







TURNING CLIQUES INTO PATHS TO ACHIEVE PLANARITY

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Hybrid Representations of Graphs

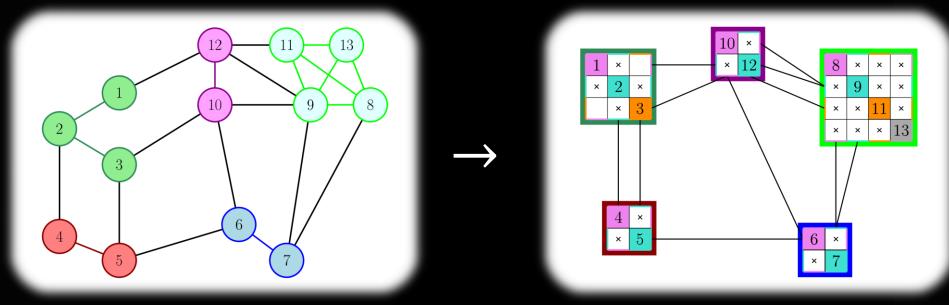


- Global structure of the network → Node-link paradigm
- Dense subgraphs (clusters) → A different representation paradigm

Hybrid Representations of Graphs

- Global structure of the network → Node-link paradigm
- Dense subgraphs (clusters) → Adjacency matrices

NodeTrix representation [Henry et al., 2007]

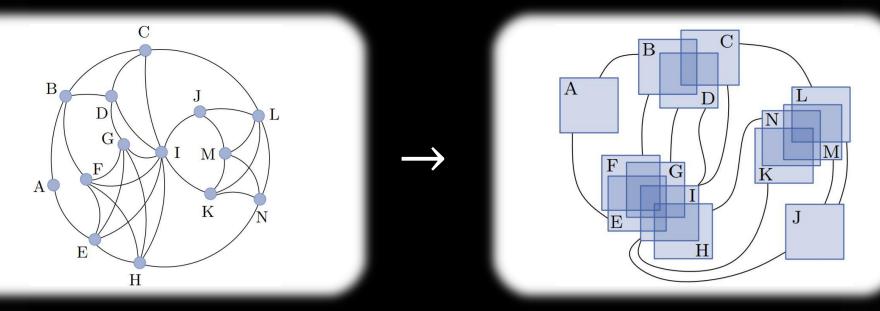


Hybrid Representations of Graphs



• Dense subgraphs (clusters) → Intersections between objects

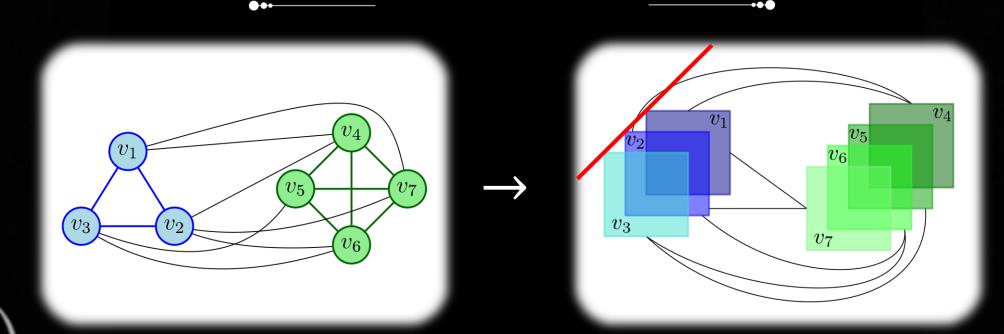
Intersection-link representation [Angelini et al., 2017]



Clique-planar Representations



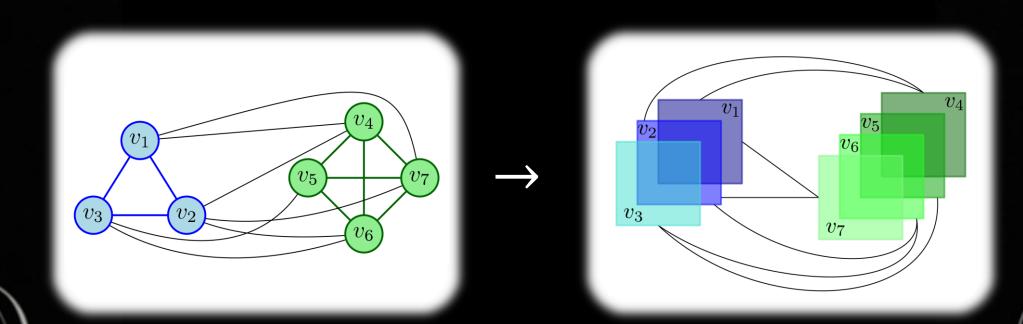
- Crossing-free intersection-link representations
- Objects: Isothetic rectangles
- Clusters are given: Vertex-disjoint cliques



Clique Planarity



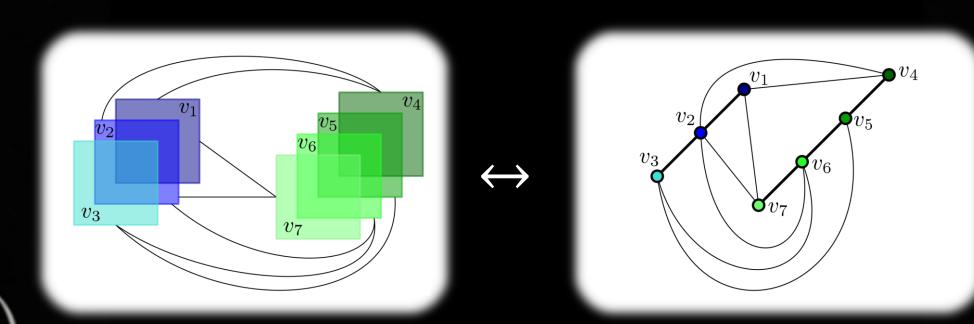
- Input: A clustered graph G = (V, E) such that each cluster is a clique
- <u>Problem</u>: Does *G* admit a clique-planar representation?
- Clique Planarity is NP-complete [Angelini et al., 2017]



Clique Planarity



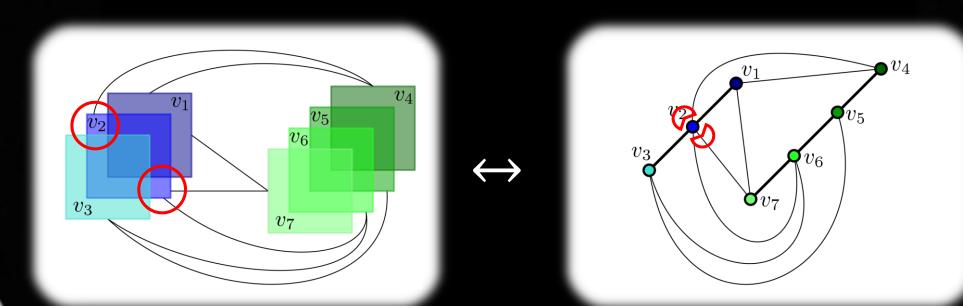
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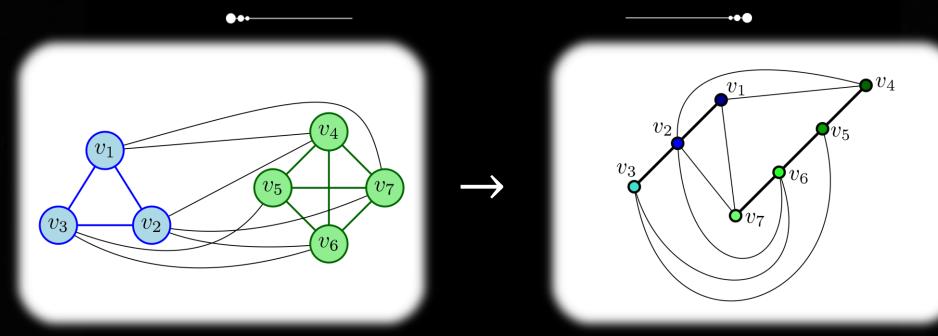
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Two attachments for the central vertices

Clique Planarity - Another Formulation

- Input: A graph G=(V,E) and a partition of V into subsets such that each subset induces a clique
- <u>Problem</u>: Does G contain a planar subgraph such that each clique is replaced by a path spanning its vertices?



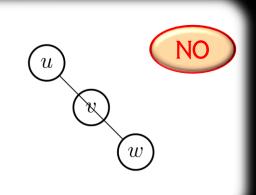
Input:

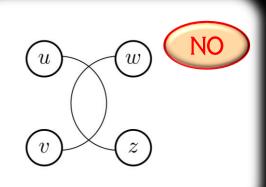
- A simple topological graph G = (V, E)
- A clustering of G such that each cluster is a clique of size at most h Problem: Does G admit a clique-planar representation?

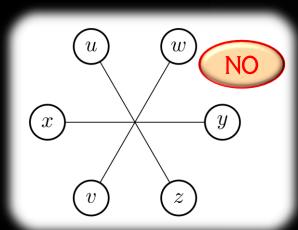
Input:

- A simple topological graph G = (V, E)
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Simple topological graph





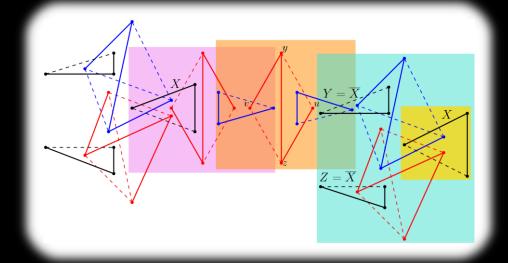


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Known results [Kindermann et al., 2018]

- 4-C2PP is NP-complete for geometric graphs
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- 3-C2PP can be solved in polynomial time for geometric graphs
 - 2SAT formulation that can be extended to solve 3-C2PP for simple topological graphs
- 4-C2PP is NP-complete for <u>simple topological</u> 4-plane graphs
- 3-C2PP can be solved in polynomial time for <u>simple topological</u> graphs

We study the complexity of h-C2PP in relation to k-planar graphs

Our results

- 4-C2PP is NP-complete for simple topological 3-plane graphs
- $h\text{-}\mathsf{C2PP}$ can be solved in linear time for simple topological 1-plane graphs, for any value of h

Suspense There are always negative results...



h-C2PP \in NP

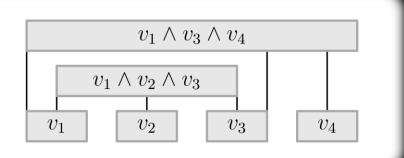
Planarity for a simple topological graph can be tested in linear time

 $h ext{-C2PP}$ for simple topological $k ext{-plane}$ graphs belongs to NP, for all values of h and k



Planar Positive 1-in-3-SAT

- Variables appear only with their positive literal
- Each clause has at most three variables
- The graph obtained by connecting variables with clauses is planar
- Find a truth assignment such that, for each clause, exactly one variable is set to True

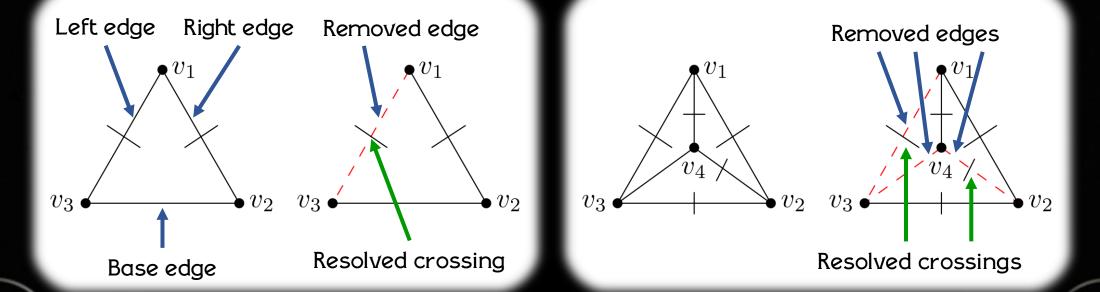




Planar Positive 1-in-3-SAT

3-cliques

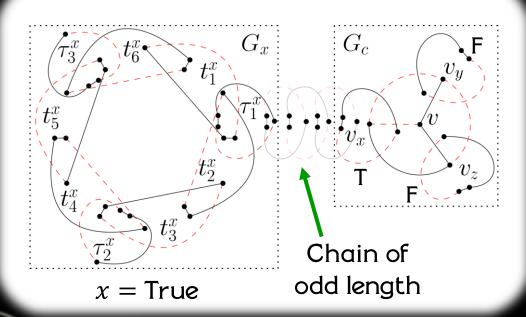
4-cliques

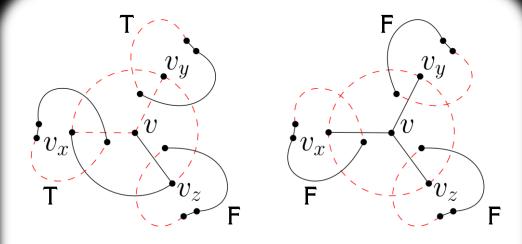


Planar Positive 1-in-3-SAT

Variable Gadget

Clause Gadget



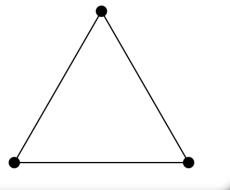


It is not possible to resolve all crossings

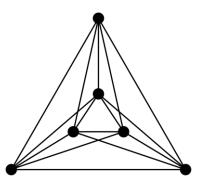
Intermission ... but also positive results...

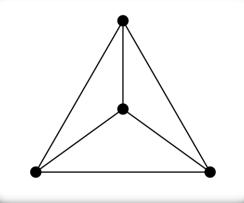


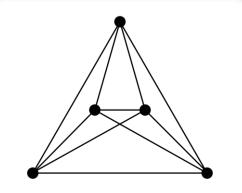




1-plane cliques



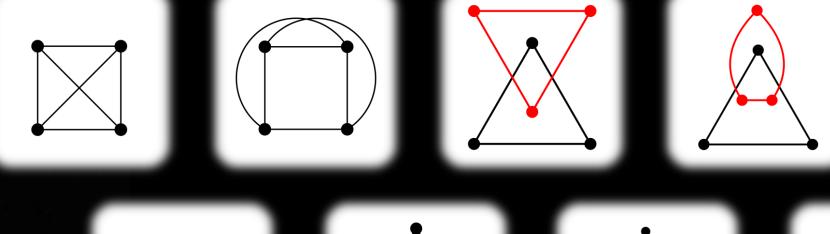


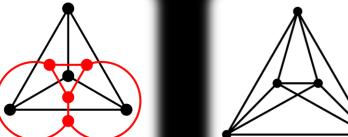


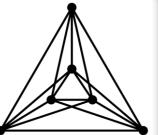




1-plane graphs involving one or more cliques









Open Problems



Study the complexity of h-Clique2Path Planarity for:

- Simple topological 2-plane graphs
- Geometric 2-plane and 3-plane graphs
 - 4-C2PP is NP-complete for geometric 4-plane graphs
- Abstract graphs
 - ullet Equivalent to CliquePlanarity, but h is bounded by a constant
 - CliquePlanarity is NP-complete (when $h \in O(n)$)

The End

The Cast















