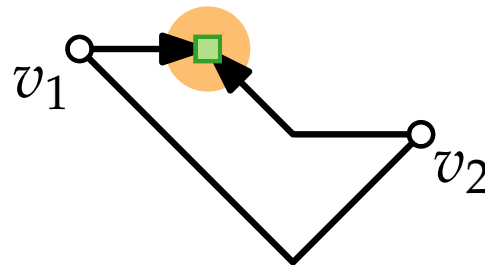
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



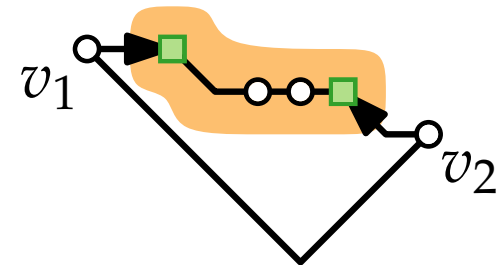
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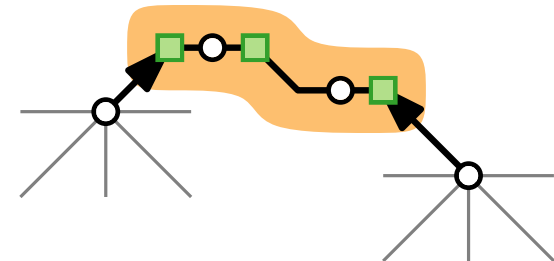
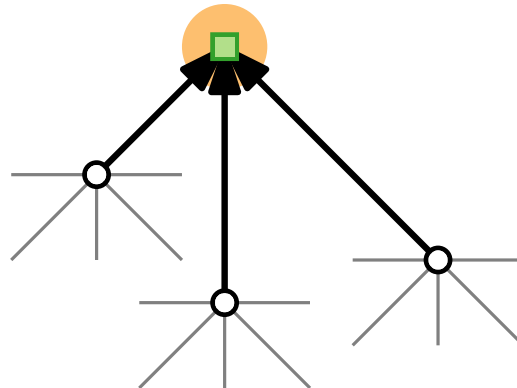
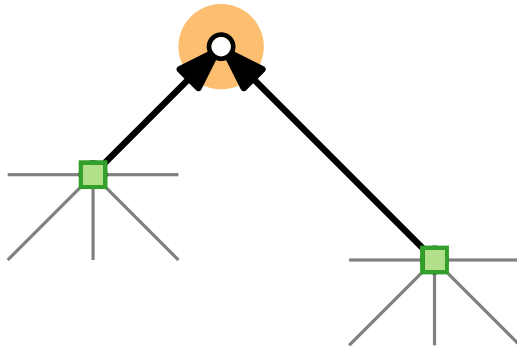
dummy vertex



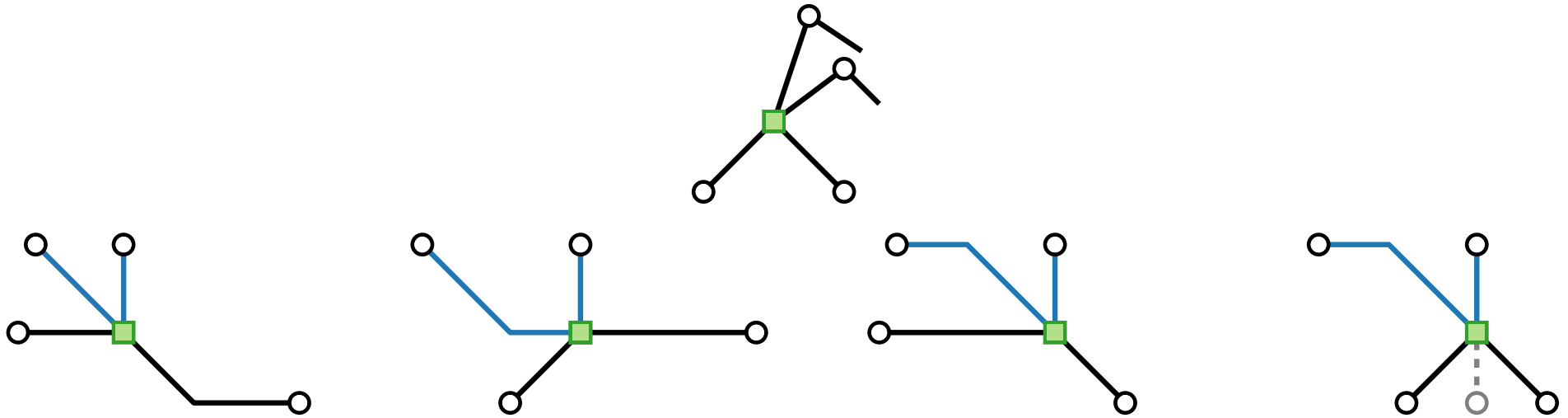
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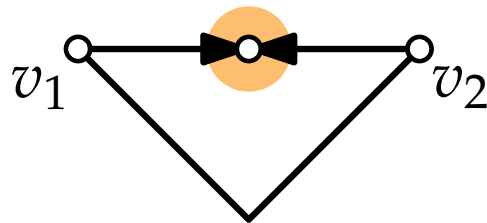
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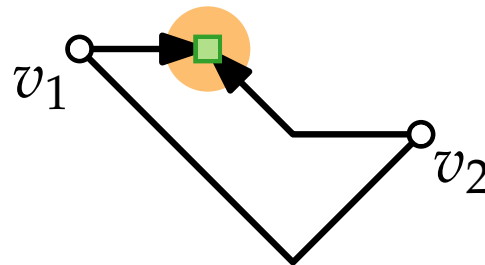
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



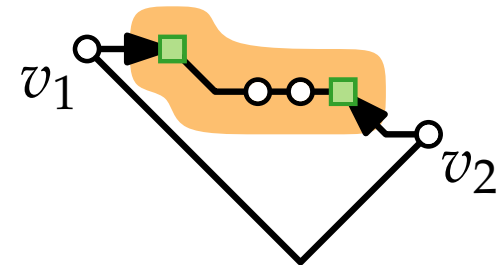
\mathcal{V}_2 : real vertex



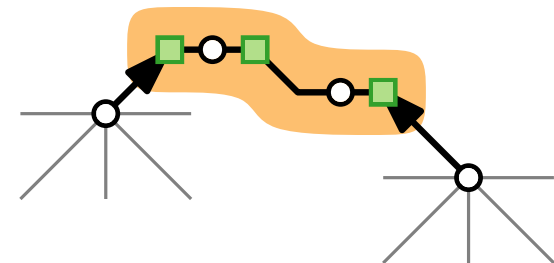
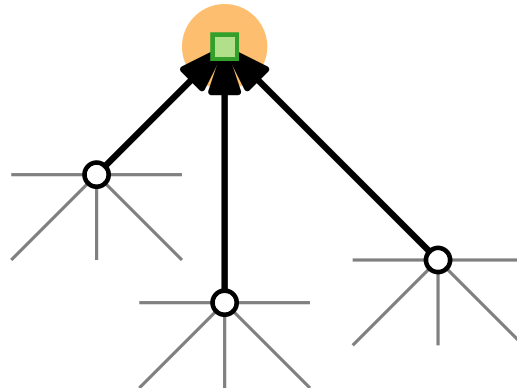
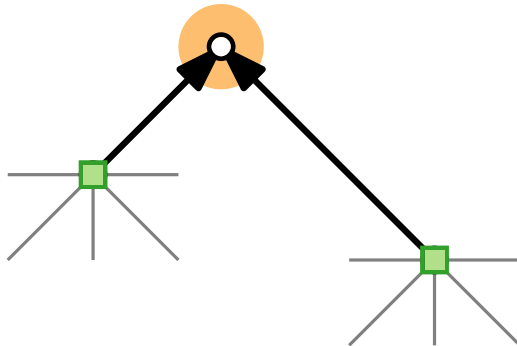
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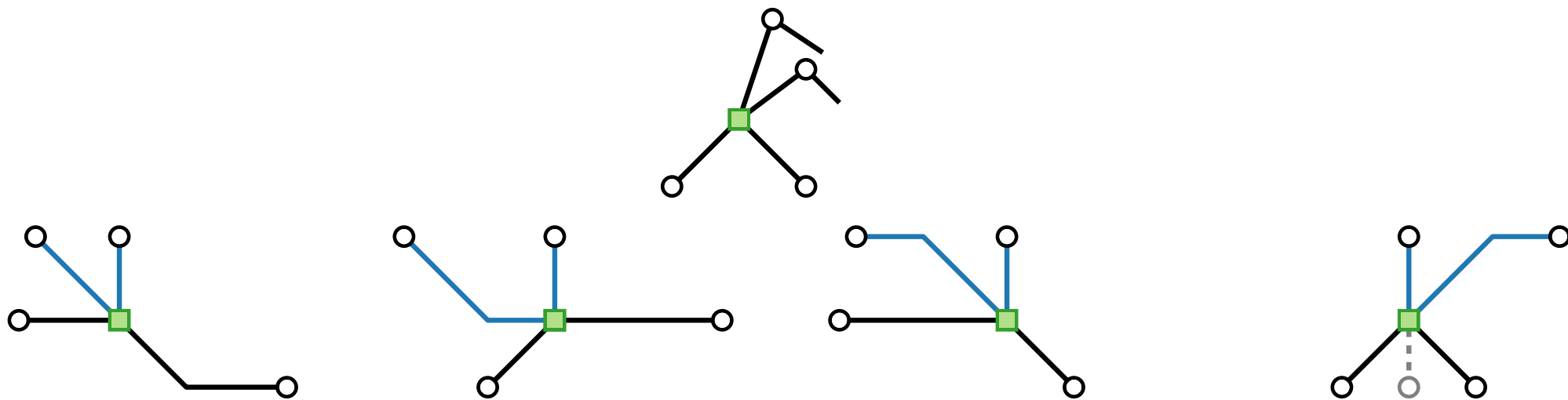
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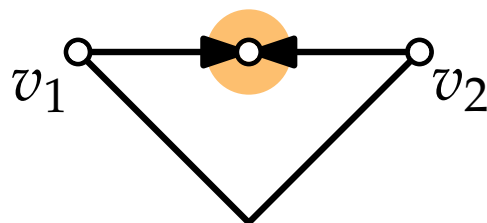
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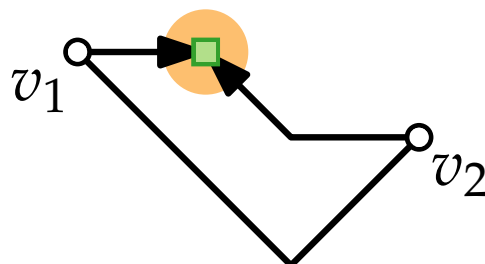
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



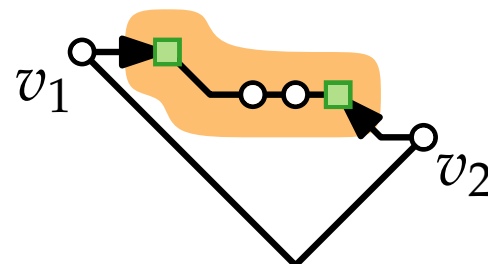
\mathcal{V}_2 : real vertex



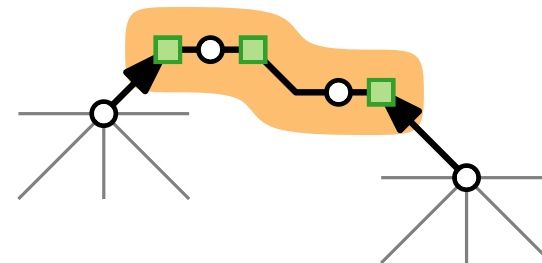
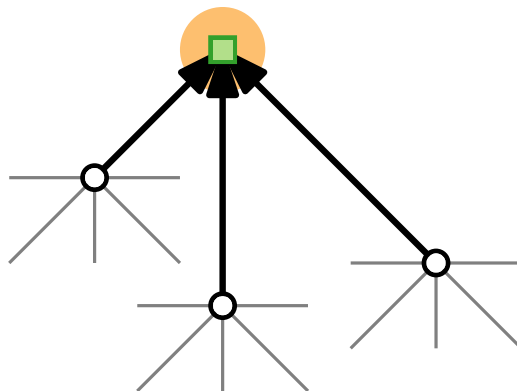
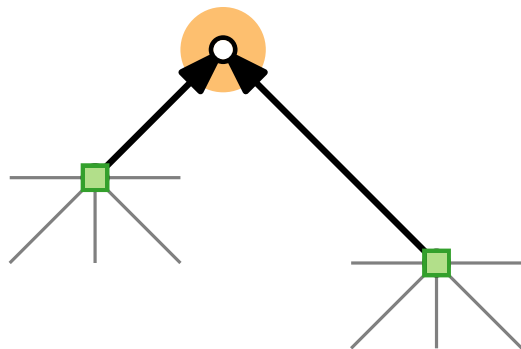
dummy vertex



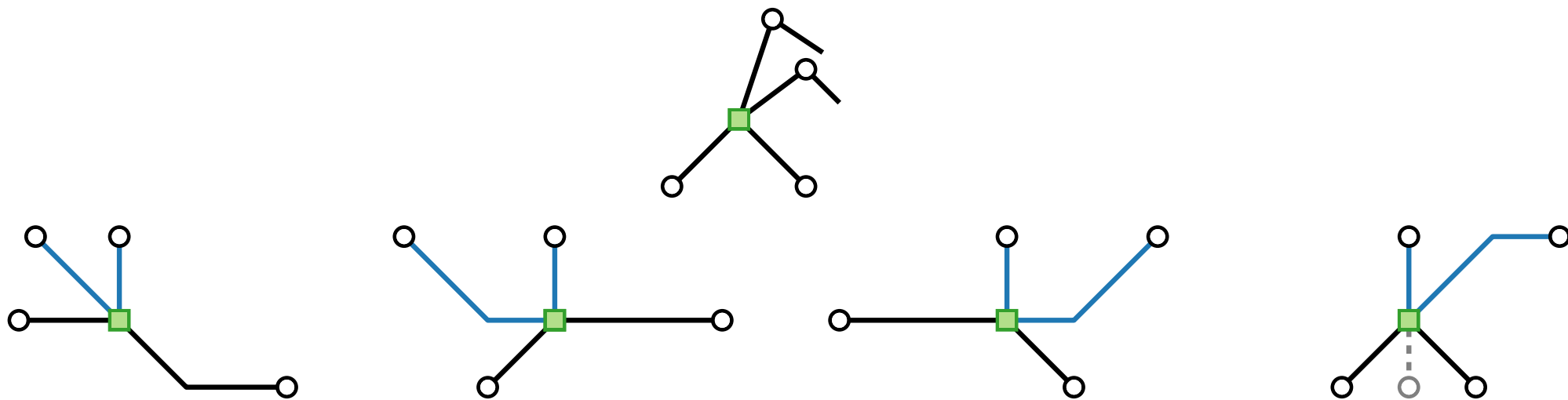
chain



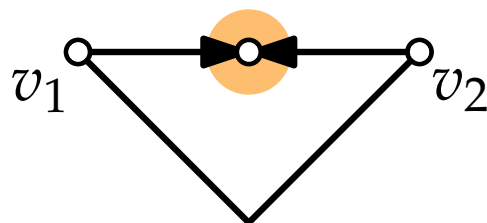
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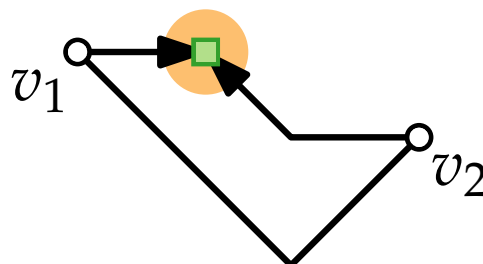
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



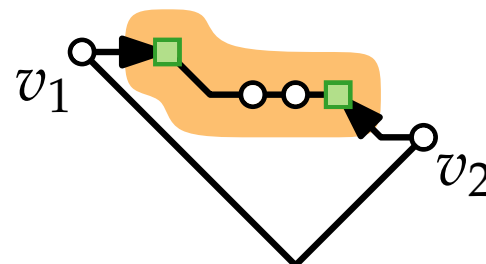
\mathcal{V}_2 : real vertex



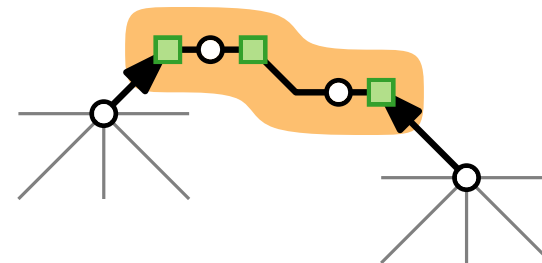
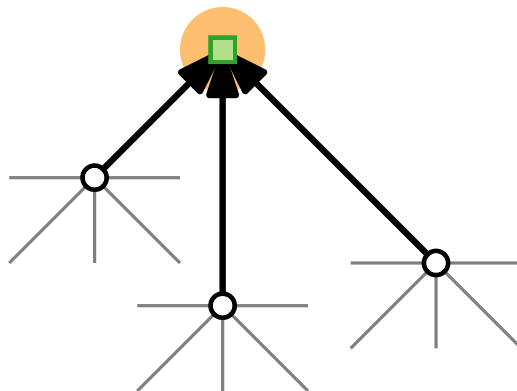
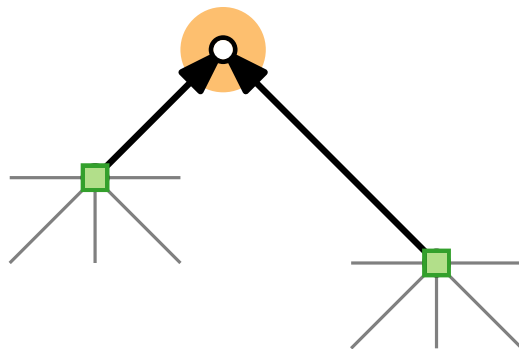
dummy vertex



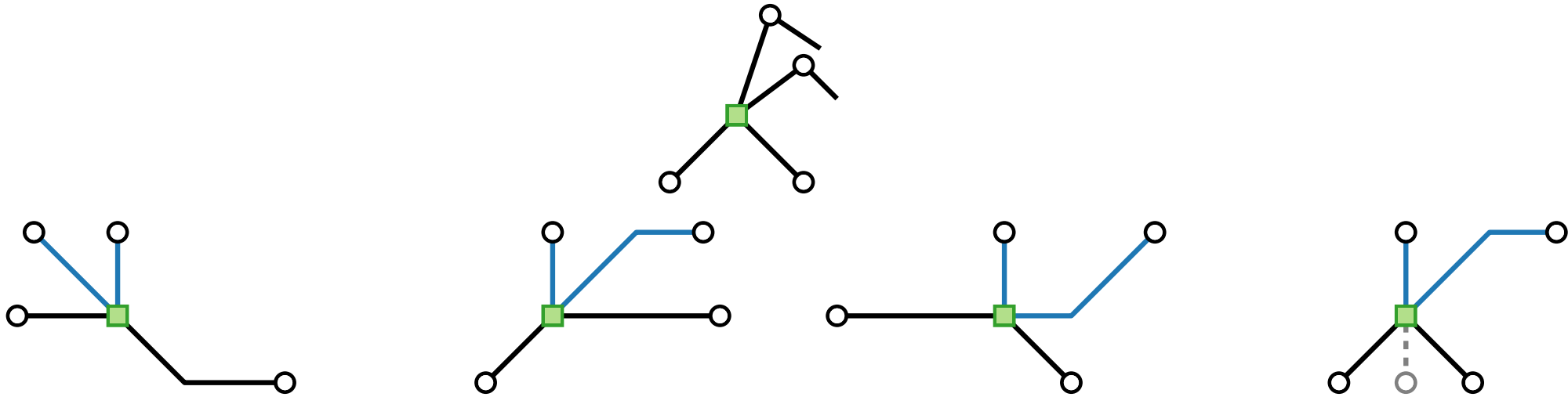
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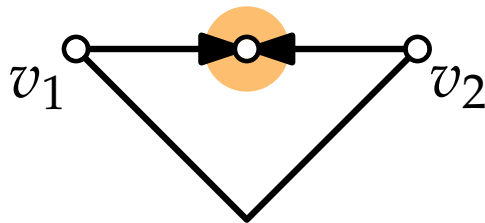
\mathcal{V}_i :



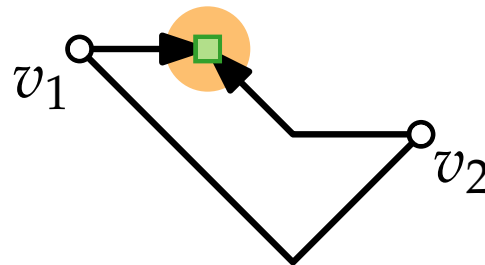
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



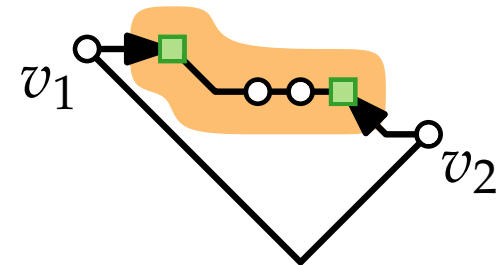
\mathcal{V}_2 : real vertex



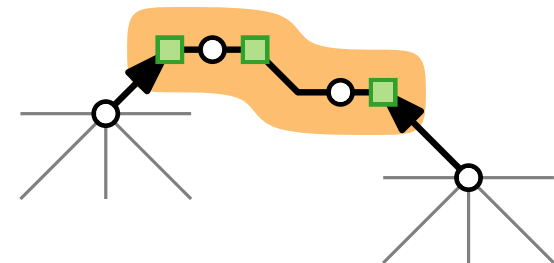
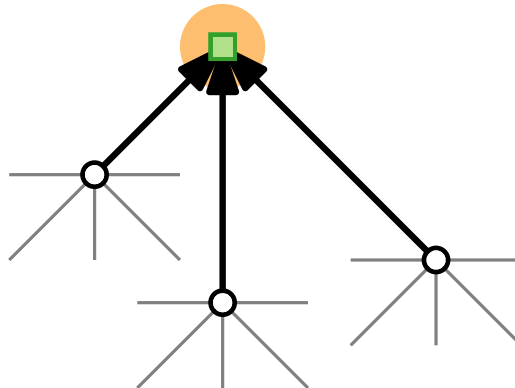
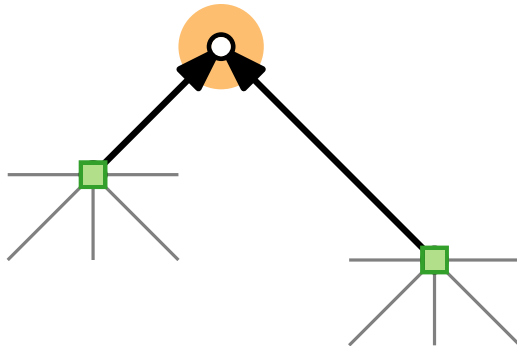
dummy vertex



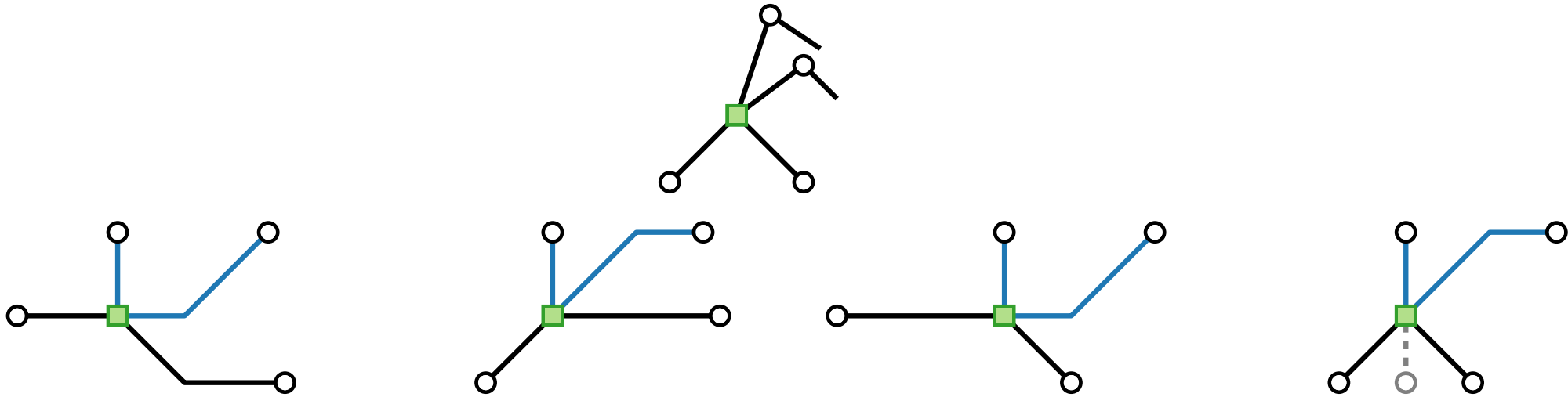
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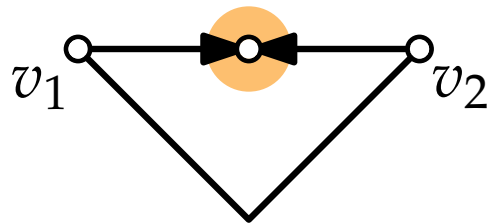
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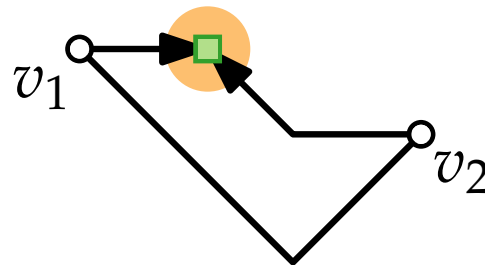
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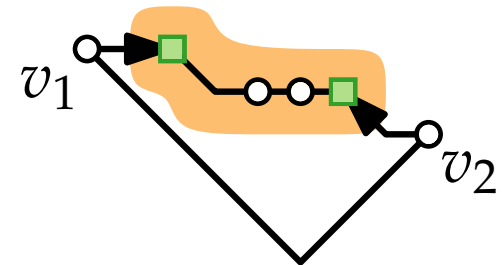
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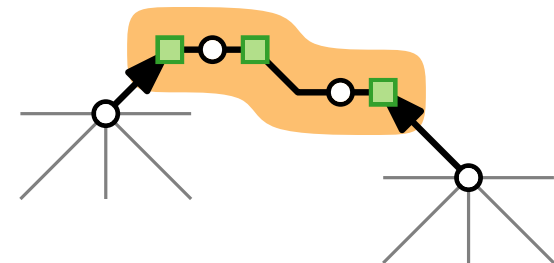
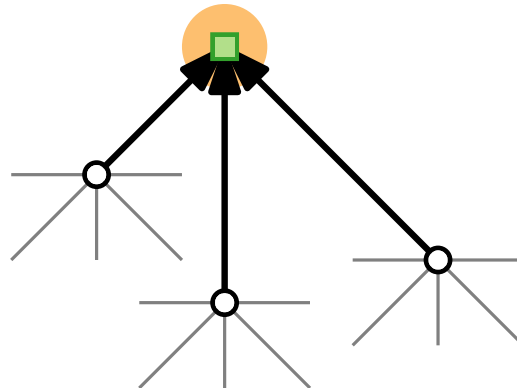
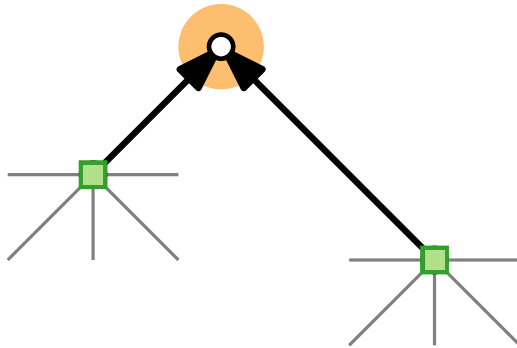
dummy vertex



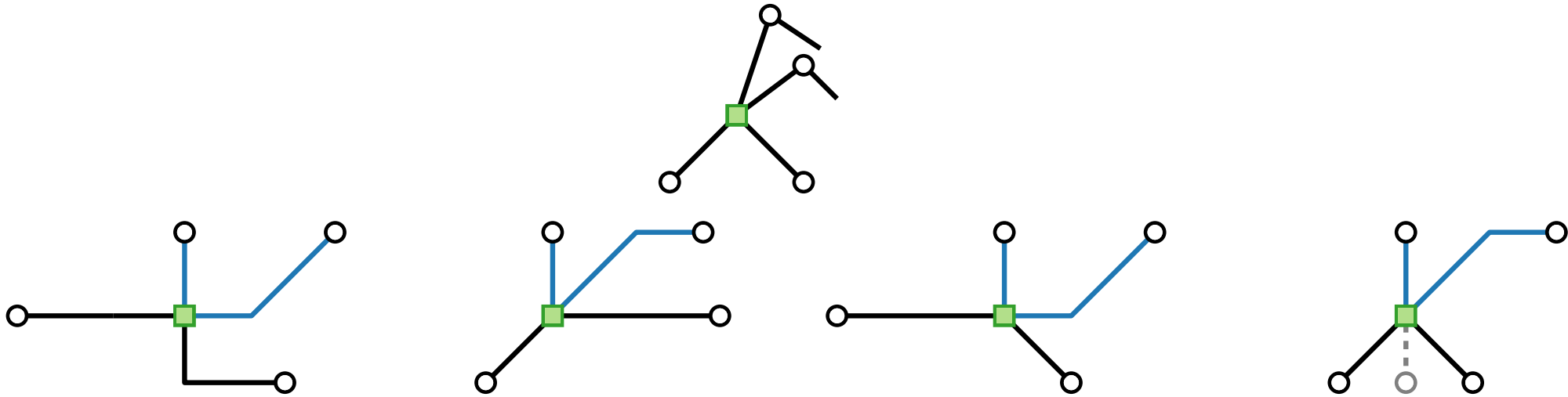
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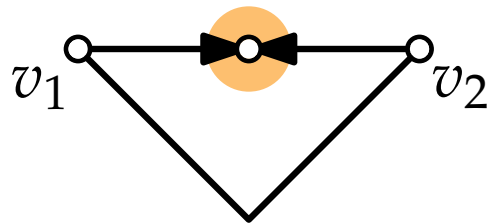
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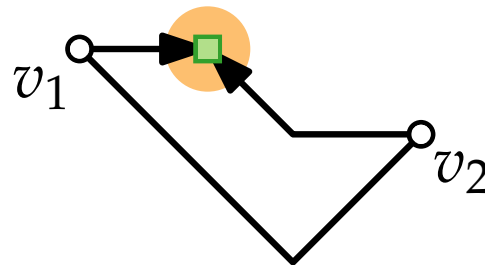
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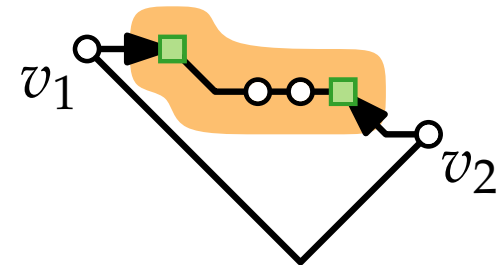
\mathcal{V}_2 : real vertex



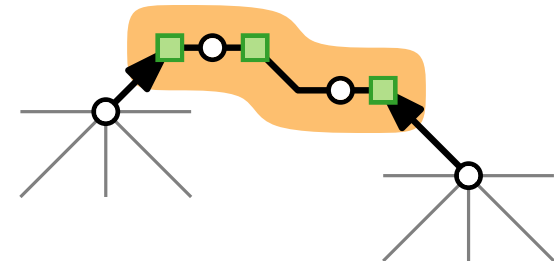
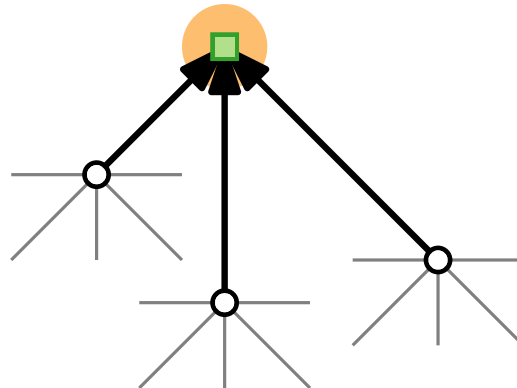
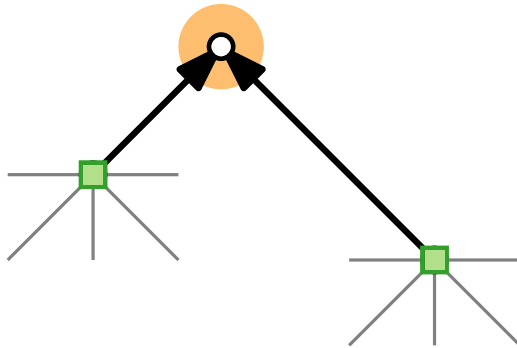
dummy vertex



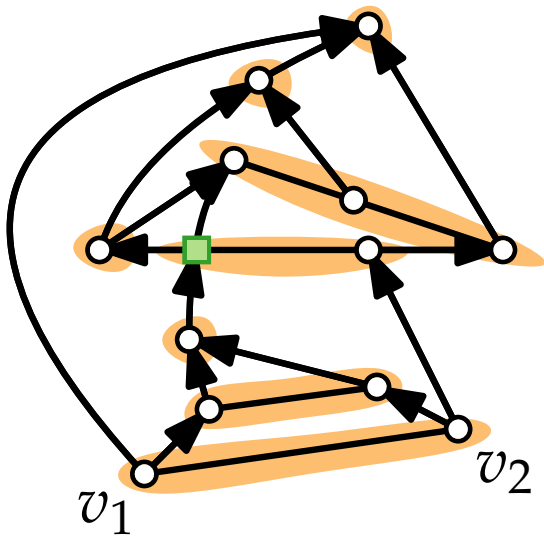
chain



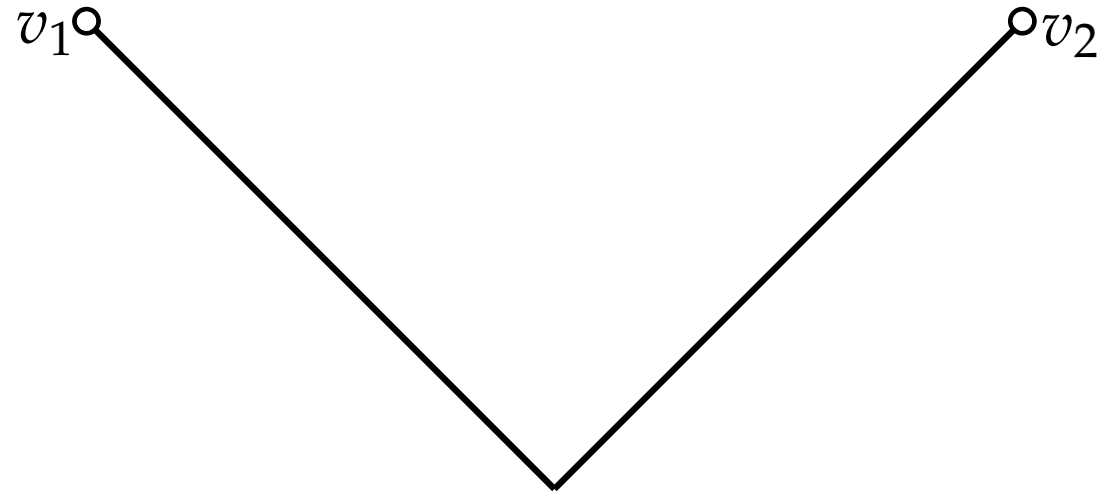
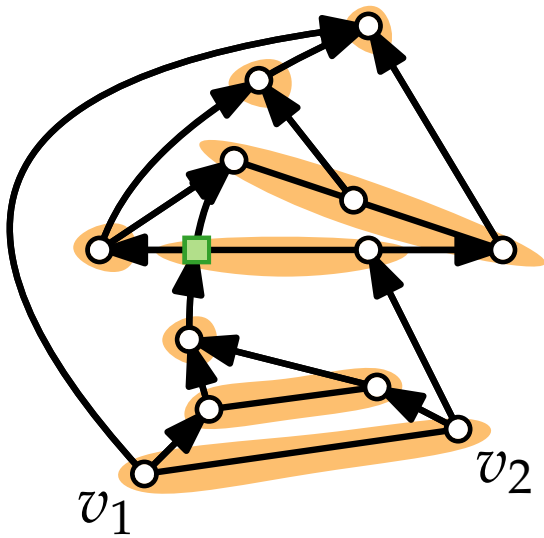
\mathcal{V}_i :



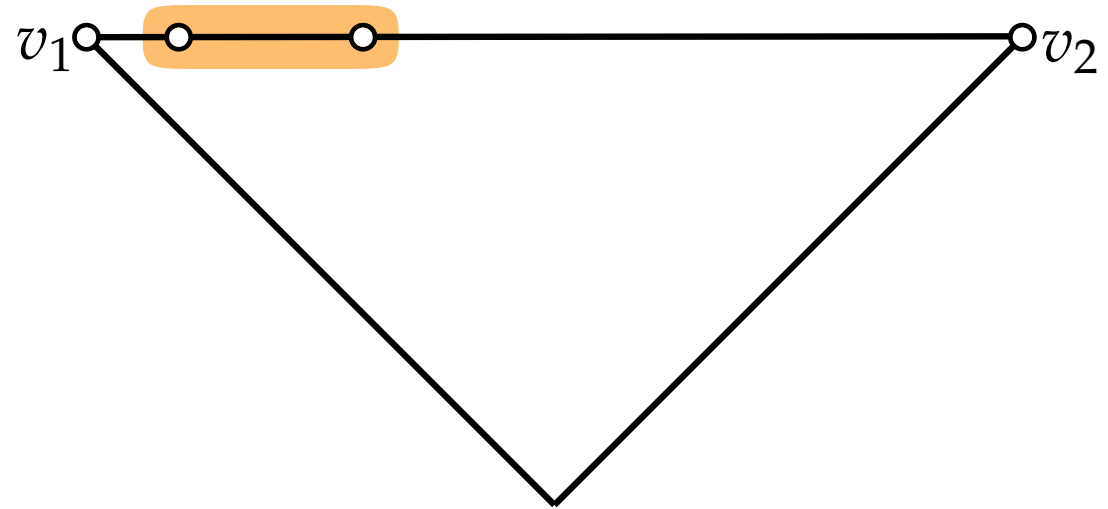
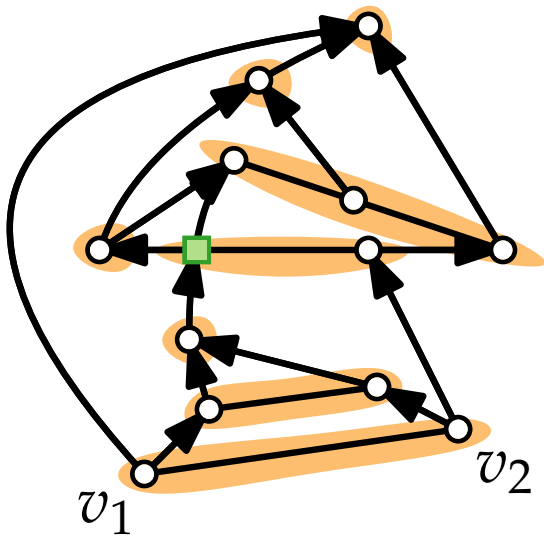
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



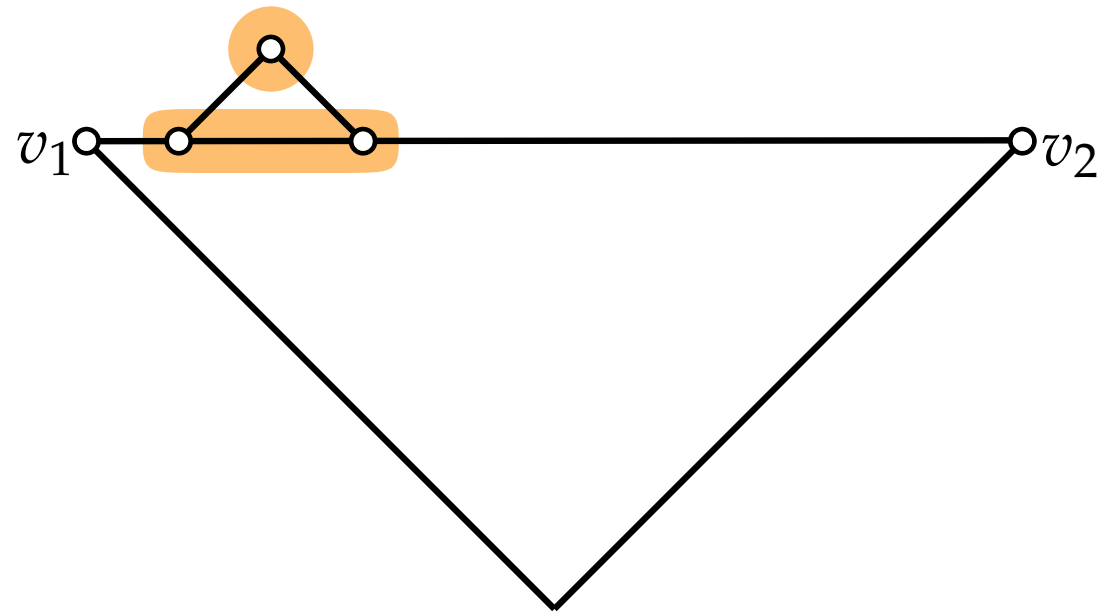
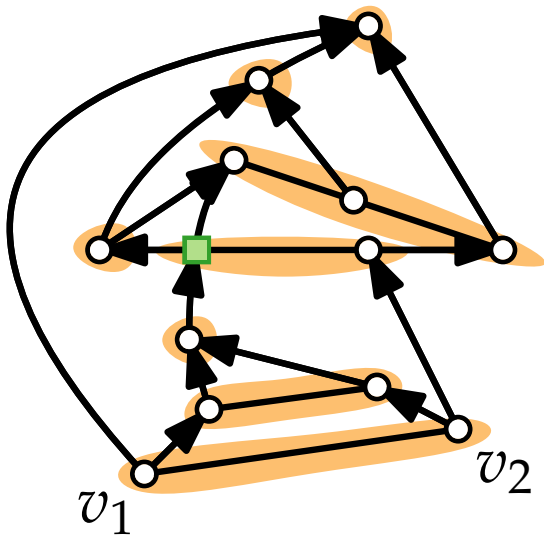
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



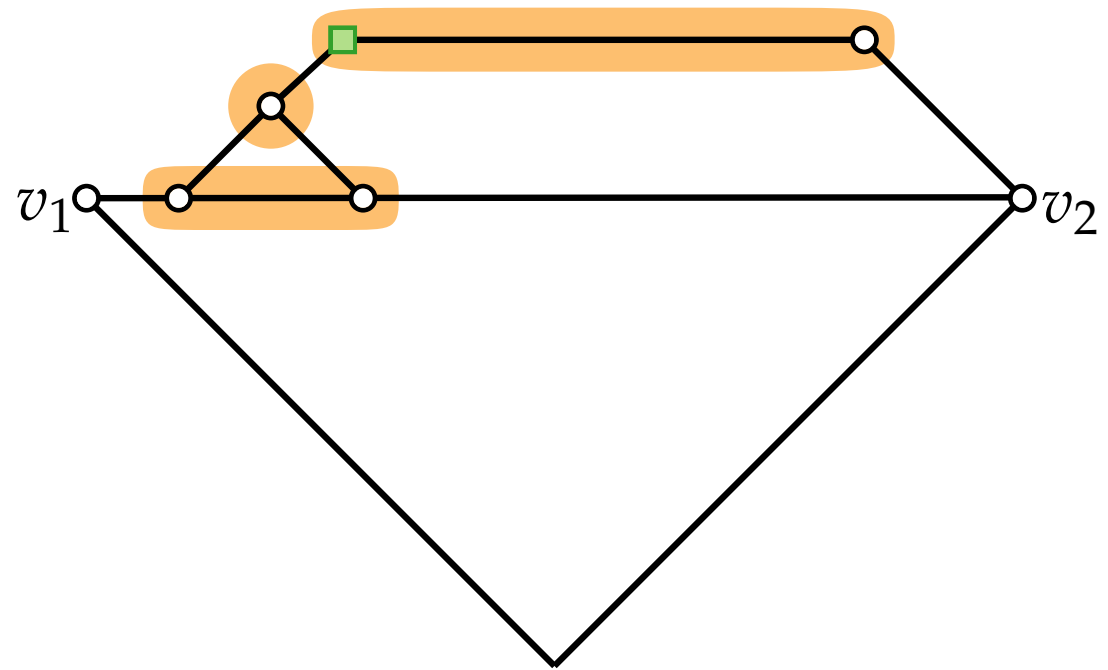
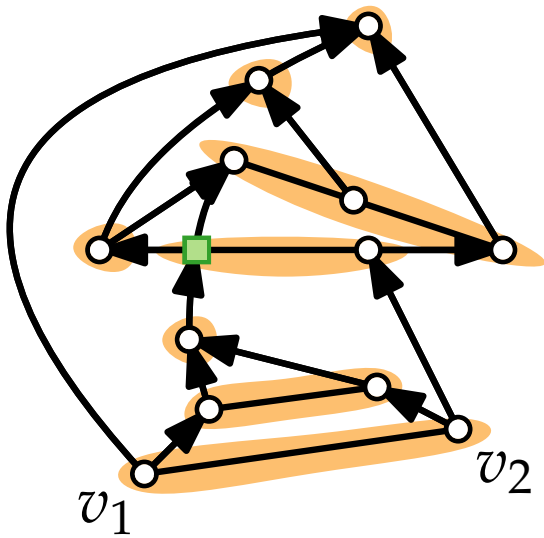
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



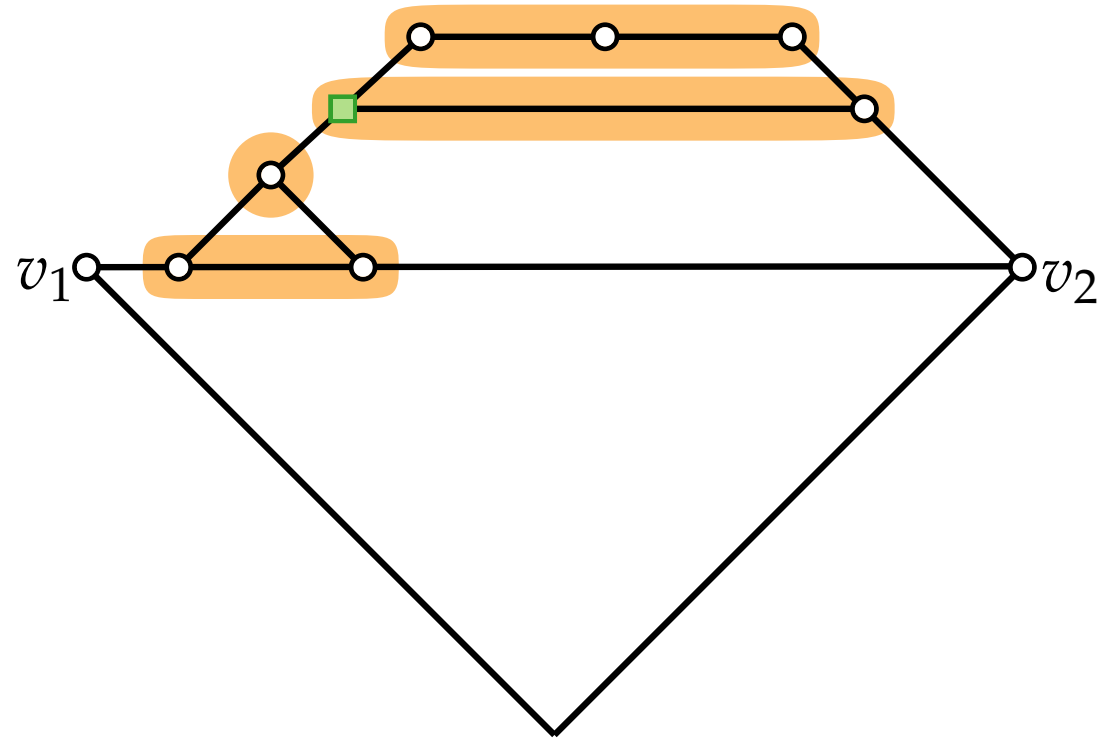
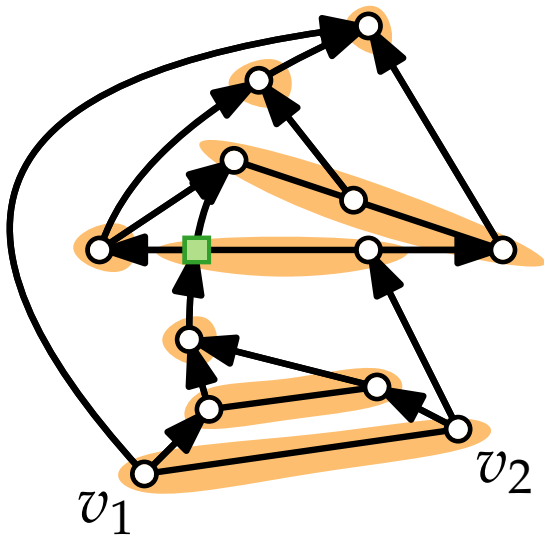
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



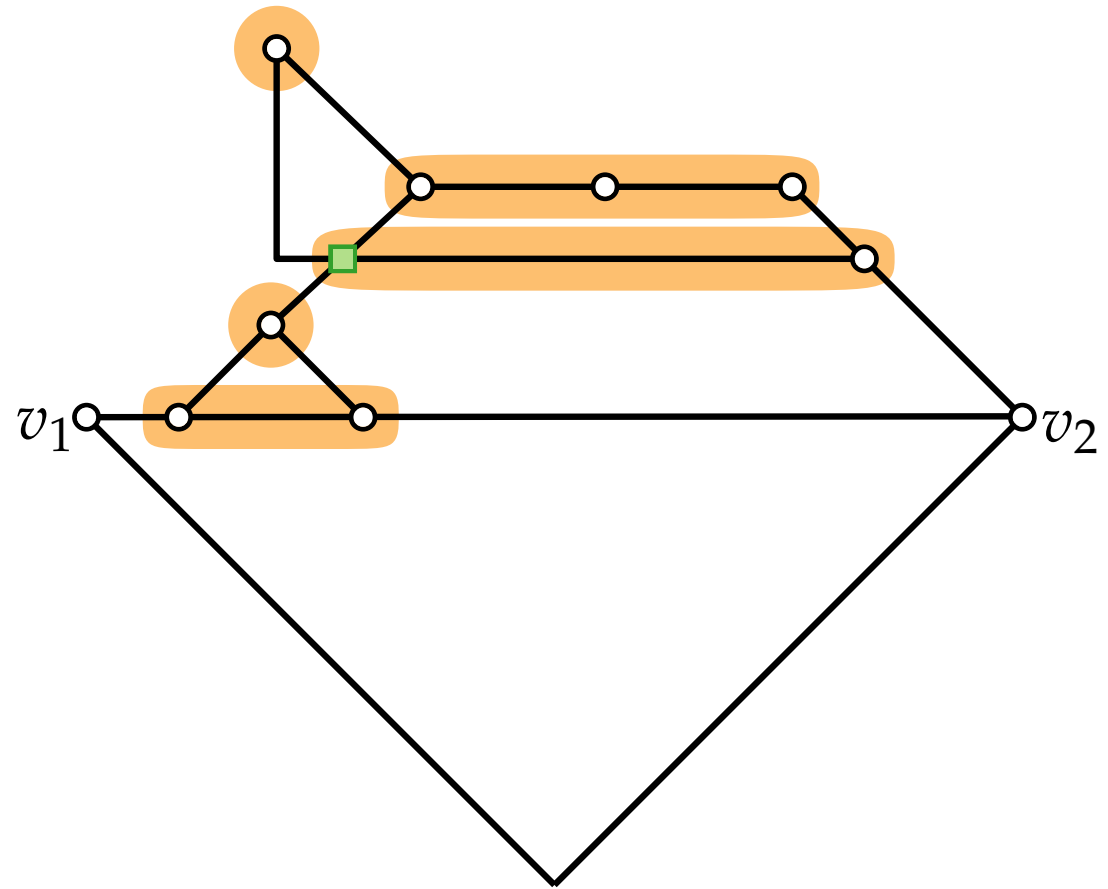
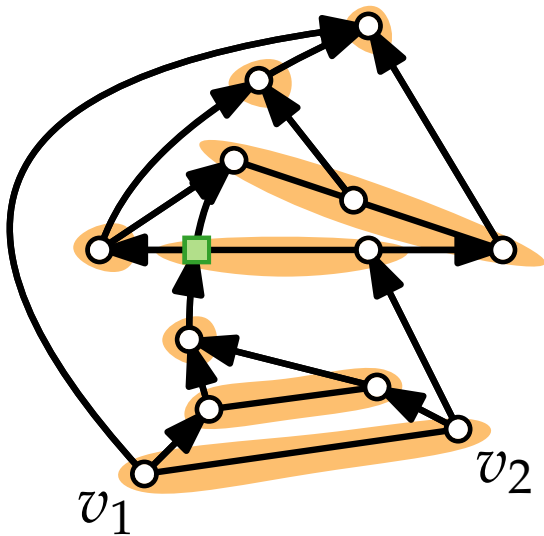
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



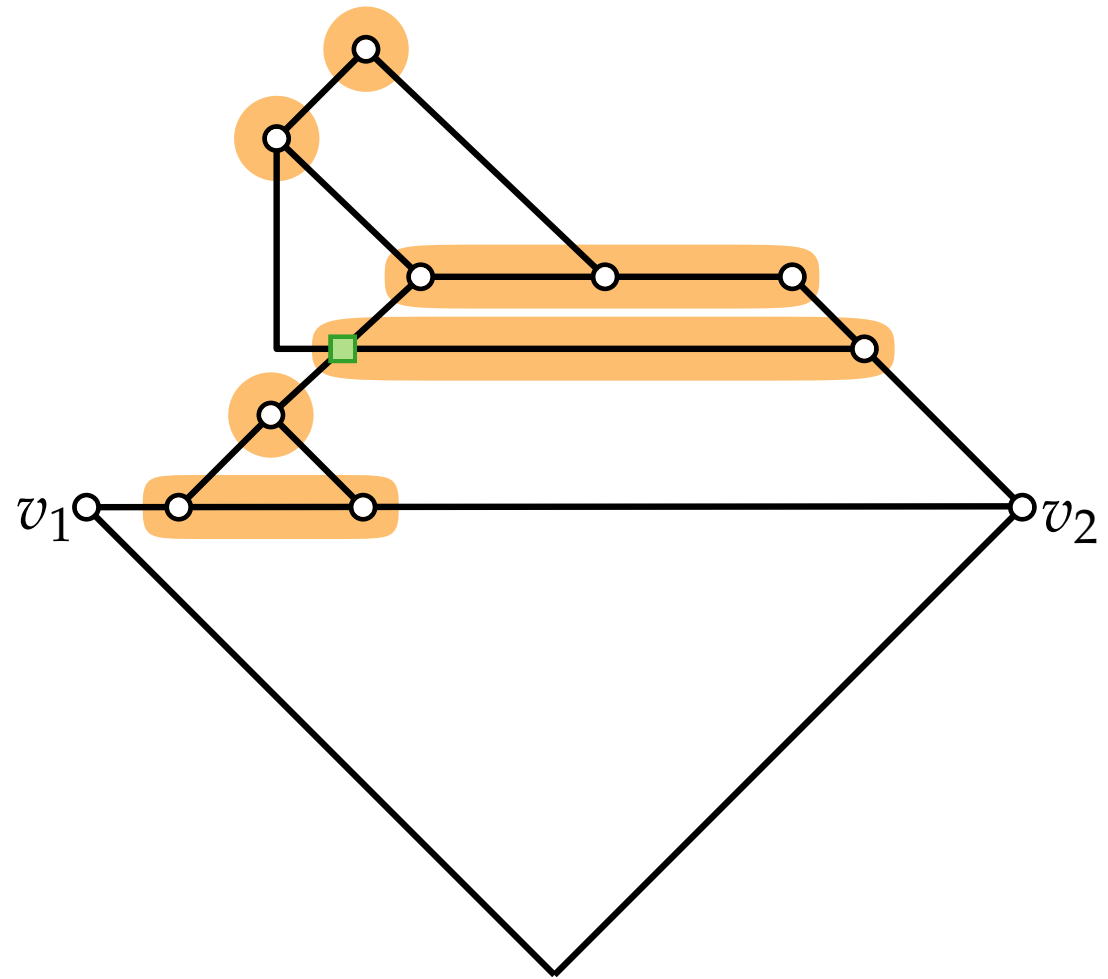
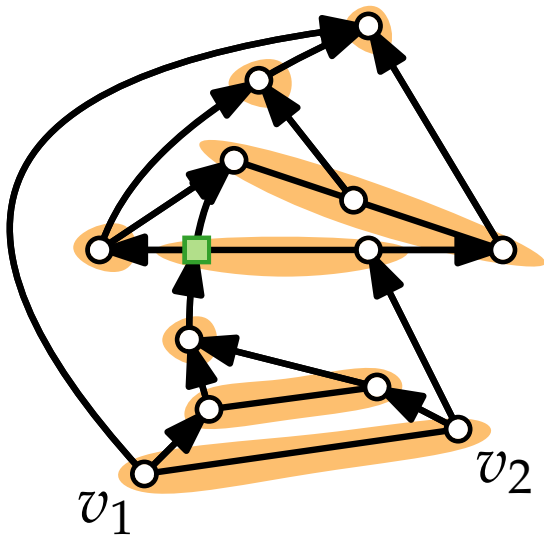
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



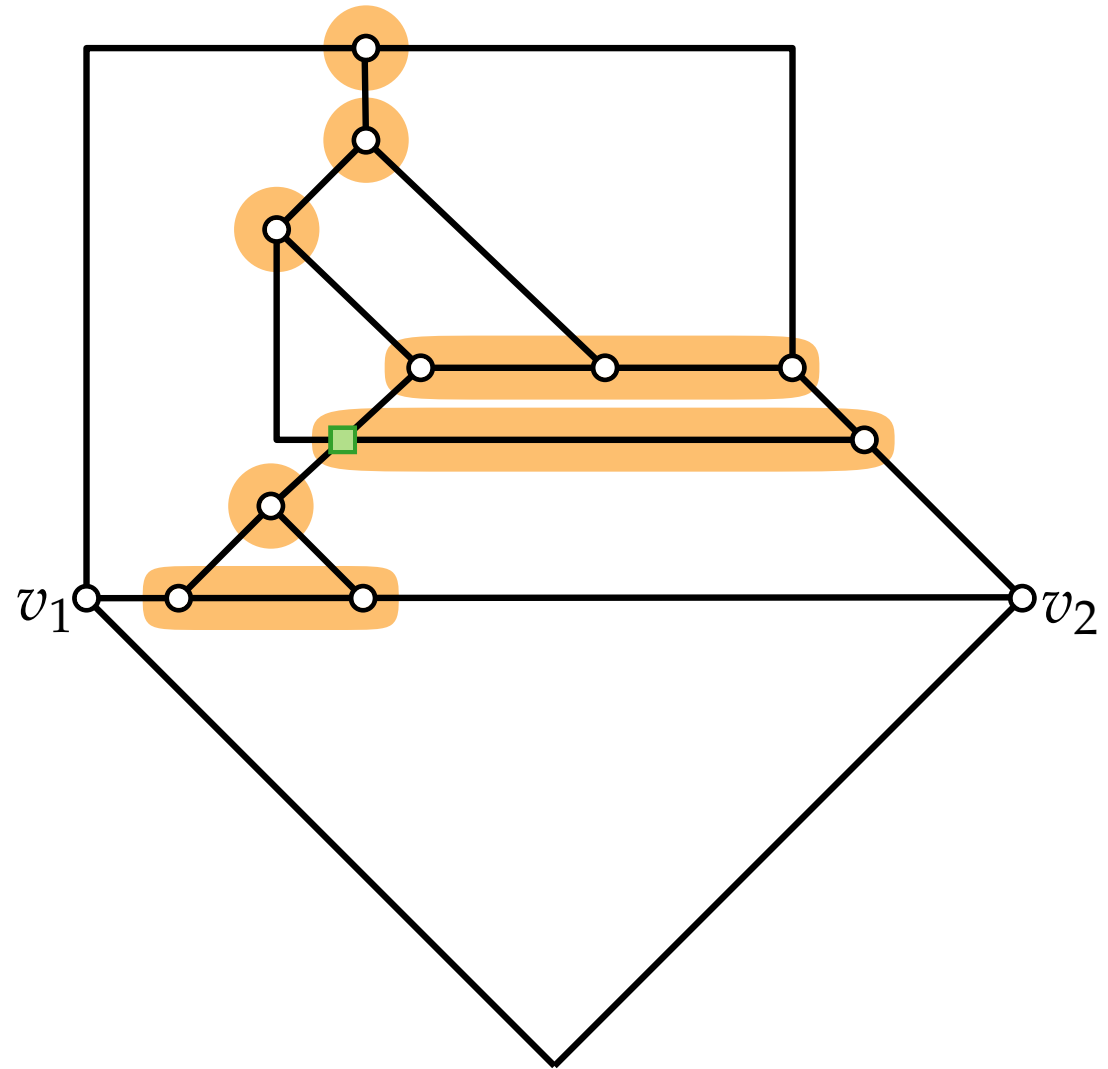
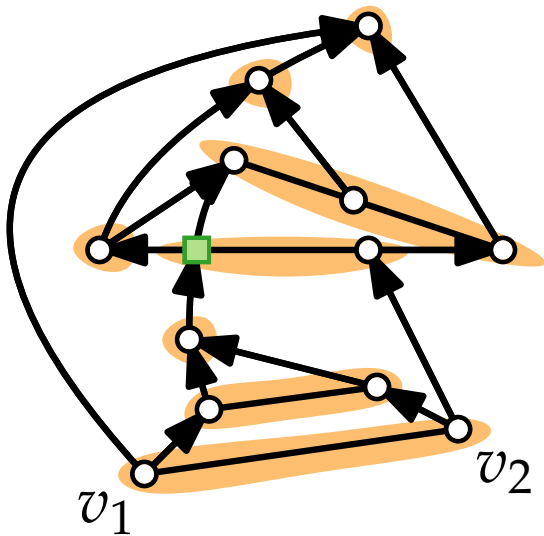
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



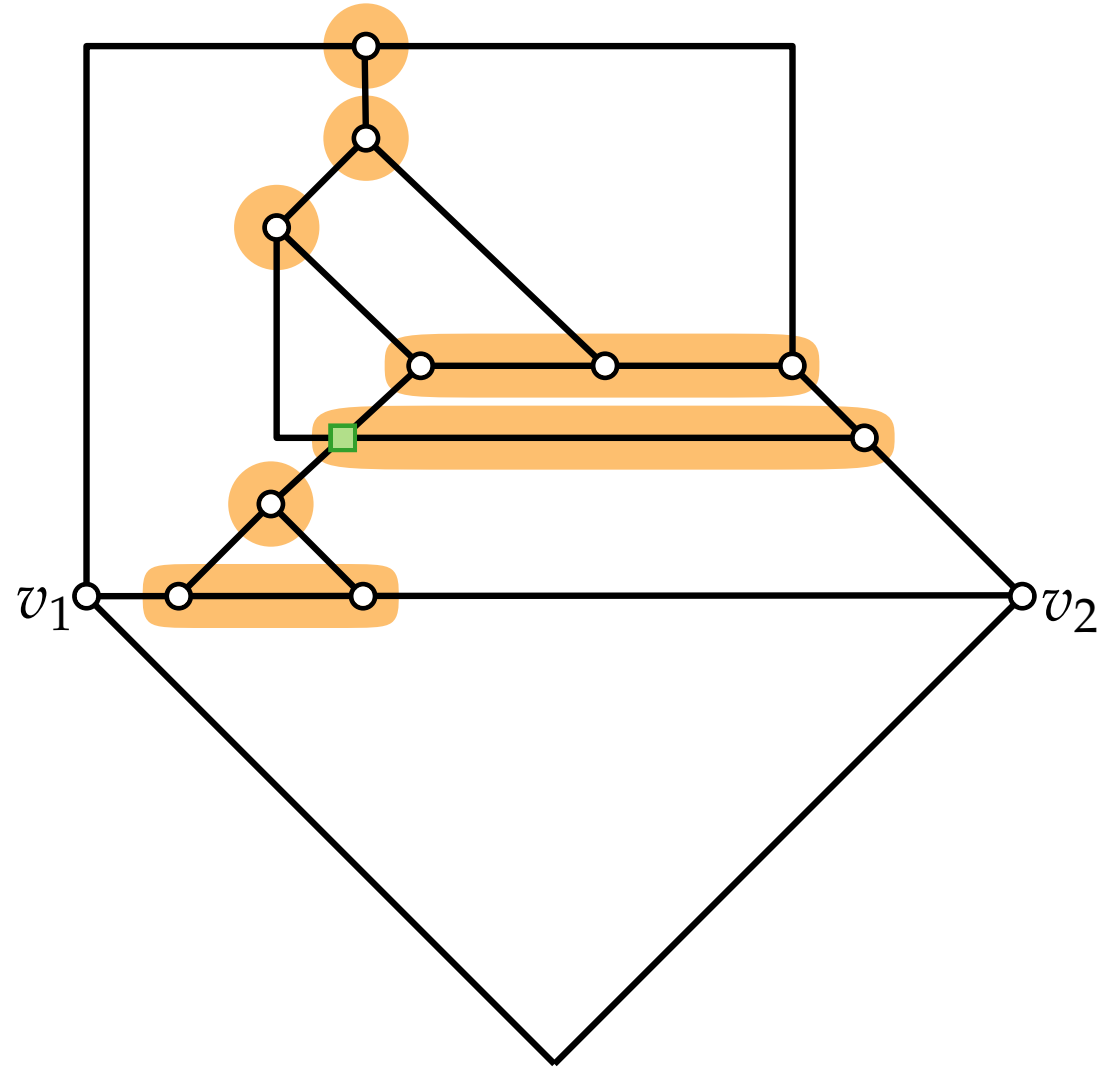
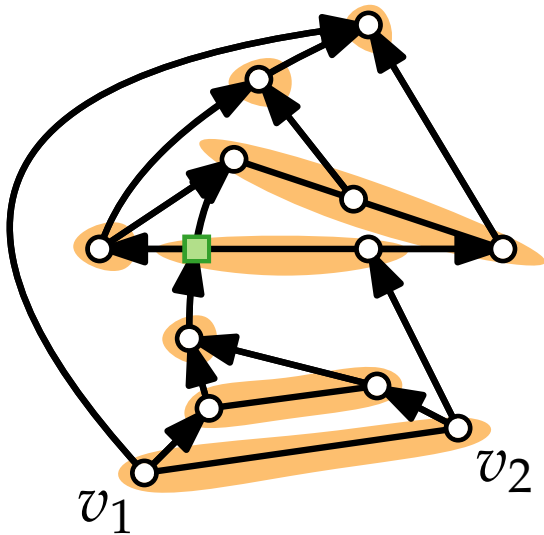
Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes



Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes

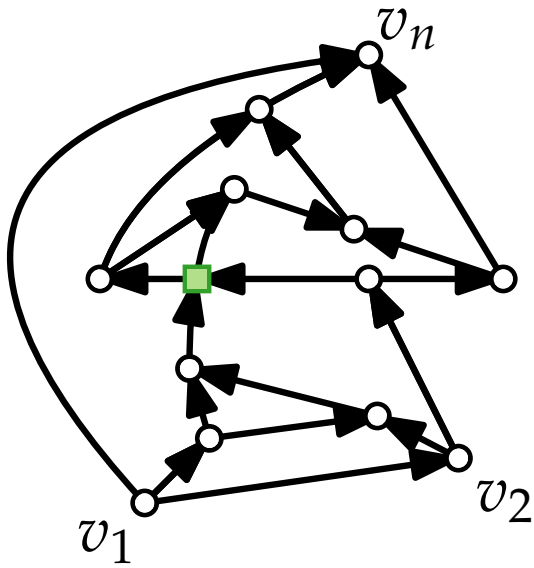


Cubic 3-Connected \Rightarrow 1 Bend, 4 Slopes

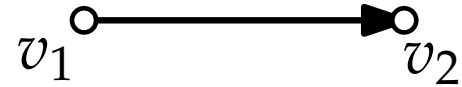
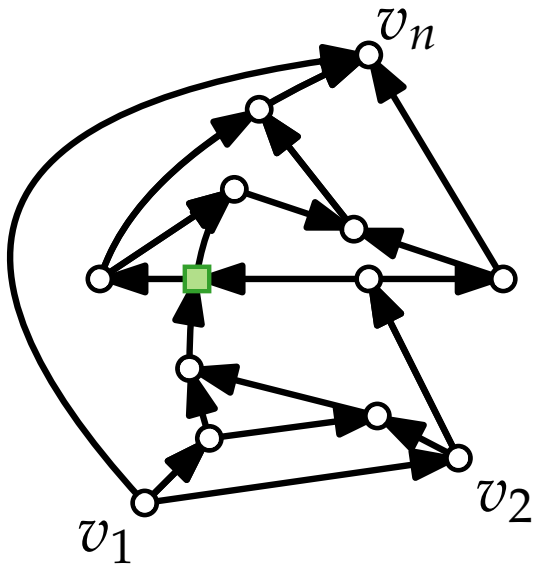


cubic 3-connected 1-plane 1-bend $\Rightarrow \leq 4$ slopes

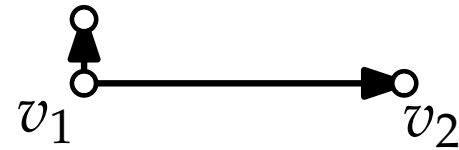
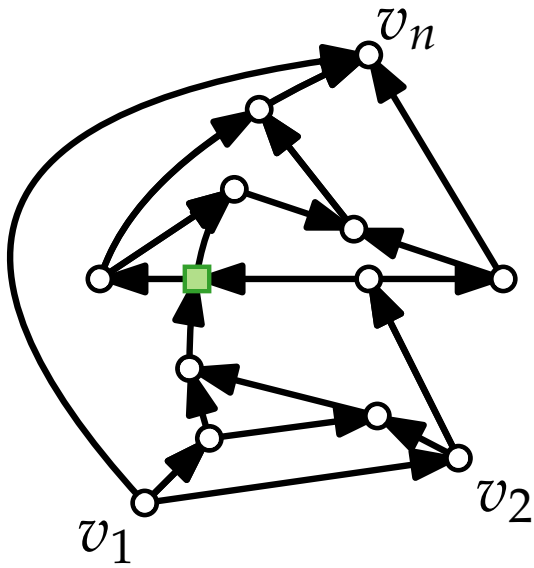
Subcubic \Rightarrow 2 Bends, 2 Slopes



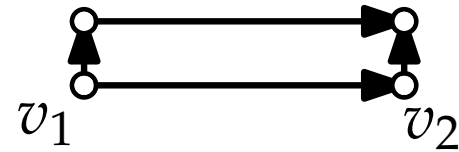
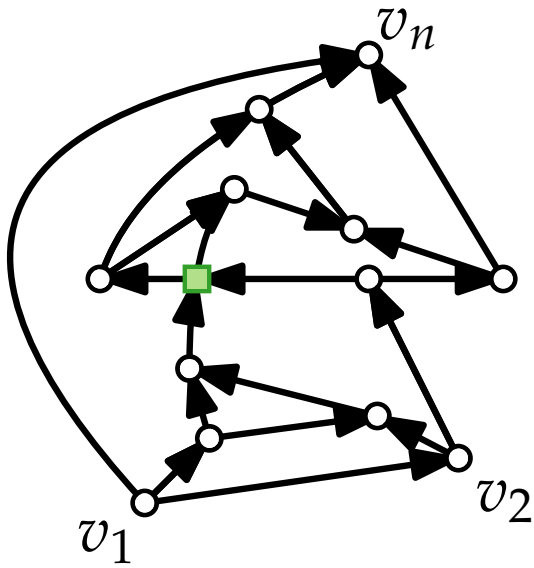
Subcubic \Rightarrow 2 Bends, 2 Slopes



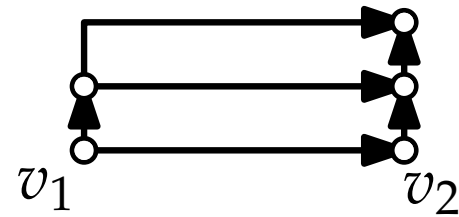
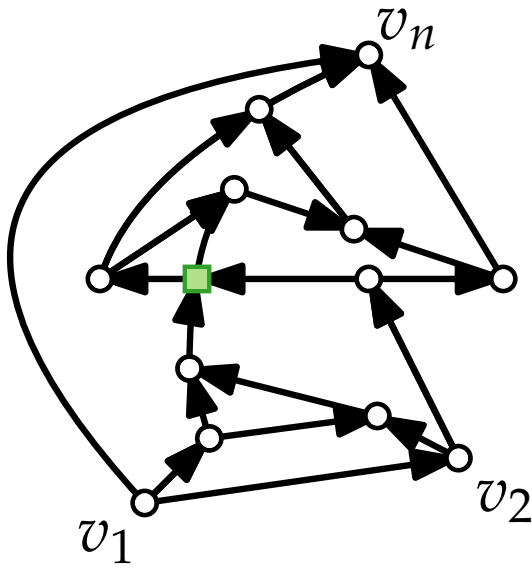
Subcubic \Rightarrow 2 Bends, 2 Slopes



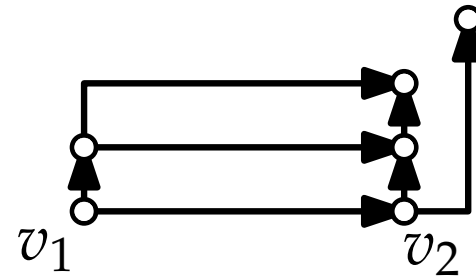
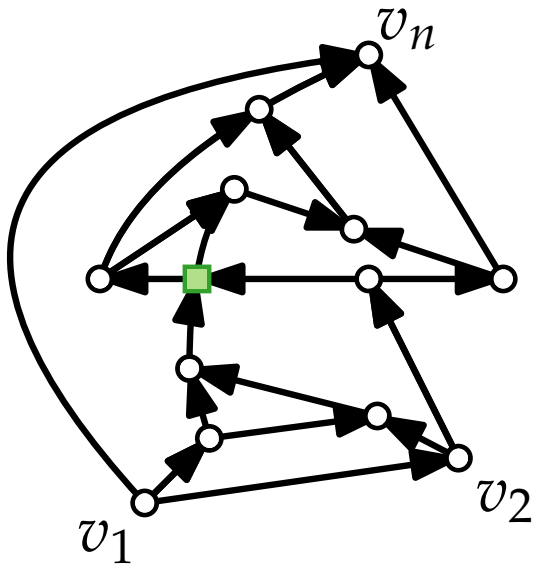
Subcubic \Rightarrow 2 Bends, 2 Slopes



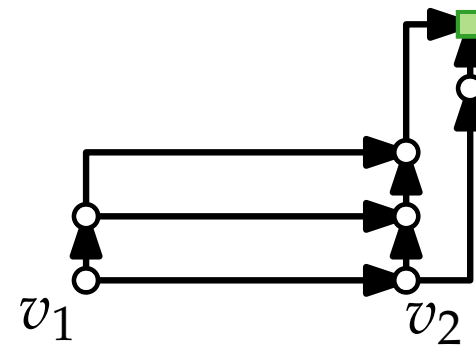
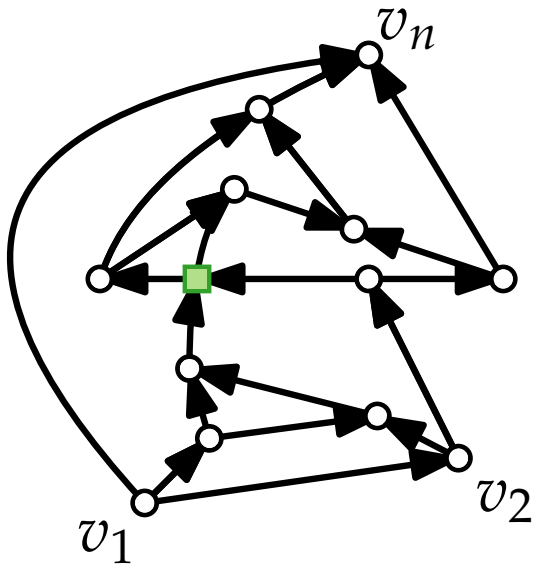
Subcubic \Rightarrow 2 Bends, 2 Slopes



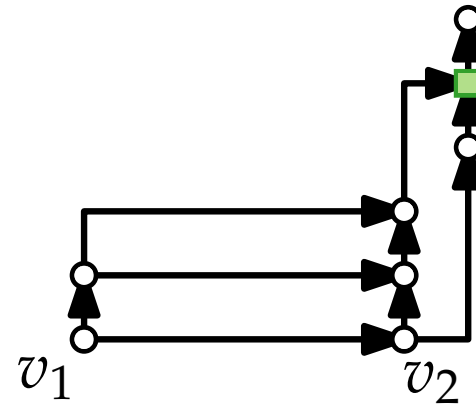
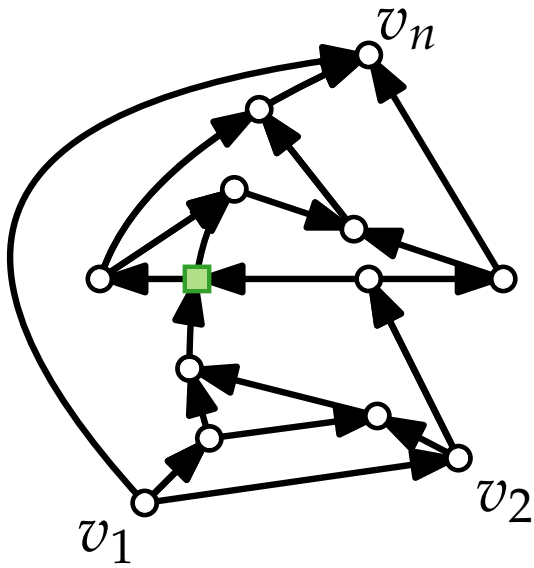
Subcubic \Rightarrow 2 Bends, 2 Slopes



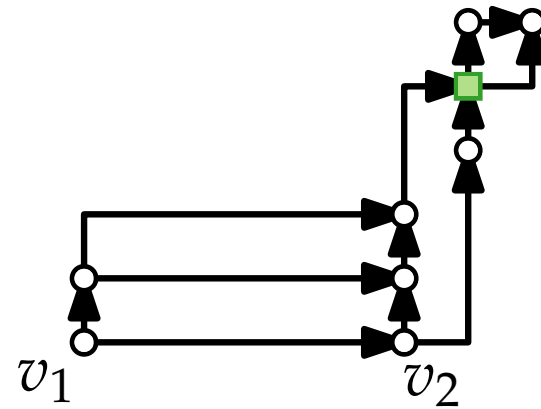
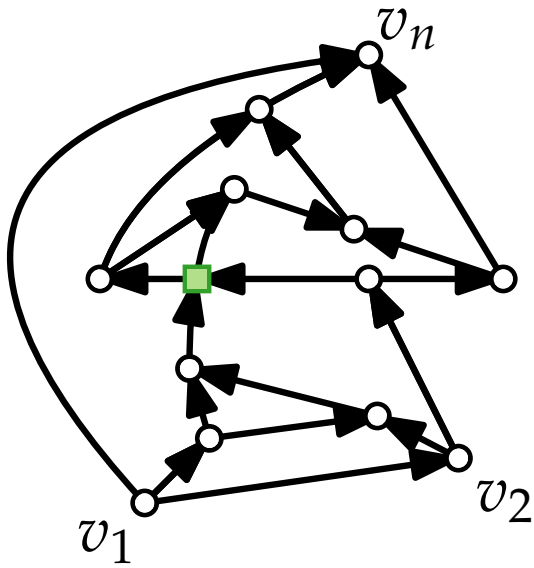
Subcubic \Rightarrow 2 Bends, 2 Slopes



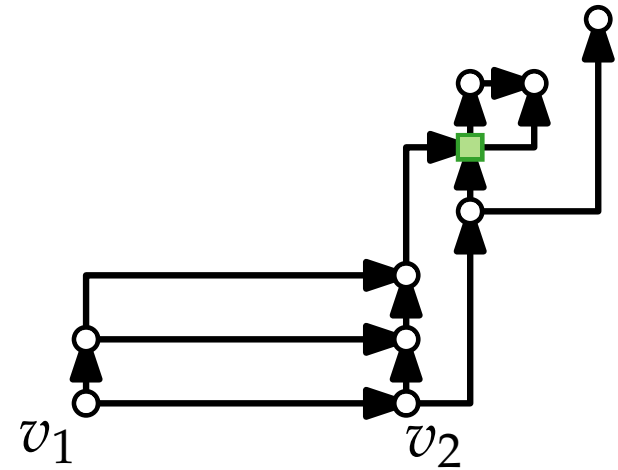
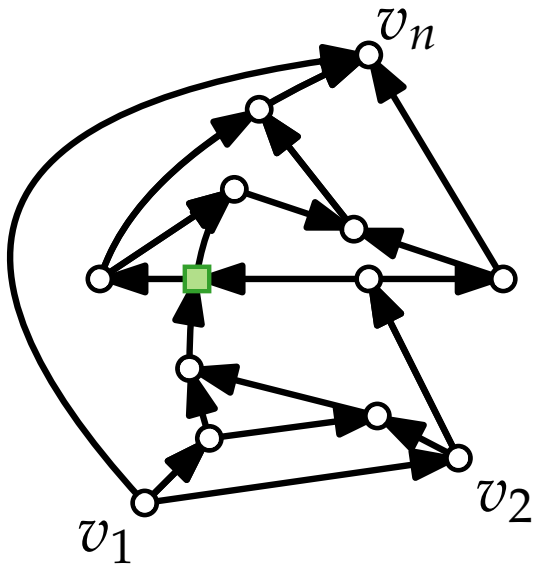
Subcubic \Rightarrow 2 Bends, 2 Slopes



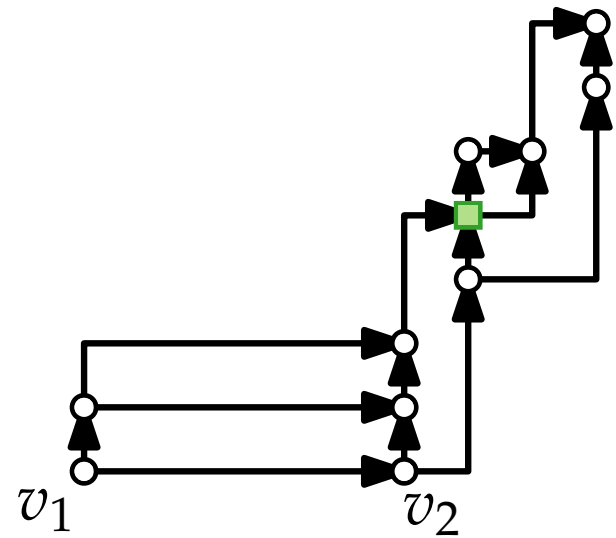
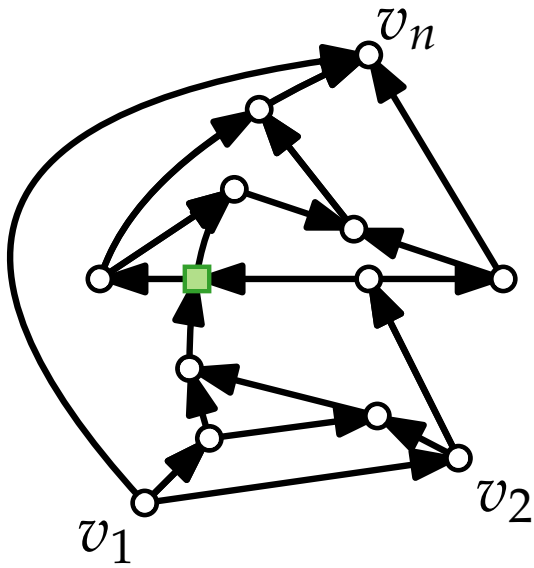
Subcubic \Rightarrow 2 Bends, 2 Slopes



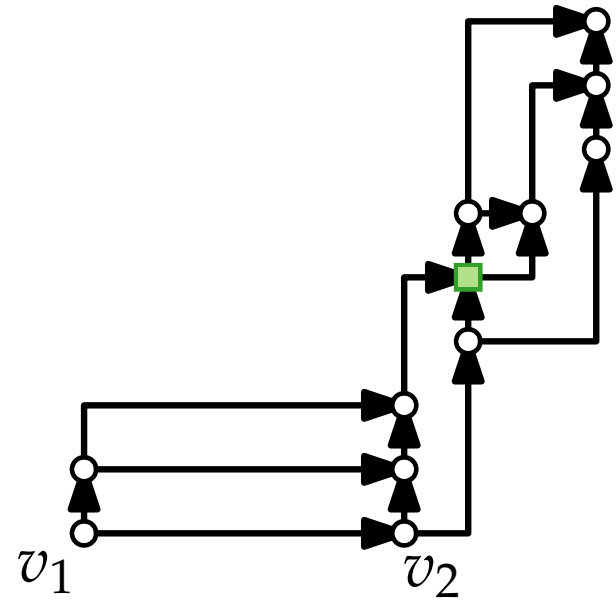
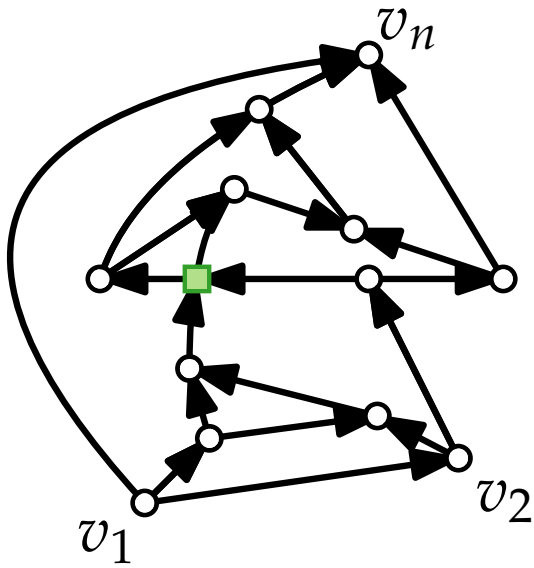
Subcubic \Rightarrow 2 Bends, 2 Slopes



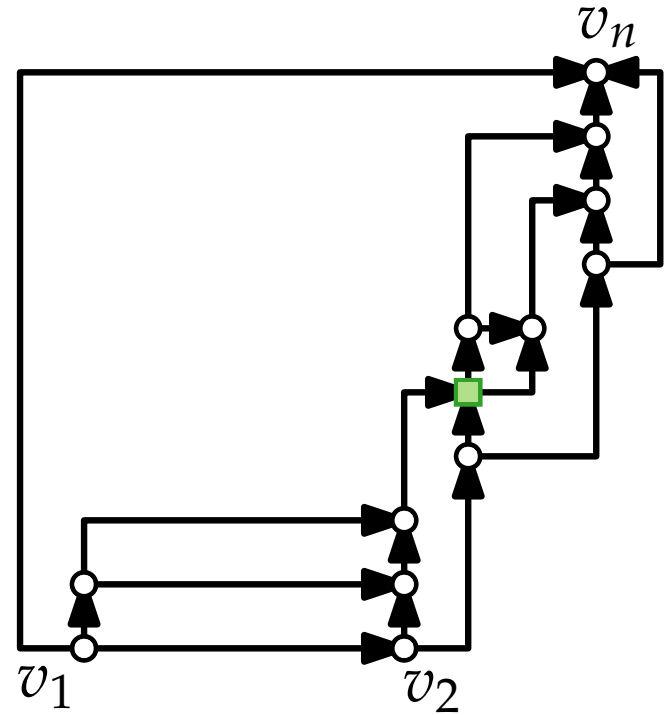
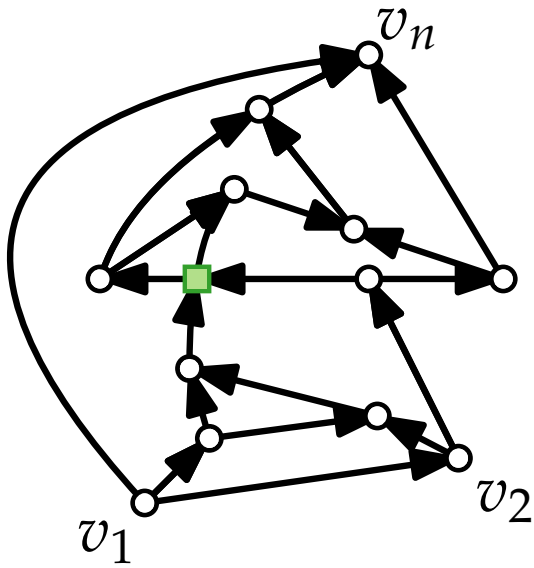
Subcubic \Rightarrow 2 Bends, 2 Slopes



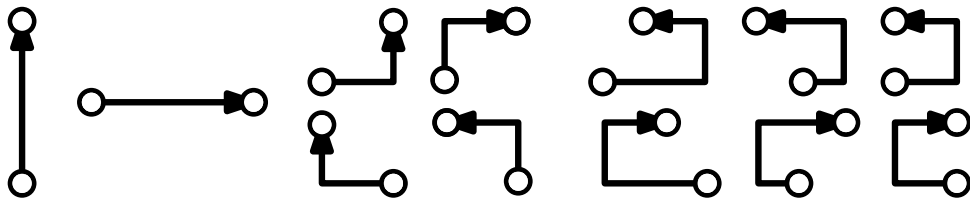
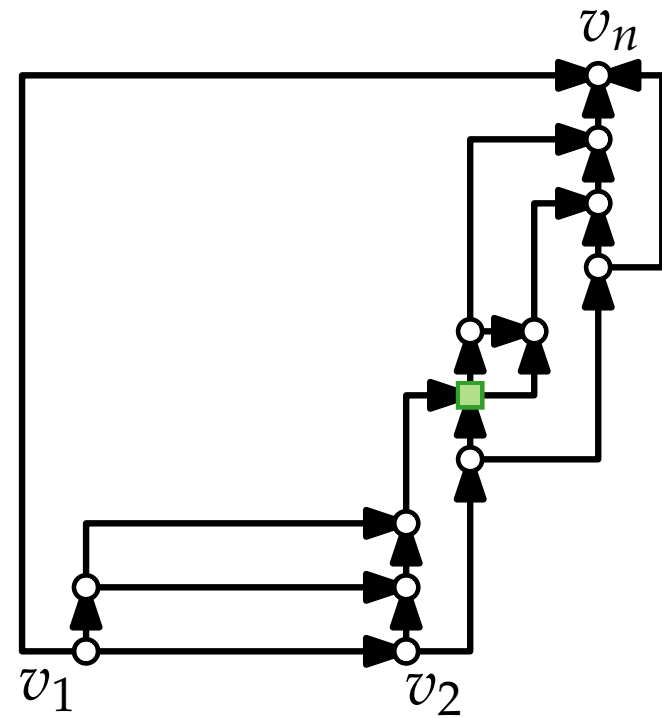
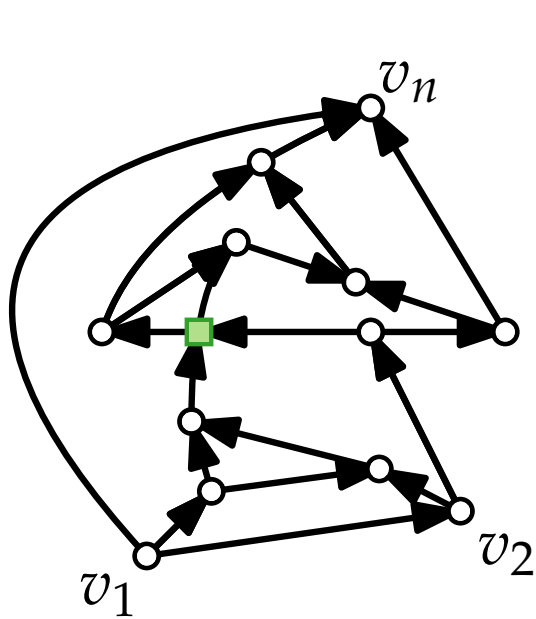
Subcubic \Rightarrow 2 Bends, 2 Slopes



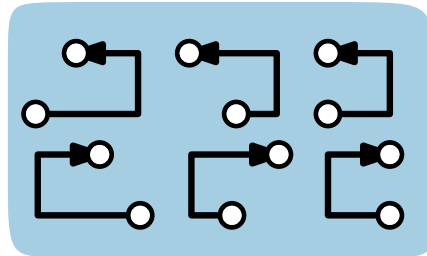
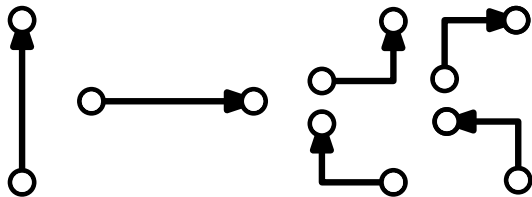
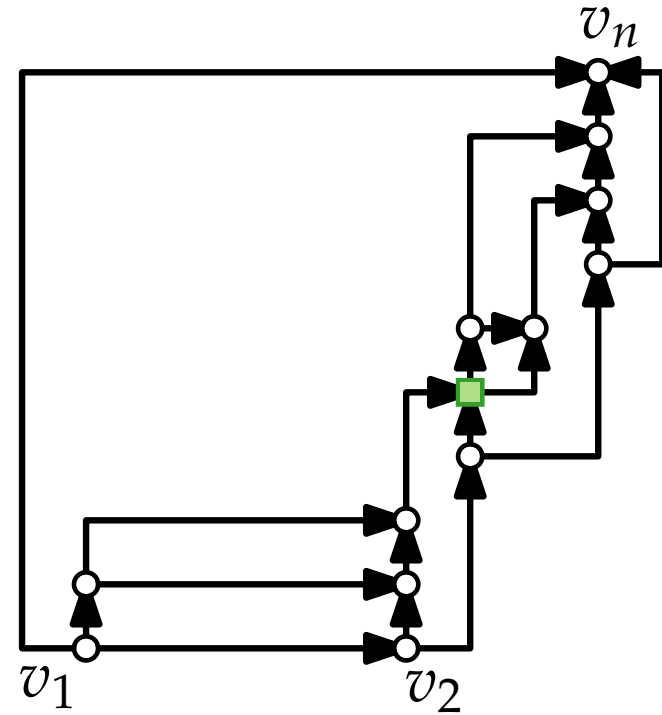
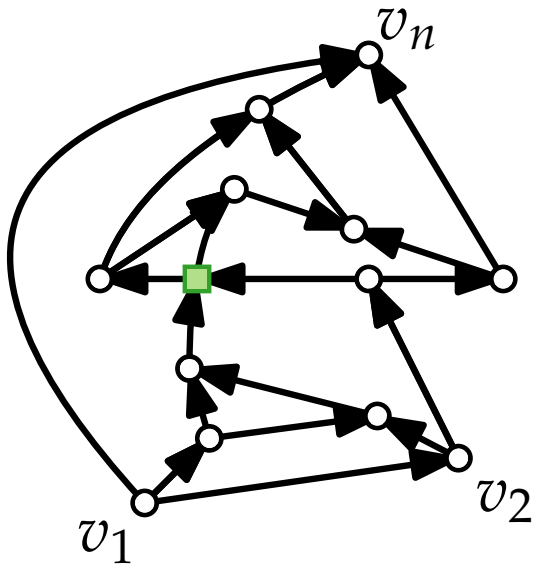
Subcubic \Rightarrow 2 Bends, 2 Slopes



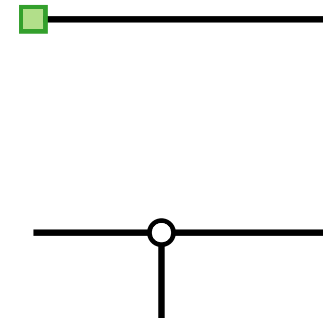
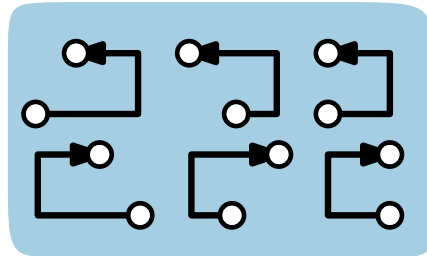
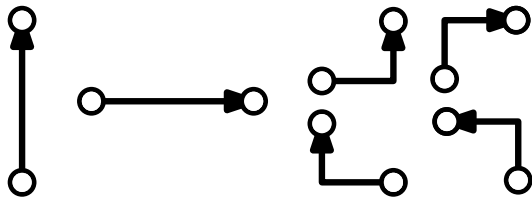
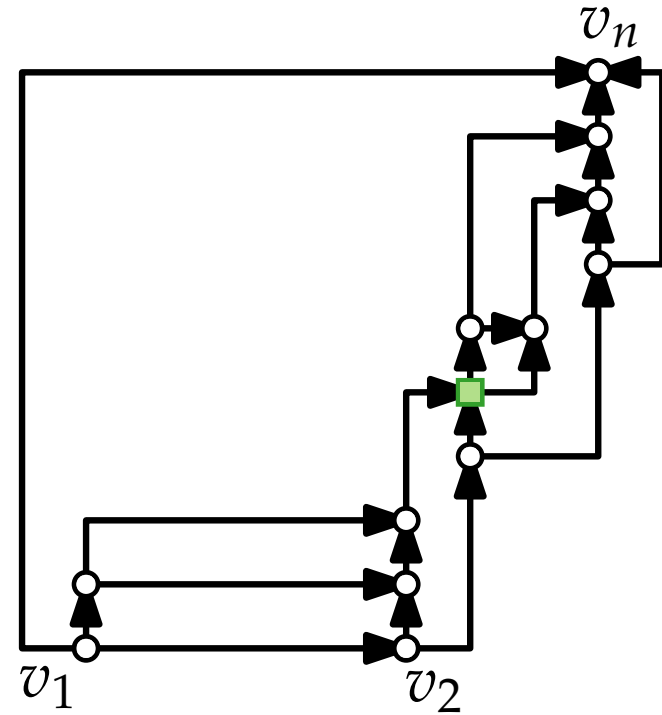
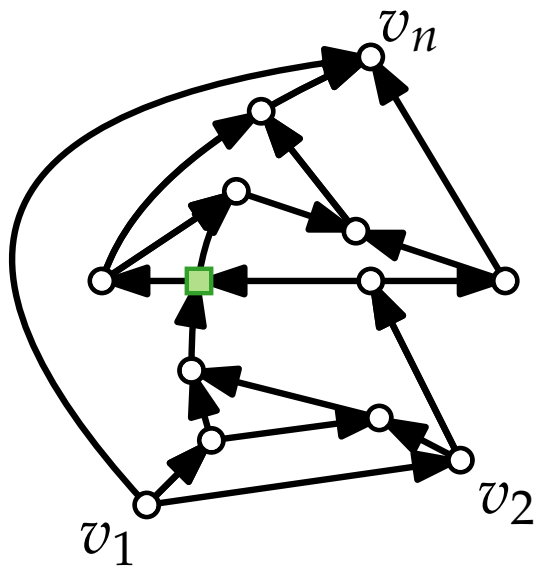
Subcubic \Rightarrow 2 Bends, 2 Slopes



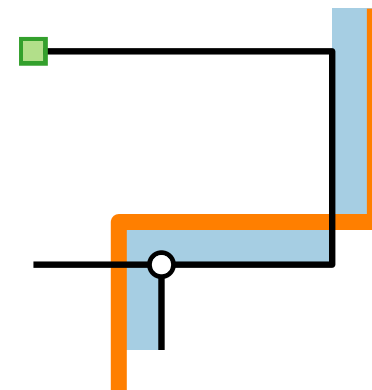
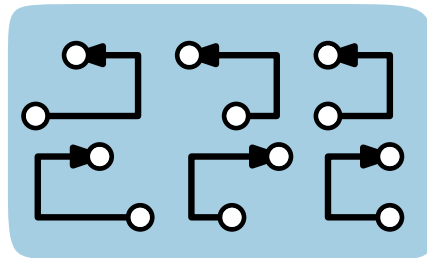
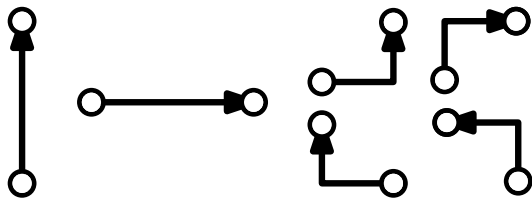
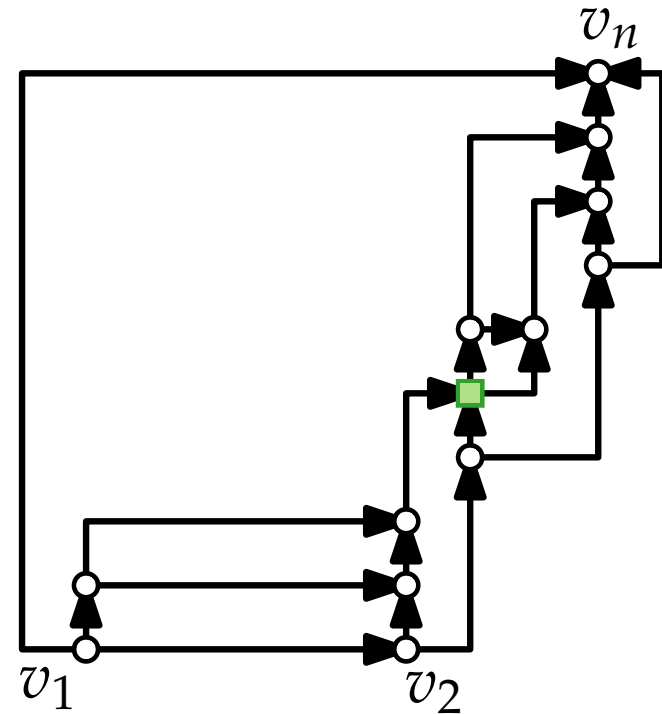
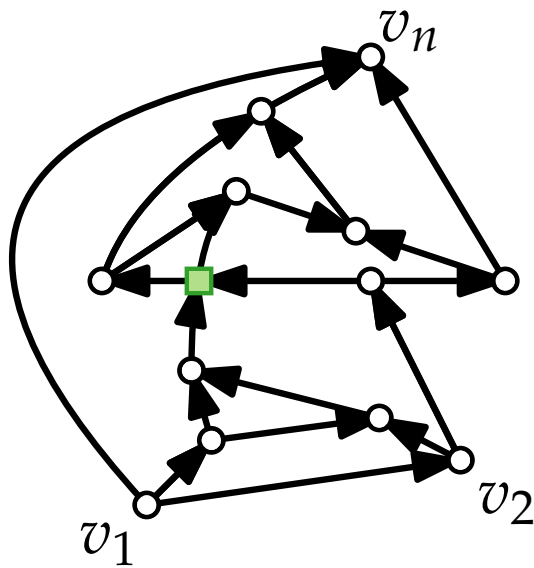
Subcubic \Rightarrow 2 Bends, 2 Slopes



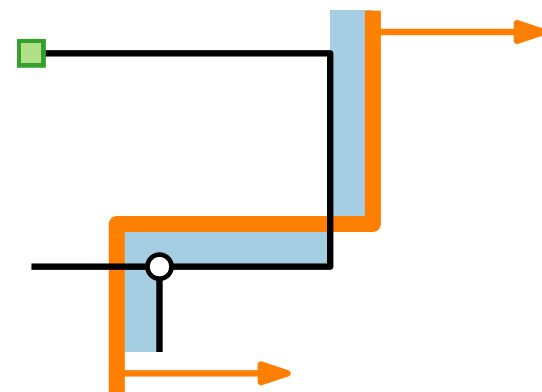
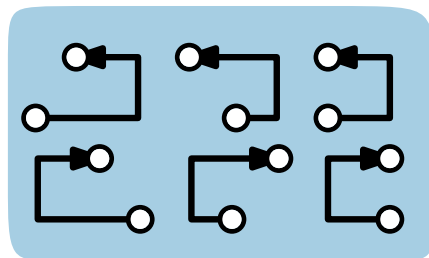
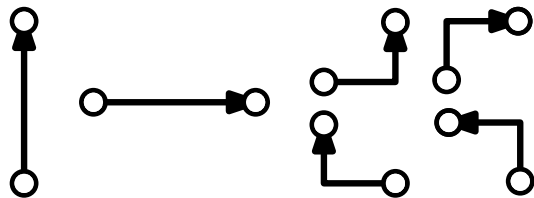
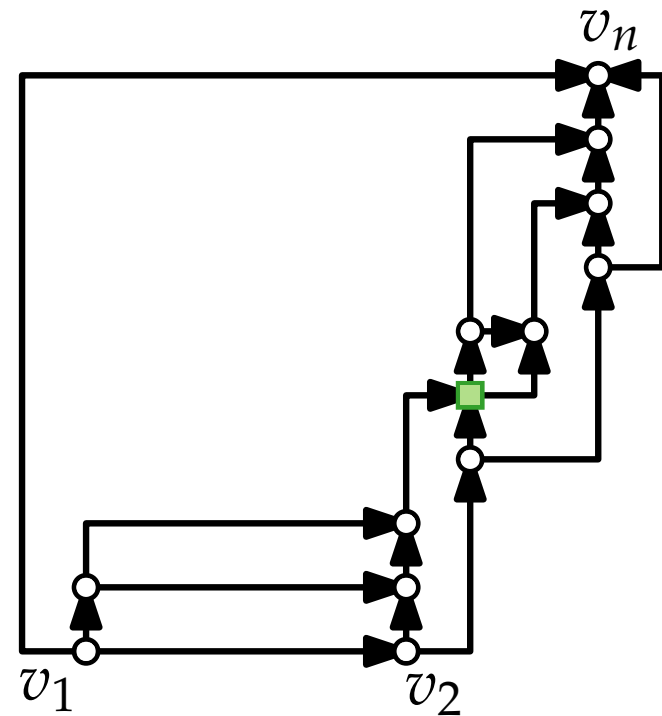
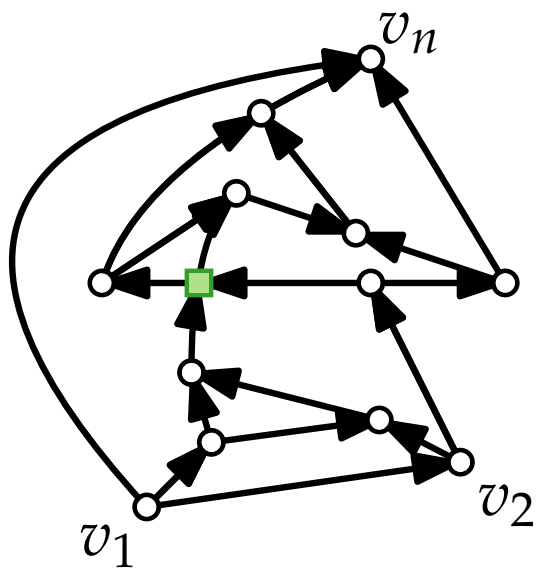
Subcubic \Rightarrow 2 Bends, 2 Slopes



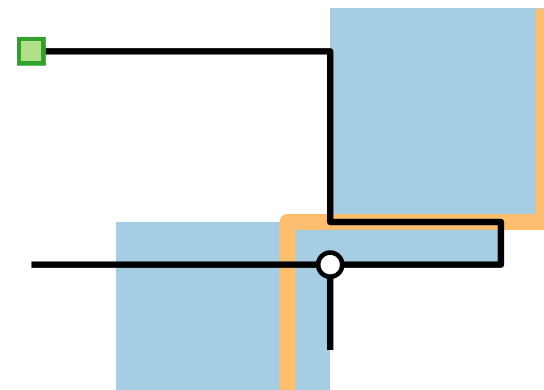
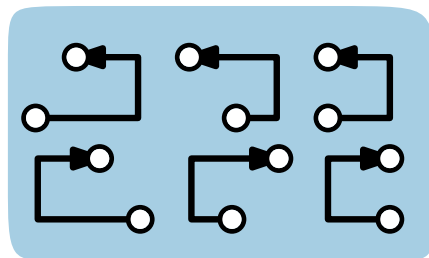
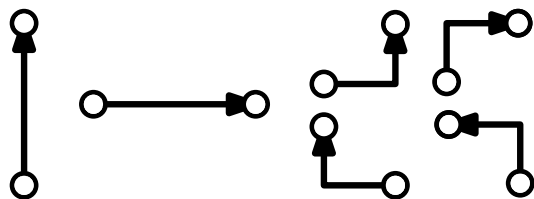
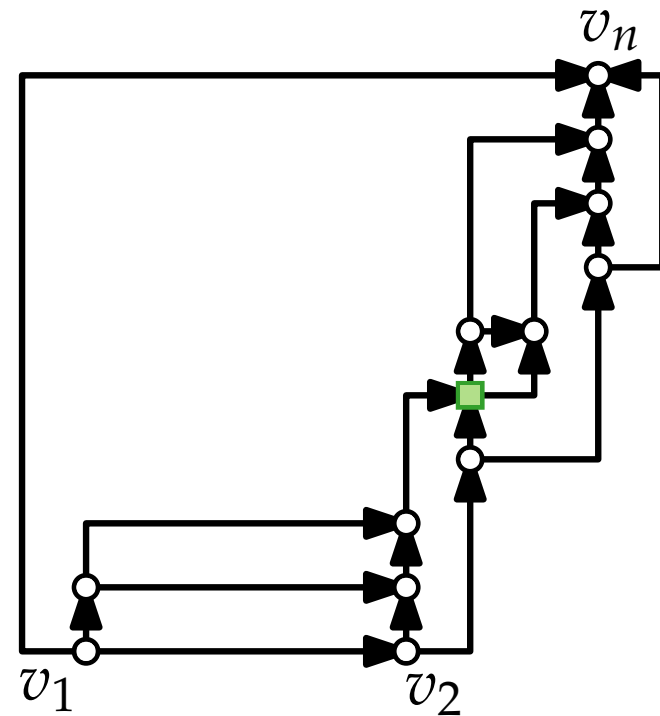
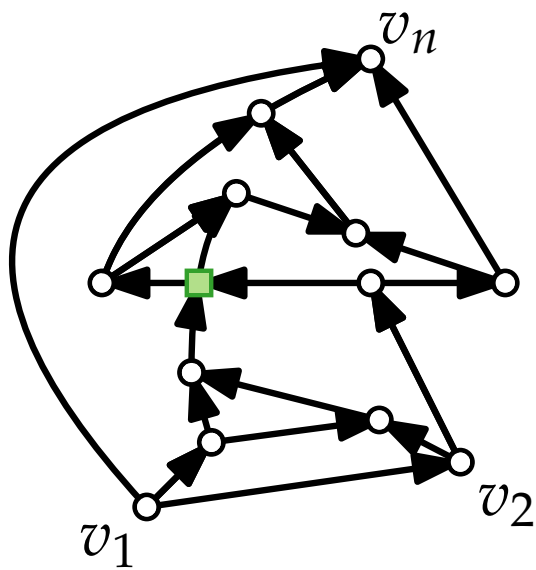
Subcubic \Rightarrow 2 Bends, 2 Slopes



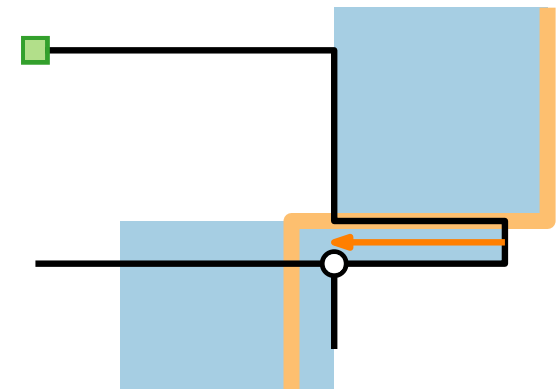
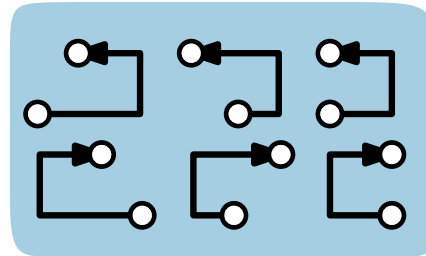
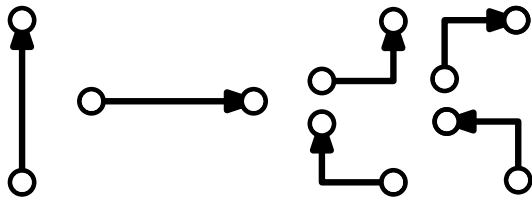
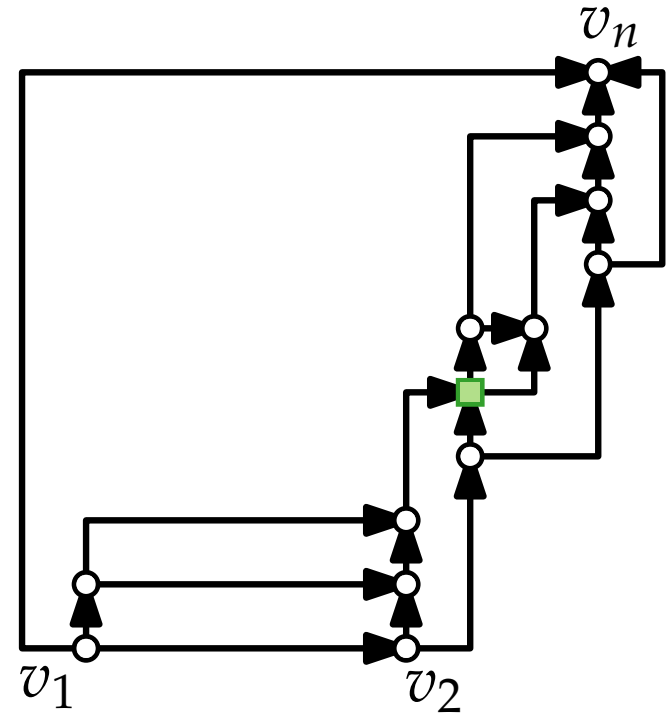
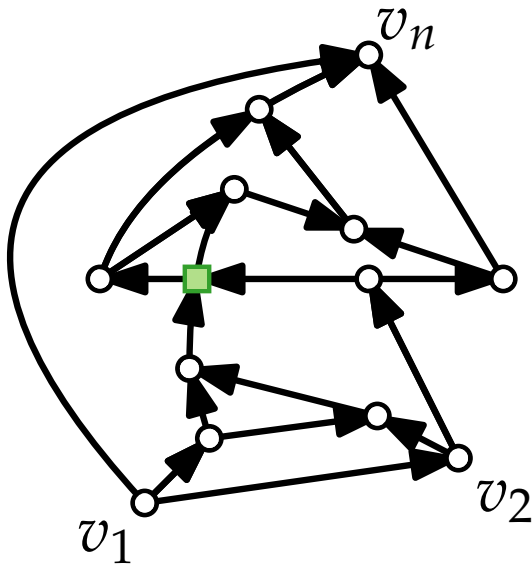
Subcubic \Rightarrow 2 Bends, 2 Slopes



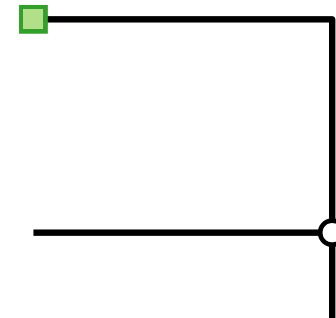
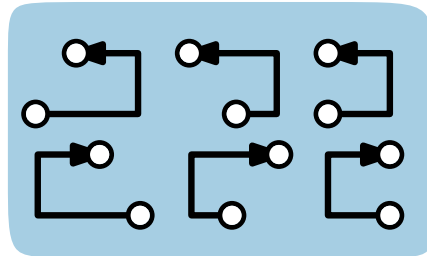
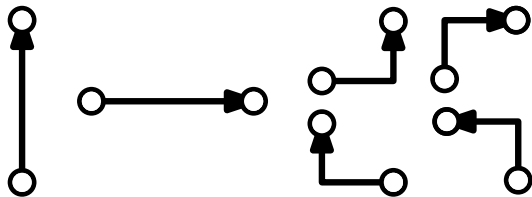
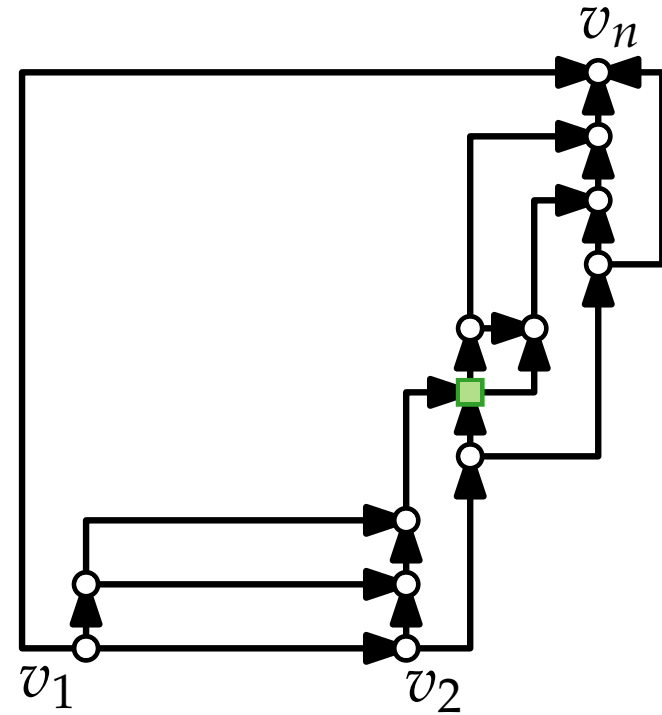
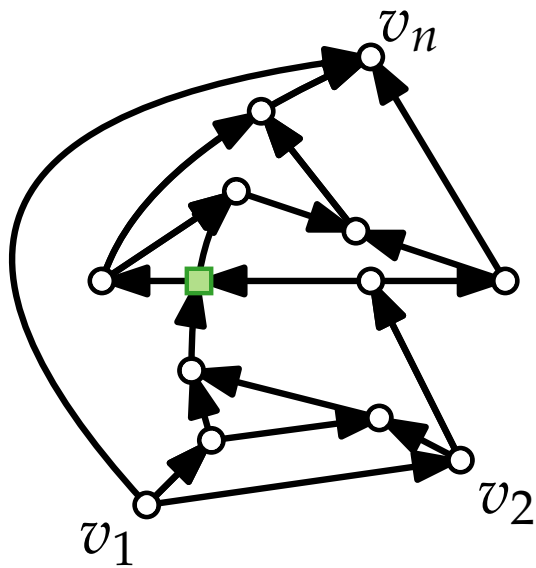
Subcubic \Rightarrow 2 Bends, 2 Slopes



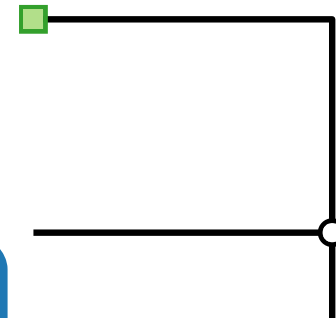
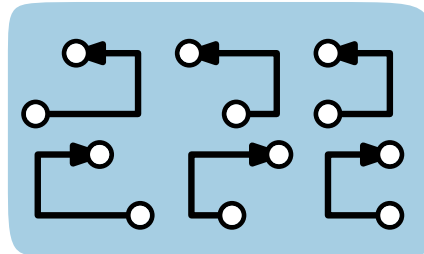
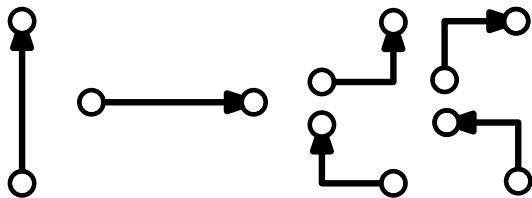
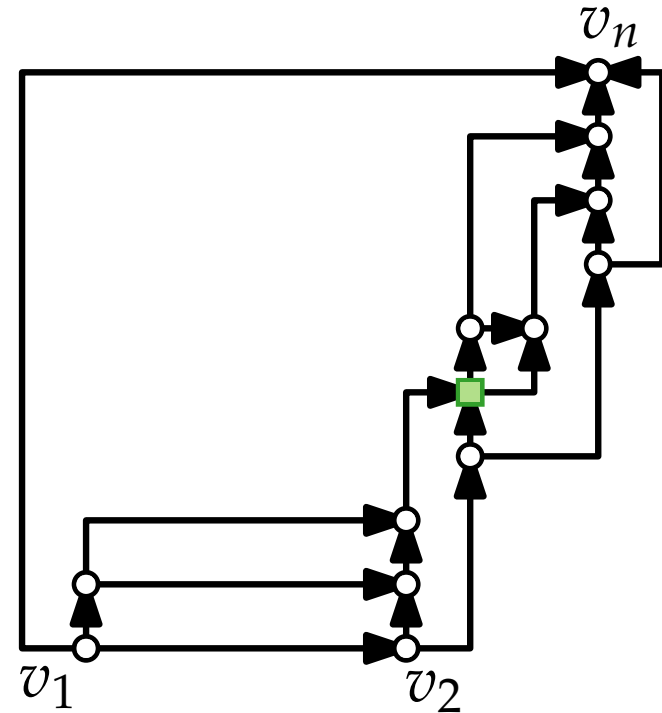
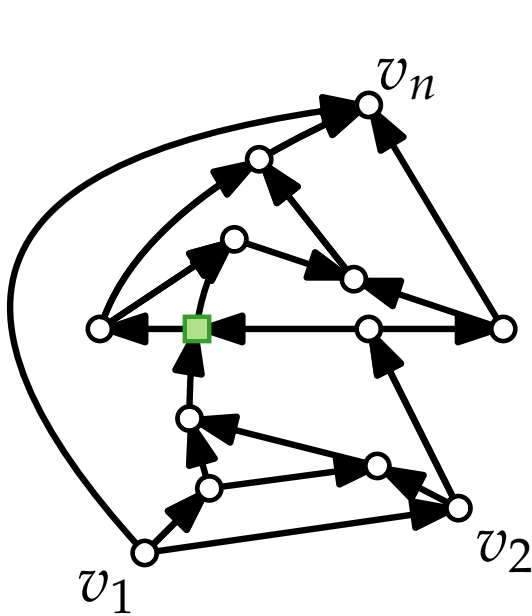
Subcubic \Rightarrow 2 Bends, 2 Slopes



Subcubic \Rightarrow 2 Bends, 2 Slopes

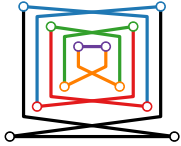


Subcubic \Rightarrow 2 Bends, 2 Slopes



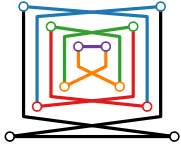
subcubic 1-plane 2-bend $\Rightarrow \leq 2$ slopes

Conclusion

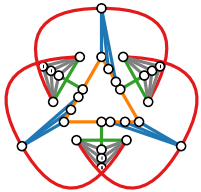


2-regular 2-connected 1-plane s-l $\Rightarrow \Omega(n)$ slopes

Conclusion

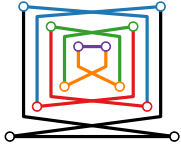


2-regular 2-connected 1-plane s-l $\Rightarrow \Omega(n)$ slopes

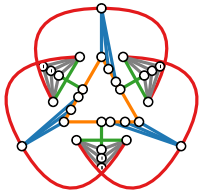


3-connected 1-plane s-l $\Rightarrow \geq 9(\Delta - 1)$ slopes

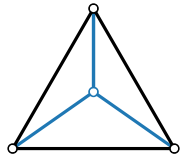
Conclusion



2-regular 2-connected 1-plane s-l $\Rightarrow \Omega(n)$ slopes

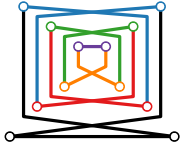


3-connected 1-plane s-l $\Rightarrow \geq 9(\Delta - 1)$ slopes

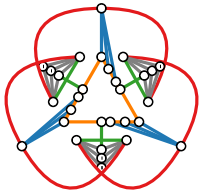


cubic 3-connected 1-plane 1-bend $\Rightarrow \geq 3$ slopes

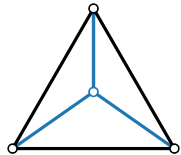
Conclusion



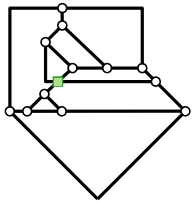
2-regular 2-connected 1-plane s-l $\Rightarrow \Omega(n)$ slopes



3-connected 1-plane s-l $\Rightarrow \geq 9(\Delta - 1)$ slopes

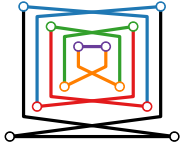


cubic 3-connected 1-plane 1-bend $\Rightarrow \geq 3$ slopes

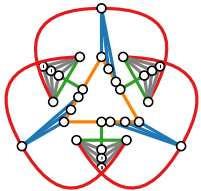


cubic 3-connected 1-plane 1-bend $\Rightarrow \leq 4$ slopes

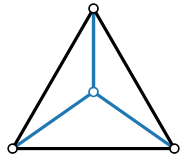
Conclusion



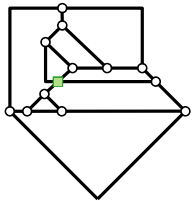
2-regular 2-connected 1-plane s-l $\Rightarrow \Omega(n)$ slopes



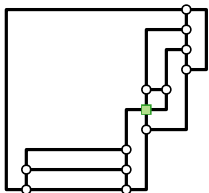
3-connected 1-plane s-l $\Rightarrow \geq 9(\Delta - 1)$ slopes



cubic 3-connected 1-plane 1-bend $\Rightarrow \geq 3$ slopes

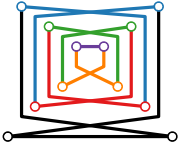


cubic 3-connected 1-plane 1-bend $\Rightarrow \leq 4$ slopes

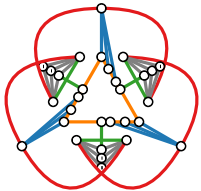


subcubic 1-plane 2-bend $\Rightarrow \leq 2$ slopes

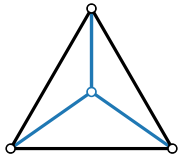
Conclusion



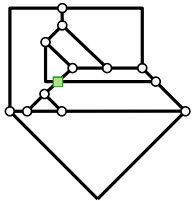
2-regular 2-connected 1-plane s-1 $\Rightarrow \Omega(n)$ slopes



3-connected 1-plane s-1 $\Rightarrow \geq 9(\Delta - 1)$ slopes

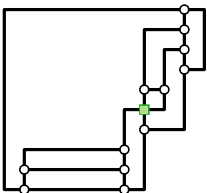


cubic 3-connected 1-plane 1-bend $\Rightarrow \geq 3$ slopes



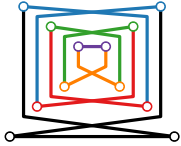
cubic 3-connected 1-plane 1-bend $\Rightarrow \leq 4$ slopes

angular and crossing resolution: $\pi/4$ and $\pi/2$

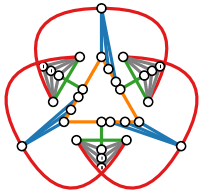


subcubic 1-plane 2-bend $\Rightarrow \leq 2$ slopes

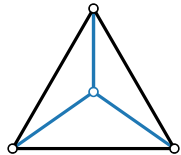
Conclusion



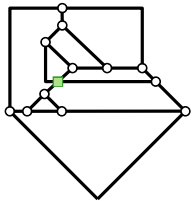
2-regular 2-connected 1-plane s-l $\Rightarrow \Omega(n)$ slopes



3-connected 1-plane s-l $\Rightarrow \geq 9(\Delta - 1)$ slopes

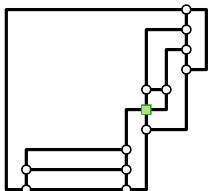


cubic 3-connected 1-plane 1-bend $\Rightarrow \geq 3$ slopes



cubic ~~3-connected~~ 1-plane 1-bend $\Rightarrow \leq 4$ slopes

angular and crossing resolution: $\pi/4$ and $\pi/2$



subcubic 1-plane 2-bend $\Rightarrow \leq 2$ slopes