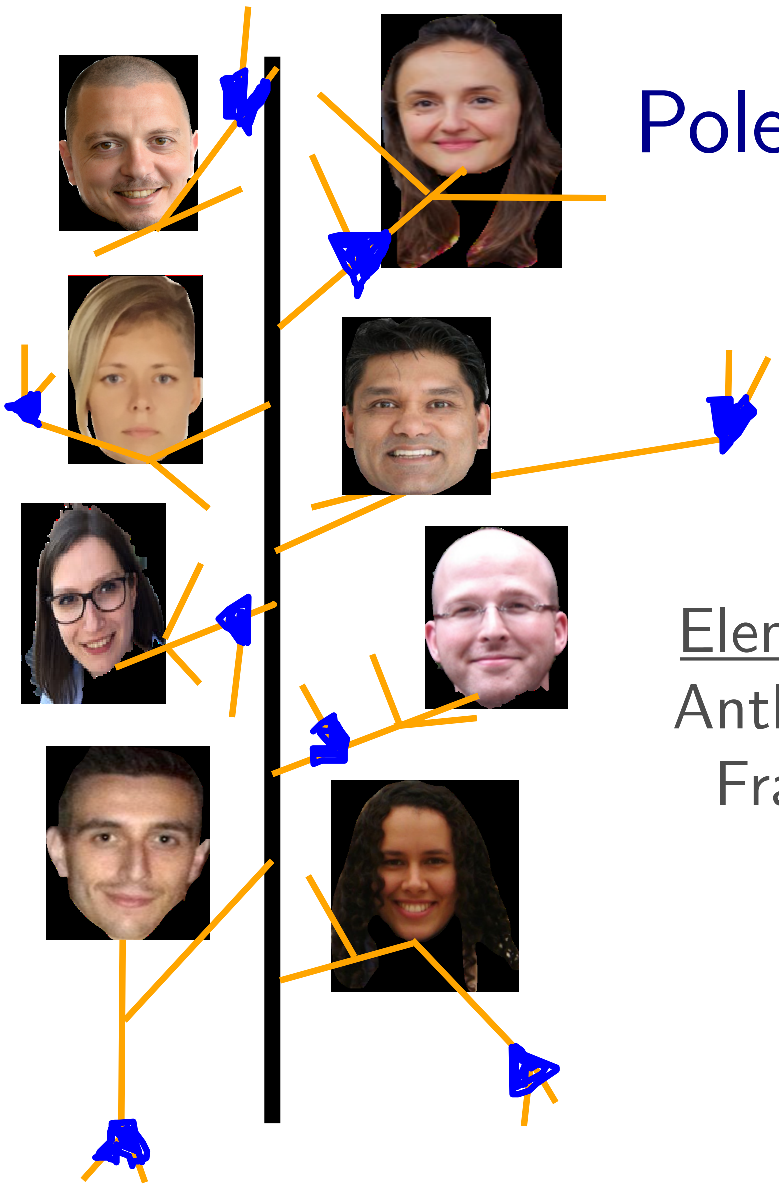
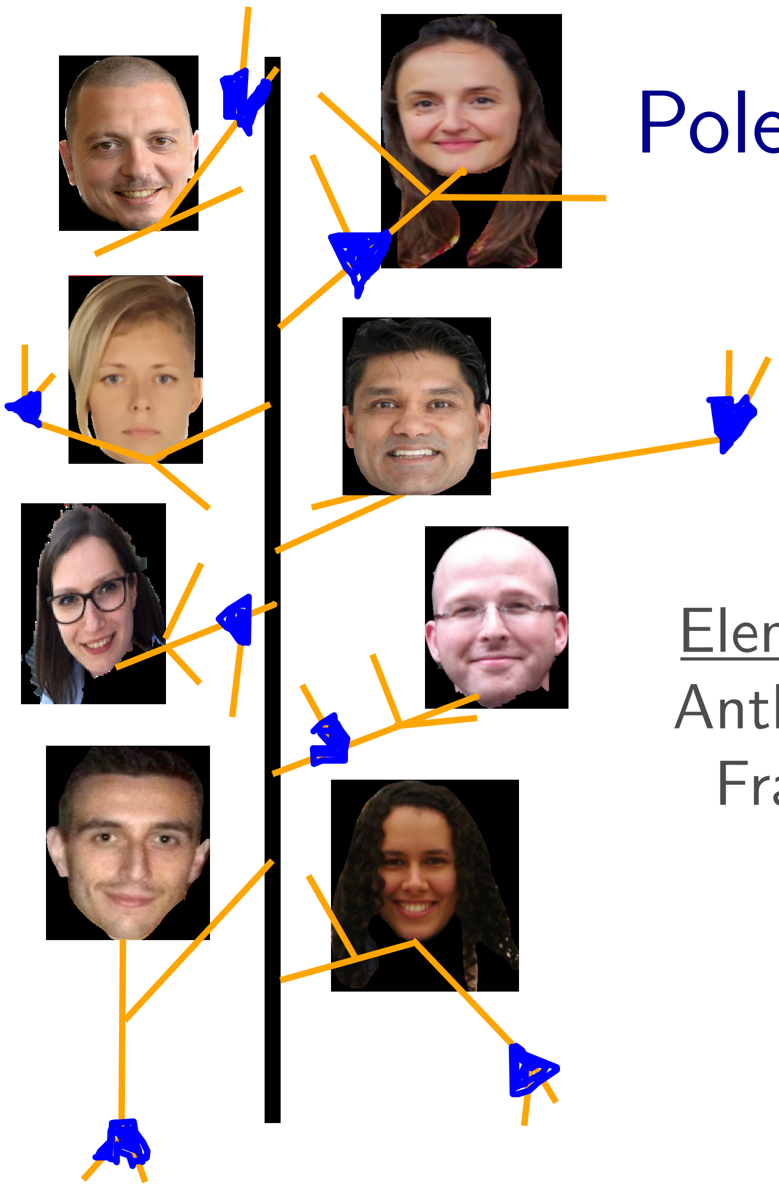


Pole Dancing: 3D Morphs for tree Drawings

Elena Arseneva, Prosenjit Bose, Pilar Cano, Anthony D'Angelo, Vida Dujmović, Fabrizio Frati, Stefan Langerman, and Alessandra Tappini



Pole Dancing: 3D Morphs for tree Drawings

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Morph

Morph

Morph

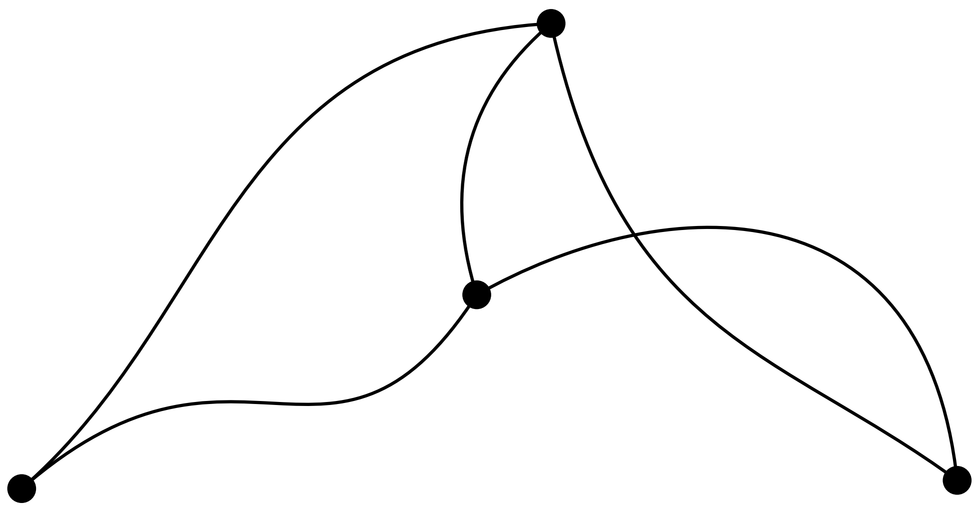
Morph

Morph

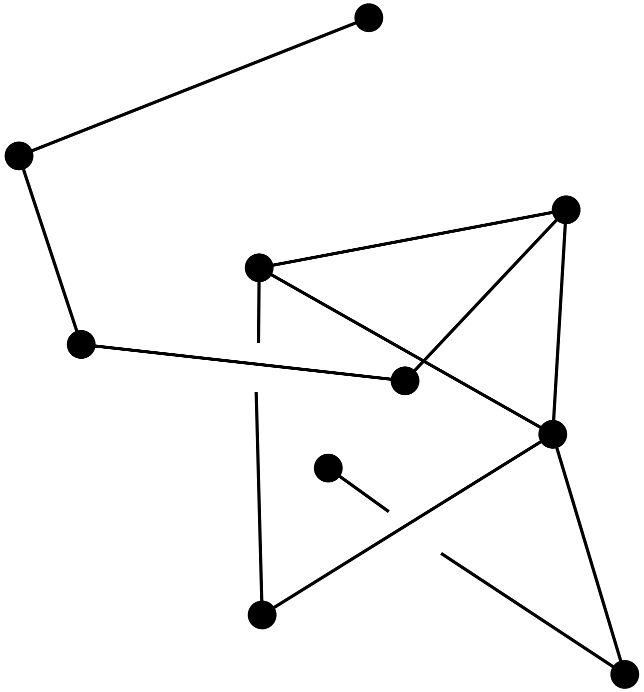
Morph

Morph

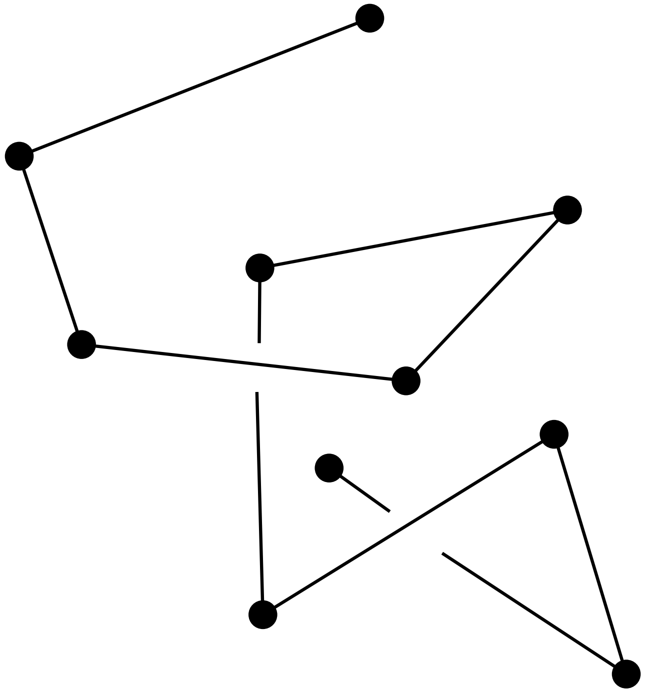
Graph drawing



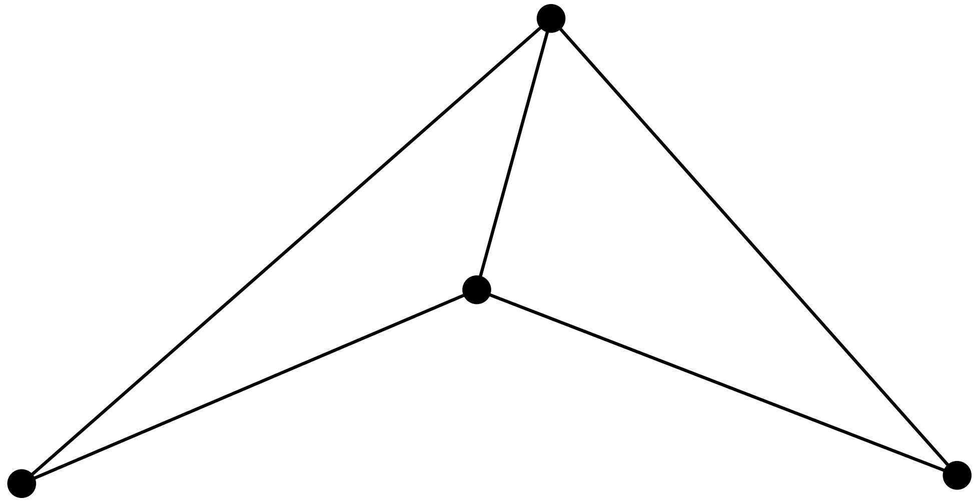
Straight-line drawing



Non–crossing straight-line drawing



Planar straight-line drawing



Morphing planar graphs

Morphing planar graphs

Morphing planar graphs

Morphing planar graphs

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Morphing planar graphs

Planar drawings: topologically equivalent or not

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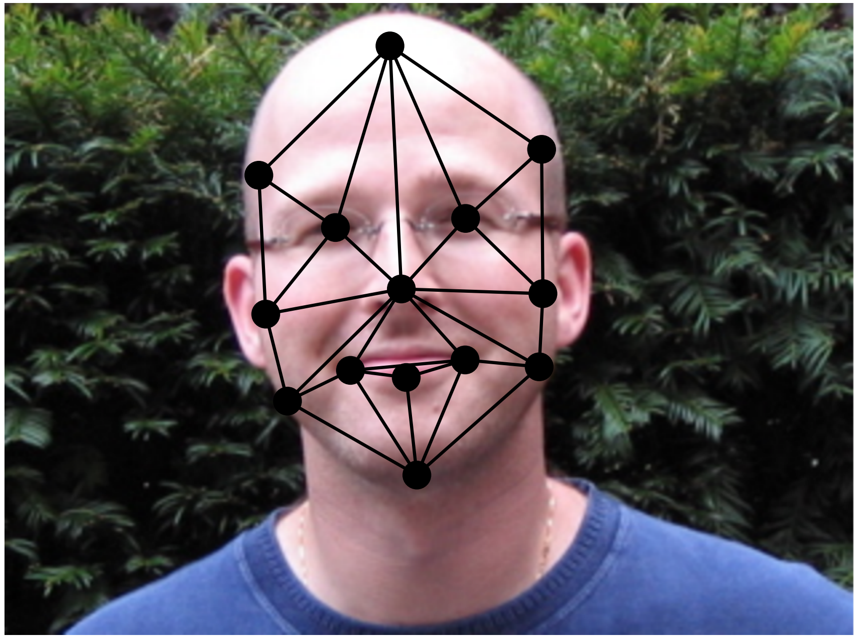
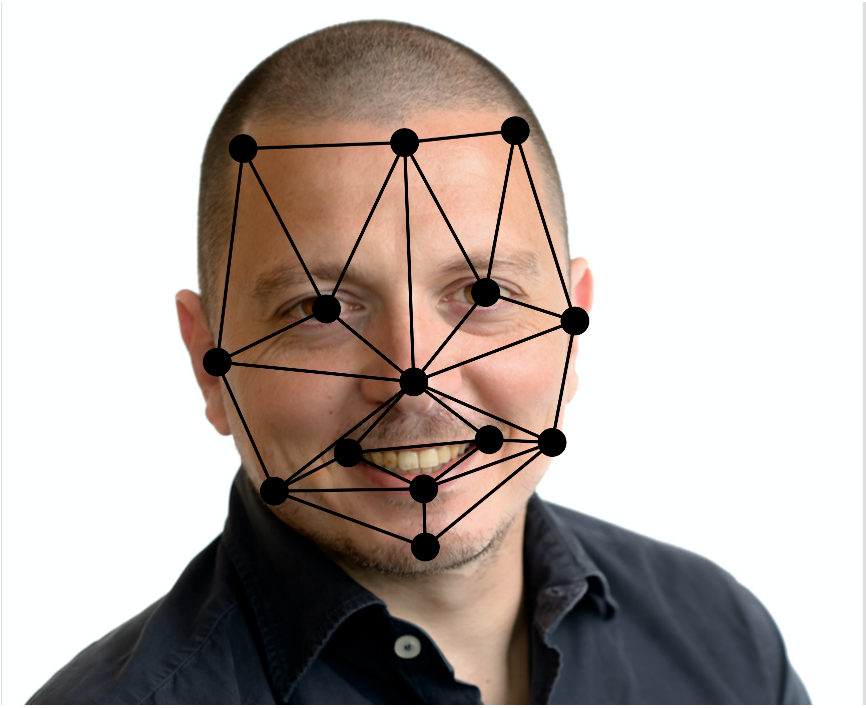
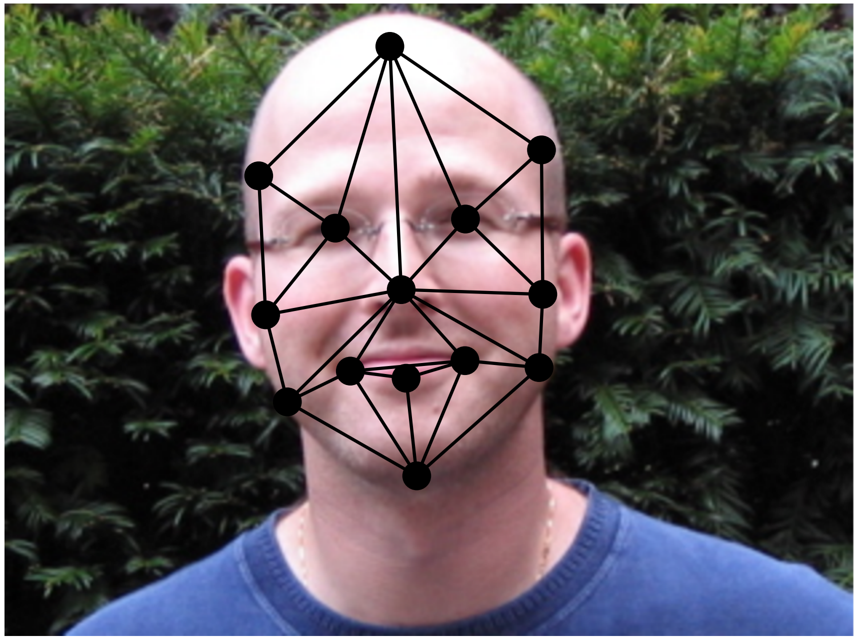


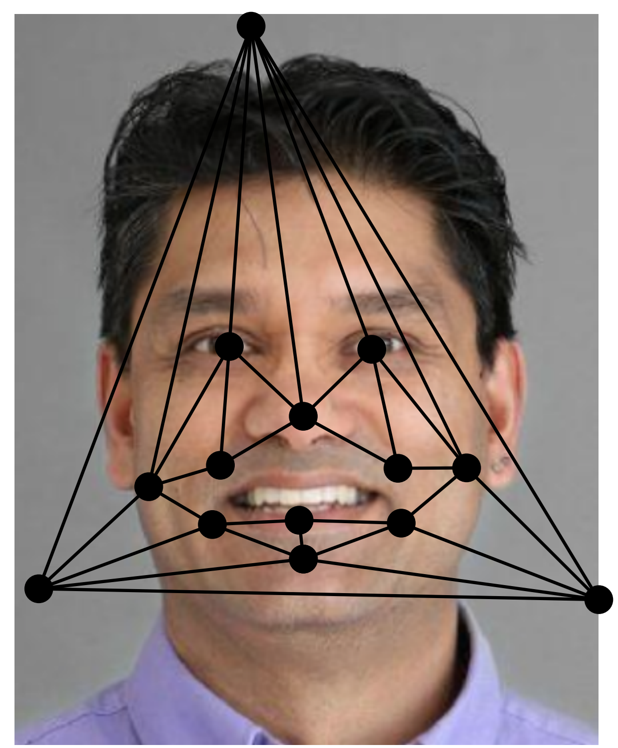
$\cong$



Planar drawings: topologically equivalent or not

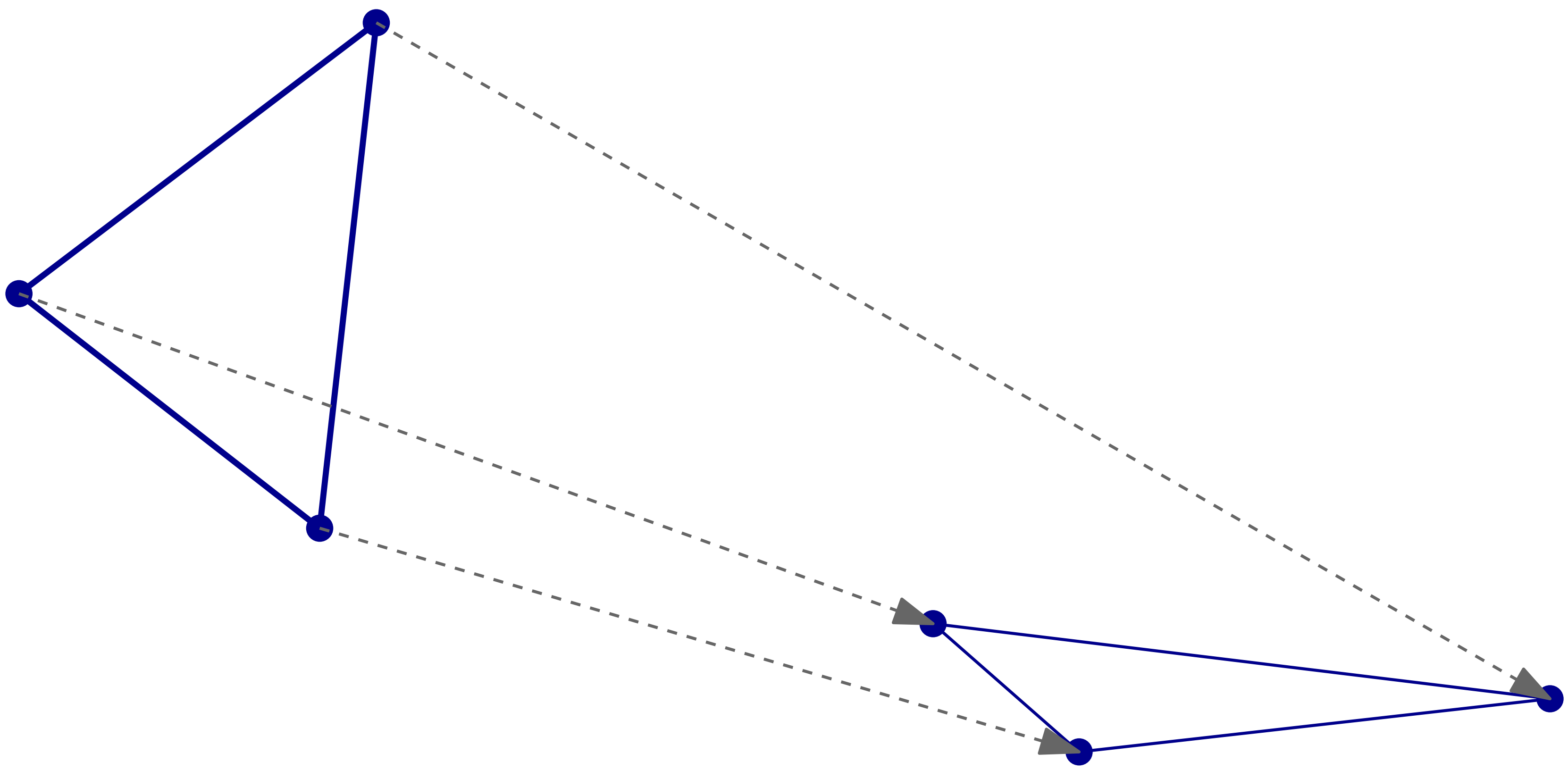


$\cancel{\cong}$

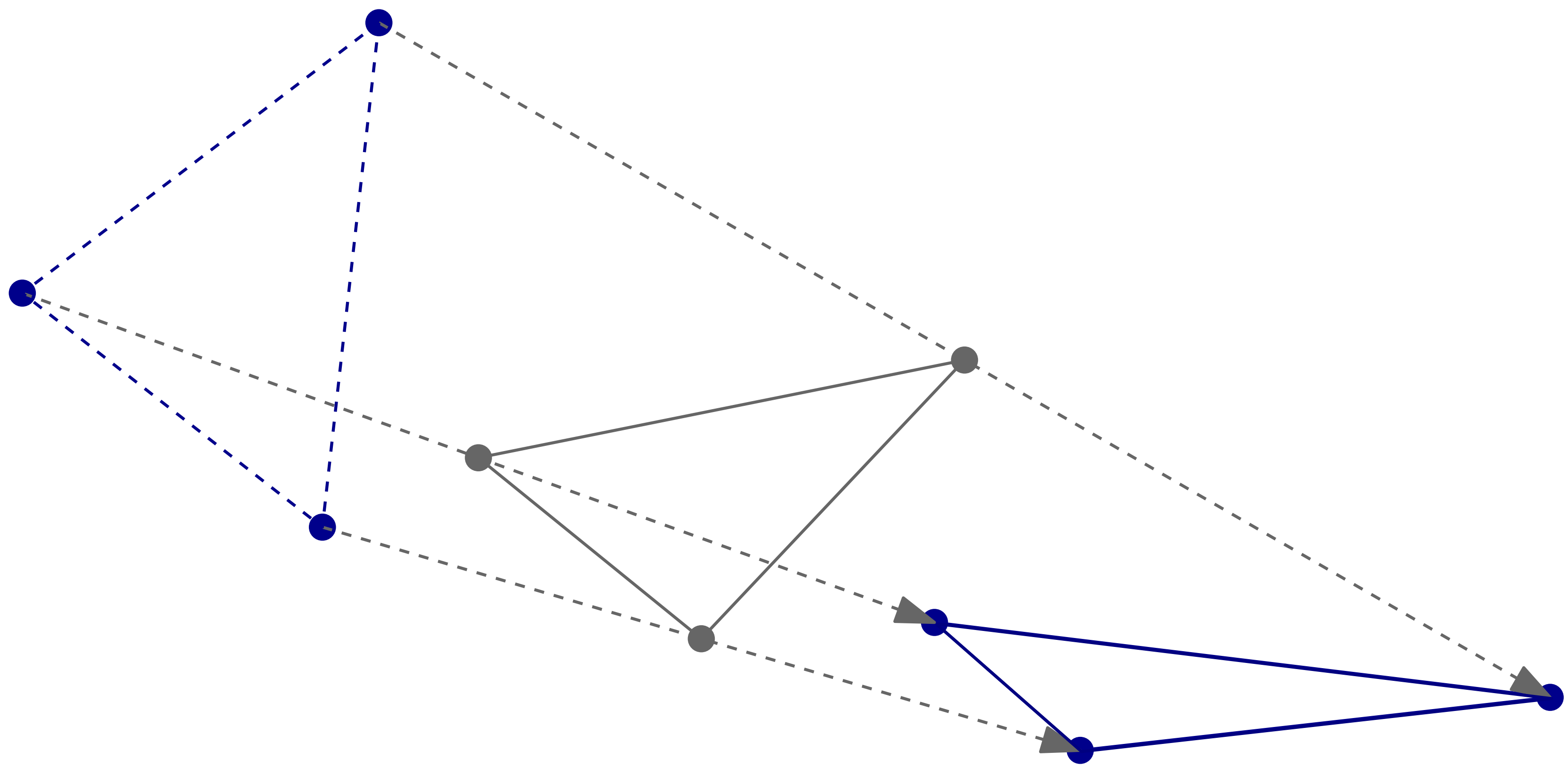


One step of a morph; linear morph

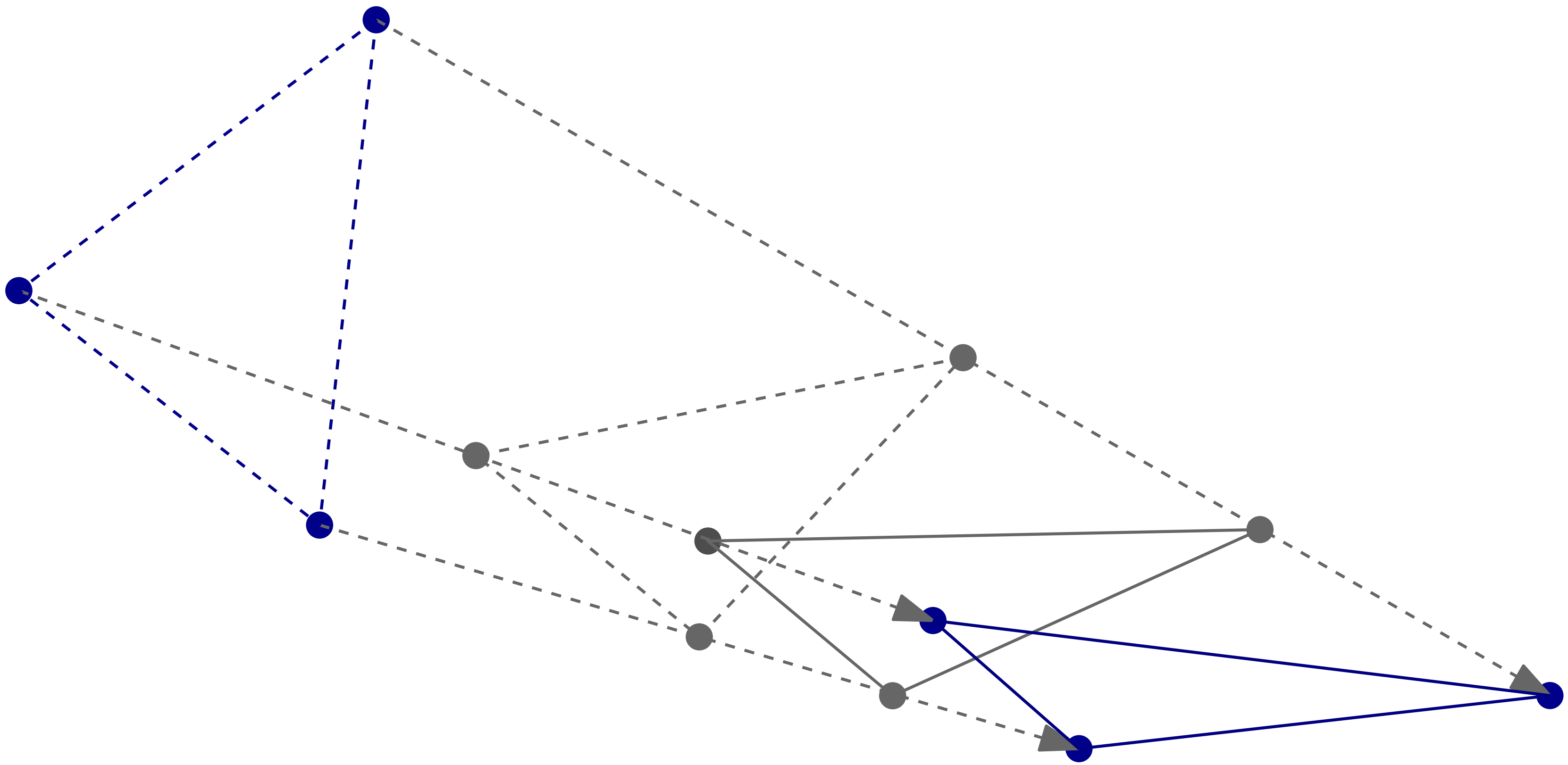
One step of a morph; linear morph



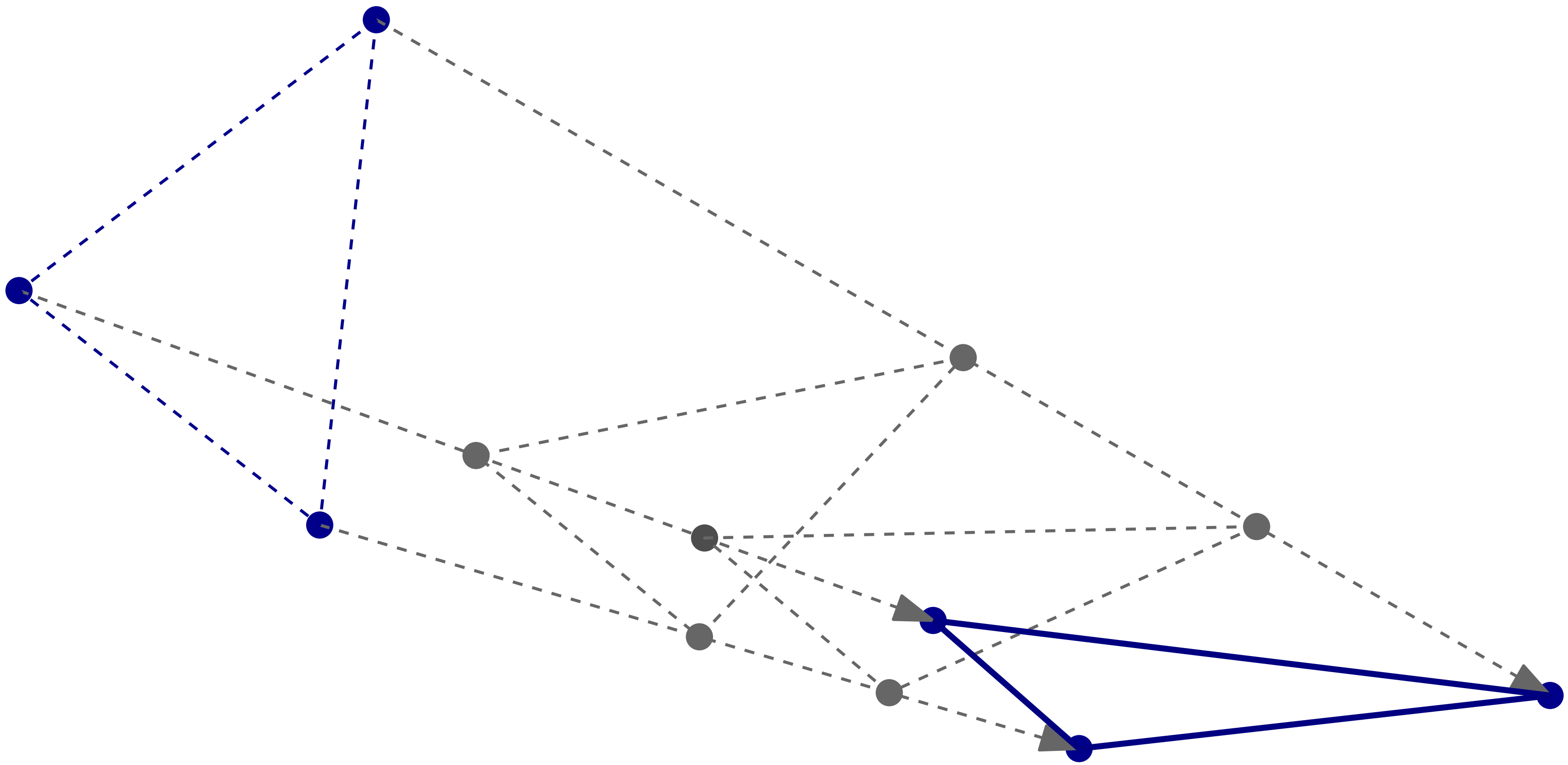
One step of a morph; linear morph



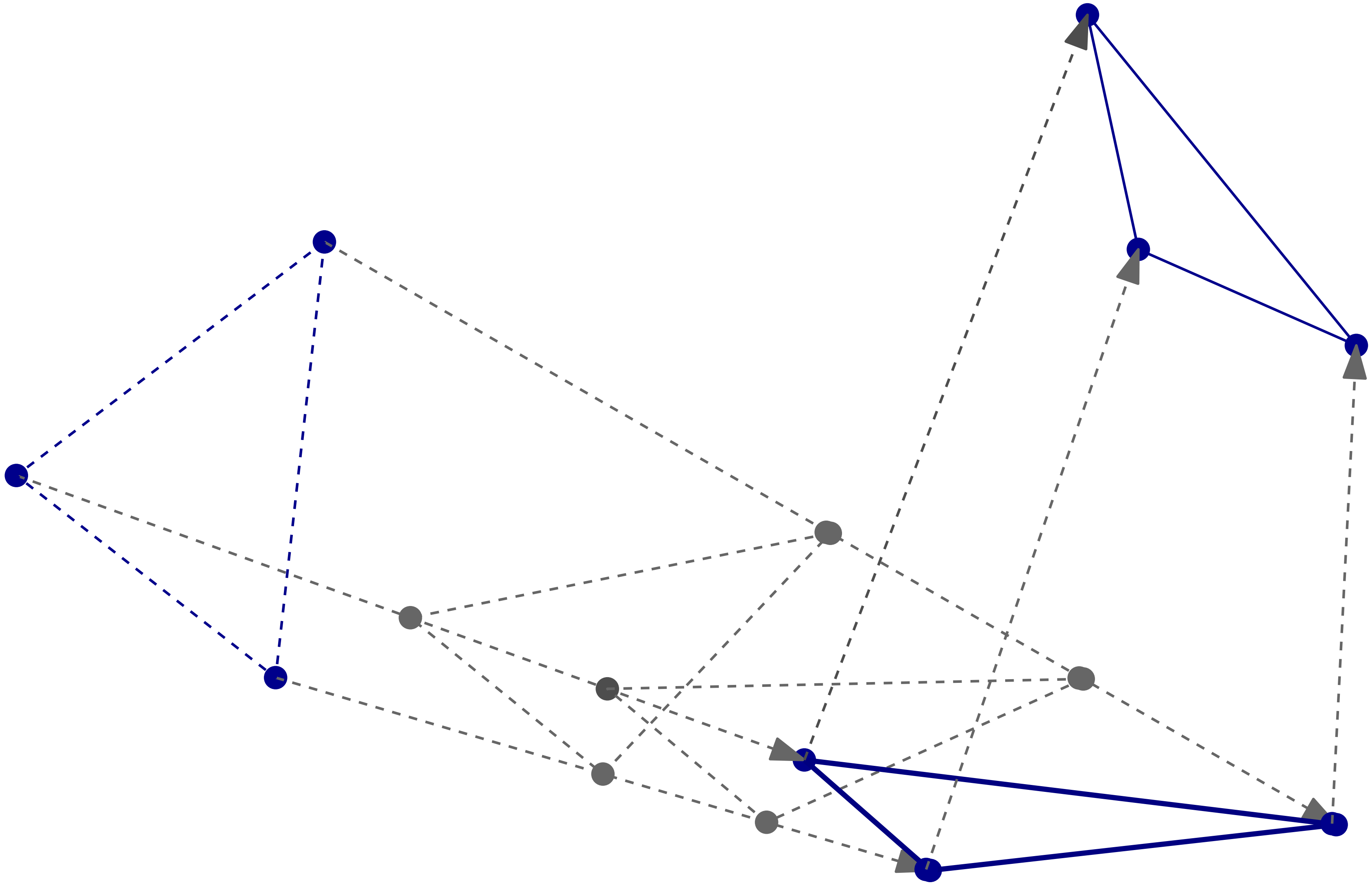
One step of a morph; linear morph



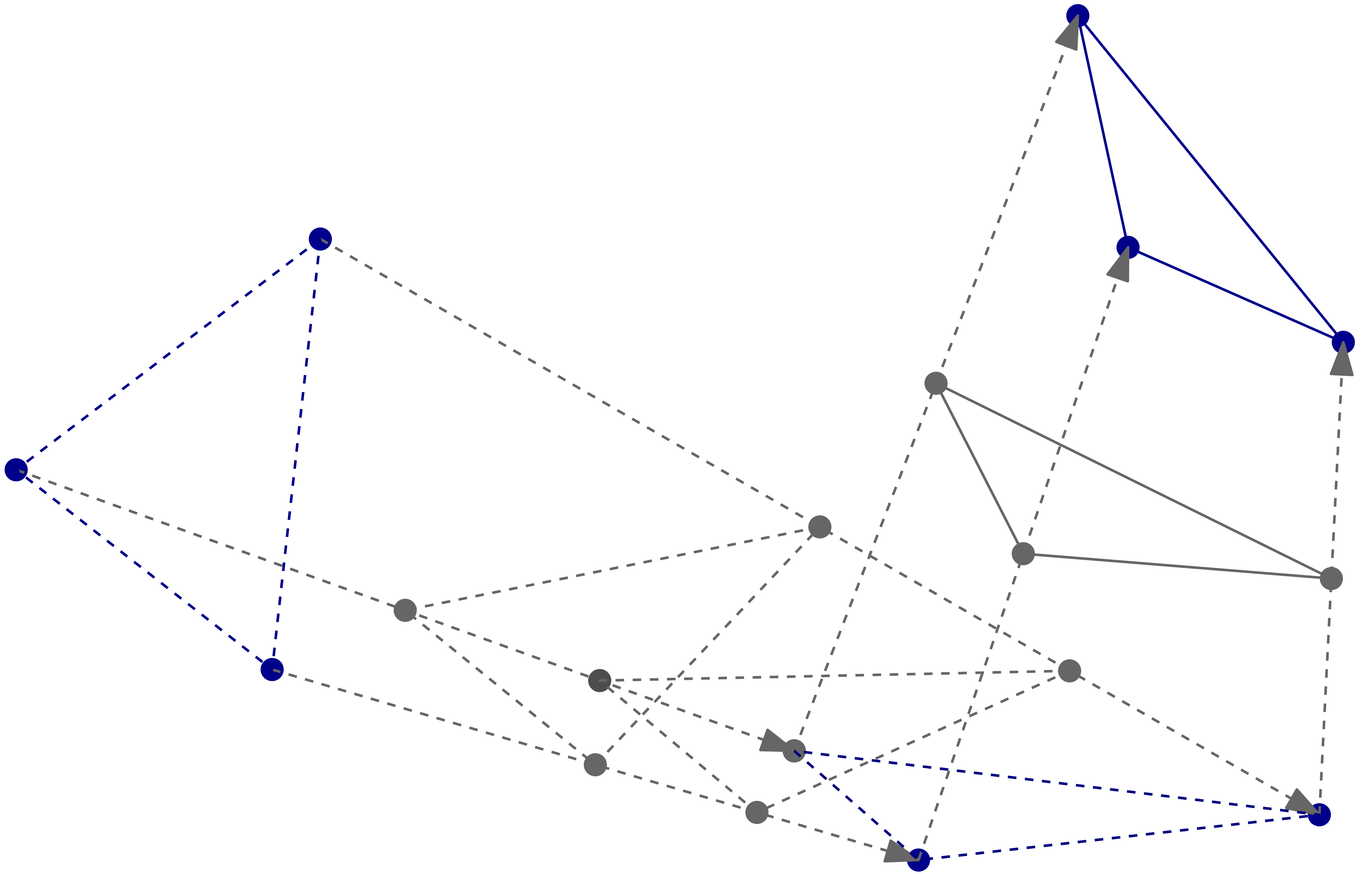
One step of a morph; linear morph



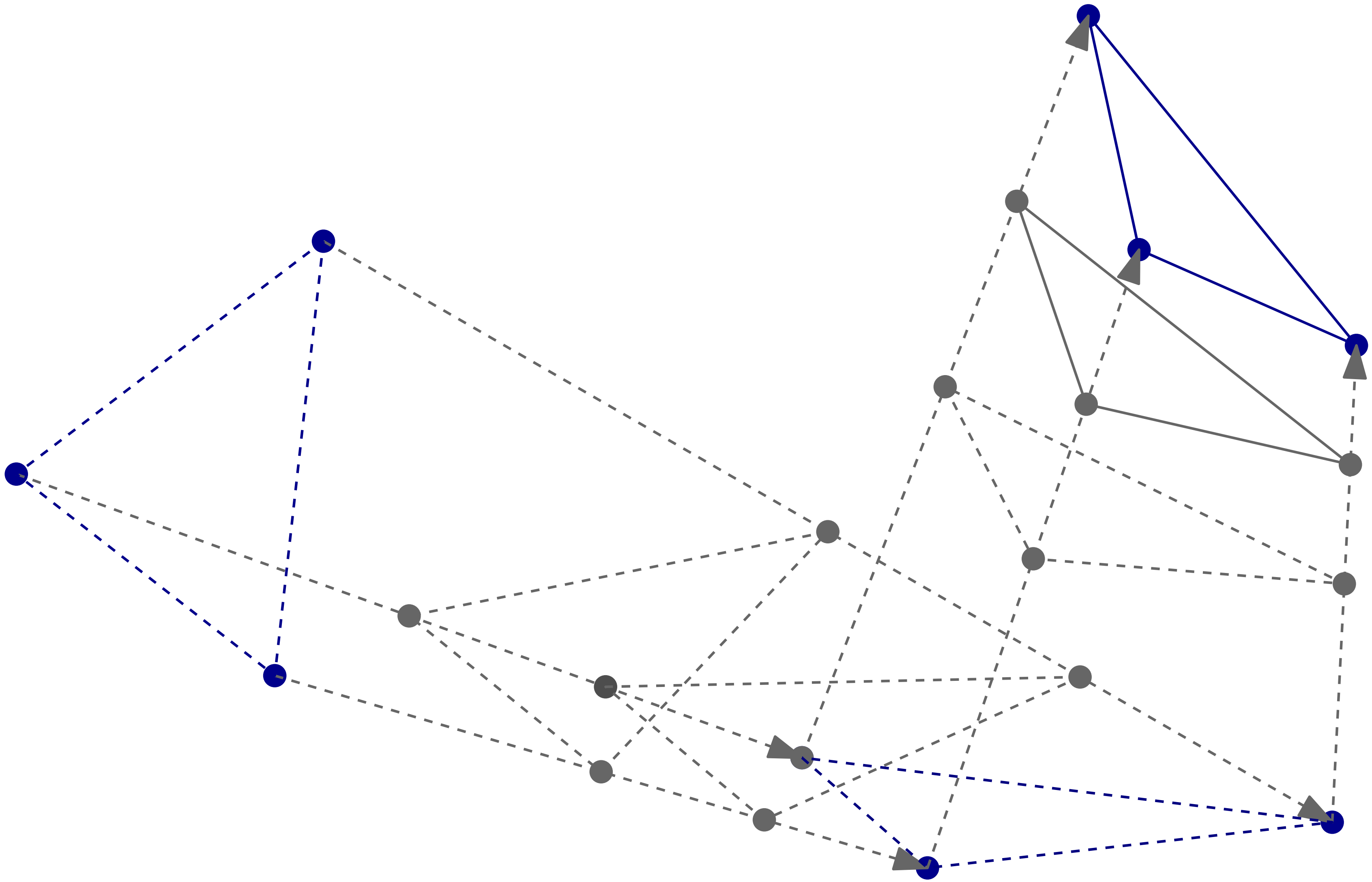
One step of a morph; linear morph



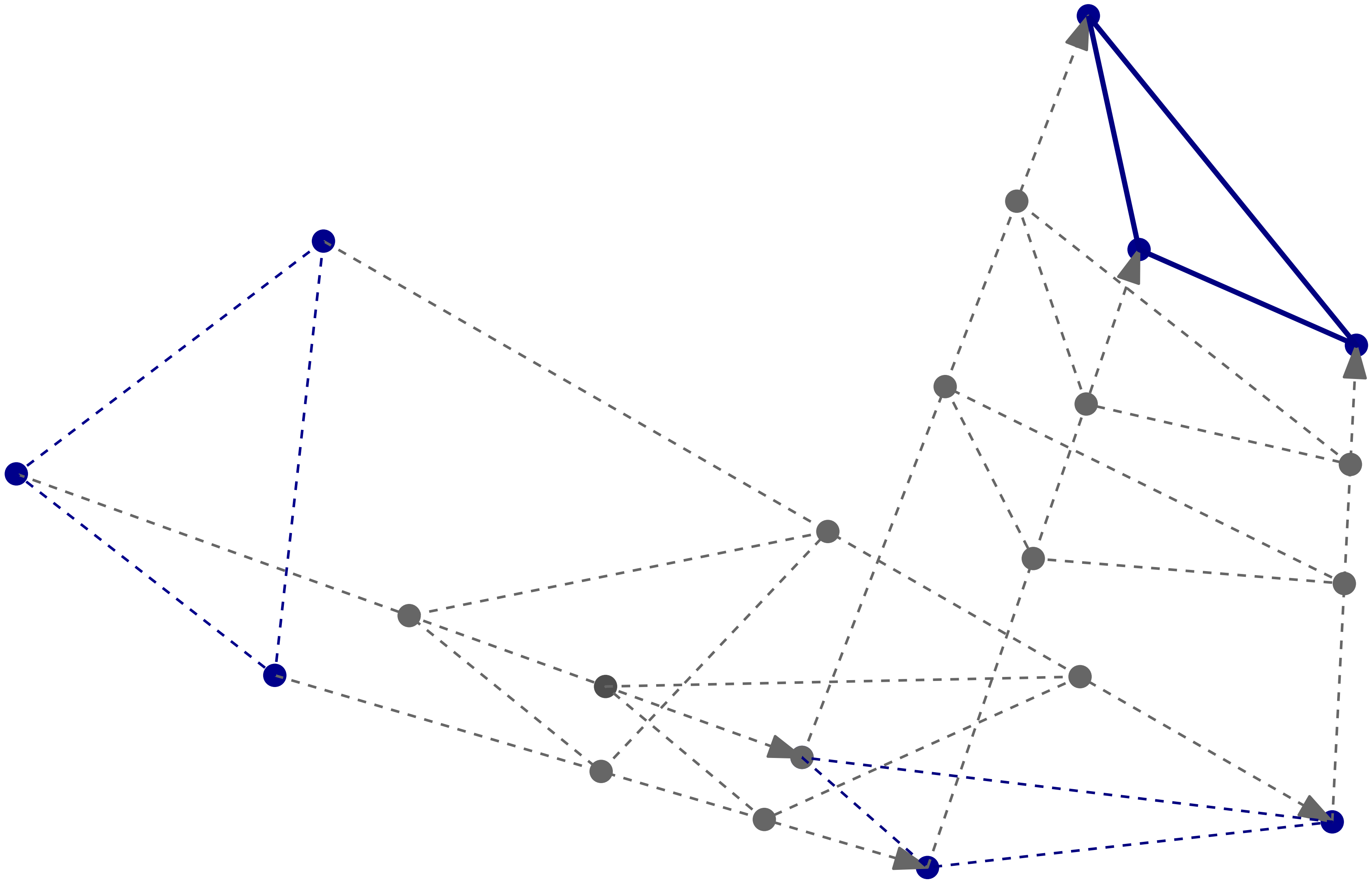
A morph of two steps



A morph of two steps



A morph of two steps



State of the art

Is it true that, for any two topologically equivalent planar drawings, there exists a morph in the plane from one to the other?

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[Cairns, 1944],
[Thomassen, 1983]:

YES!!

State of the art

Is it true that, for any two topologically equivalent planar drawings, there exists a morph in the plane from one to the other?

YES

In how many steps?

State of the art

Is it true that, for any two topologically equivalent planar drawings, there exists a morph in the plane from one to the other?

YES

In how many steps?

[Alamdari et al., 2017]:

In $O(n)$ steps

State of the art

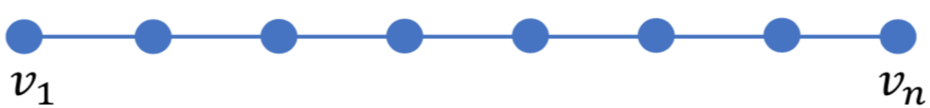
Is it true that, for any two topologically equivalent planar drawings, there exists a morph in the plane from one to the other?

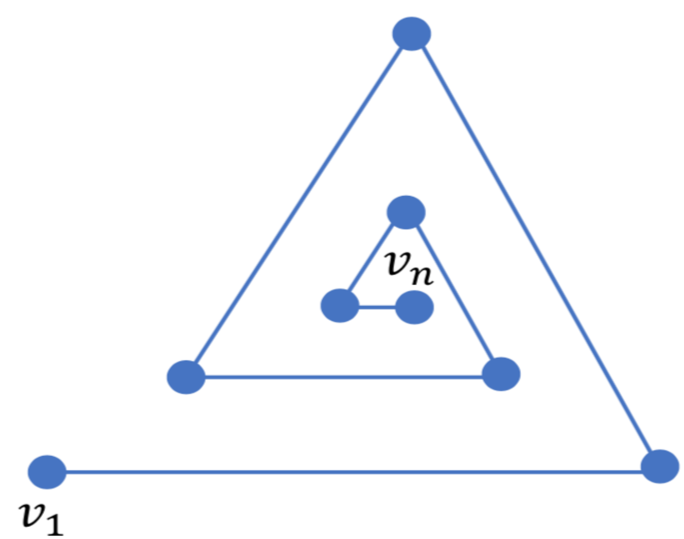
YES

In how many steps?

[Alamdari et al., 2017]:

In $\Theta(n)$ steps





3D crossing-free morph

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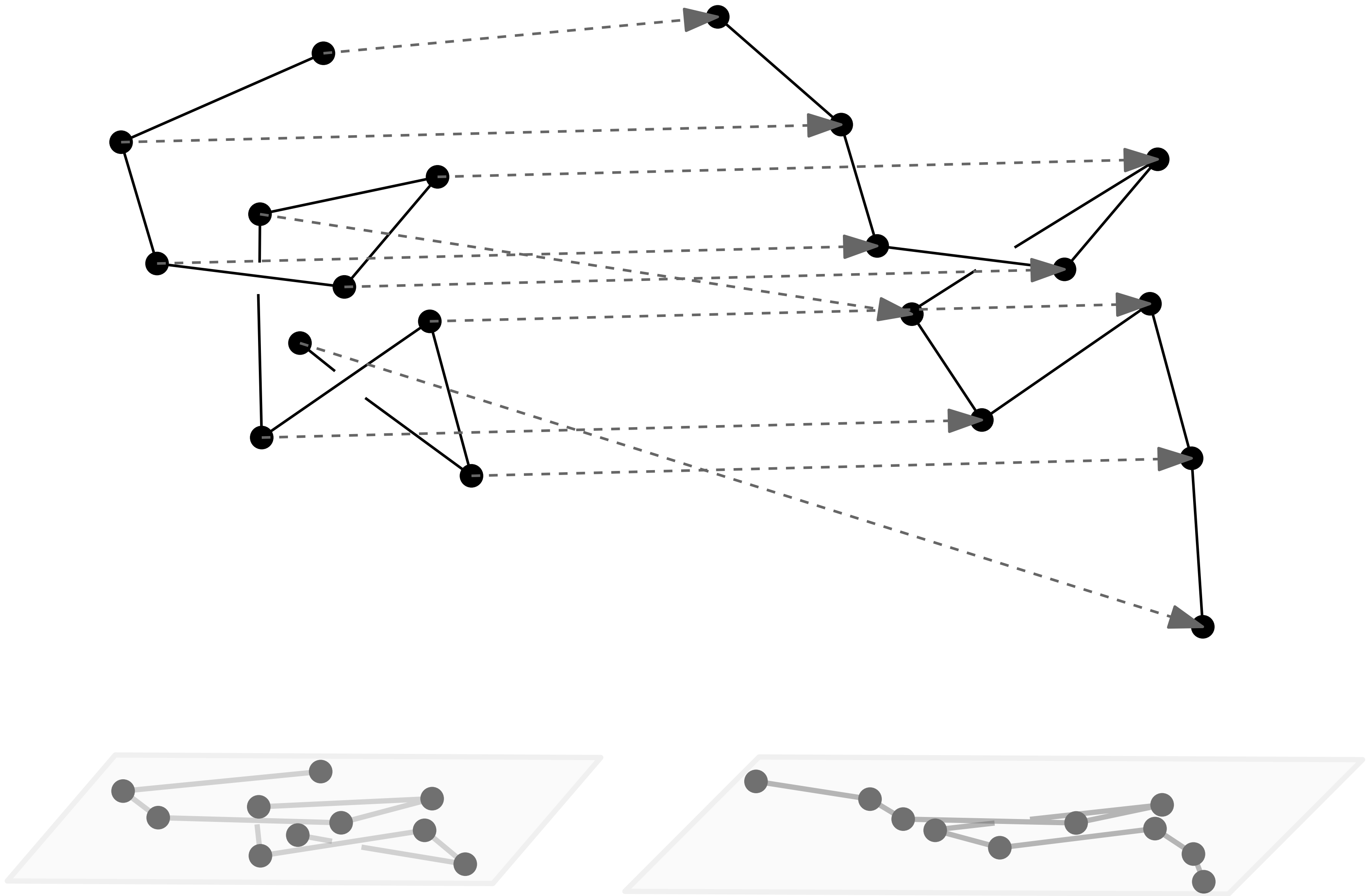
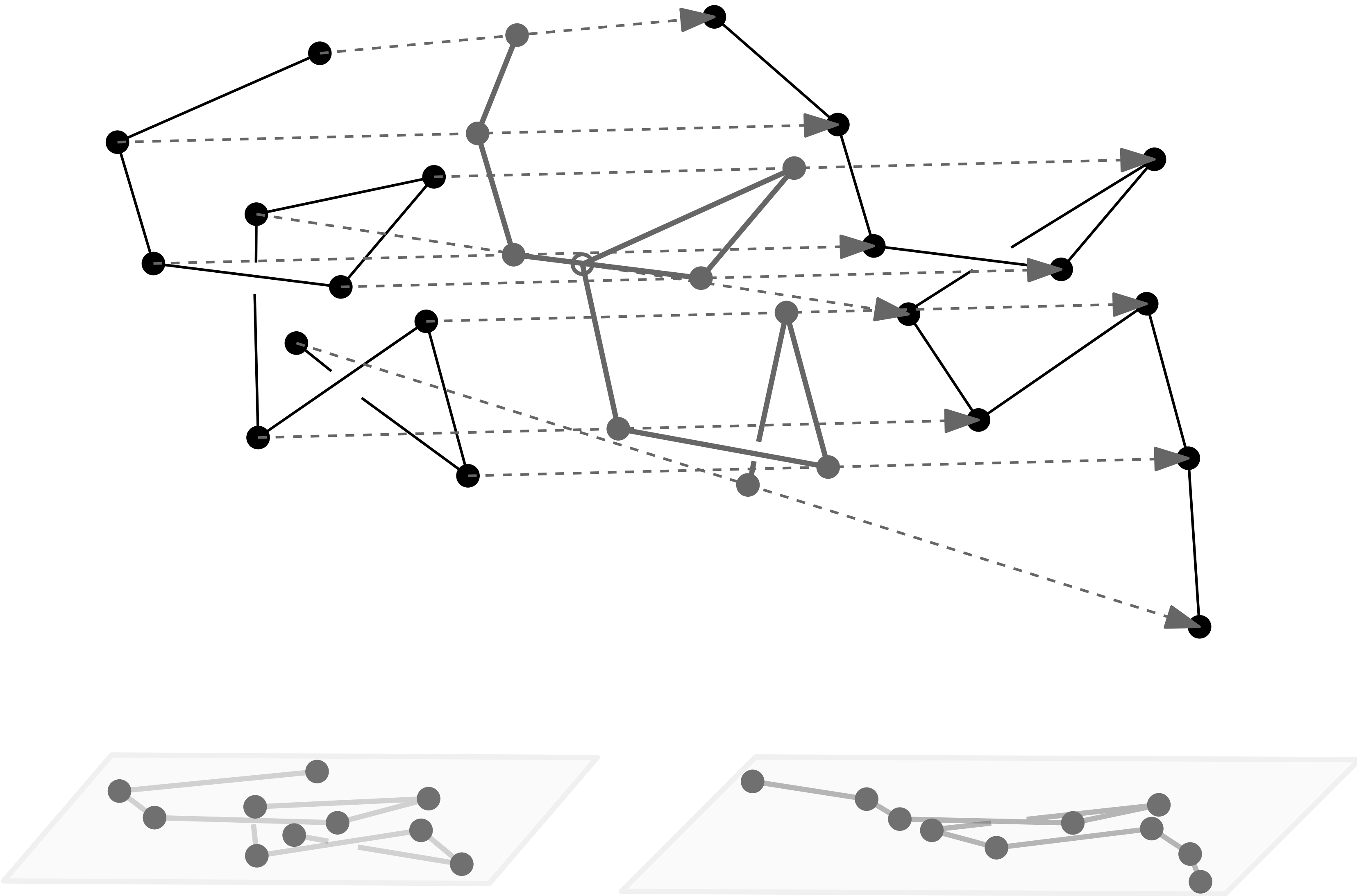
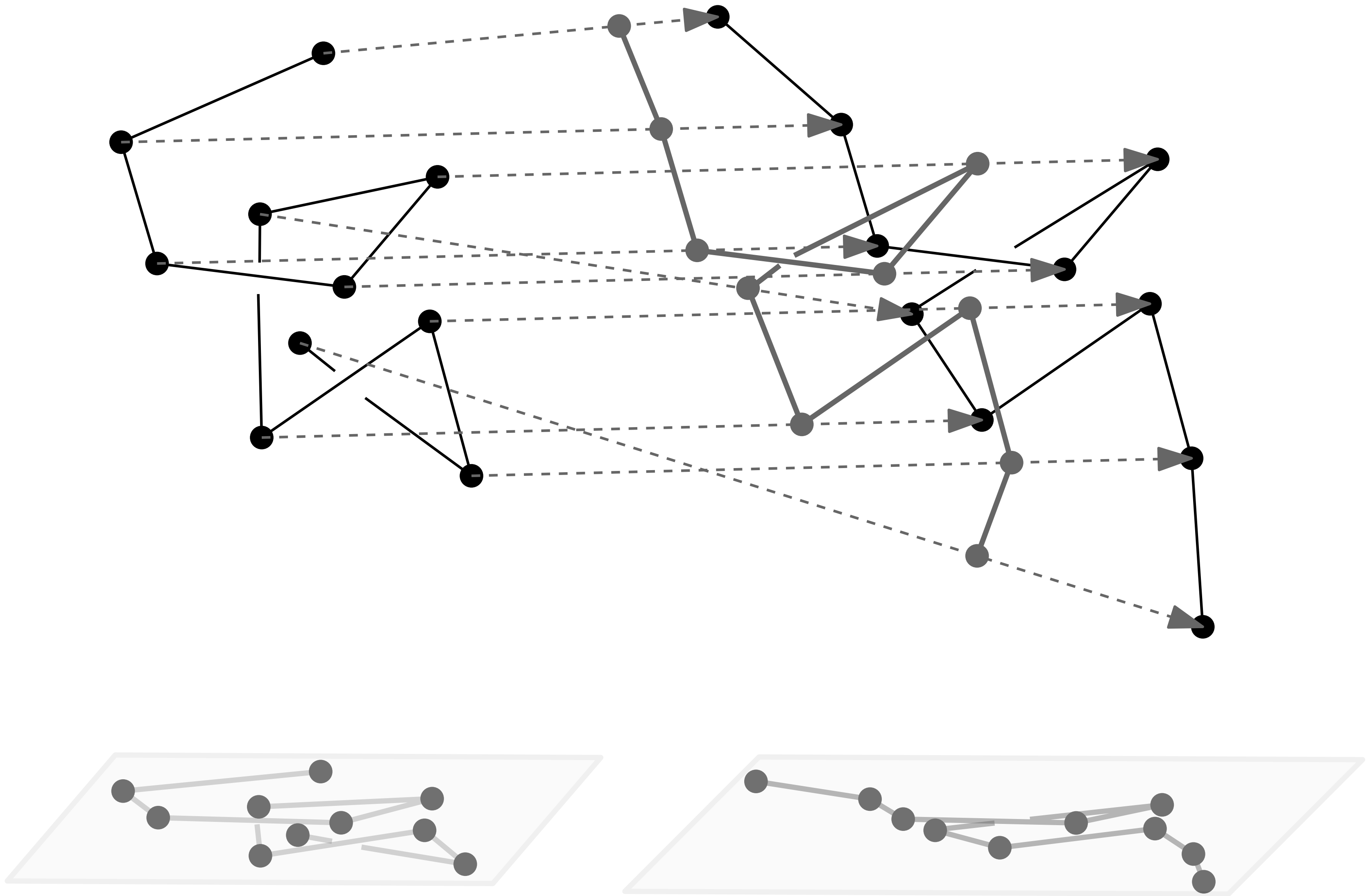


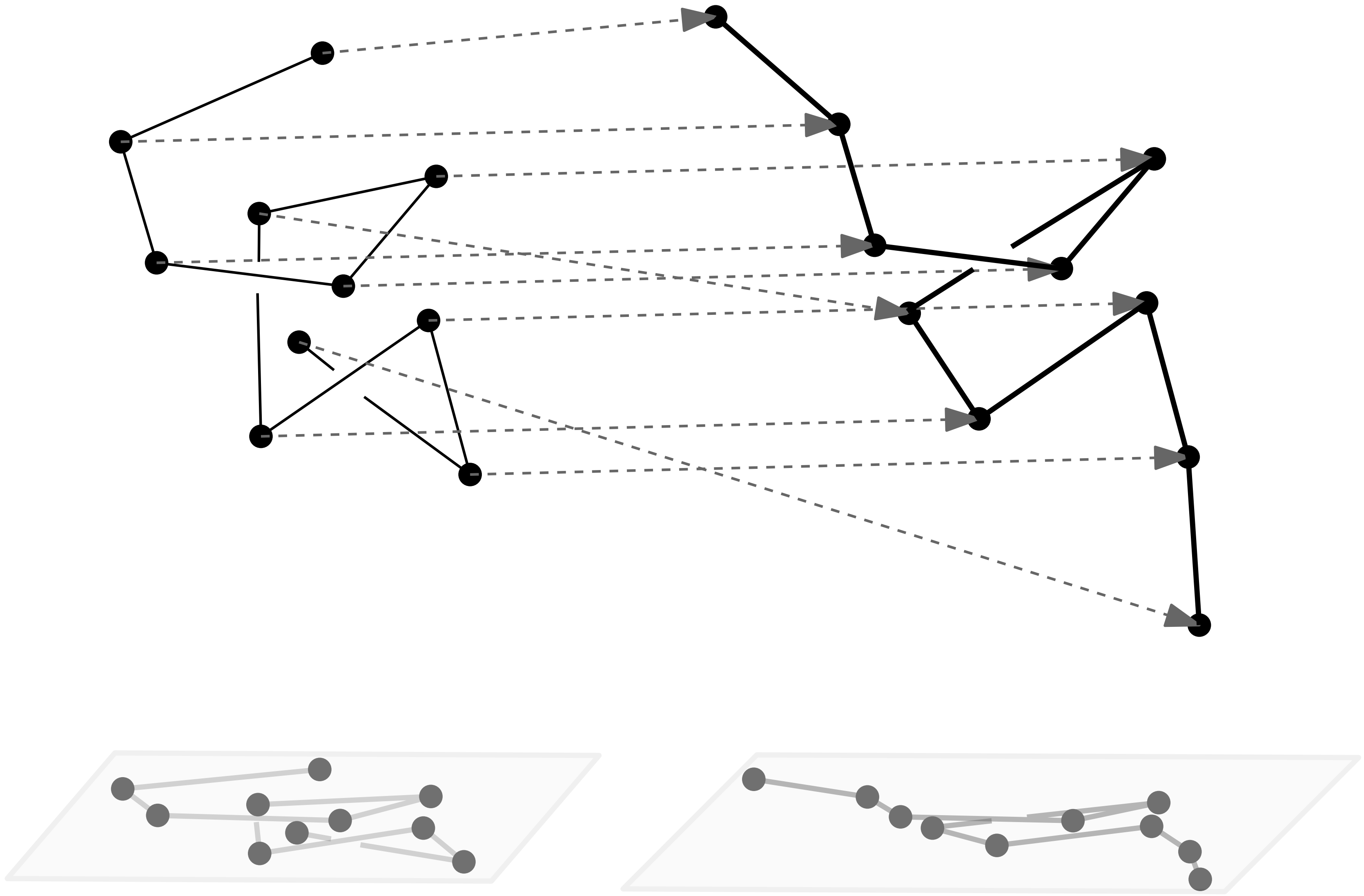
3D crossing-free morph



3D crossing-free morph



3D crossing-free morph



What about trees?

Our results

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- Any two crossing-free straight-line 3D drawings of an n-vertex tree can be morphed into each other in $O(n)$ steps.

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- Sometimes $\Theta(n)$ steps are necessary.

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- Any two crossing-free straight-line 3D drawings of an n-vertex tree can be morphed into each other in $O(n)$ steps.

- Sometimes $\Theta(n)$ steps are necessary.

- For any two planar straight-line drawings of the same n-vertex tree, there is a crossing-free 3D morph between them of $O(\log n)$ steps.

Morphs of 3D drawings of trees

Morphs of 3D drawings of trees

Can we find a 3D non-crossing morph for any two 3D drawings of the same tree?

Morphs of 3D drawings of trees

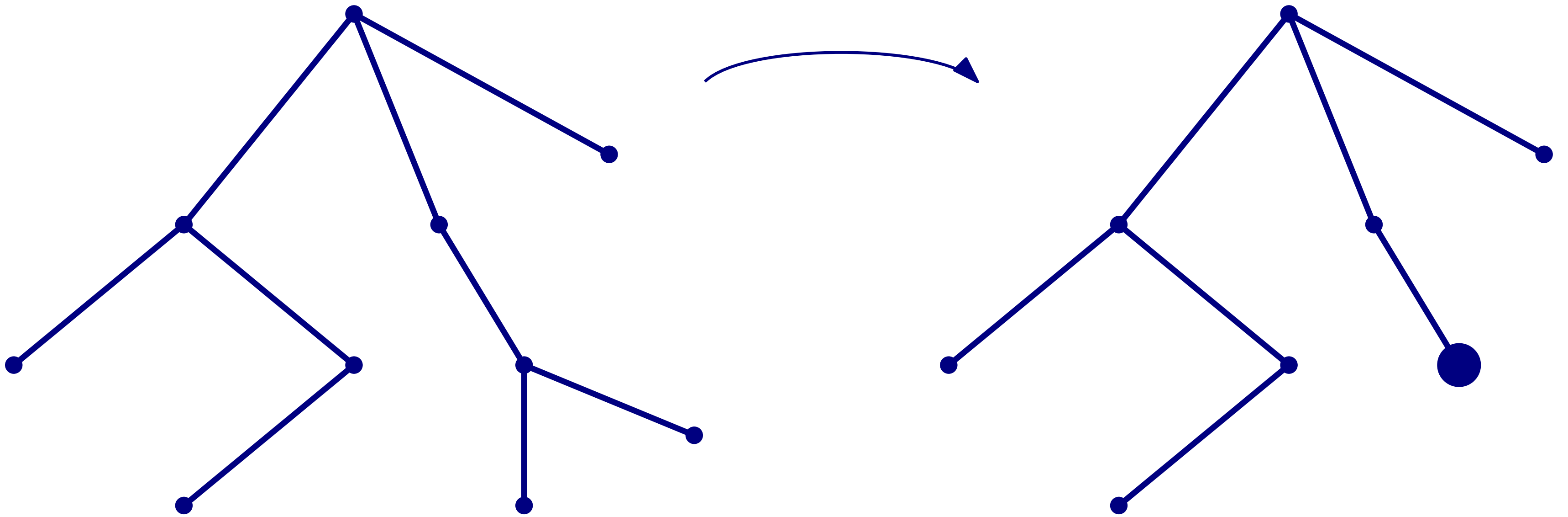
Can we find a 3D non-crossing morph for any two 3D drawings of the same tree?

yes

… in $O(n)$ steps

Morphs of 3D drawings of trees

Theorem. *For any two crossing-free straight-line 3D drawings of an n-vertex tree, there exists a crossing-free 3D morph between them of $O(n)$ steps.*



Main idea:
contract edges one by one

Morphs of 3D drawings of trees

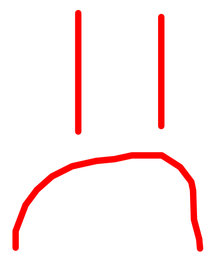
Theorem. *For any two crossing-free straight-line 3D drawings of an n-vertex tree, there exists a crossing-free 3D morph between them of $O(n)$ steps.*

Can we do better?

Morphs of 3D drawings of trees

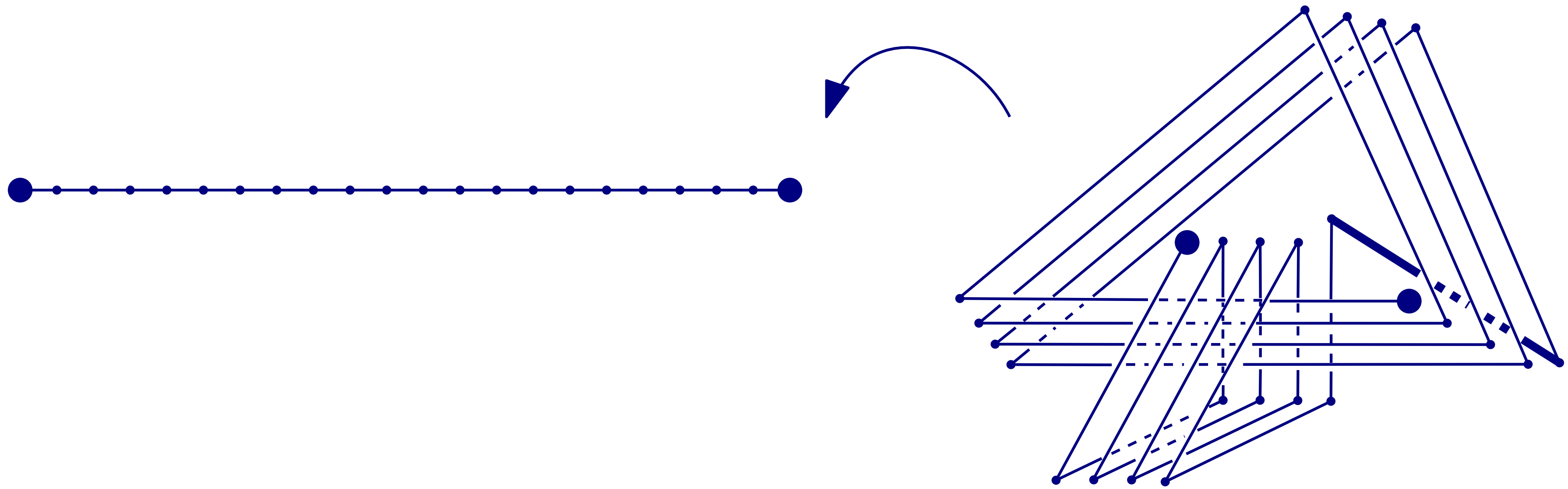
Theorem. *For any two crossing-free straight-line 3D drawings of an n-vertex tree, there exists a crossing-free 3D morph between them of $O(n)$ steps.*

Can we do better?

no 

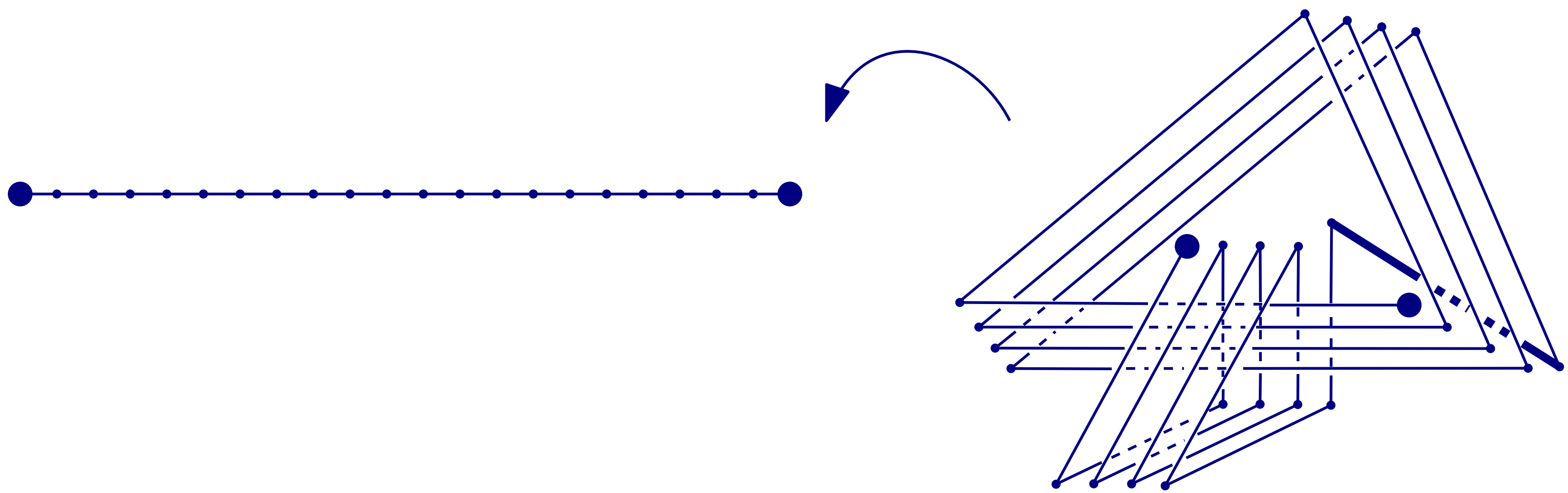
Morphs of 3D drawings of trees

Theorem. *There exist two crossing-free straight-line 3D drawings* Γ, Γ' *of an* n*-vertex path* P *such that any crossing-free 3D morph from* Γ *to* Γ' *consists of* $\Omega(n)$ *steps.*



Morphs of 3D drawings of trees

Theorem. *There exist two crossing-free straight-line 3D drawings Γ, Γ' of an n-vertex path P such that any crossing-free 3D morph from Γ to Γ' consists of $\Omega(n)$ steps.*



Main idea:
look at the LINKING NUMBER
of each drawing

Morphing two planar drawings of a path in 3D

Morphing two planar drawings of a path in 3D

Theorem. *For any two planar straight-line drawings* Γ *and* Γ' *of an* n*-vertex path* P*, there exists a crossing-free 3D morph with 2 steps.*

Morphing two planar drawings of a path in 3D

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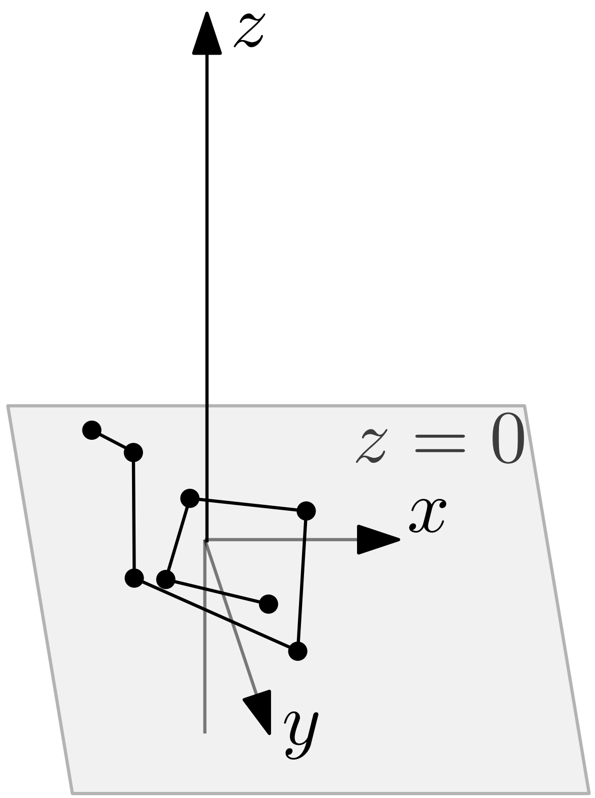
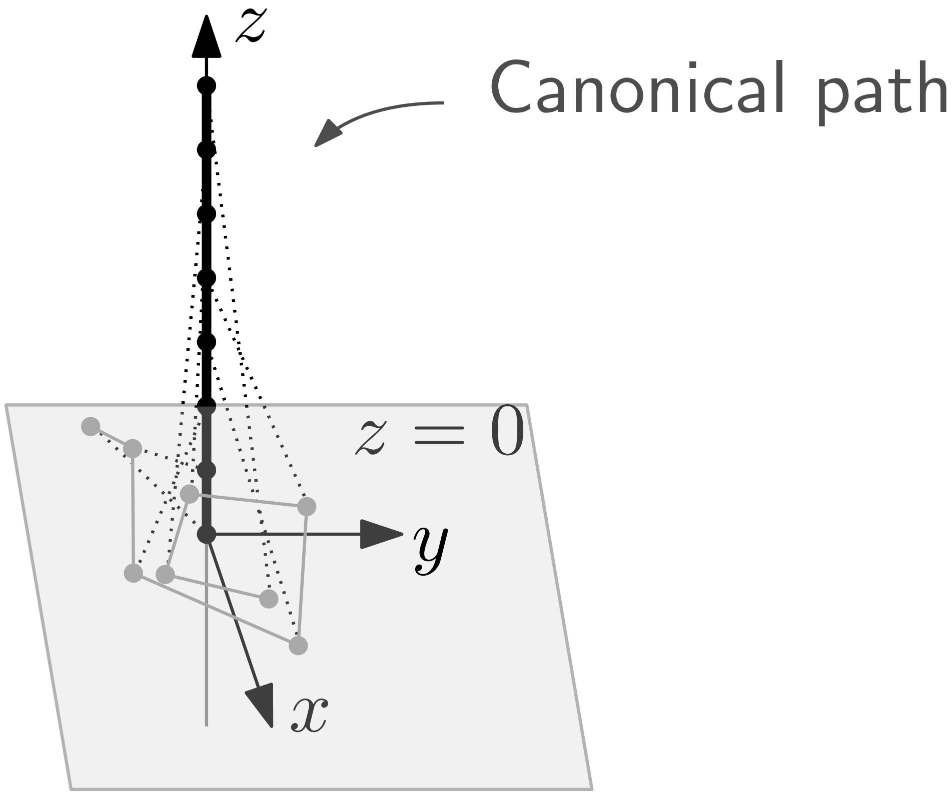


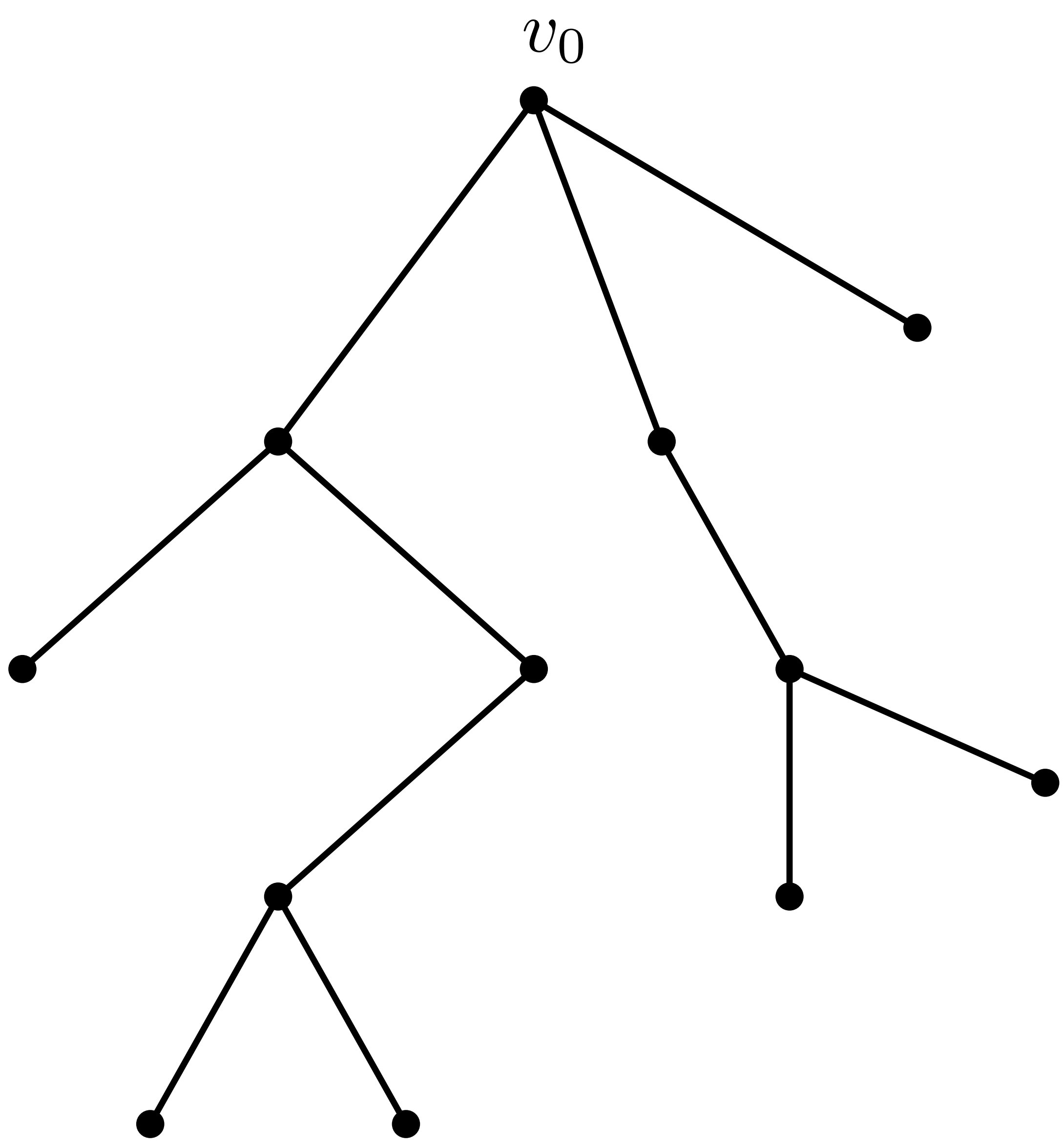
Morphing two planar drawings of a path in 3D

Theorem. *For any two planar straight-line drawings Γ and Γ' of an n-vertex path P, there exists a crossing-free 3D morph with 2 steps.*

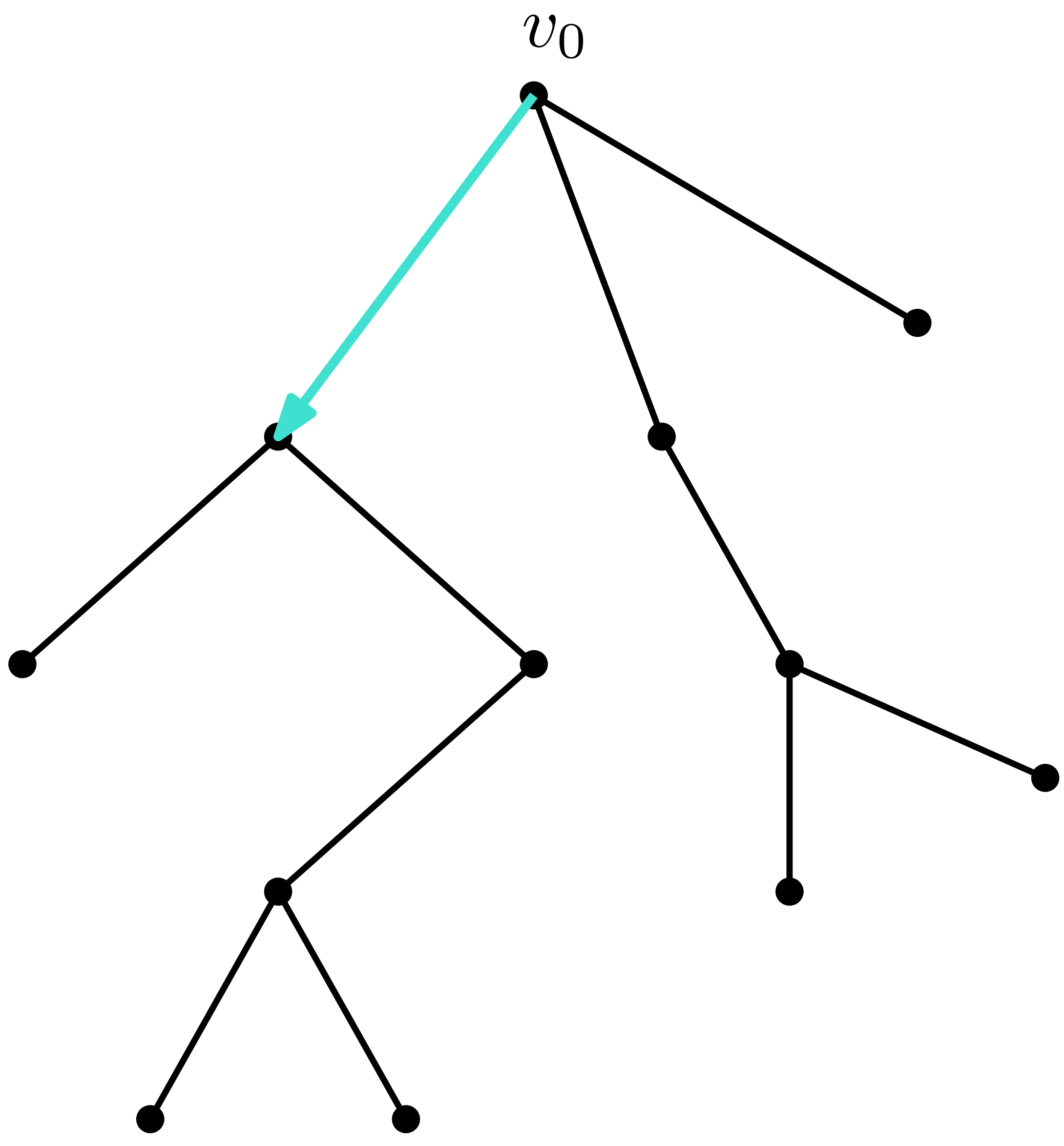


Heavy-path decomposition

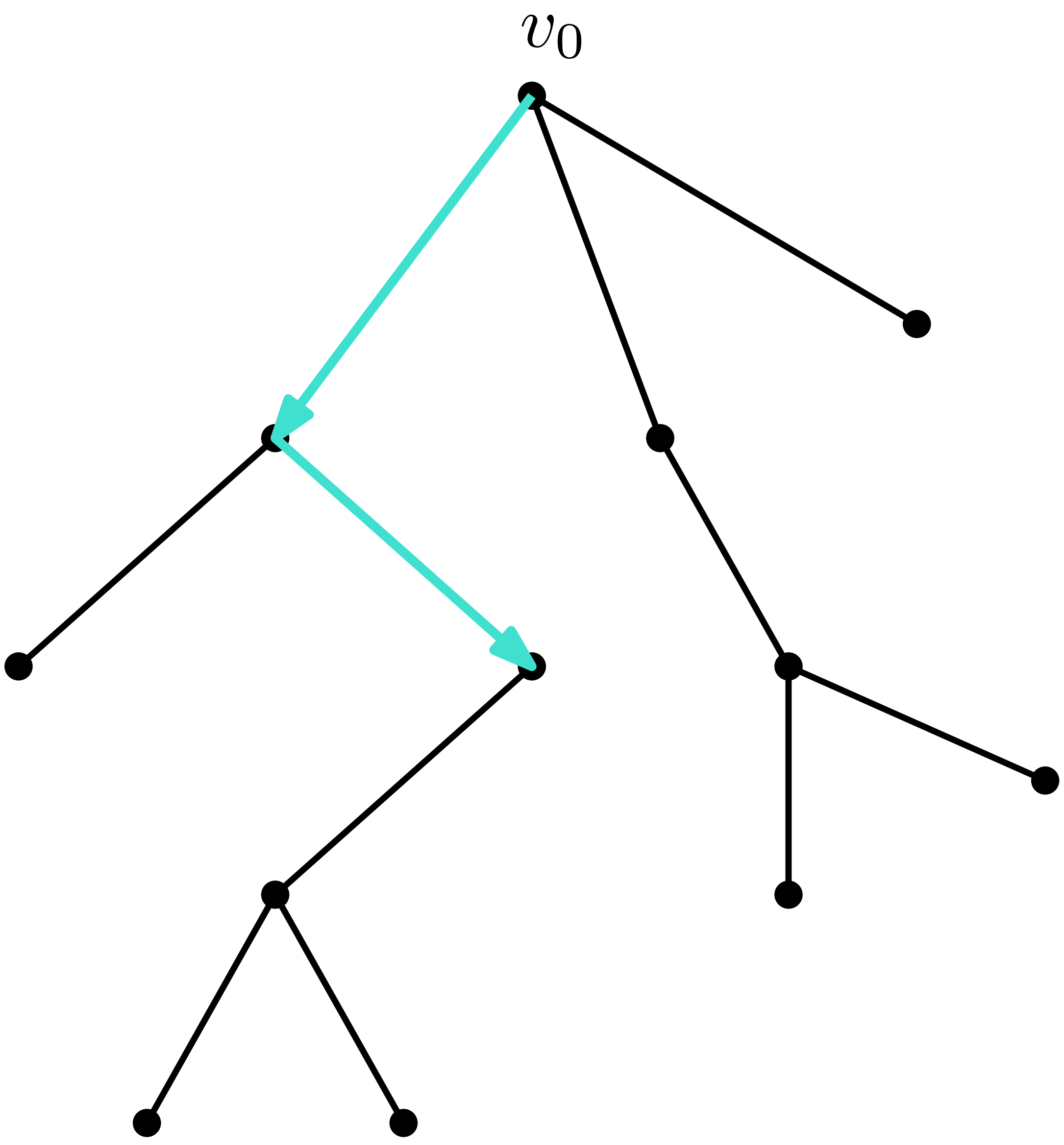
Heavy-path decomposition



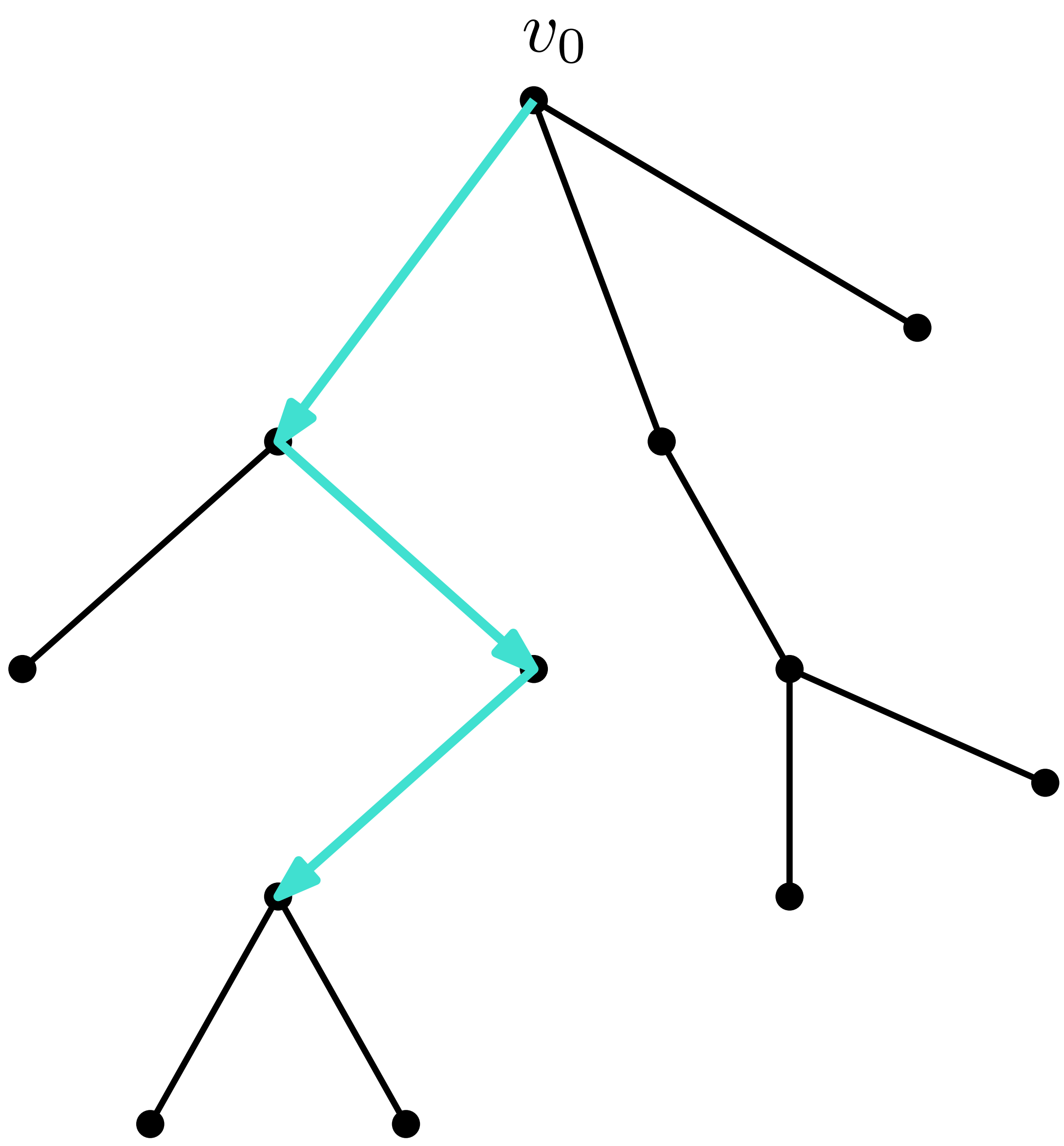
Heavy-path decomposition



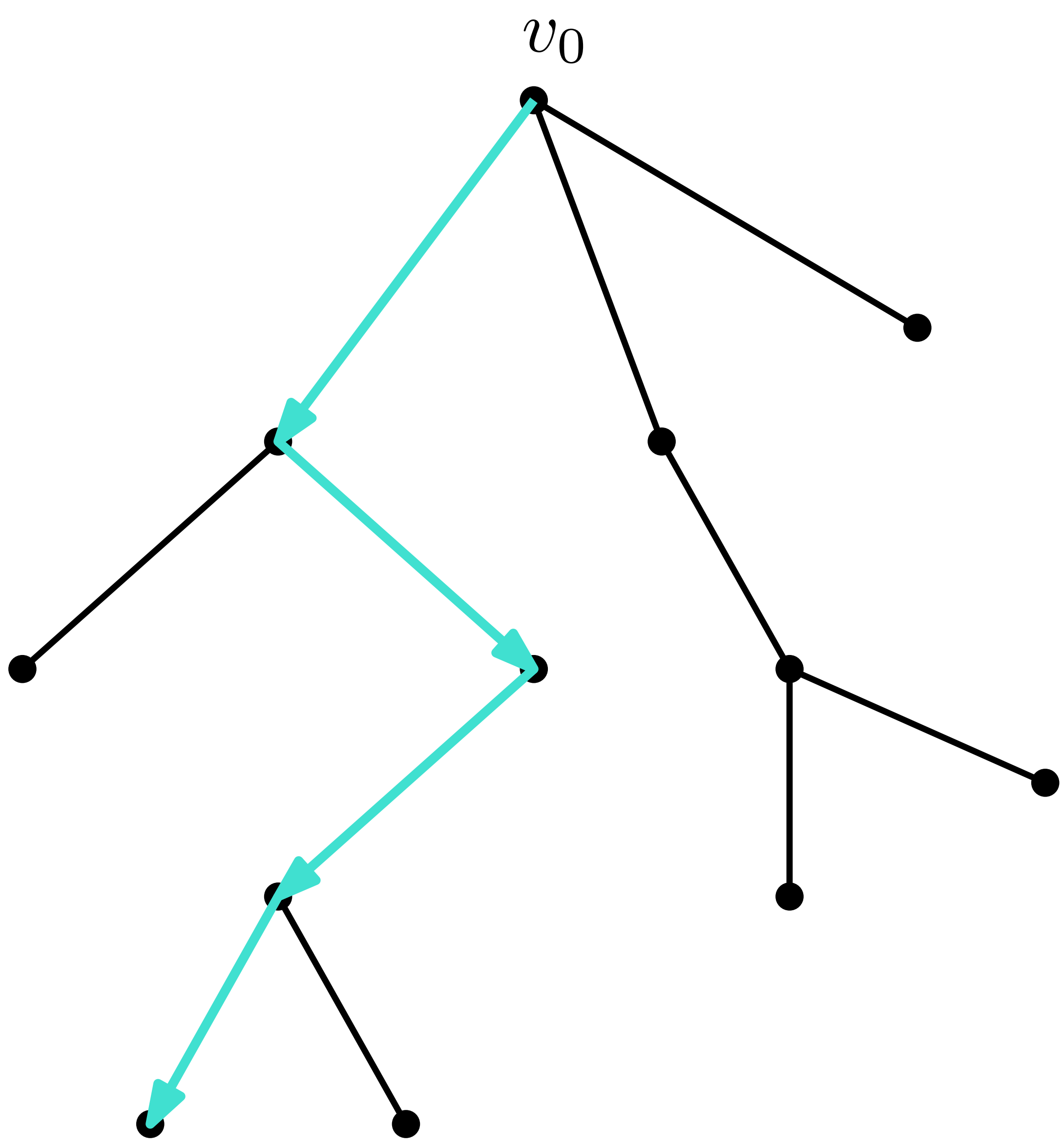
Heavy-path decomposition



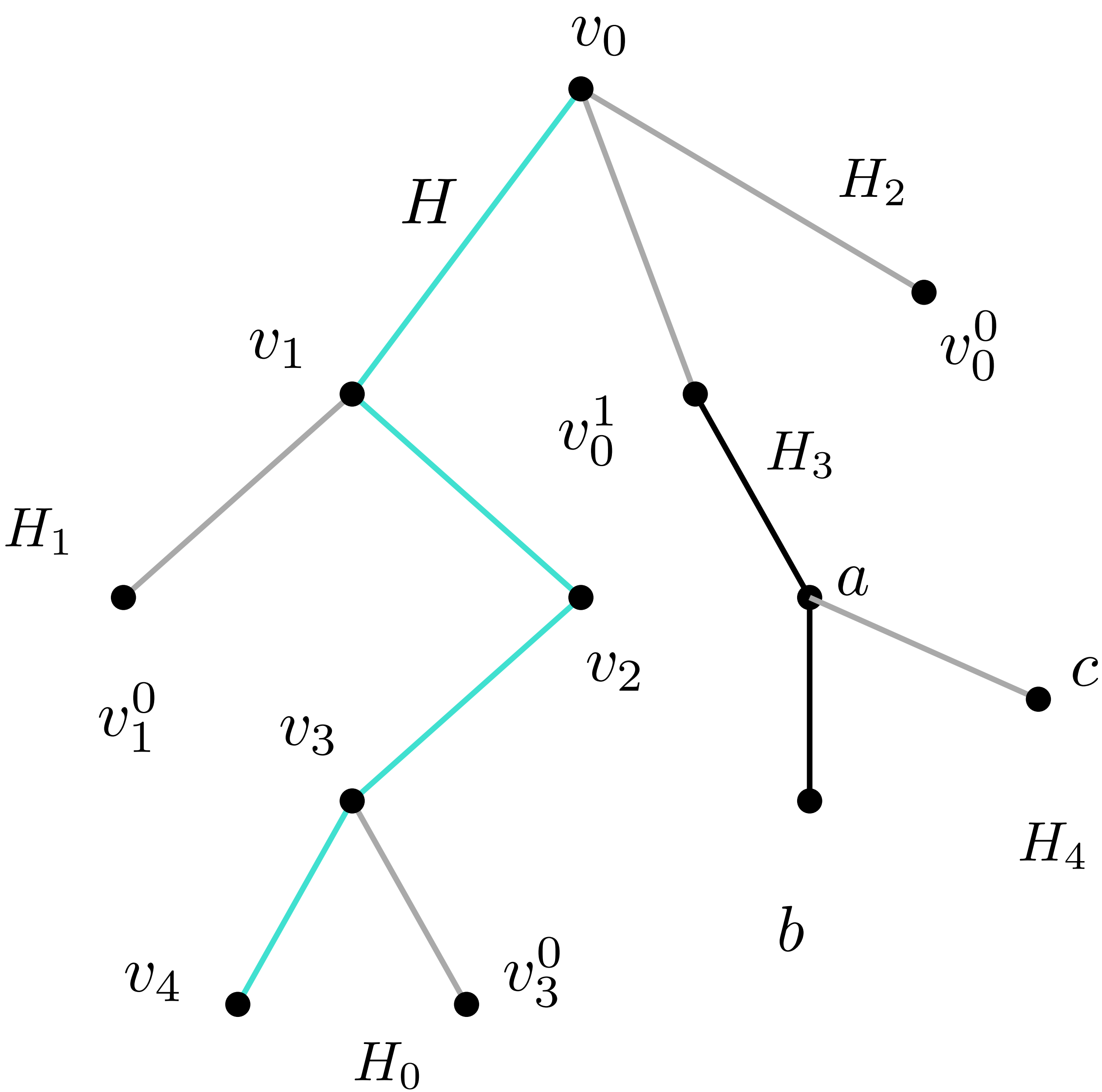
Heavy-path decomposition



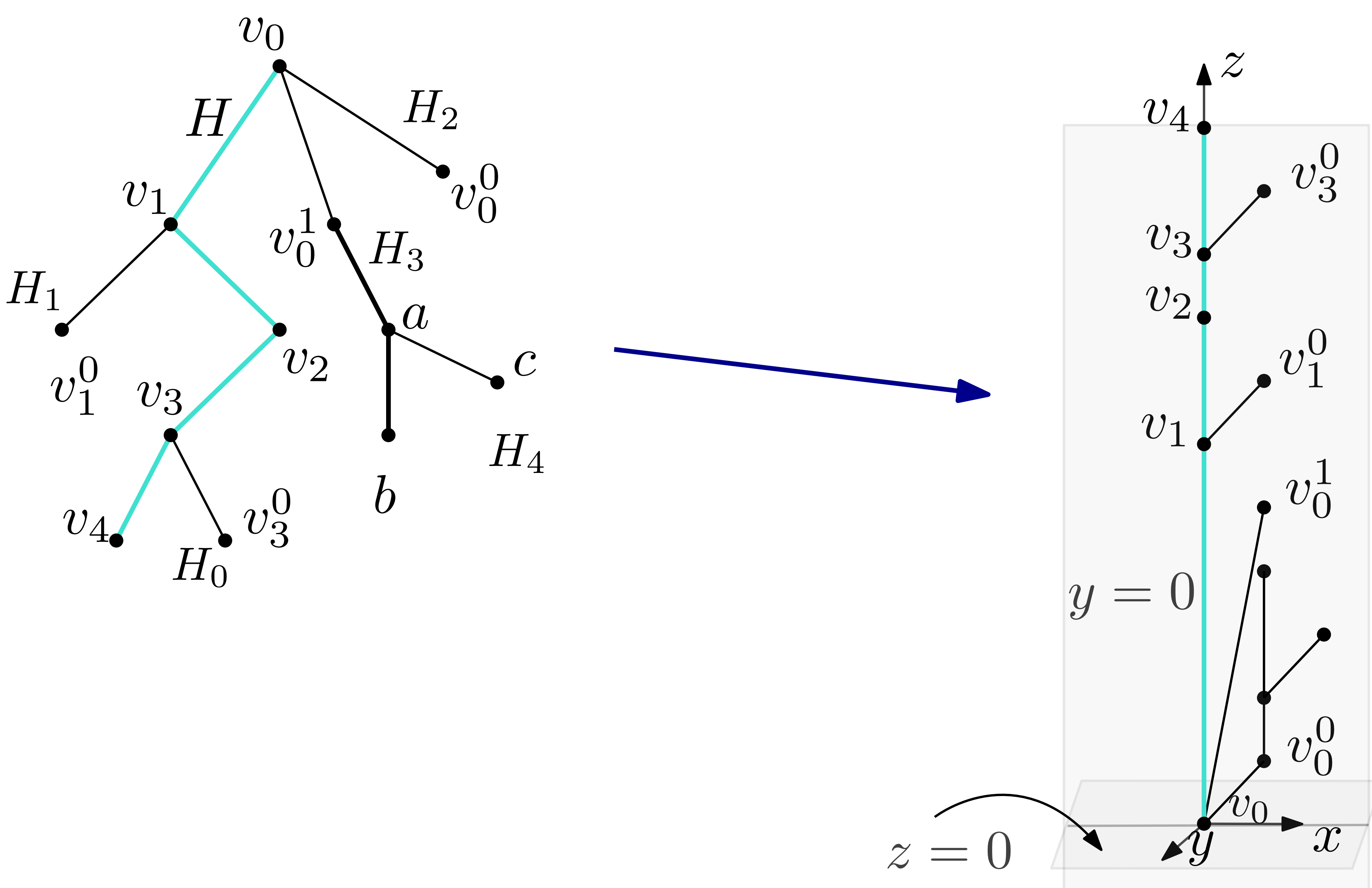
Heavy-path decomposition



Heavy-path decomposition

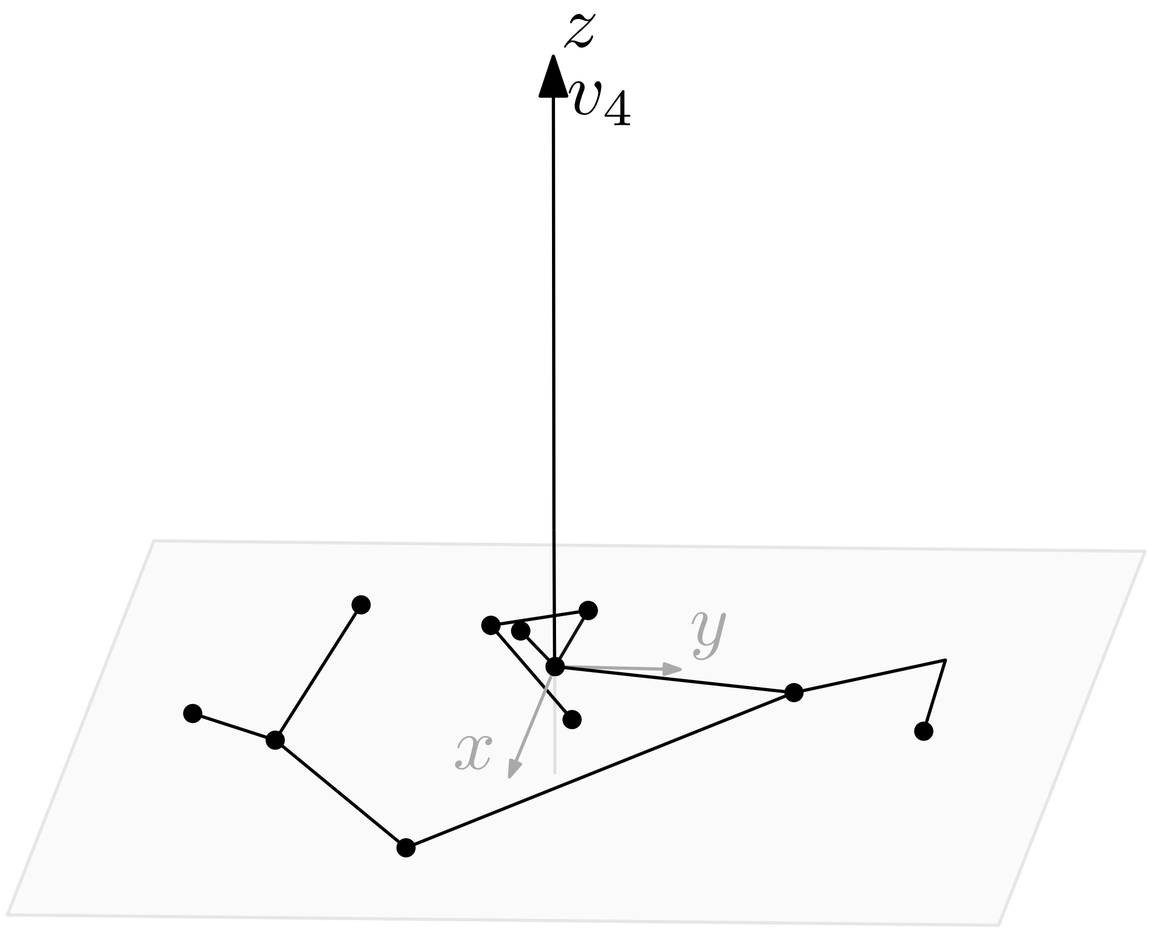


Canonical 3D drawing of a tree



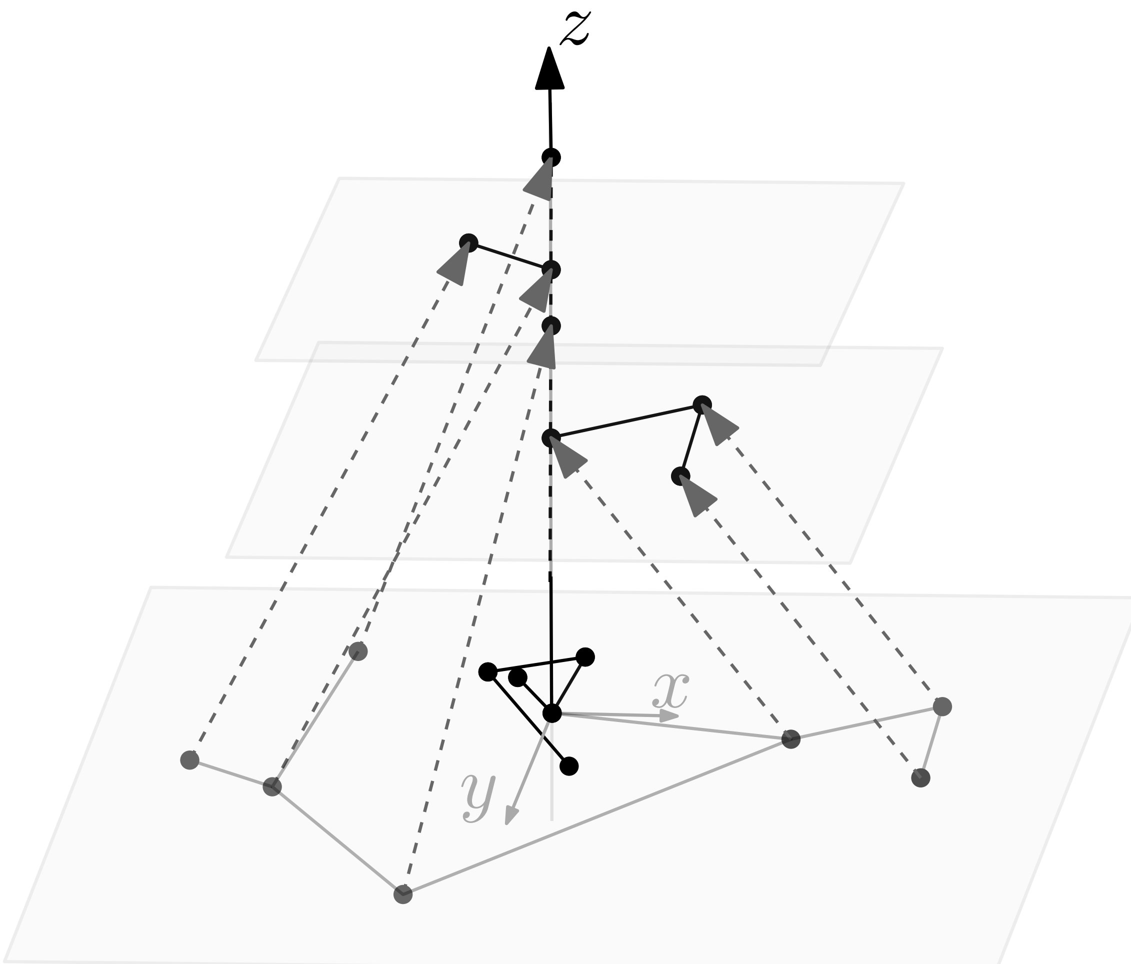
Morphing two planar drawings of a tree in 3D

Morphing two planar drawings of a tree in 3D



Morphing two planar drawings of a tree in 3D

Step 1: Set the pole



Morphing two planar drawings of a tree in 3D

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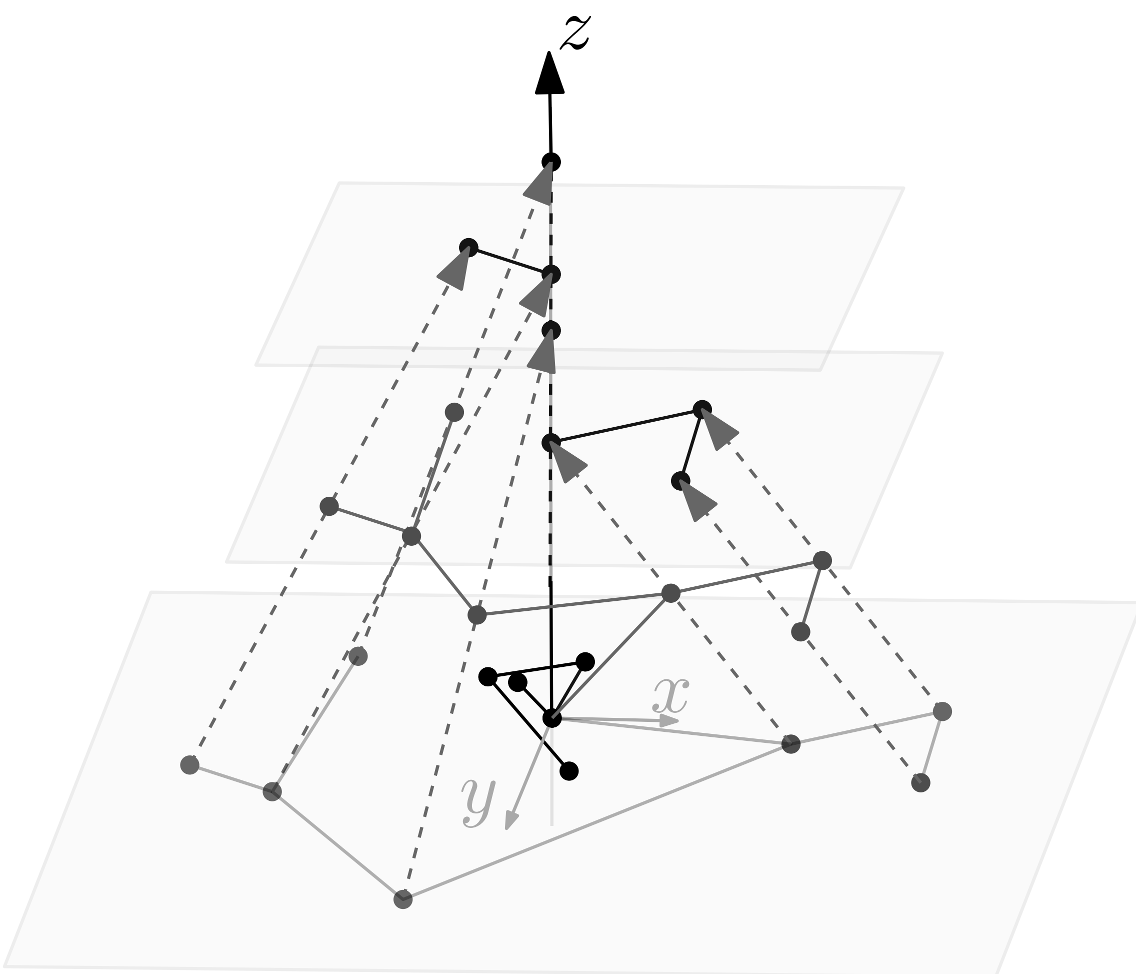

Morphing two planar drawings of a tree in 3D

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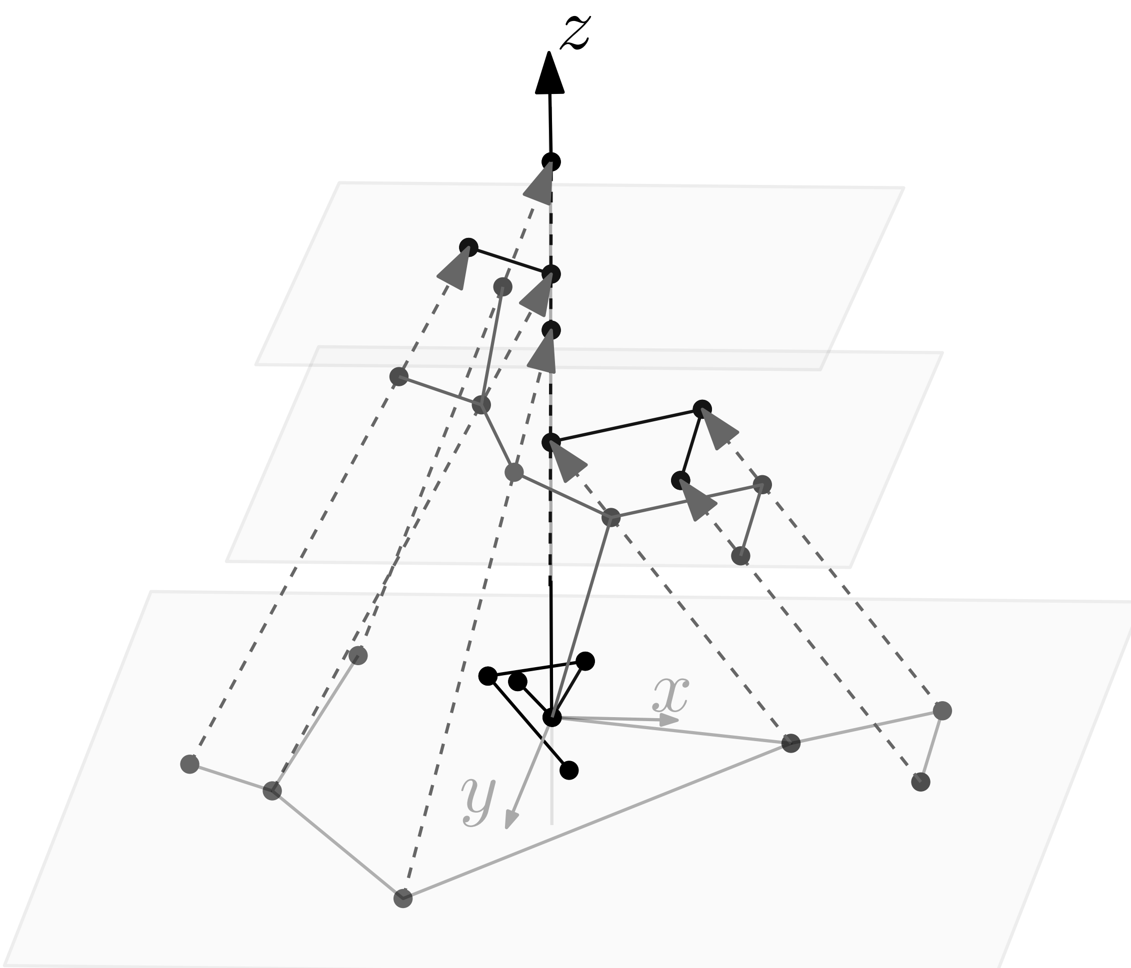

Morphing two planar drawings of a tree in 3D

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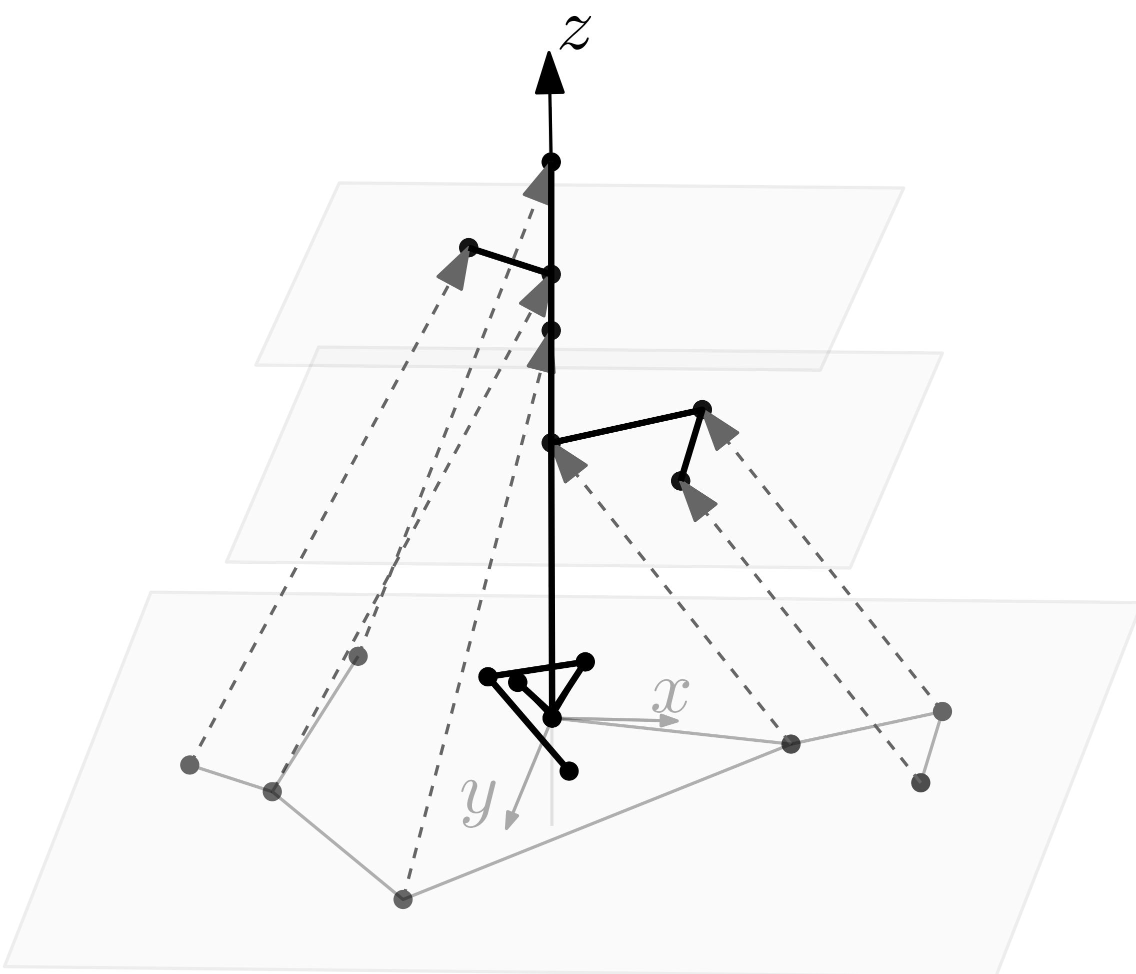
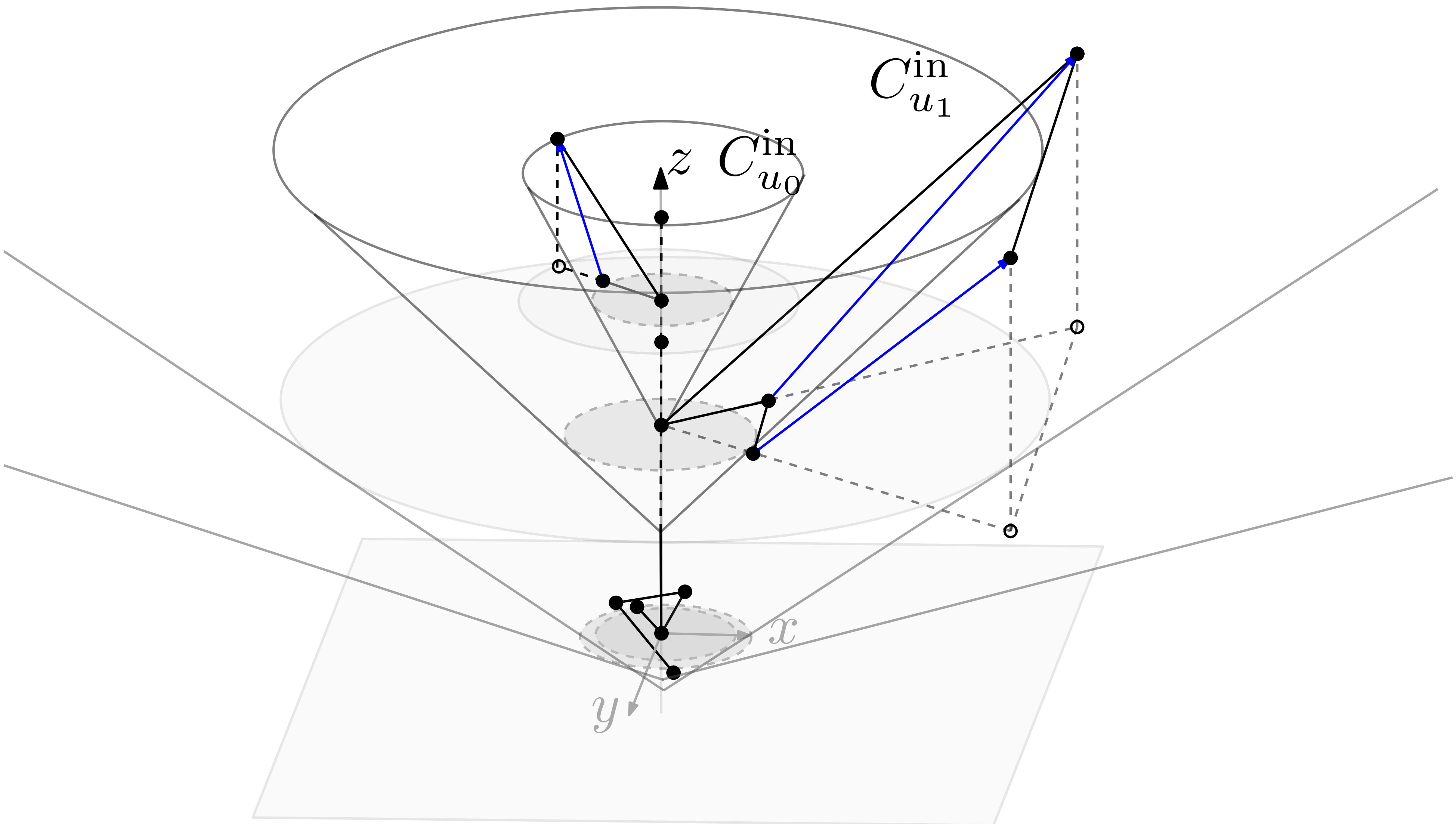


Morphing two planar drawings of a tree in 3D

Step 2: Lift the subtrees



Morphing two planar drawings of a tree in 3D

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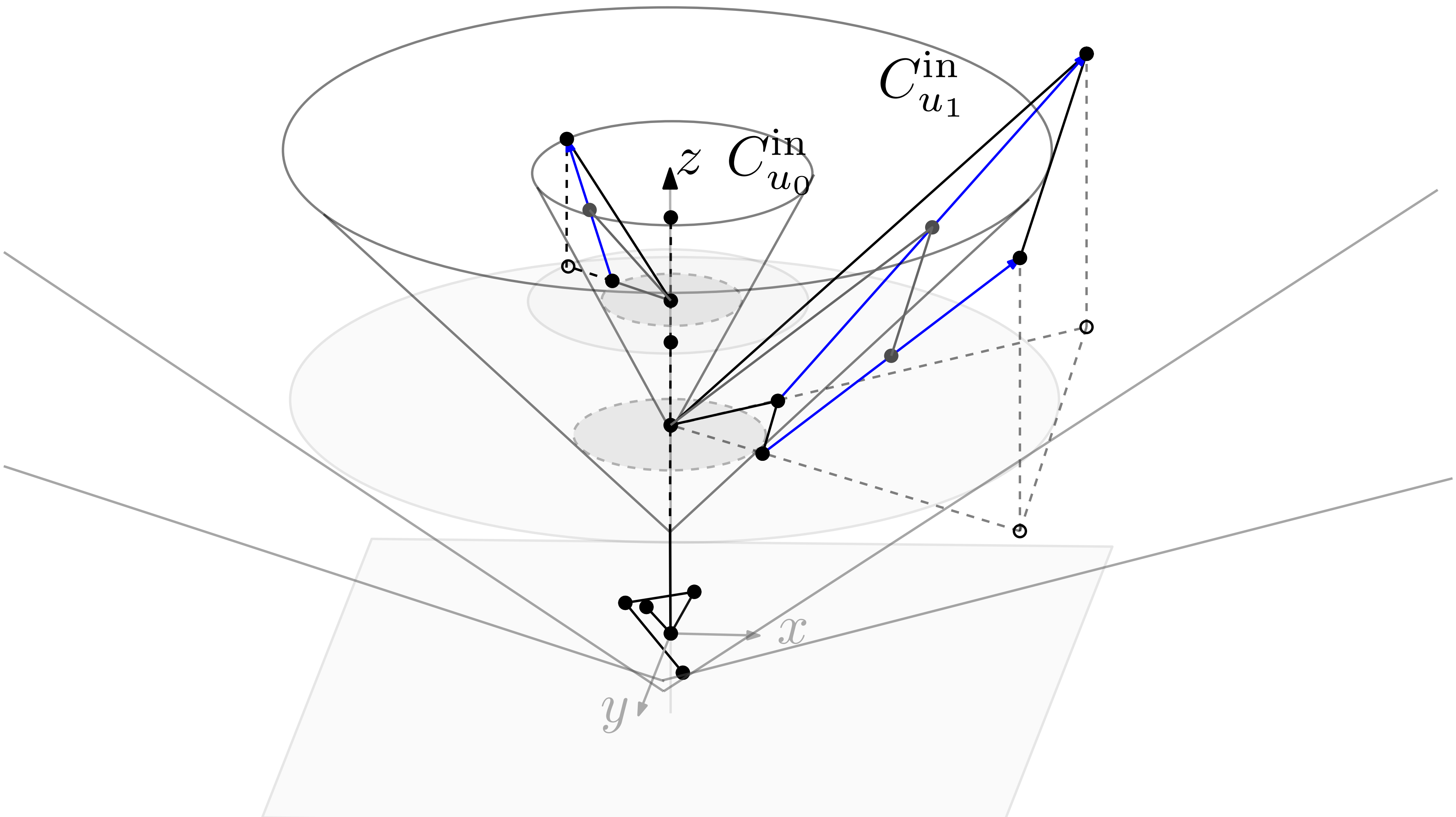

Morphing two planar drawings of a tree in 3D

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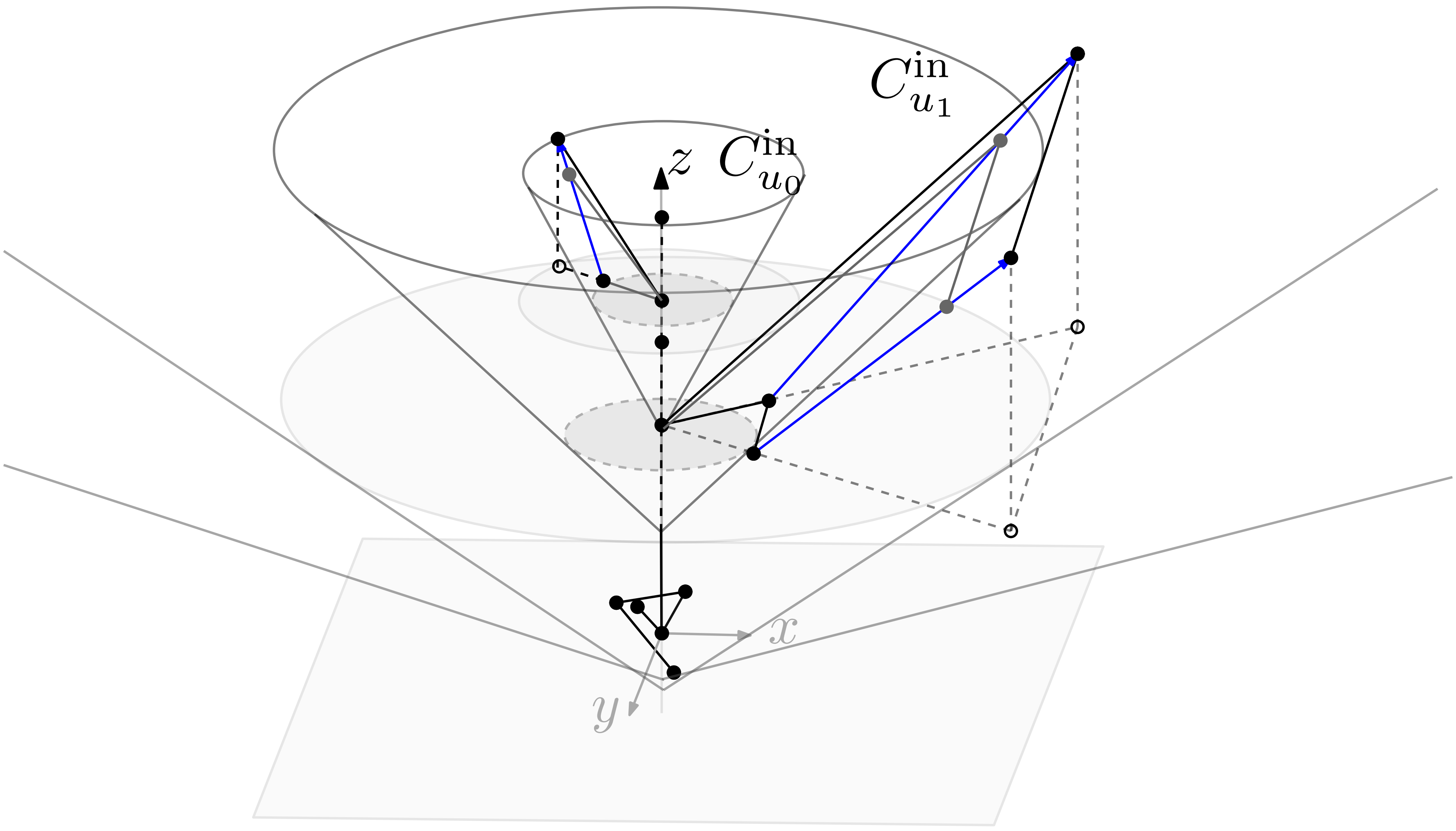

Morphing two planar drawings of a tree in 3D

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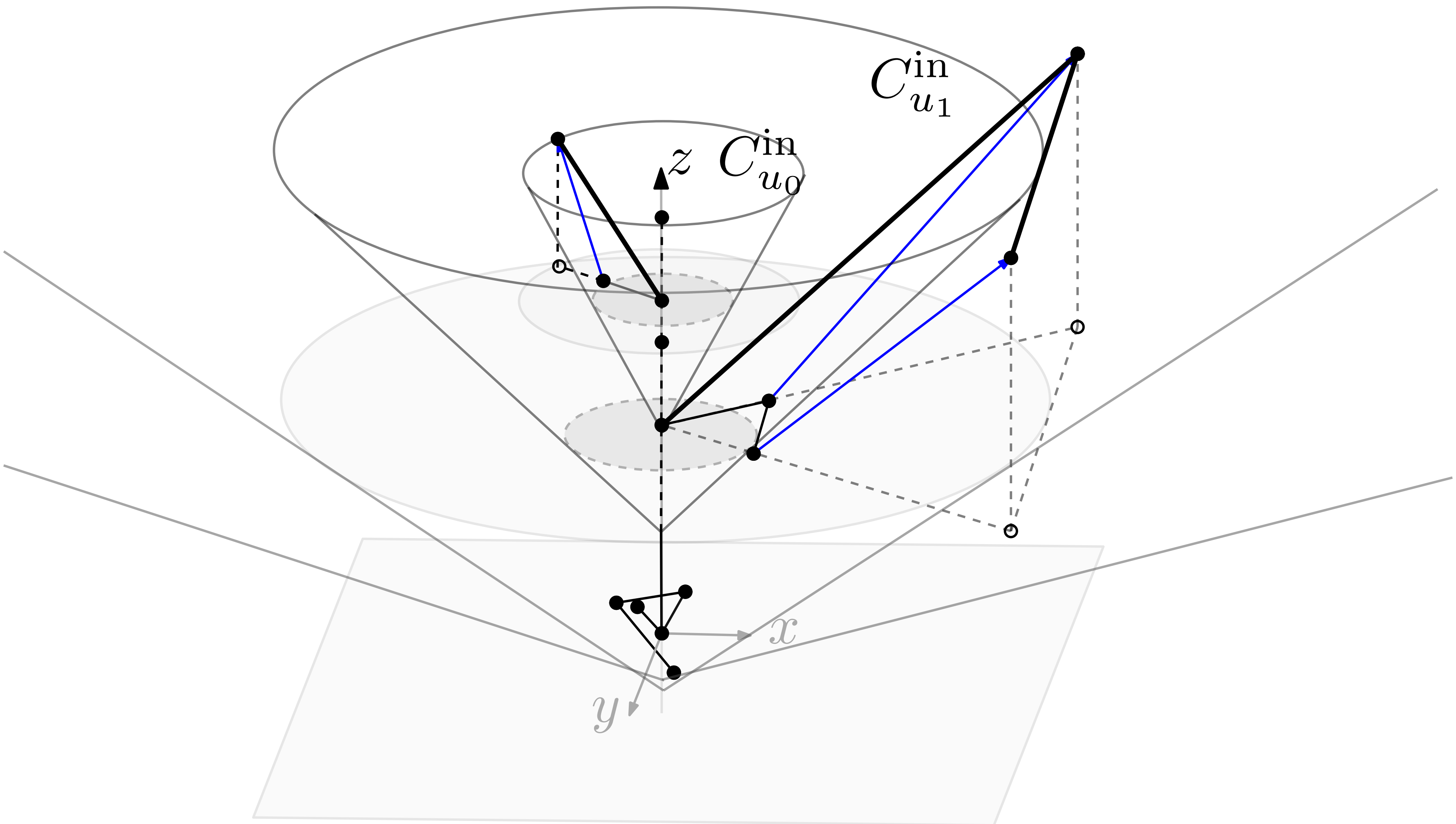
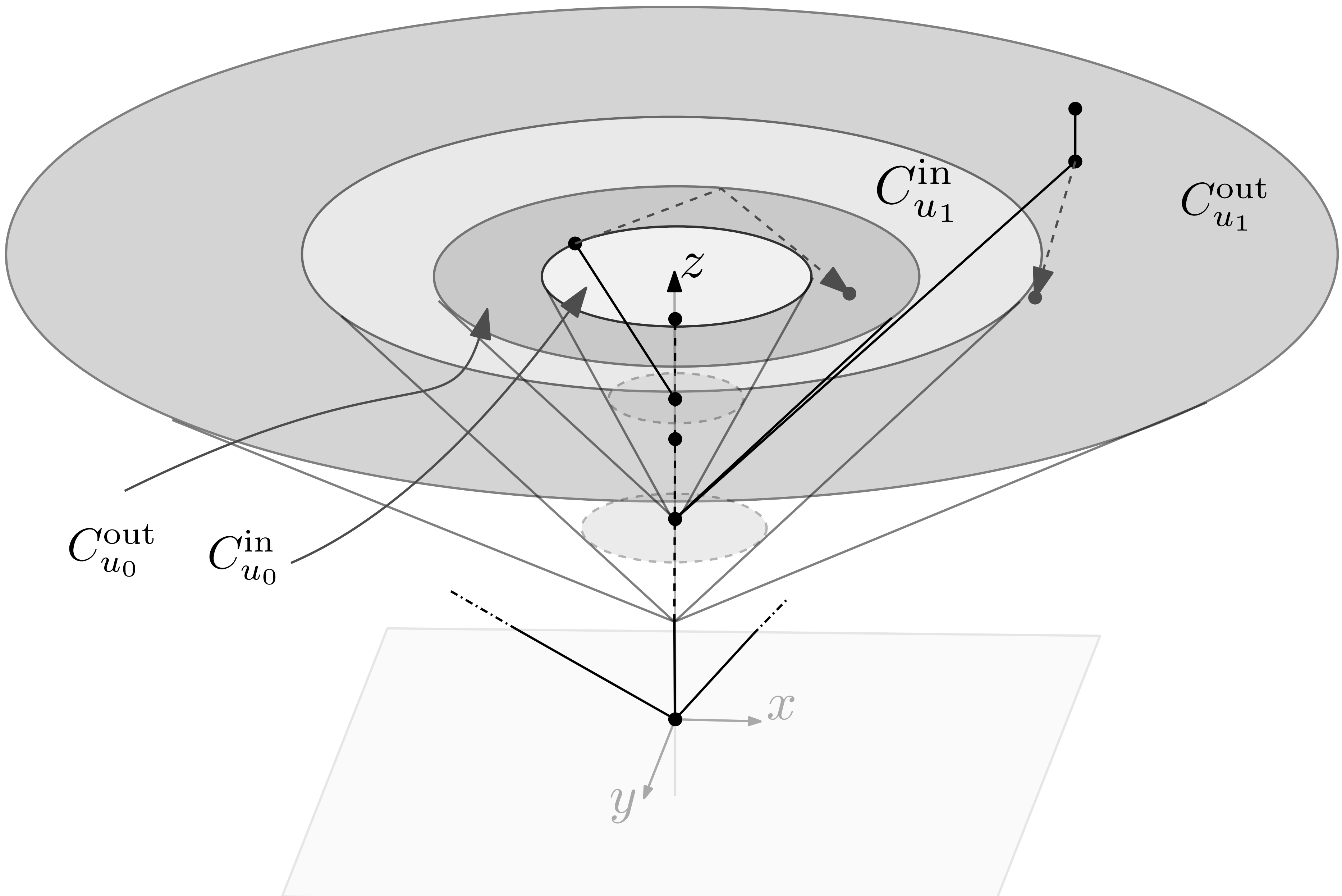


Morphing two planar drawings of a tree in 3D

Step 3: RECURSE AT EACH SUBTREE LIFTED

Morphing two planar drawings of a tree in 3D

Step 4: "Rotate" clockwise



Morphing two planar drawings of a tree in 3D

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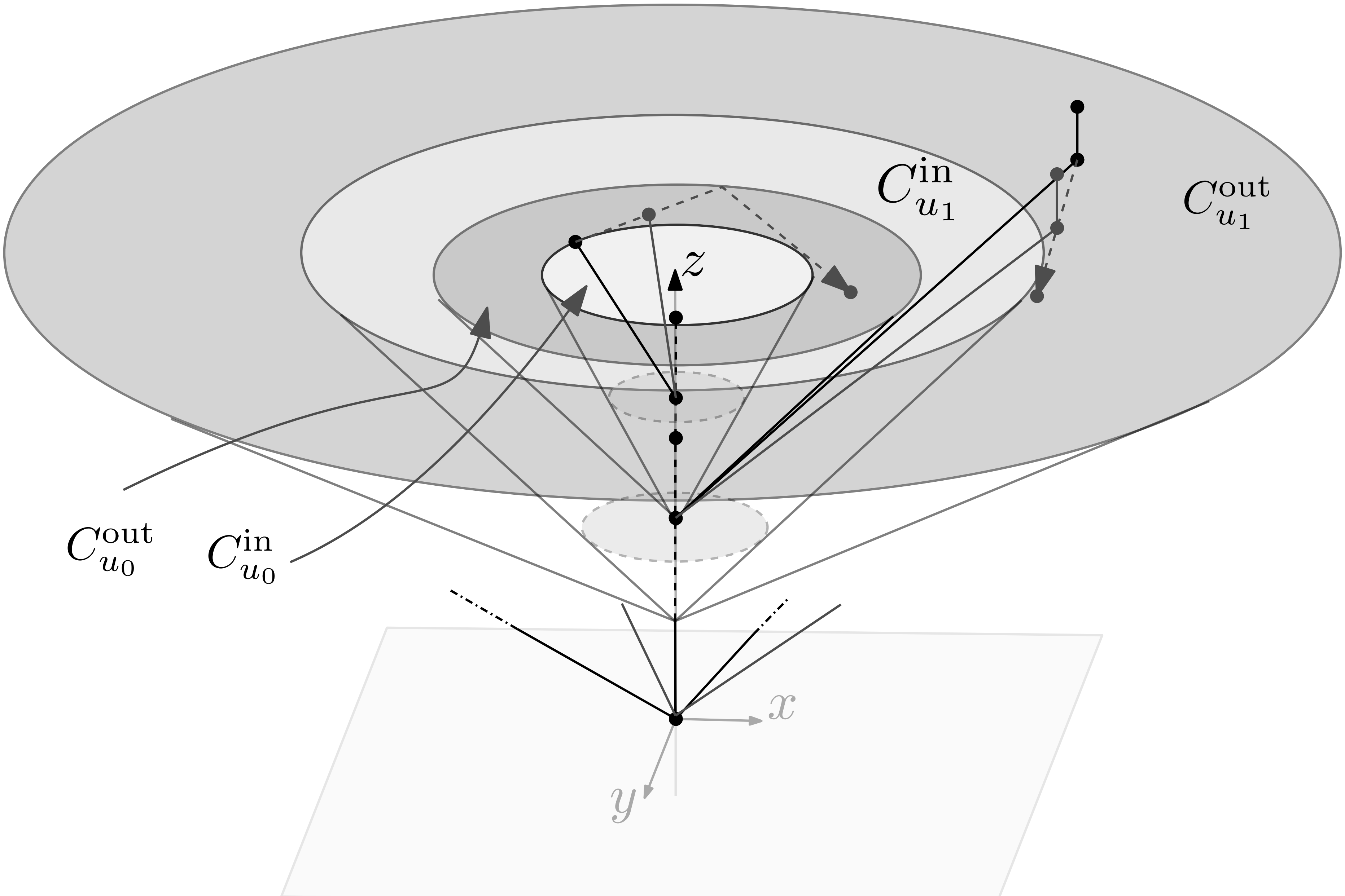

Morphing two planar drawings of a tree in 3D

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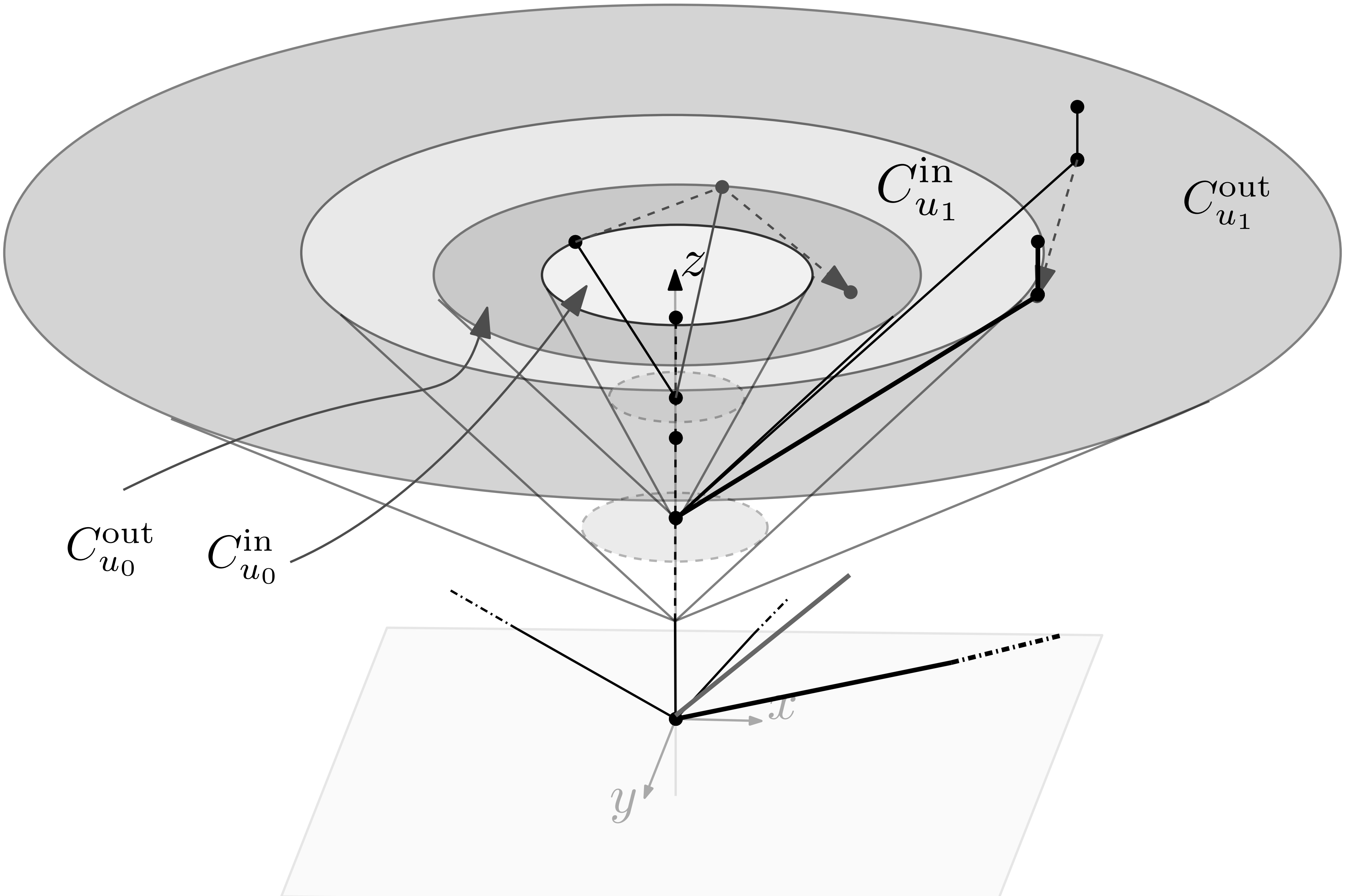
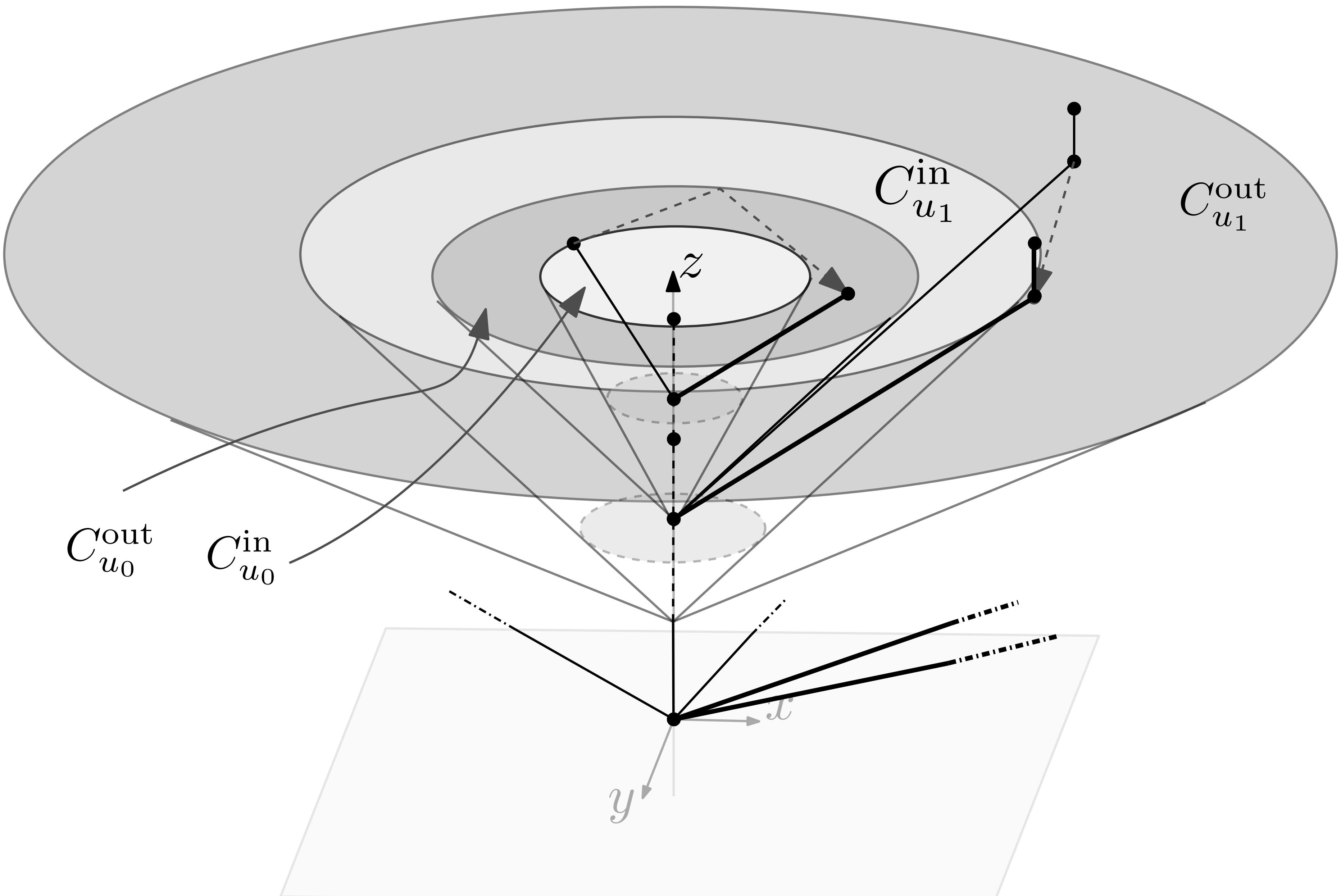


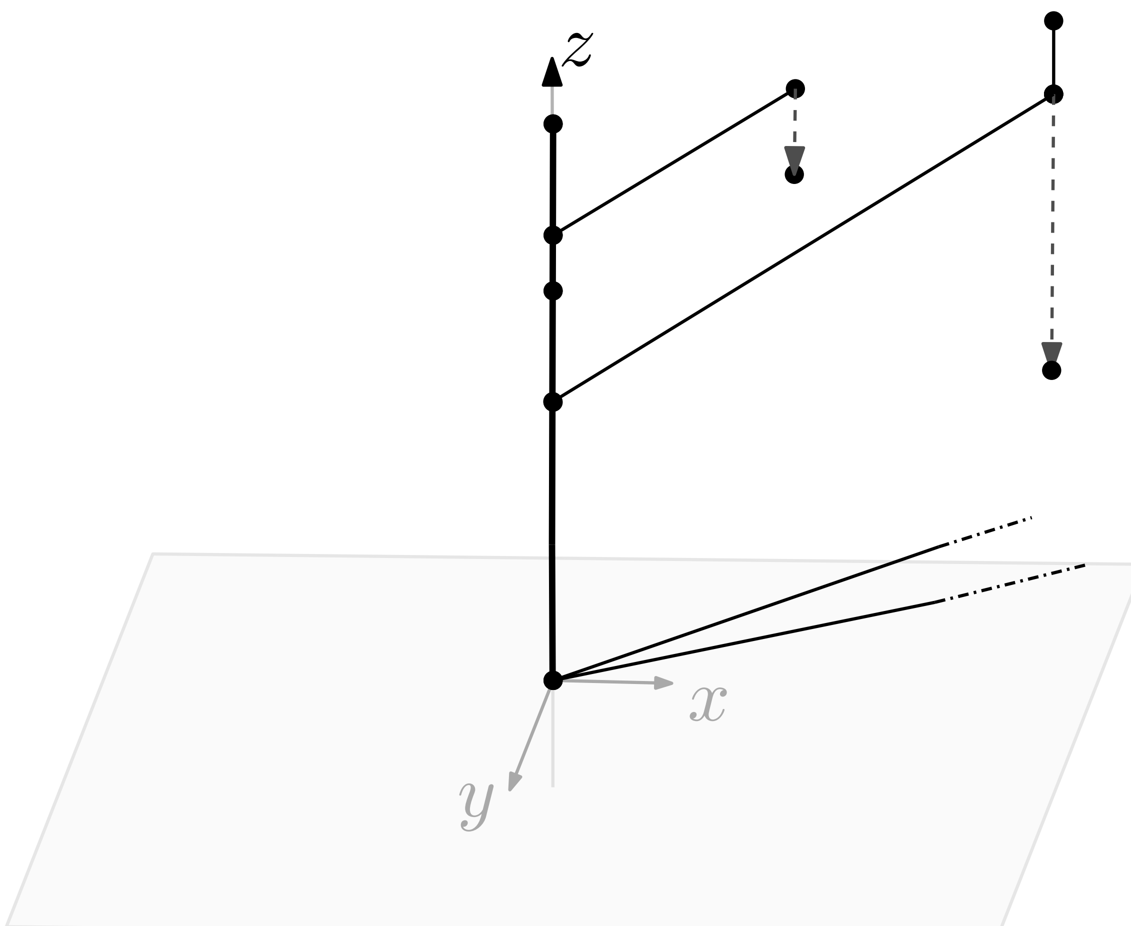
Morphing two planar drawings of a tree in 3D

Step 4: "Rotate" clockwise



Morphing two planar drawings of a tree in 3D

Step 5: Go down



Morphing two planar drawings of a tree in 3D

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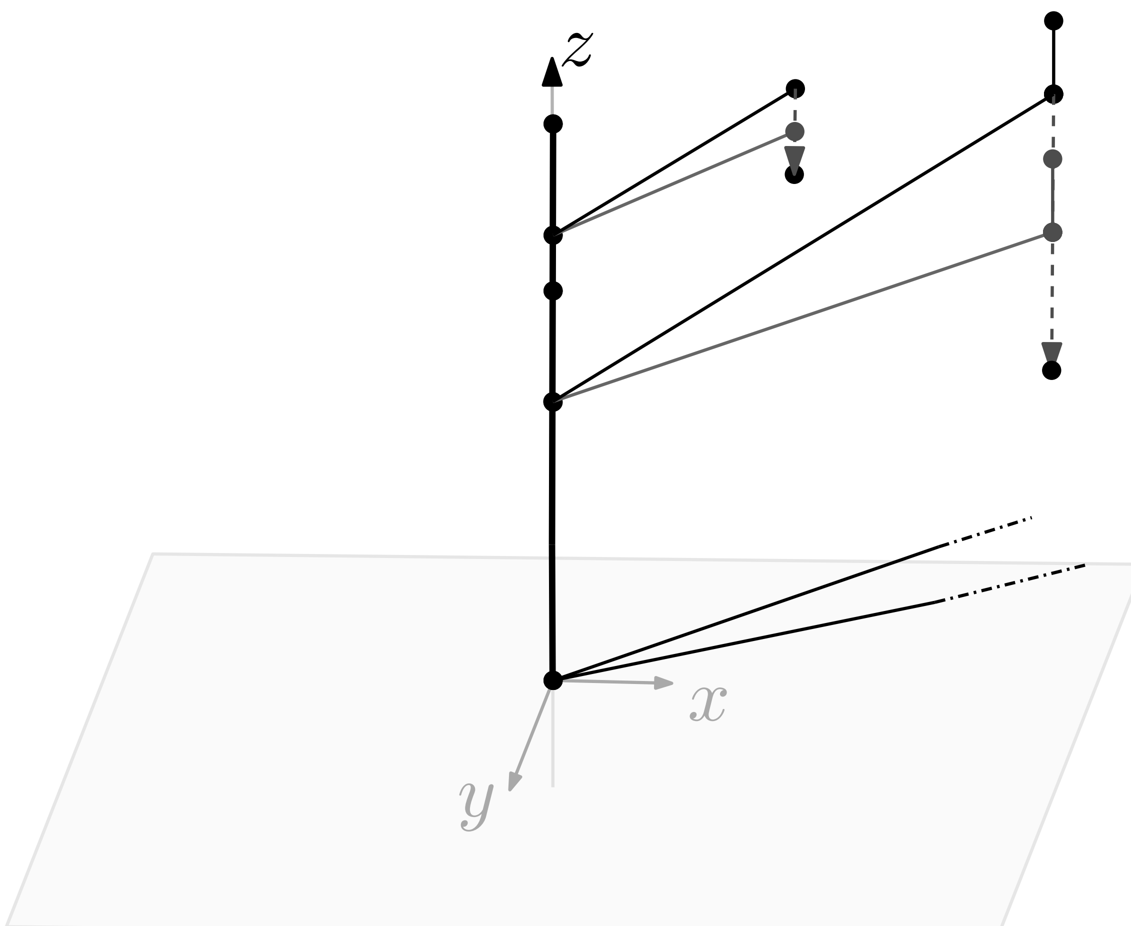

Morphing two planar drawings of a tree in 3D

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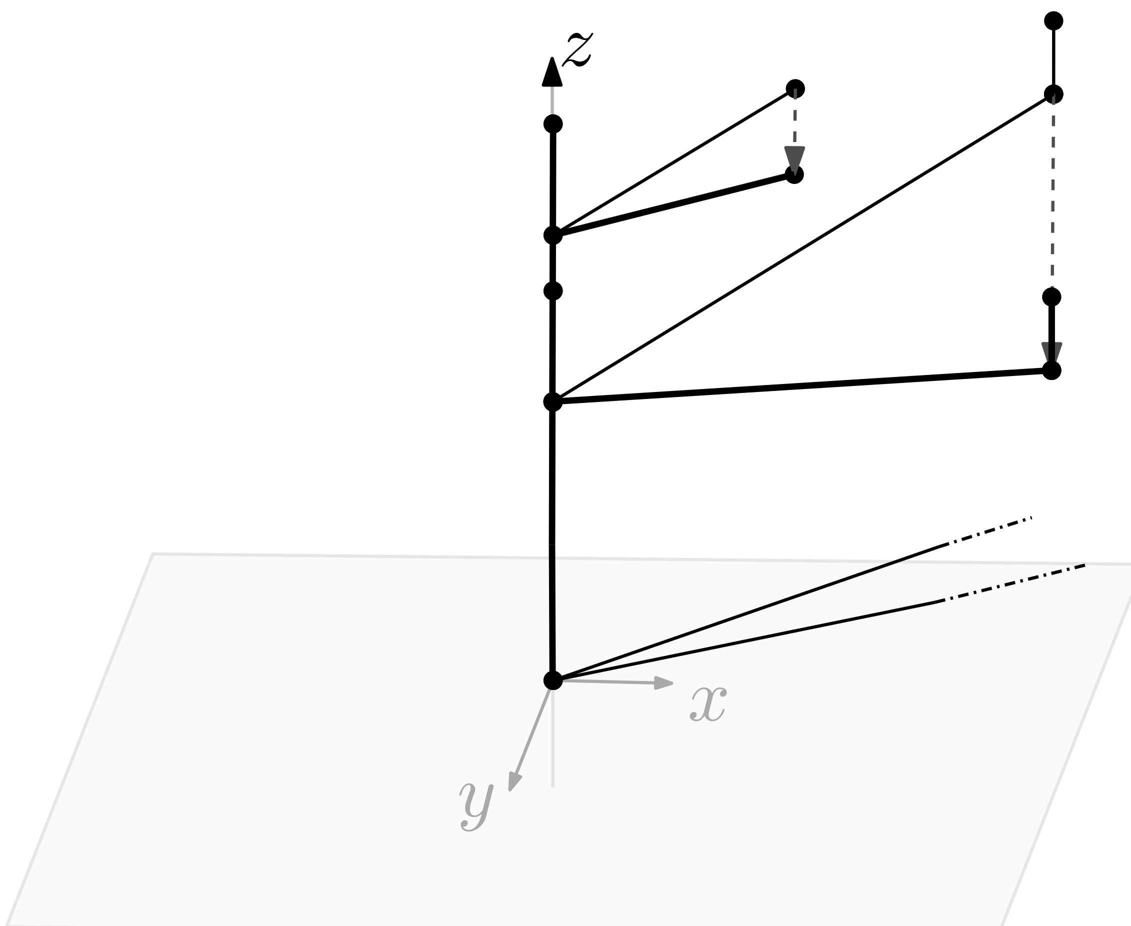
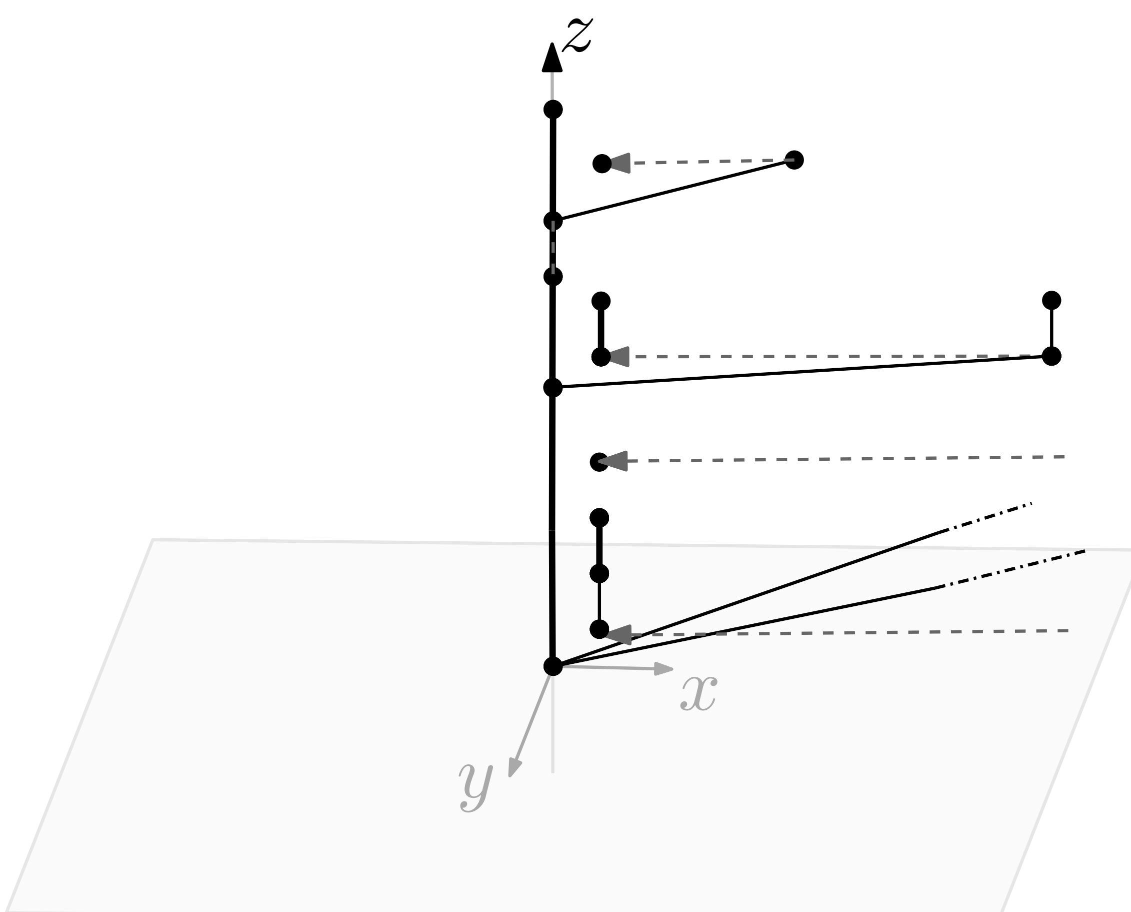


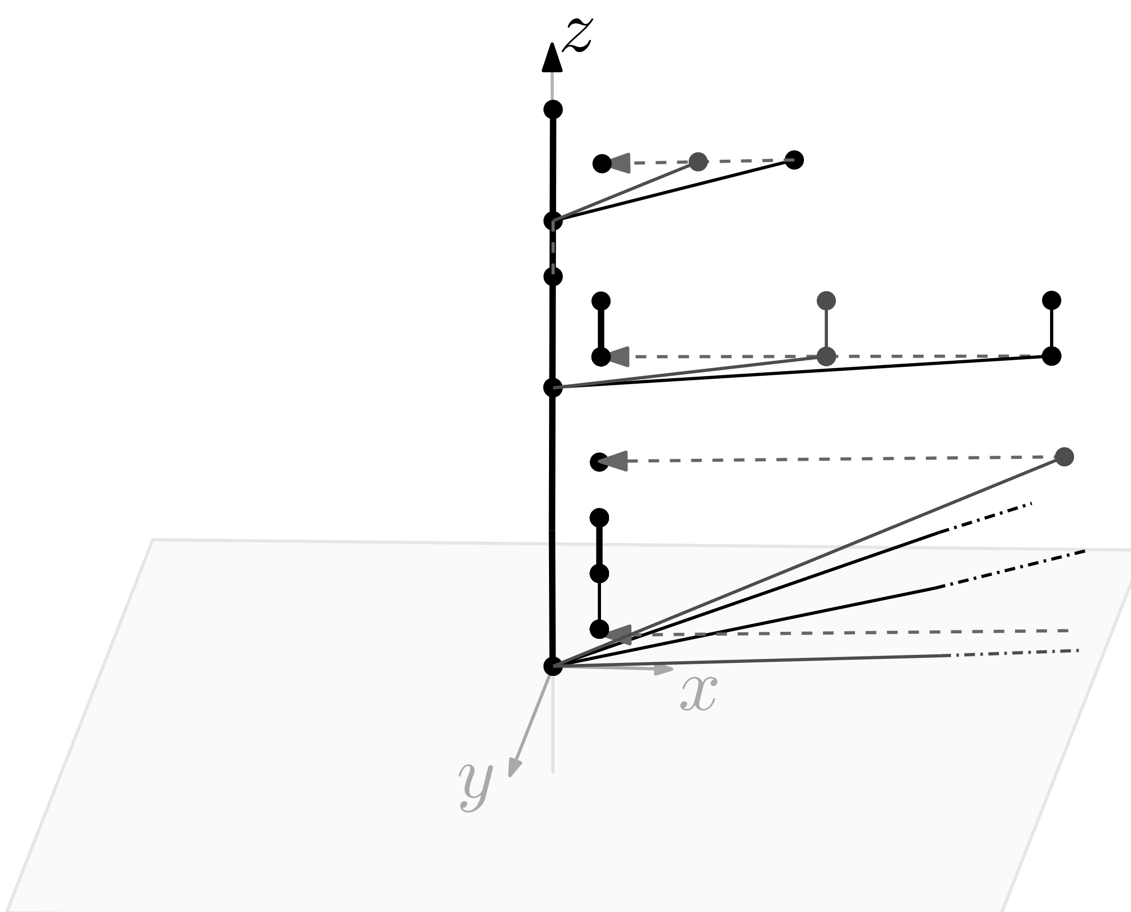
Morphing two planar drawings of a tree in 3D

Step 6: Go left



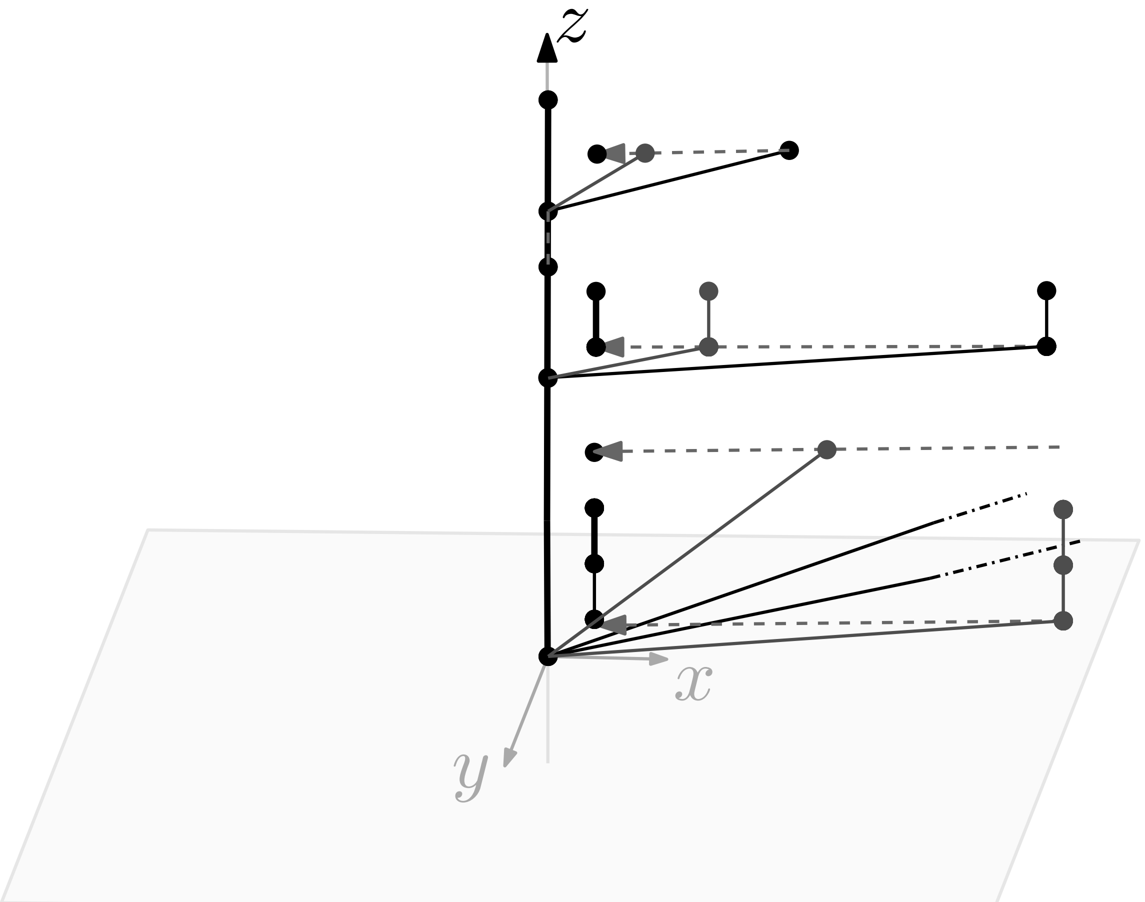
Morphing two planar drawings of a tree in 3D

Step 6: Go left



Morphing two planar drawings of a tree in 3D

Step 6: Go left



Morphing two planar drawings of a tree in 3D

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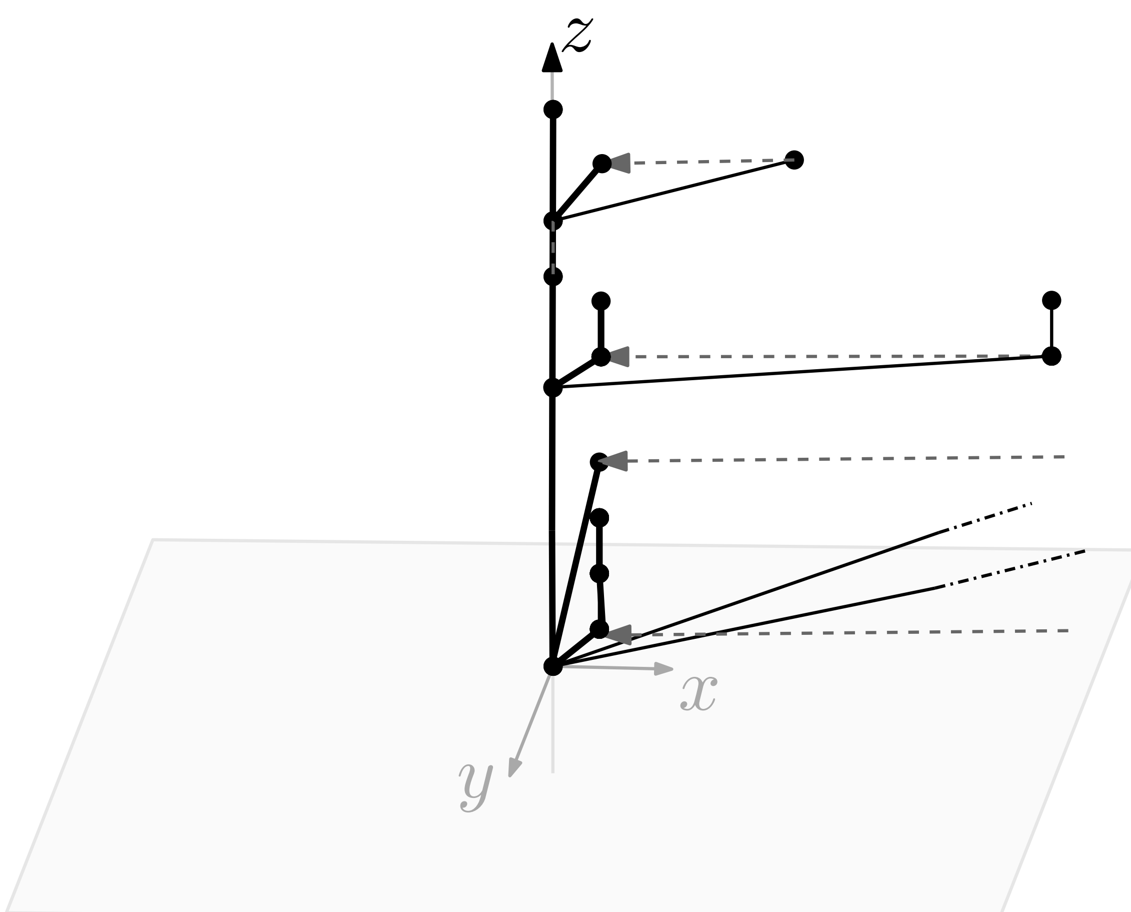


Morphing two planar drawings of a tree in 3D

Theorem. *For any two plane straight-line drawings* Γ, Γ' *of an* n*-vertex tree* T*, there exists a crossing-free 3D morph from* Γ *to* Γ' *with* $O(\log n)$ *steps.*

Morphing two planar drawings of a tree in 3D

Theorem. *For any two plane straight-line drawings Γ, Γ' of an n-vertex tree T, there exists a crossing-free 3D morph from Γ to Γ' with $O(\log n)$ steps.*

Why do not you use the rooted pathwidth decomposition instead of the heavy-path decomposition?

Morphing two planar drawings of a tree in 3D

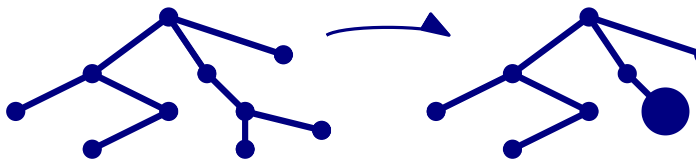
Theorem. *For any two plane straight-line drawings Γ, Γ' of an n-vertex tree T, there exists a crossing-free 3D morph from Γ to Γ' with* ~~$O(\log n)$ *steps.*~~ $O(p)$ steps, where p is the pathwidth of T.

Why do not you use the rooted pathwidth decomposition instead of the heavy-path decomposition?

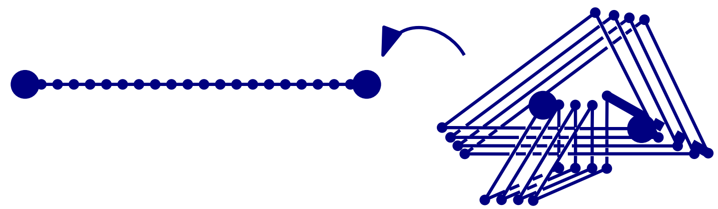
Summary

Summary

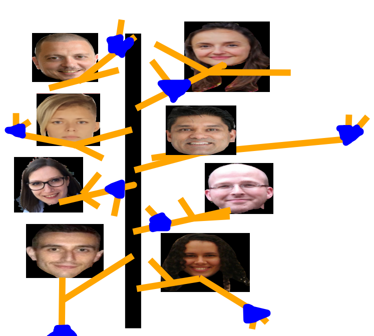
- Any two crossing-free straight-line 3D drawings of an n-vertex tree can be morphed into each other in $O(n)$ steps.



- Sometimes $\Theta(n)$ steps are necessary.



- For any two planar straight-line drawings of the same n-vertex tree, there is a crossing-free 3D morph between them of $O(\log n)$ steps.



Summary

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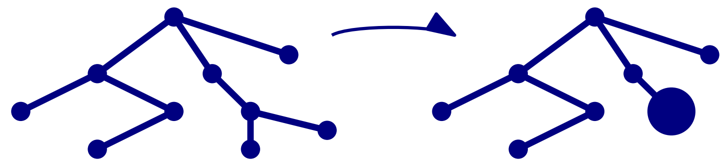
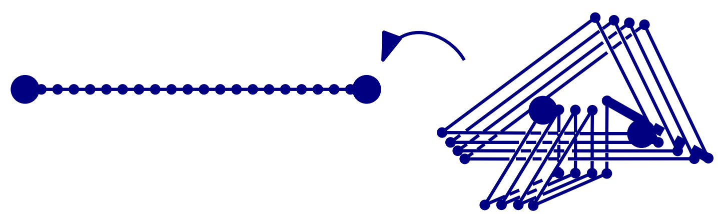


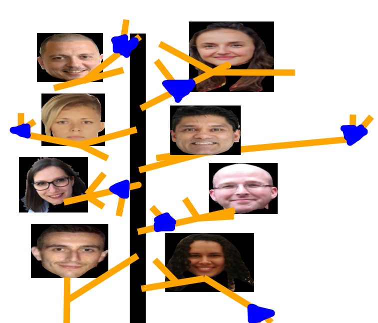
OPEN: generalize

- Sometimes $\Theta(n)$ steps are necessary.



OPEN: bounded size of coordinates

- For any two planar straight-line drawings of the same n-vertex tree, there is a crossing-free 3D morph between them of $O(\log n)$ steps.



Summary

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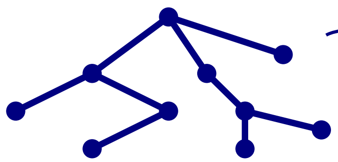

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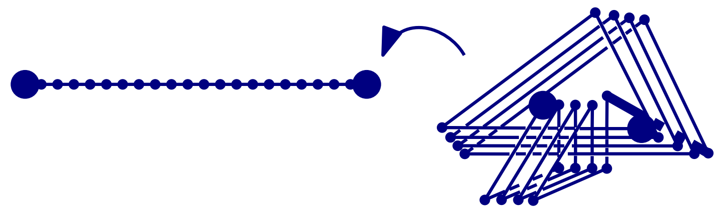

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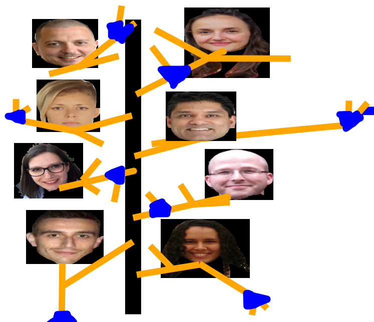

THANK YOU!